Title: Lecture - Gravitational Physics, PHYS 636

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Collection/Series: Gravitational Physics (Elective), PHYS 636, January 6 - February 5, 2025

Subject: Cosmology, Strong Gravity

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L10 WARPED COMPACTIFICATIONS

Recall for KK

$$-\frac{1}{16\pi G_5}\int \sqrt{g_5}\, R_5\, d^5x \longrightarrow -\frac{L}{16\pi G_5}\int d^4x \sqrt{\tilde{g}}\, e^{\sigma} \tilde{R} \cdots$$

$$\frac{1}{(8\pi)\, M_p^2} \leftarrow G_4 = G_5/L \rightarrow \frac{1}{(8\pi)\, M_5^3 L}$$

If $L \neq L_5$ (5D Planck scale)

then the Planck scales in 4 & 5D

are different.

In general

$$\underset{(4D)}{M_p^2} = \underset{(5D)}{M_D^{D-2}} \underset{\text{compactification scale}}{L^{\overbrace{D-4}^{n}}}$$

Planck scale)

scales in 4&SD

$$\overset{n}{\overbrace{D-4}}$$

L

compactification scale.

"Large Extra Dimensions" (LED)

$$\text{or } L \sim \underbrace{\left(\frac{M_P}{M_D}\right)^{\frac{D-2}{D-4}}}_{\left(\frac{10^{16}\,\text{TeV}}{M_D}\right)^{1+\frac{2}{n}}} M_P^{-1}$$

$n = D-4$

$$L_P$$

$$\sim \left(\frac{\text{TeV}}{M_D}\right)^{1+\frac{2}{n}} 10^{\frac{32}{n}-17}\ \text{cm}.$$

eg $n=6$, $L = 10^{-13}\,\text{cm} \rightarrow M_D \sim O(10\,\text{TeV})$

For gravity, this is not measurable

For pp not so much, so have to find

a mechanism to make the Standard Model

remain effectively 4D.

eg $n=6$, $L=10^{-13}$ cm $\rightarrow$ $M_D \sim O(10\,\text{TeV})$

For gravity, this is not measurable

For P.P. not so much, so have to find

a mechanism to make the Standard Model

remain effectively 4D - invoke confinement

in dimensionality.

~ O(10TeV)

asurable

are to find

andard Model

onfinement

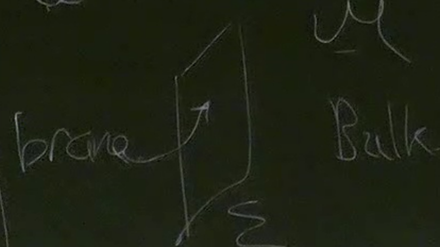
Toy model: Randall-Sundrum

- 5D, $\Lambda < 0$, domain wall

example of warped compactification

/ braneworld. ↔ "Flatlanders"

i.e. we live on a submanifold



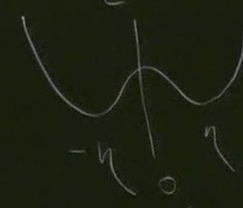
De

um

pactification

Domain Wall: A 'static' solution of field theory – a vacuum defect – protected by topology

$$\mathcal{L}_\phi = \frac{1}{2}(\partial\phi)^2 - V(\phi) \hookrightarrow \frac{\lambda}{2}(\phi^2-\eta^2)^2$$



$\phi \in \mathbb{R}$

in dimensionality.

ϕ e.o.m. $\quad \Box\phi + 2\lambda\phi(\phi^2-\eta^2)=0.$

Look for an interpolating soln $\phi \underset{z\to\pm\infty}{\to} \pm\eta$

(cartesian coords flat space)

