Title: Lecture - Gravitational Physics, PHYS 636

Speakers: Ruth Gregory

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Subject: Cosmology, Strong Gravity

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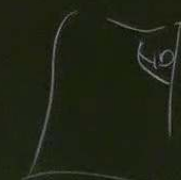
L 9 Extra Dimensions - KK

Various motivations: pure curiosity, string theory, hierarchy problem.

Main goal: Why is gravity effectively 4D?

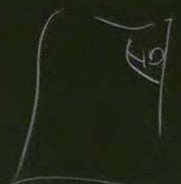
- KK Extra dims small
- Branes



KK

, string

likely 4D?



In K-K theory, extra dims are compact : S^1

$$ds^2 = \underbrace{\overset{\text{"}t,x,y,z\text{"}}{ds_4^2}}_{\text{indep of } \chi} - L^2 d\chi^2$$

Scalar field $\Phi(x^\mu, \chi) \sim \varphi(x^\mu) e^{i\chi}$

$$\nabla^2 \Phi \to \left(\Box_4 \varphi + \frac{1}{L^2} \varphi \right)$$

eory, extra dims

: S^1

"t, x, y, z"

$\underbrace{ds_4^2}_{\text{indep of } \chi} - L^2 d\chi^2$

ld $\Phi(x^\mu, \chi) \sim \varphi(x^\mu) e^{i\chi n}$

$\nabla^2 \Phi \to \left(\Box_4 \varphi + \frac{n^2}{L^2} \varphi\right)$

Interpret as field in 4D with mass $m^2 = n^2/L^2$

If L small, these modes depending on χ have high eff. mass, so are not relevant for low energy physics

Look at gravity

$$ds^2 = \hat{g}_{\mu\nu}\, dx^\mu dx^\nu - e^{2\sigma}\left[d\psi + A_\mu dx^\mu\right]^2$$

$\hat{g}, \sigma, A_\mu$ depend only on X^μ $(\mu = 0,1,2,3)$

Using Cartan method, compute curvatures.

$$\underset{\sim}{\omega}^a = \hat{e}^a{}_\mu dx^\mu, \quad \underset{\sim}{\omega}^5 = e^\sigma\left[d\psi + A_\mu dx^\mu\right]$$

Example with "off diagonal" terms

$$\underline{d}\,\underline{\omega}^a = -\underline{\theta}_0{}^a{}_b \wedge \underline{\omega}^b$$

$$\underline{d}\,\underline{\omega}^5 = \sigma_{,a}\,\underline{\omega}^a \wedge \underline{\omega}^5 + e^\sigma \underline{F}$$

$$\rightarrow \quad \underline{\theta}^5{}_a = \sigma_{,a}\,\underline{\omega}^5 + \frac{1}{2} e^\sigma F_{ab}\,\underline{\omega}^b$$

$$\underset{\sim}{\theta}{}^a{}_b = \underset{\sim}{\theta}_0{}^a{}_b + \frac{1}{2} e^\sigma F^a{}_b\,\underline{\omega}^5$$

From

onal" terms

$+ e^\sigma F$

$\frac{1}{2} e^\sigma F_{ab} \underline{\omega}^b$

$e^\sigma F^a{}_b \underline{\omega}^5$

From this (Cartan 2) get

$$R^{\hat a}{}_{\hat b \hat c \hat d} = \tilde R^{\hat a}{}_{\hat b \hat c \hat d} + \frac{1}{4} e^\sigma \left(F^a{}_c F_{bd} - F^a{}_d F_{bc} + 2F^a{}_b F_{cd} \right)$$

$$R^{\hat 5}{}_{\hat a \hat 5 \hat b} = -\sigma_{,\hat a \hat b} - \sigma_{,\hat a}\, \sigma_{,\hat b} - \frac{1}{4} e^{2\sigma} F_{ac} F_b{}^c$$

$$\omega^a = \tilde{e}^a{}_\mu dx^\mu, \quad \omega^5 = e^\sigma [d\psi + A_\mu dx^\mu]$$

$$\Rightarrow \quad R_5 = \breve{R}_\circ + \frac{1}{4} e^{2\sigma} F^2 - 2 e^{-\sigma} \breve{\Box} e^\sigma$$

$$\sqrt{g_5} = e^\sigma \sqrt{\breve{g}}$$

$$\hookrightarrow \quad S_5 = -\frac{1}{16\pi G_5} \int d^4x \, d\psi \, e^\sigma \sqrt{\breve{g}} \left(\breve{R}_\circ + e^{2\sigma} F^2 + e^{-\sigma} \breve{\Box} e^\sigma \right)$$

$$= -\frac{L}{16\pi G_5} \int d^4x \sqrt{\breve{g}} \left(e^\sigma \breve{R} + e^{3\sigma} F^2 \right) + \text{bdry term}$$

