Title: Lecture - Gravitational Physics, PHYS 636

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Subject: Cosmology, Strong Gravity

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L8 Black Hole Thermodynamics

Consider the Kerr b.h

$$ds^2 = \frac{\Delta}{\Sigma}\left[dt - a\sin^2\theta\, d\varphi\right]^2 - \frac{\Sigma}{\Delta}dr^2$$

$$- \frac{\sin^2\theta}{\Sigma}\left[(r^2+a^2)\,d\varphi - a\,dt\right]^2 - \Sigma\, d\theta^2 .$$

$\Delta = r^2 + a^2 - 2GMr$

$\Sigma = r^2 + a^2\cos^2\theta$

$\Delta = 0$ is horizon

$r_+ = GM + \sqrt{G^2M^2 - a^2}$

$a = J/M$

Now

Now consider the impact of absorbing a particle

M & J potentially change, as will r_+, but $\Delta = 0$ determines horizon.

Area of horizon: $\int (r^2 + a^2) \sin\theta = 4\pi (r_+^2 + a^2)$

$$\Rightarrow \quad \delta A = 8\pi (r_+ \delta r_+ + a\,\delta a)$$

$$\rightarrow \quad \delta r_+ = \frac{\delta A}{8\pi r_+} - \frac{a\,\delta a}{r_+}$$

Also

$$\delta a = \frac{M\delta J - J\delta M}{M^2}$$

Horizon is still $\Delta(r_+ + \delta r_+, M+\delta M, J+\delta J) = 0$

$$0 = \delta r_+ \Delta'(r_+) - 2Gr_+\delta M + 2a\,\delta a$$

$$= 2(r_+ - GM)\frac{\delta \mathcal{A}}{8\pi G} - 2Gr_+\delta M + \frac{2a\,\delta a}{r_+}\left[r_+ - (r_+ - GM)\right]$$

now use expression for δa

$0 = (r_+ - G$

$$0 = (r_+ - GM)\frac{\delta A}{4\pi r_+} - \frac{2G\,\delta M}{r_+}\left[r_+^2 + \frac{aJ}{M}\right] + \frac{2GMa}{r_+}\frac{\delta J}{M}$$

$$= \frac{2G}{r_+}(r_+^2+a^2)\left[-\delta M + \frac{a\,\delta J}{r_+^2+a^2} + \frac{(r_+-GM)}{2\pi(r_+^2+a^2)}\frac{\delta A}{4G}\right]$$

The angular velocity of the horizon is $\lim_{r\to r_+} \frac{g_{t\varphi}}{g_{\varphi\varphi}} = \frac{a}{r_+^2+a^2} = \Omega$

now use expression for δa

$$T = \frac{(r_+ - GM)}{2\pi(r_+^2 + a^2)}$$

$$\underline{\delta M} = \underline{\Omega \delta J} + \underline{T \delta S}$$

1st Law

$r_+ - GM)]$

The angular velocity of the horizon is $\lim_{r \to r_+} \frac{g_{t\varphi}}{g_{\varphi\varphi}} = \frac{a}{r_+^2 + a^2} = \Omega$

Alternatively

$$J^2 = a^2 M^2 = \frac{a^2 (r_+^2 + a^2)^2}{4 r_+^2 G^2} = \frac{a^2 S^2}{4\pi^2 r_+^2}$$

$$\Rightarrow \frac{4\pi^2 J^2}{S^2} + 1 = \frac{a^2}{r_+^2} + 1 = \frac{r_+^2 + a^2}{r_+^2} = \frac{4G^2M^2}{(r_+^2 + a^2)}$$

$$\rightarrow M^2 = \frac{S}{4\pi G}\left[1 + \frac{4\pi^2 J^2}{S^2}\right]$$

Christodoulou - Ruffini

"Chemical expression"

$$\Omega = \left.\frac{\partial M}{\partial J}\right|_S = \left(\frac{S}{4\pi G} \times \frac{8\pi^2 J}{S^2}\right) \Big/ 2M$$