Look for $\phi = \phi(z)$

$$-\phi'' + 2\lambda\phi(\phi^2-\eta^2)=0$$

soln

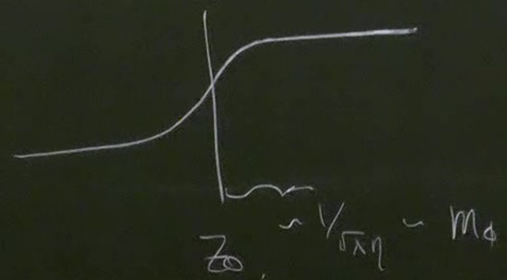
Σ $\qquad$ Z_2 symmetry

$= 0.$

$\pm\eta$

$z \to \pm\infty$

solved by $\phi = \eta \tanh[\sqrt{\lambda}\,\eta(z - z_0)]$



"$\lambda\phi^4$ kink"

Energy momentum $T_{\mu\nu} = \phi_{,\mu}\phi_{,\nu} - g_{\mu\nu}\left(\frac{1}{2}(\partial\phi)^2 - V\right)$

$$= \phi'^2 \delta^z_\mu \delta^z_\nu + g_{\mu\nu}\left(V + \frac{1}{2}\phi'^2\right)$$

$\mathbb{Z}_2$ symmetry

$\ldots h[\sqrt{\lambda}\eta(z-z_0)]$

"$\lambda\phi^4$ kink"

$\ldots_\nu - g_{\mu\nu}\left(\frac{1}{2}(\partial\phi)^2 - V\right)$

$\partial_\mu^z \partial_\nu^z + g_{\mu\nu}\left(V + \frac{1}{2}\phi'^2\right)$

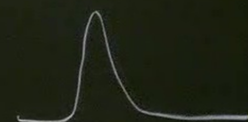
$$\phi'^2 = \lambda\eta^4 \operatorname{sech}^4[\sqrt{\lambda}\eta(z-z_0)] = \lambda(\phi^2-\eta^2)^2 = 2V$$

The surface $z = z_0$ has normal $\underline{n} = \frac{\partial}{\partial z}$

$$T_{\mu\nu} = 2V\Big[\underbrace{g_{\mu\nu} + n_\mu n_\nu}_{h_{\mu\nu} \text{ of } \Sigma_{z_0}}\Big]$$

indep of D

2V is strongly localized around z_0



$\rightarrow$ add gravity $\quad \int T_{00}\, dz = \frac{4}{3}\sqrt{\lambda}\eta^3 = \sigma$

σ = energy / tension (per unit area)

is finite, but wall has ∞ area.

$$ds^2 = \underset{\text{WARP FACTOR}}{A^2(z)}\,\underbrace{\gamma_{\mu\nu}\,dx^\mu dx^\nu}_{\text{Lorentzian (D-1) metric}} - \underset{\text{proper distance from wall}}{dz^2}$$

Since $T_{\mu\nu} \propto h_{\mu\nu}$, expect γ to be const. curv.

have
$$R^z{}_z = (D-1)A''/A$$

const &

$$R^\mu{}_\nu = \frac{1}{A^2}\boxed{R_\gamma{}^\mu{}_\nu} + \left[(D-2)\frac{A'^2}{A^2} + \frac{A''}{A}\right]\delta^\mu{}_\nu$$

$R_{\mu\nu}$

2V strong

localized

$$R_{\mu\nu} = 8\pi G\left[T_{\mu\nu} - \frac{T}{D-2} g_{\mu\nu}\right]$$

$$\sim 8\pi G \cdot 2V \underbrace{\left[h_{\mu\nu} - \frac{D-1}{D-2} g_{\mu\nu}\right]}_{-\frac{h_{\mu\nu}}{D-2} + \frac{(D-1)}{(D-2)} n_\mu n_\nu.}$$

2V strongly localized so A'' strongly localized. $\propto$ $A'^2 \sim -R_0{}^t{}_t/(D-2)$.

$$A'^2 = \frac{\kappa(D-2)}{\ell^2(D-2)}$$

$$A'^2 = \frac{\kappa(D-2)}{\ell^2(D-2)} = \frac{\kappa}{\ell^2}$$ $\leftarrow$ constant curv

length scale ℓ

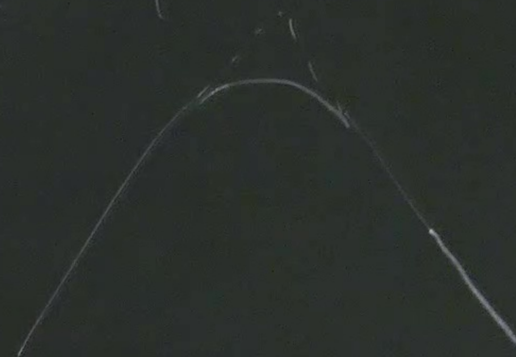
$\kappa = +1$ de-Sitter
0 flat
-1 adS

i.e. $\kappa = 1$ here

(taking $\Lambda_{bulk} = 0$)