$\Omega^a{}_b = \Omega_0{}^a{}_b + \frac{1}{2} e^{\sigma} F^a{}_b \omega$

$\frac{1}{4} e^{\sigma} F_{ac} F^c{}_b$

If S^1 has fixed size, is $\sigma = \text{const}$ – gravity and electromagnetism are geometrically unified.

Here we don't quite have Einstein action

$\tilde{g}_{ab} \leftrightarrow e^{\sigma}\tilde{R}$ is called the Jordan frame

multiplying R by a scalar is like making G position dependent – scalar-tensor gravity (Brans Dicke)

+ bdry term

Conventionally, transform to

<u>Einstein frame</u>

$$\tilde{g}_{ab} = \Omega^2(x^\mu)\, g_{ab}$$

$$\tilde{R} = \Omega^{-2}\left(R - 6\Omega^{-1}\Box\Omega\right)$$

Setting $\Omega = e^{-\sigma/2}$,

$\sqrt{\tilde{g}}\,\tilde{R} \to$

$\Omega)$

$$\sqrt{\hat{g}}\,\hat{R} \to e^{-\sigma}\sqrt{g}\left(R + 3\Box\sigma - \frac{3}{2}(\nabla\sigma)^2\right)$$

Finally $\phi = \sigma/\sqrt{3}$ gives

$$S_5 = \frac{1}{16\pi G_5/L}\int\left[-R + \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}e^{\sqrt{3}\phi}F^2\right]\sqrt{g}\,d^4x$$

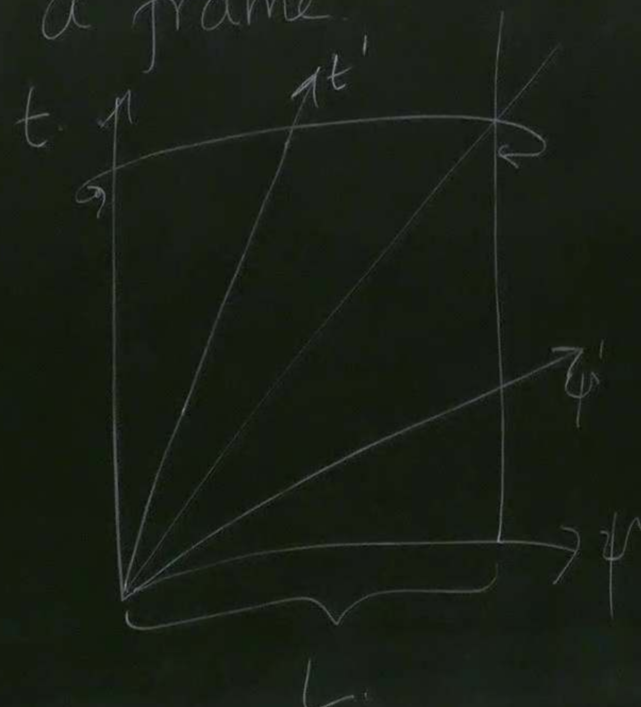
$$ds_5^2 = e^{-\sqrt{3}\phi}g_{\mu\nu}dx^\mu dx^\nu - e^{2\sqrt{3}\phi}\left[d\psi + A_\mu dx^\mu\right]^2$$

$\ldots(\nabla\sigma)^2\big)$

$e^{\sqrt{3}\phi} F^2\big]\sqrt{g}\, d^4x$

$\big[d\psi + A_\mu dx^\mu\big]^2$

Note: compactification picks a frame.



