

Title: Lecture - Gravitational Physics, PHYS 636

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Subject: Cosmology, Strong Gravity

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L 9 Extra Dimensions - KK

Various motivations: pure curiosity, string theory, hierarchy problem.

Main goal: Why is gravity effectively 4D?

• KK Extra dims

Small

• Branes



KK
string

only 4D?



In K-K theory, extra dims
are compact S^1

$$ds^2 = \underbrace{ds_4^2}_{\text{indep of } \chi} - L^2 d\chi^2$$

Scalar field $\Phi(x^m, \chi) \sim \varphi(x^m) e^{i\chi}$

$$\nabla^2 \Phi \rightarrow \left(\square_4 \varphi + \frac{1}{L^2} \varphi \right)$$

low, extra dims

S^1

" t, x, y, z "

$$dS_4 = L^2 d\chi^2$$

indep of χ

$$\Phi(x^m, \chi) \sim \varphi(x^m) e^{i n \chi}$$

$$\square \Phi \rightarrow \left(\square_4 \varphi + \frac{n^2}{L^2} \varphi \right)$$

Interpret as field in 4D
with mass $m^2 = n^2 / L^2$

If L small, these modes
depending on χ have high
eff. mass, so are not relevant
for low energy physics.

Look at gravity.

$$ds^2 = \tilde{g}_{\mu\nu} dx^\mu dx^\nu - e^{2\sigma} [d\psi + A_\mu dx^\mu]^2$$

\tilde{g}, σ, A_μ depend only on X^μ ($\mu=0,1,2,3$)

Using Cartan method, compute curvatures.

$$\underline{\omega}^a = \tilde{E}^a_\mu dx^\mu, \quad \underline{\omega}^5 = e^\sigma [d\psi + A_\mu dx^\mu]$$

Example with "off diagonal" terms

From

$$\underline{d}\underline{\omega}^a = -\underline{\Theta}_0^a \wedge \underline{\omega}^b$$

$$\underline{d}\underline{\omega}^5 = \sigma_{,a} \underline{\omega}^a \wedge \underline{\omega}^5 + e^5 F$$

$$\rightarrow \underline{\Theta}_a^5 = \sigma_{,a} \underline{\omega}^5 + \frac{1}{2} e^5 F_{ab} \underline{\omega}^b$$

$$\underline{\Theta}_b^a = \underline{\Theta}_0^a \wedge \underline{\omega}^b + \frac{1}{2} e^a F^a_b \underline{\omega}^5$$

onal terms

$$+ e^{\sigma} F$$

$$- \frac{1}{2} e^{\sigma} F_{ab} \omega^b$$

$$e^{\sigma} F^a_b \omega^s$$

From this (Cartan 2) get

$$R^{\hat{a}}_{\hat{b}\hat{c}\hat{d}} = \tilde{R}^{\hat{a}}_{\hat{b}\hat{c}\hat{d}} + \frac{1}{4} e^{\sigma} \begin{pmatrix} F^a_c F_{bd} \\ -F^a_d F_{bc} \\ +2F^a_b F_{cd} \end{pmatrix}$$

$$R^{\hat{s}}_{\hat{a}\hat{s}\hat{b}} = -\sigma_{,ab} - \sigma_{,a} \sigma_{,b} - \frac{1}{4} e^{2\sigma} F_{ac} F_b^c$$

$$\underline{\omega}^a = \tilde{e}^a{}_{\mu} dx^{\mu}, \quad \underline{\omega}^5 = e^{\sigma} [d\psi + A_{\mu} dx^{\mu}]$$

$$\Rightarrow R_5 = \tilde{R}_0 + \frac{1}{4} e^{2\sigma} F^2 - 2e^{-\sigma} \tilde{\square} e^{\sigma}$$

$$\sqrt{g_5} = e^{\sigma} \sqrt{\tilde{g}}$$

$$\begin{aligned} \hookrightarrow S_5 &= -\frac{1}{16\pi G_5} \int d^4x d\psi e^{\sigma} \sqrt{g} \left(\tilde{R}_0 + e^{2\sigma} F^2 + e^{-\sigma} \tilde{\square} e^{\sigma} \right) \\ &= -\frac{L}{16\pi G_5} \int d^4x \sqrt{\tilde{g}} \left(e^{\sigma} \tilde{R} + e^{3\sigma} F^2 \right) + \text{bdry term} \end{aligned}$$

$$\underline{Q}_b = \underline{Q}_0{}_b + \frac{1}{2} e^a F^a{}_b \underline{\omega}^a$$

$$= \frac{1}{4} e^a \dots$$

If S^1 has fixed size, i.e. $\sigma = \text{const}$ - gravity and electromagnetism are geometrically unified.