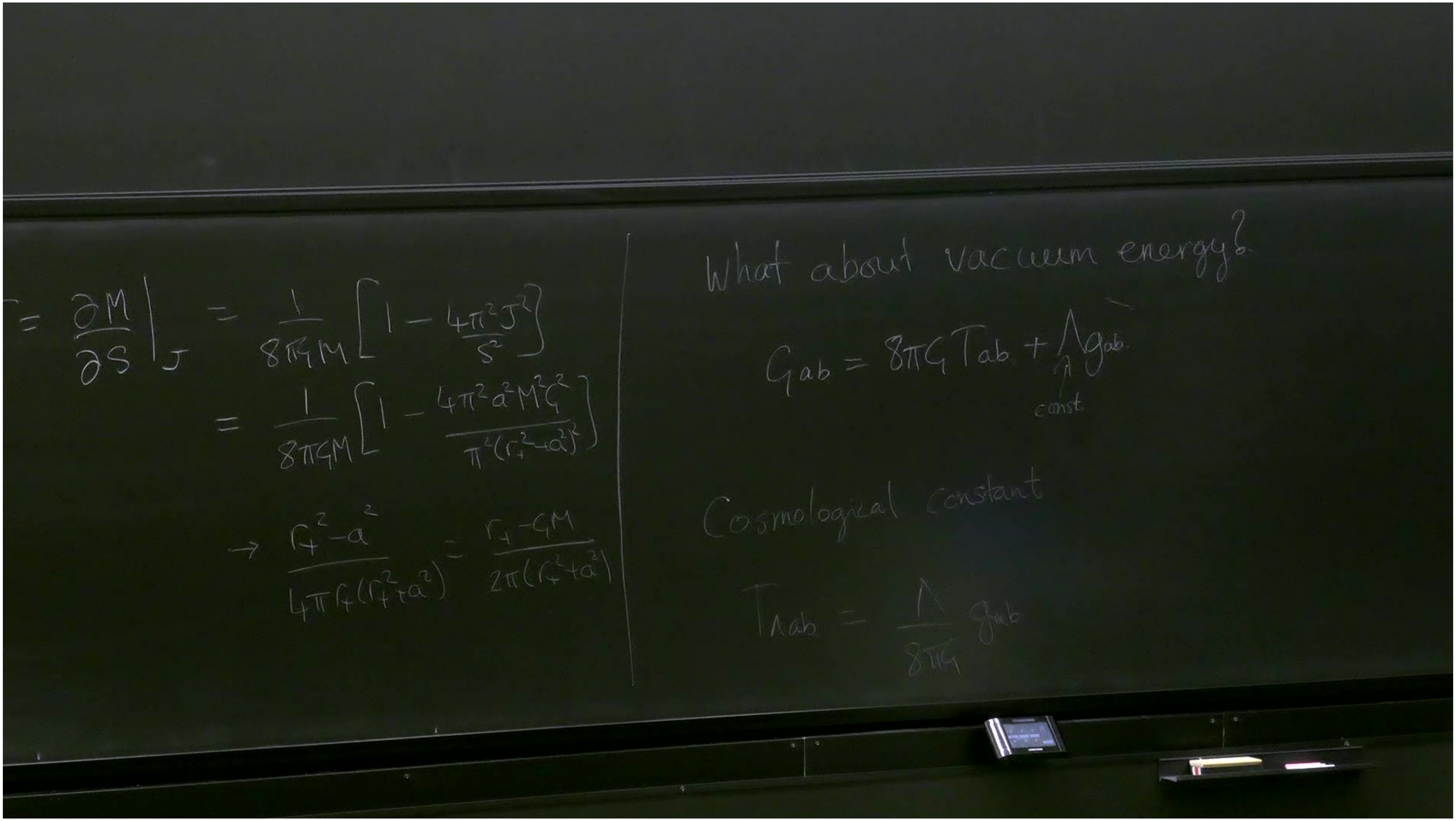
$$= \frac{\pi J}{GMS} \rightarrow \frac{a}{r_+^2 + a^2}.$$

$$T = \frac{\partial M}{\partial S}\bigg|_J = \frac{1}{8\pi G M}\left[1 - \frac{4\pi^2 J^2}{S^2}\right]$$