$\psi \sim \psi + L$

at const t

$t' \sim t' - v\gamma L$

$\psi' \sim \psi' + \gamma L$

In 5D have a black string soln

$$ds^2 = \left(1 - \frac{r_+}{r}\right)\underbrace{dt^2}_{\frac{(dt+v\,dz)^2}{1-v^2}} - \frac{dr^2}{1-\frac{r_+}{r}} - r^2 d\Omega_{II}^2 - \underbrace{dz^2}_{\frac{(dz+v\,dt)^2}{1-v^2}}$$

If compactify on z, get SCH

Instead: boost, t & z parts of metric pick up boost terms.

soln.

$- dz^2$

$\frac{(dz + v\,dt)^2}{1-v^2}$

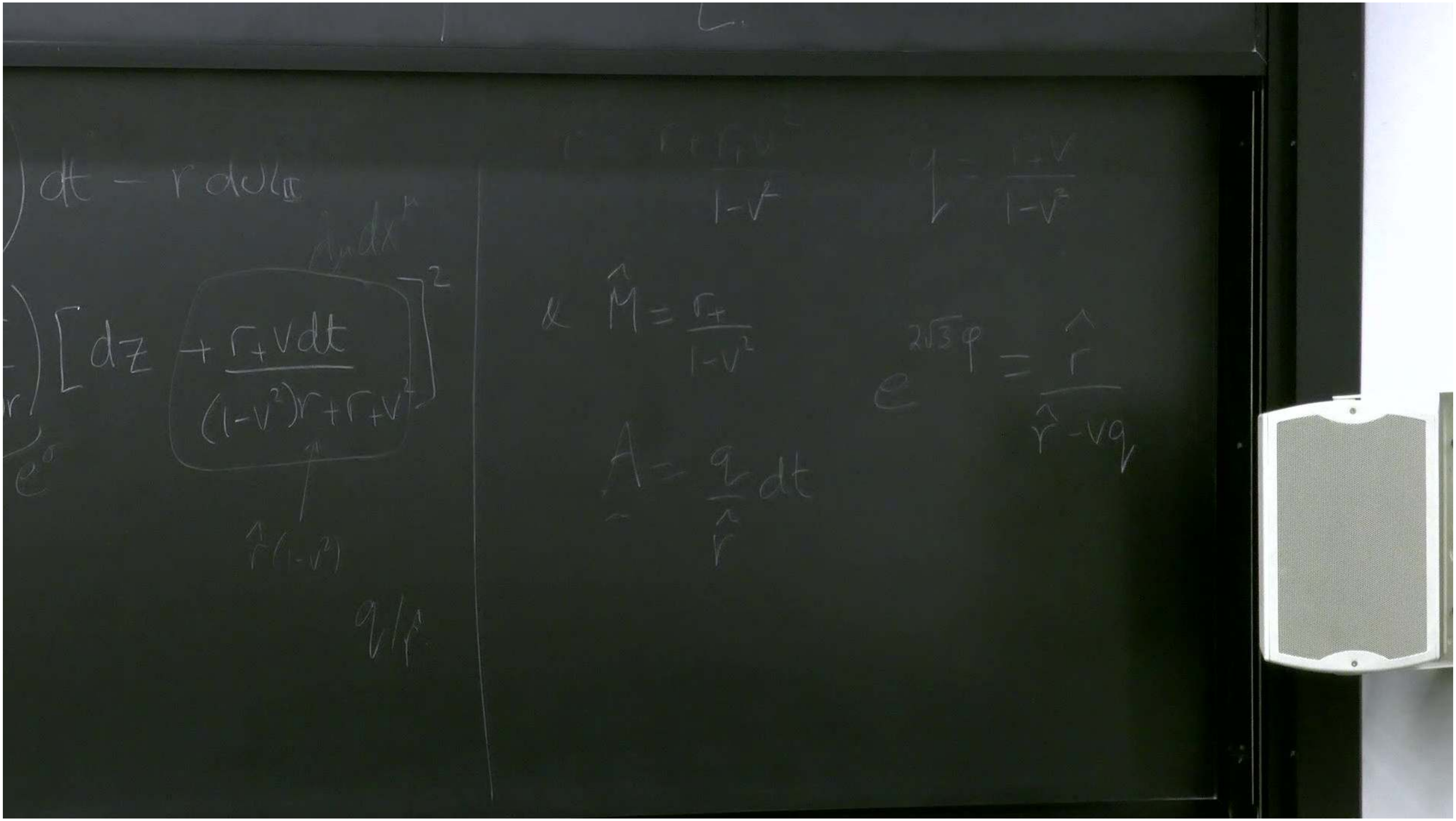
etric pick

$$ds^2 = \underbrace{\left(1 - \frac{r_+}{(1-v^2)r + r_+ v^2}\right)}_{} dt^2 - r^2 d\Omega_{II}^2 - \frac{dr^2}{\left(1 - \frac{r_+}{r}\right)} - \underbrace{\left(1 + \frac{r_+ v^2}{(1-v^2)r}\right)}_{e^{\sigma}} \Big[dz \overbrace{+ \frac{r_+ v\, dt}{(1-v^2)r + r_+ v^2}}^{A_\mu dx^\mu} \Big]^2$$

Identify $z \sim z + L$.