$A' = \pm 1/\ell$ outside wall

$$A \simeq \left(1 - |z|/\ell\right)$$

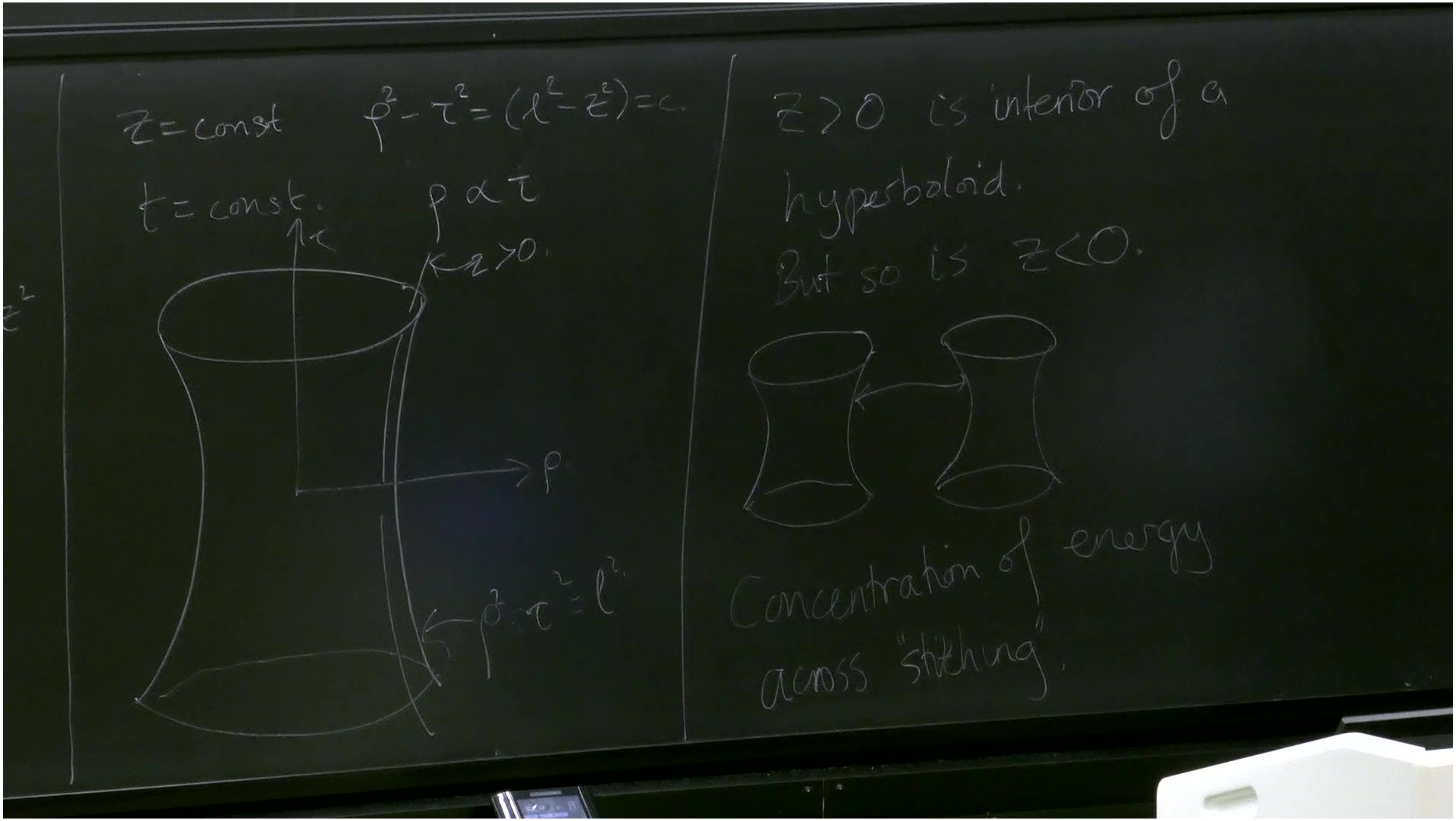


$$\ell = \frac{(D-2)}{4\pi G \sigma} \quad \text{(from integrating } A''\text{)}$$