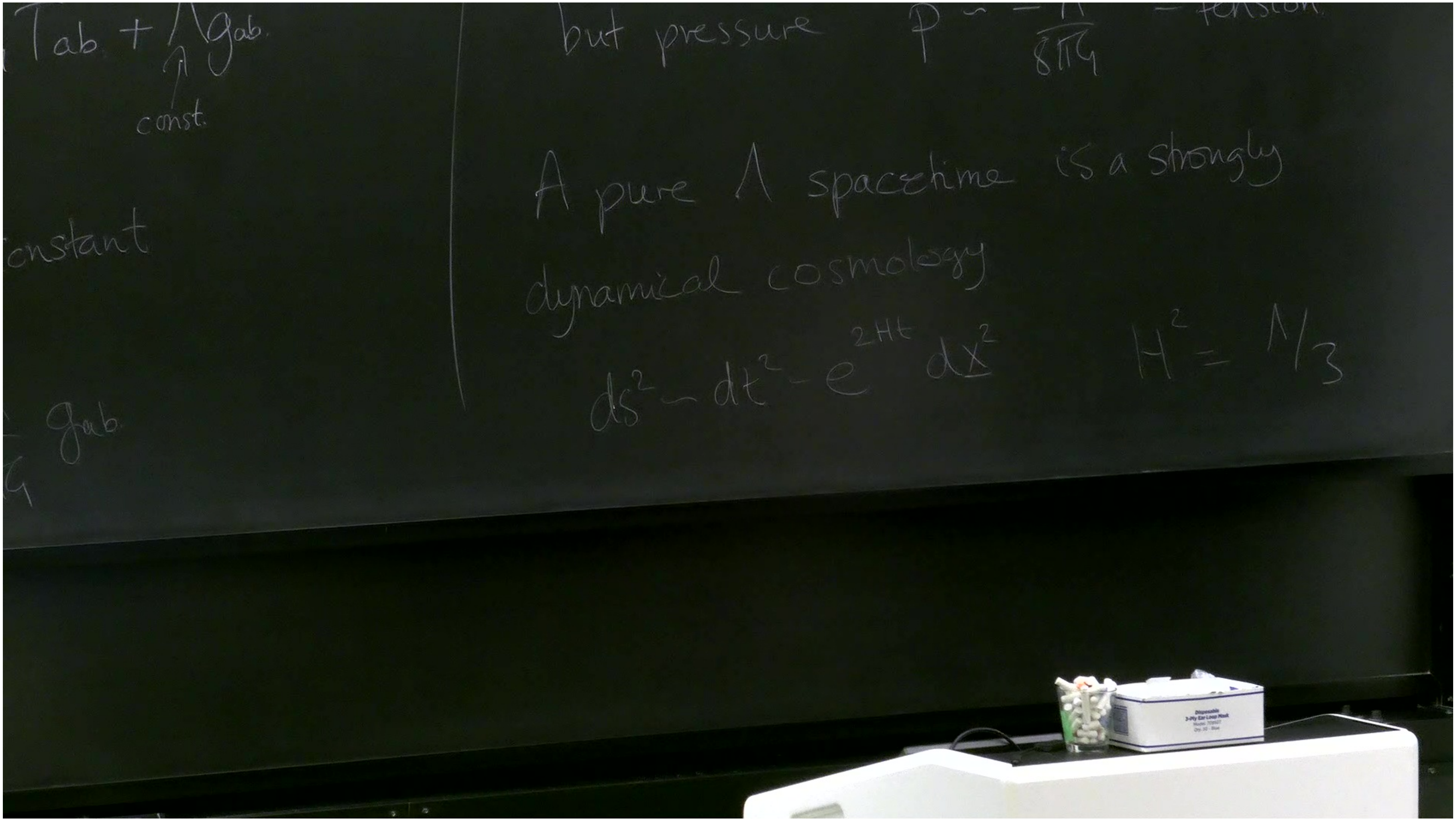
$$= \frac{1}{8\pi G M}\left[1 - \frac{4\pi^2 a^2 M^2 G^2}{\pi^2 (r_+^2 + a^2)^2}\right]$$

$$\rightarrow \frac{r_+^2 - a^2}{4\pi r_+ (r_+^2 + a^2)} = \frac{r_+ - GM}{2\pi (r_+^2 + a^2)}$$