$\hat{r} =$

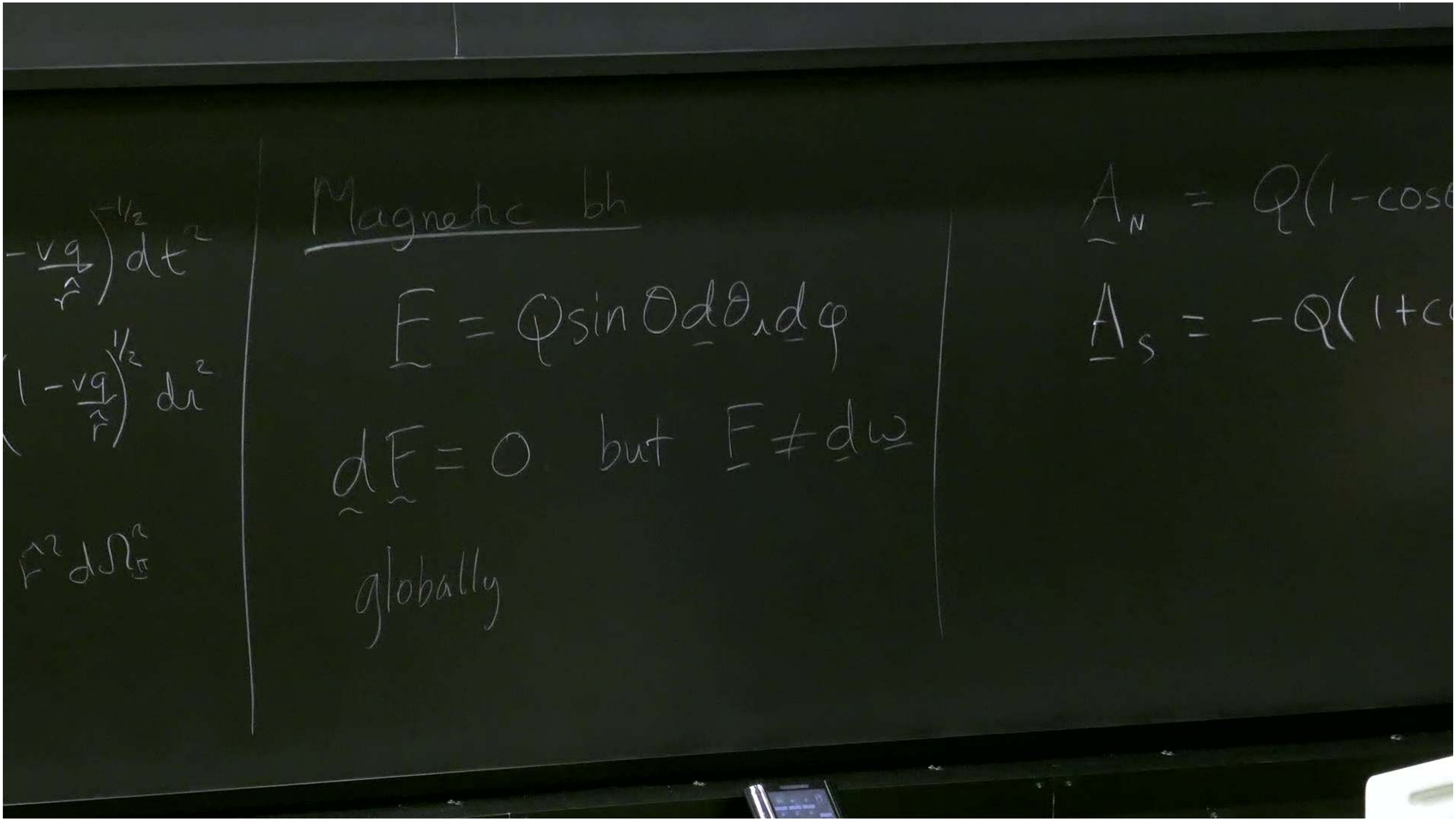
$\&\ \hat{\mu}$


dt − r dω_II
A_μ dx^μ
[dz + r_+ v dt / ((1−v²)r + r_+ v²)]²
r̂(1−v²)
q/r̂
1−v²
& M̂ = r_+ / (1−v²)
A = q/r̂ dt
e^{2√3 φ} = r̂ / (r̂ − vq)

$$g_{\mu\nu}\,dx^\mu dx^\nu = \left(1-\frac{2G\hat{M}}{\hat{r}}\right)\left(1-\frac{vq}{\hat{r}}\right)^{-1/2} dt^2 - \left(1-\frac{2G\hat{M}}{\hat{r}}\right)^{-1}\left(1-\frac{vq}{\hat{r}}\right)^{1/2} dr^2 - \left(1-\frac{vq}{\hat{r}}\right)^{3/2} \hat{r}^2\, d\Omega^2$$

electric KK bh

Magnetic


Magnetic bh
F = Q sin θ dθ ∧ dφ
dF = 0 but F ≠ dω
globally
A_N = Q(1-cos
A_S = -Q(1+c

$$\ldots\theta \wedge \underline{d}\varphi$$

$$\ldots F \neq \underline{d}\underline{\omega}$$

$$\underline{A}_N = Q(1-\cos\theta)\,\underline{d}\varphi \qquad \text{regular on NP}$$

$$\underline{A}_S = -Q(1+\cos\theta)\,\underline{d}\varphi \qquad \text{" " SP.}$$

$$\boxed{\underline{A}_S = \underline{A}_N - 2Q\,\underline{d}\varphi}$$

With this Ansatz:

$$ds_5^2 = \left(\frac{r-r_+}{r-r_-}\right)dt^2 - \frac{dr^2}{\left(1-\frac{r_+}{r}\right)} - r(r-r_+)\,d\Omega_{\pi}^2$$

$$-\left(1-\frac{r_+}{r}\right)\left[d\psi_N + \sqrt{r_+ r_-}\,(1-\cos\theta)\,d\varphi\right]^2$$

$$Q = \sqrt{r_+ r_-}$$

$$\Psi_S = \psi_N + 2Q\varphi$$

$$= \psi_N + 2Q(\varphi + 2n\pi) = \psi_S + nL$$

$$\Rightarrow 4\pi Q = nL$$

Charge is quantised

Let $r_+ \to r_-$

$(\pi) = \psi_S + i L$

ed

Let $r_+ \to r_-$

$$ds^2 = dt^2 - \frac{dr^2}{1-\frac{Q}{r}} - \left(1 - \frac{Q}{r}\right)\left[r^2 d\Omega_{\mathrm{II}}^2 + Q^2 (d\chi_N + A_N)^2 \right]$$

$r \to Q$?

$$\rho^2 = 4Q(r-Q)$$

$$\to dt^2 - d\rho^2 - \frac{\rho^2}{4}\left[d\theta^2 + \sin^2\theta\, d\varphi^2 + \left(d\chi + (1-\cos\theta)\, d\varphi\right)^2 \right]$$

Origin of $\mathbb{R}^4$ in Euler angles

NP

SP.

$$x + iy = \rho \cos\frac{\theta}{2}\, e^{i\chi/2}$$

$$z + iw = \rho \sin\frac{\theta}{2}\, e^{i(\phi + \chi/2)}$$

HOPF FIBRATION

of S^2 by S^1 giving S^3.