Here we don't quite have Einstein action

$\check{g}_{ab} \leftrightarrow e^a \check{R}$ is called the Jordan frame

multiplying R by a scalar is like making G position dependent - scalar-tensor gravity (Brans Dicke)

+ body term

Conventionally, transform to
Einstein frame

$$\tilde{g}_{ab} = \Omega^2(x^\mu) g_{ab}$$

$$\tilde{R} = \Omega^{-2} (R - 6\Omega^{-1} \square \Omega)$$

Setting $\Omega = e^{-\sigma/2}$,

$$\sqrt{\tilde{g}} \tilde{R} \rightarrow$$

$$\sqrt{g} \tilde{R} \rightarrow e^{-\sigma} \sqrt{g} \left(R + 3 \square \sigma - \frac{3}{2} (\nabla \sigma)^2 \right)$$

Finally $\phi = \sigma / \sqrt{3}$ gives

$$S_5 = \frac{1}{16\pi G_5/L} \int \left[-R + \frac{1}{2} (\partial \phi)^2 - \frac{1}{4} e^{\sqrt{3}\phi} F^2 \right] \sqrt{g} dx^4$$

$$ds_5^2 = e^{-\sqrt{3}\phi} g_{\mu\nu} dx^\mu dx^\nu - e^{2\sqrt{3}\phi} [d\psi + A_\mu dx^\mu]^2$$

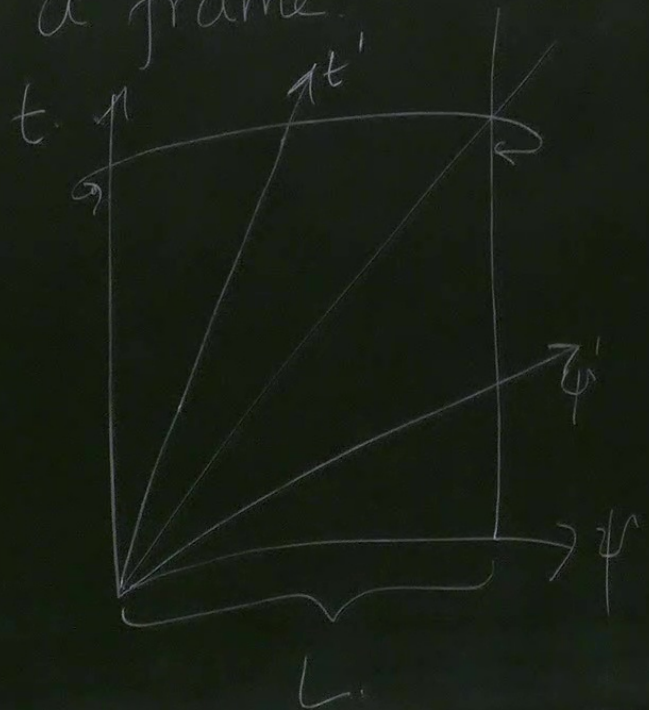
$(\sqrt{5})^2$

$$e^{\sqrt{3}\phi} F^2 \int \sqrt{g} dx^4$$

$$[d\psi + A_\mu dx^\mu]^2$$

Note: compactification picks

a frame



$$\psi \sim \psi + L$$

at const t

$$t' \sim t - v\gamma L$$

$$\psi' \sim \psi + \gamma L$$

In 5D have a black string soln.

$$ds^2 = \left(1 - \frac{r_+}{r}\right) dt^2 - \frac{dr^2}{\left(\frac{dt+vdz}{1-v^2}\right) \left(1 - \frac{r_+}{r}\right)} - r^2 d\Omega_2^2 - \frac{dz^2}{\left(\frac{dz+vdt}{1-v^2}\right)^2}$$

If compactly on z , get SCH

Instead: boost, t & z parts of metric pick up boost terms.

soln.