What about vacuum



$= \frac{\partial M}{\partial S}\Big|_J = \frac{1}{8\pi G M}\left[1 - \frac{4\pi^2 J^2}{S^2}\right]$
$= \frac{1}{8\pi G M}\left[1 - \frac{4\pi^2 a^2 M^2 G^2}{\pi^2 (r_+^2 + a^2)^2}\right]$
$\rightarrow \frac{r_+^2 - a^2}{4\pi r_+ (r_+^2 + a^2)} = \frac{r_+ - GM}{2\pi (r_+^2 + a^2)}$
What about vacuum energy?
$G_{ab} = 8\pi G T_{ab} + \Lambda g_{ab}$
const.
Cosmological constant
$T_{\Lambda ab} = \frac{\Lambda}{8\pi G} g_{ab}$


Tab + Λ gab.
const.
onstant
gab.
but pressure p = −Λ/8πG = tension.
A pure Λ spacetime is a strongly dynamical cosmology
ds² = dt² − e^{2Ht} dx²
H² = Λ/3

de Sitter $\Lambda > 0$

const. curv. spacetime

$$\frac{1}{\ell} = \sqrt{\frac{\Lambda}{3}}$$

anti de Sitter $\Lambda < 0$.

const negatively curved spacetime

$$\frac{1}{L} = \sqrt{-\Lambda/3}$$

$\Lambda > 0$

pacetime

er $\Lambda < 0$.

ively curved spacetime

$-\Lambda/3$

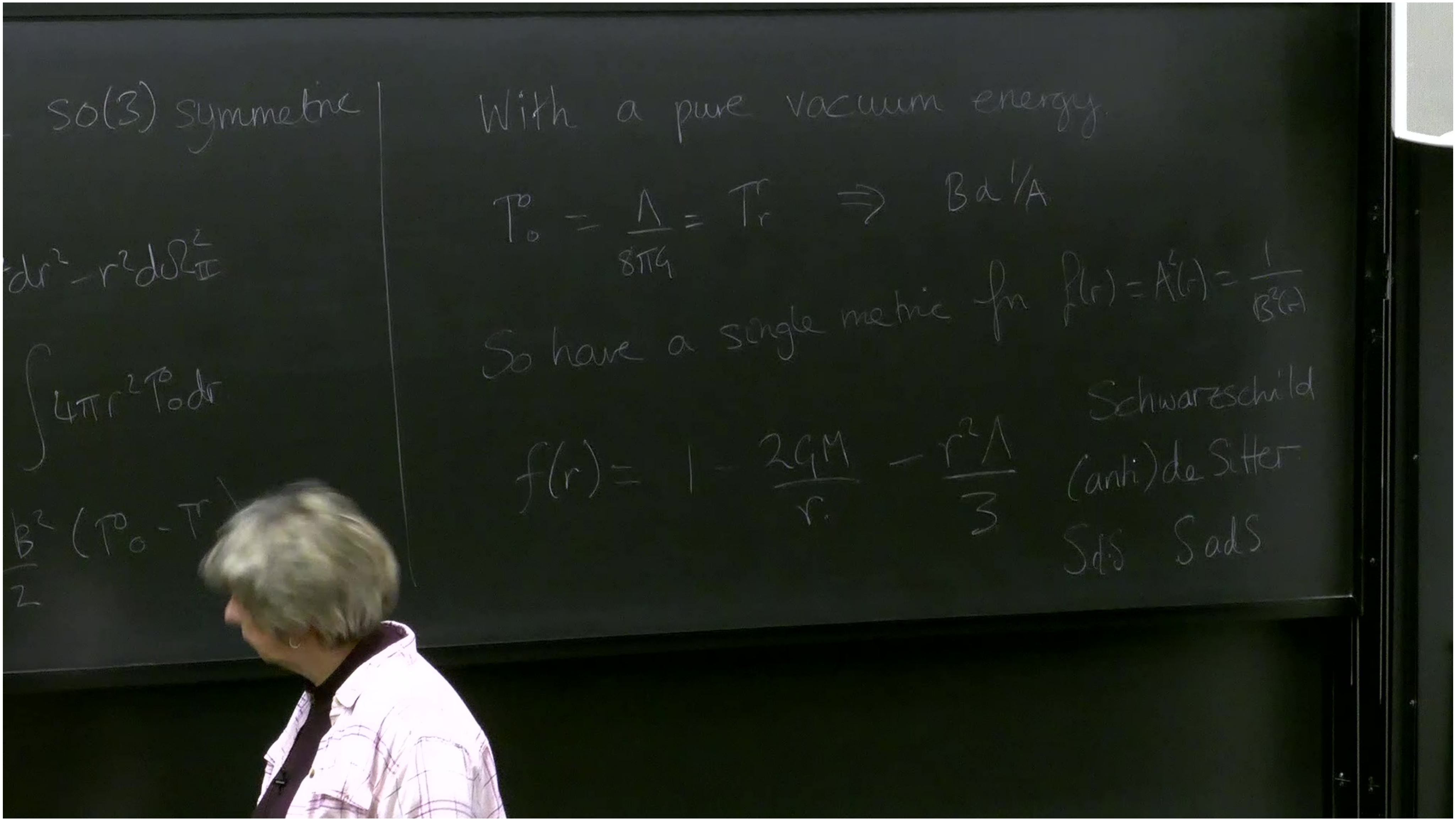
Recall the SO(3) symmetric static metric