$$ds^2 = \left(1 - \frac{|z|}{\ell}\right)^2 \left[dt^2 - \ell^2 \cosh^2\frac{t}{\ell}\, d\Omega_{II}^2\right] - dz^2$$

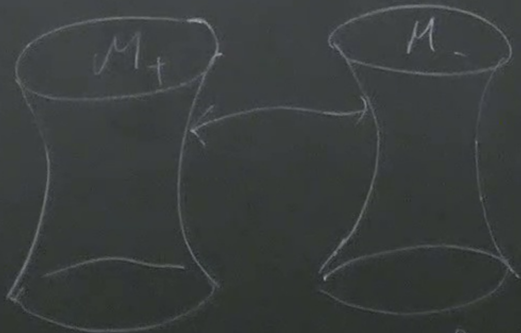
Looks curved, but

$$\left.\begin{aligned} \rho &= (\ell - z)\cosh t/\ell \\ \tau &= (\ell - z)\sinh t/\ell \end{aligned}\right\} \quad \begin{aligned} &z > 0 \\ &\underline{\underline{\underline{\text{FLAT}}}} \end{aligned}$$


z = const
$\rho^2 - \tau^2 = (l^2 - z^2) = c$.
t = const.
$\rho \propto \tau$
τ
$z > 0$.
ρ
$\rho^2 - \tau^2 = l^2$
$z > 0$ is interior of a
hyperboloid.
But so is $z < 0$.
Concentration of energy
across "stitching".

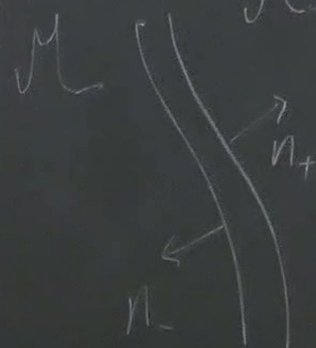
hyperboloid.

But so is $z<0$.



Concentration of energy across "stitching".

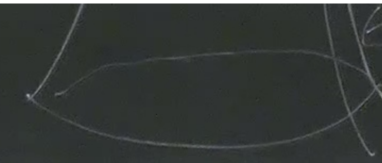
Look at from submanifold perspective



$$S = -\frac{1}{16\pi G}\int_{\mathcal{M}_+\cup\mathcal{M}_-} R\sqrt{g} + \frac{1}{8\pi G}\int_{\partial\mathcal{M}_+\cup\partial\mathcal{M}_-} \sqrt{h}\,K$$

$$\frac{\delta S}{\delta g^{ab}} = \underset{\text{EINSTEIN}}{\text{BULK}} + \int_{\partial\mathcal{M}_+\cup\partial\mathcal{M}_-} [K_{ab} - K h_{ab}]\,\delta h^{ab}.$$

$\tau = (\ell - \epsilon)\sinh \ldots$



Flip n_- to get picture of shell $\quad \xrightarrow[n_-]{} \,||\, \xrightarrow[n_+]{}$

$$\underbrace{\Delta K_{ab}}_{K_{+ab}-K_{-ab}} - \Delta K\, h_{ab} = 8\pi G\, S_{ab}$$

$$S_{ab} \downarrow \int_-^+ T_{ab}$$

$$\to 8\pi G \sigma h_{ab}$$

FOR WALL

ISRAEL EQNS

Flip n_- to get picture of shell $\;\xrightarrow[n_-]{}\;\Big|\Big|\;\xrightarrow[n_+]{}$

$$\underbrace{\Delta K_{ab}}_{K_{+ab}-K_{-ab}} - \Delta K\, h_{ab} = 8\pi G\, S_{ab}$$