$$-dz^2 \quad \text{or} \quad \frac{(dz+vdz)^2}{1-v^2}$$

metric pick

$$ds^2 = \left(1 - \frac{r_+}{(1-v^2)r + r_+ v^2} \right) dt^2 - r^2 d\phi^2 - \frac{dr^2}{(1-\frac{v^2}{r})} - \underbrace{\left(1 + \frac{r_+ v^2}{(1-v^2)r} \right)}_{e^{\alpha}} \left[dz + \frac{r_+ v dt}{(1-v^2)r + r_+ v^2} \right]^2$$

Identify Z w/ $Z+L$.

$$dt - r \, d\theta / c$$

$$[dz + \frac{r + v \, dt}{(1-v^2)r + r + v^2}]^2$$

↑
r(1-v)

q/r

$$r = \frac{r + r v^2}{1-v^2}$$

$$\downarrow = \frac{1-v}{1-v^2}$$

$$\hat{M} = \frac{r}{1-v^2}$$

$$e^{2\sqrt{3}\phi} = \frac{r}{r^2 - vq}$$

$$A = \frac{q}{r} dt$$

up boost terms,

$$g_{\mu\nu} dx^\mu dx^\nu = \left(1 - \frac{2GM}{\hat{r}}\right) \left(1 - \frac{vq}{\hat{r}}\right)^{-1/2} dt^2 \\ - \left(1 - \frac{2GM}{\hat{r}}\right)^{-1} \left(1 - \frac{vq}{\hat{r}}\right)^{1/2} dr^2 \\ - \left(1 - \frac{vq}{\hat{r}}\right)^{3/2} \hat{r}^2 d\Omega_{\hat{r}}^2$$

electric KK bh

Magnetic

$$-\frac{vq}{r} dt^2$$

$$\left(1 - \frac{vq}{r}\right)^{1/2} dr^2$$

$$r^2 d\Omega^2$$

Magnetic bh

$$F = q \sin \theta d\theta \wedge d\varphi$$

$$dF = 0 \quad \text{but} \quad F \neq d\omega$$

globally

$$A_N = Q(1 - \cos \theta)$$

$$A_S = -Q(1 + \cos \theta)$$

$\theta \cdot d\varphi$
 $E \neq d\omega$

$$\underline{A}_N = Q(1 - \cos\theta) d\varphi$$

$$\underline{A}_S = -Q(1 + \cos\theta) d\varphi$$

$$\underline{A}_S = \underline{A}_N - 2Q d\varphi$$

regular on NP

" " SP.

With this Ansatz:

$$ds^2 = \left(\frac{r-r_+}{r-r_-} \right) dt^2 - \frac{dr^2}{\left(1-\frac{r_+}{r}\right)} - r(r-r_+) d\Omega_{\mathbb{S}^2}$$

$$- \left(1-\frac{r_+}{r}\right) \left[d\psi_N + \sqrt{r_+ r_-} (1-\cos\theta) d\varphi \right]^2$$

$$Q = \sqrt{r_+ r_-}$$

$$\begin{aligned}\psi_S &= \psi_N + 2Q\varphi \\ &= \psi_N + 2Q(\varphi + 2\pi) = \psi_S + L\end{aligned}$$

$$\Rightarrow 4\pi Q \equiv L$$

Charge is quantised

Let $r_+ \rightarrow r_-$

$$(\pi) = \psi_S + L$$

Let $r_+ \rightarrow r$

$$ds^2 = dt^2 - \frac{dr^2}{1-\frac{Q}{r}} - \left(1 - \frac{Q}{r}\right) \left[r^2 d\Omega_{\mathbb{S}^2}^2 + Q^2 (d\chi_N + A_N)^2 \right]$$

$r \rightarrow Q$?

$$p^2 = 4Q(r-Q)$$

$$\rightarrow dt^2 - dp^2 - \frac{p^2}{4} \left[d\theta^2 + \sin^2\theta d\varphi^2 + (d\chi + (1-\cos\theta)d\varphi)^2 \right]$$

Origin of \mathbb{R}^4 in Euler angles

VP
SP.



$$x + iy = \rho \cos \frac{\theta}{2} e^{i\theta/2}$$

$$z + iw = \rho \sin \frac{\theta}{2} e^{i(\theta/2)}$$

HOPF FIBRATION

of S^2 by S^1 giving S^3