$$ds^2 = A^2 dt^2 - B^2 dr^2 - r^2 d\Omega_{II}^2$$

$$B^{-2} = 1 - \frac{2G}{r}\int 4\pi r^2 T^0_0 \, dr$$

$$\frac{A'}{A} + \frac{B'}{B} = \frac{rB^2}{2}\left(T^0_0 - T^r_r\right) 8\pi G$$

With a pure vacuum


SO(3) symmetric
$dr^2 - r^2 d\Omega^2_{II}$
$\int 4\pi r^2 T^0_0 dr$
$\frac{B^2}{2}(T^0_0 - T^r$
With a pure vacuum energy
$T^0_0 = \frac{\Lambda}{8\pi G} = T^r_r \Rightarrow B \propto 1/A$
So have a single metric fn $f(r) = A^2(r) = \frac{1}{B^2(r)}$
$f(r) = 1 - \frac{2GM}{r} - \frac{r^2\Lambda}{3}$
Schwarzschild
(anti) de Sitter
SdS SadS

$$\frac{1}{L} = \sqrt{-\Lambda/3}$$

$$\frac{A'}{A} + \frac{B'}{B} = \frac{rB^2}{2}\left(T^0_{\ 0} - T^r_{\ r}\right)$$

SdS. $1 - \frac{2GM}{r} - \frac{r^2}{\ell^2}$

2 horizons in general.

$r \simeq 2GM \,, \quad r = \ell = \sqrt{\frac{3}{\Lambda}} \quad (GM \ll \ell)$

$$\frac{1}{L} = \sqrt{\frac{-\Lambda}{3}}$$

$$\frac{A'}{A} + \frac{B'}{B} = \frac{rB^2}{2}\left(T^0_0 - T^r_r\right) 8\pi G$$