$$S_{ab} \Big\downarrow \int_-^+ T_{ab}$$

$$\to 8\pi G \sigma h_{ab}$$

FOR WALL

ISRAEL EQNS

Now

ture

$$\ldots_{ab} = 8\pi G\, S_{ab}$$

$$\downarrow \int_{-}^{+} T_{ab}$$

$$\rightarrow 8\pi G \sigma h_{ab}$$

FOR WALL

Now $D=5$, $\Lambda = -6/L^2$

$$ds^2 = e^{-2|z|/L} \underbrace{\eta_{\mu\nu}\, dx^\mu dx^\nu}_{\text{4D M.S.T.}} - dz^2$$

WALL AT $z=0$.

$$K_{\mu\nu} = X^a_{,\mu} X^b_{,\nu} \nabla_a n_b$$

$$= -\Gamma^c_{ab} n_c X^a_{,\mu} X^b_{,\nu}$$

$$= +\Gamma^z_{\mu\nu} = -\frac{1}{\ell} g_{\mu\nu} \quad (z>0)$$

$-6/L^2$

$\underbrace{dx^\mu dx^\nu}_{\text{M.S.T.}} - dz^2$ WALL AT $z=0$

$\nabla_a n_b$

$X^a_{,\mu} X^b_{,\nu}$

$-\frac{1}{\ell} g_{\mu\nu} \quad (z>0)$

$$K_{+\mu\nu} = -\frac{1}{\ell}\eta_{\mu\nu} \qquad K_{-\mu\nu} = \frac{1}{\ell}\eta_{\mu\nu}$$

$$\Delta K_{\mu\nu} - \Delta K \eta_{\mu\nu} = -\frac{2}{\ell}\eta_{\mu\nu} + \frac{8}{\ell}\eta_{\mu\nu}$$

$$= \frac{6}{\ell}\eta_{\mu\nu} = 8\pi G\sigma\eta_{\mu\nu}$$

in dimensionality.

$z=0$ corr. to a domain wall with tension tuned to $\sigma = \frac{6}{8\pi G \ell}$

space warps strongly off brane Gravity remains 4D to leading order.

all

$\frac{6}{8\pi G\ell}$

ongly

ity

ing

Confinement?

Couple fermion to D.W. ϕ.

$$\mathcal{L}_{\Psi} = i\,\bar{\Psi}\,\Gamma^a \nabla_a \Psi - g\,\phi\,\bar{\Psi}\Psi$$

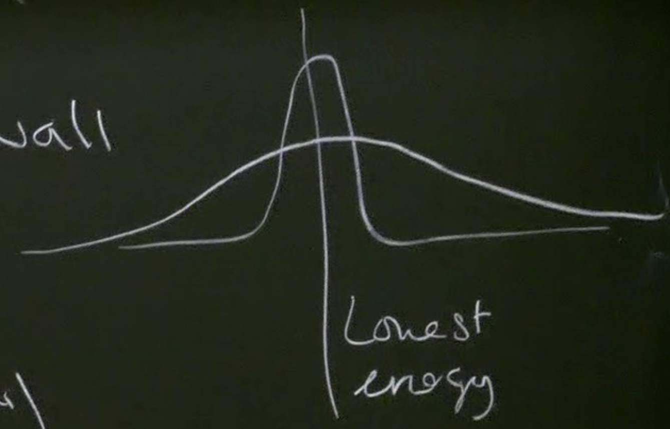
Solve

$$i\gamma^5 \Psi' = g\eta \tanh(\sqrt{\lambda}\,\eta(z-z_0))\,\Psi.$$

$$\to \Psi \propto \Psi_0 \left(\mathrm{sech}[\sqrt{\lambda}\,\eta(z-z_0)]\right)^{g/\sqrt{\lambda}}.$$

with $i\gamma^5 \psi_0 = -\psi_0$. (Chiral)

Fermion condensate on wall



Low energy excitations;

Zero modes: $\psi_0 \to \psi_0(x^\mu)$

where $\not{\partial} \psi_0 = 0$.