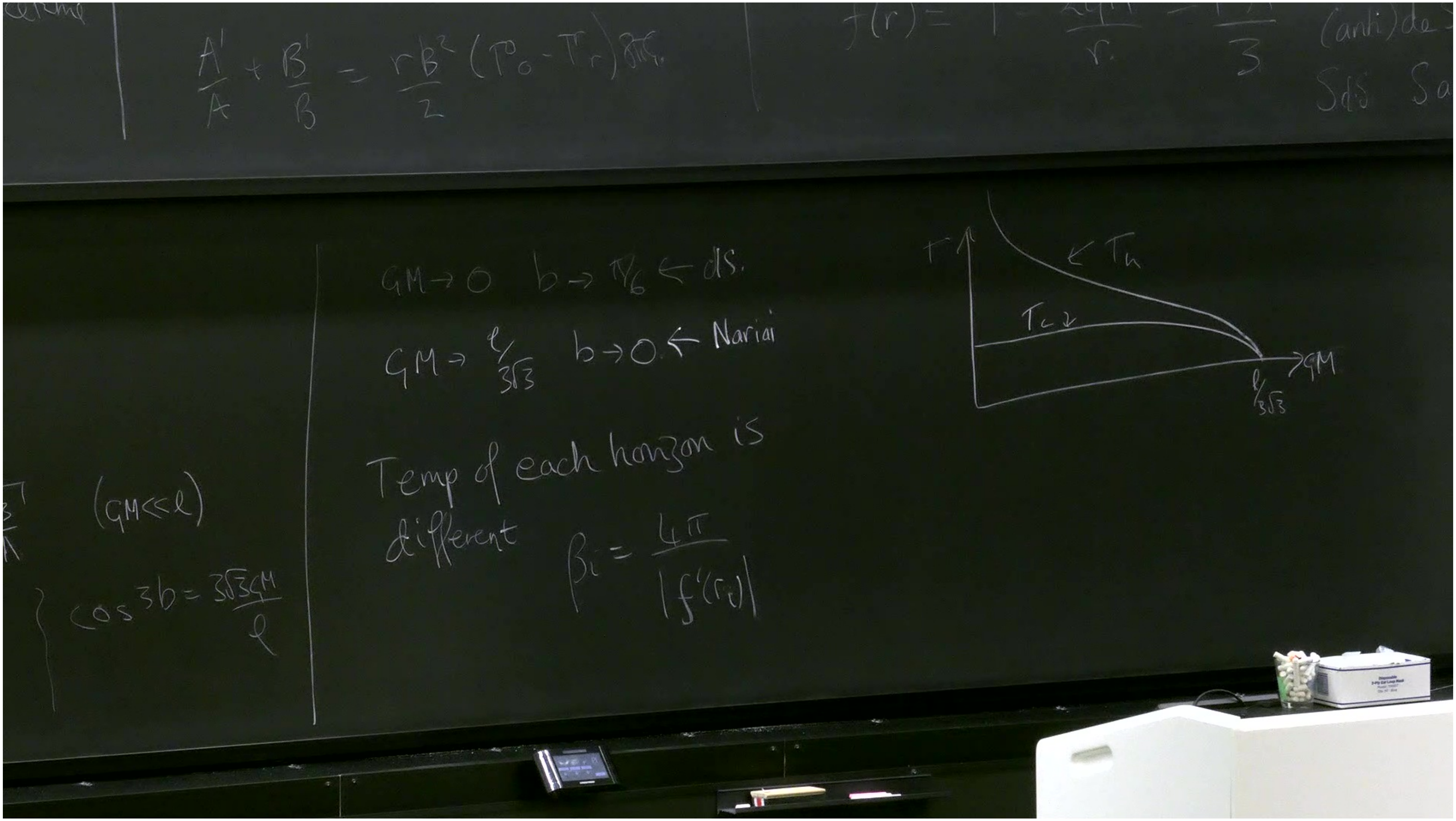
SdS. $1 - \frac{2GM}{r} - \frac{r^2}{\ell^2}$

2 horizons in general.

$r \simeq 2GM$, $r = \ell = \sqrt{\frac{3}{\Lambda}}$ $(GM \ll \ell)$

$$r_c = \frac{2}{\sqrt{3}}\,\ell \cos\left(\frac{\pi}{3} - b\right) \qquad \cos 3b = \frac{3\sqrt{3}GM}{\ell}$$

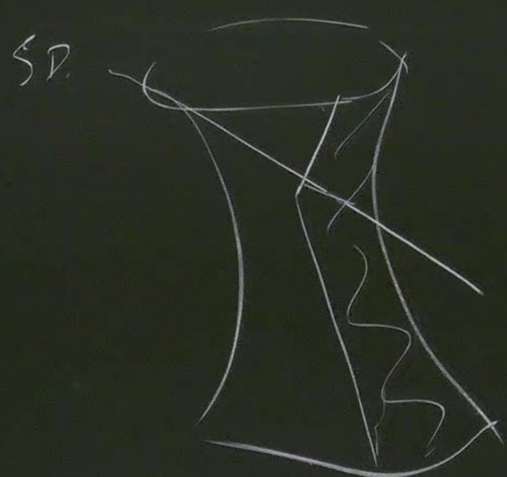
$$r_h = \frac{2}{\sqrt{3}}\,\ell \cos\left(\frac{\pi}{3} + b\right)$$



$\frac{A'}{A} + \frac{B'}{B} = \frac{rB^2}{2}(T^0_0 - T^r_r)8\pi G$
GM → 0 b → π/6 ← dS.
GM → ℓ/3√3 b → 0 ← Nariai
Temp of each horizon is different
$\beta_i = \frac{4\pi}{|f'(r_i)|}$
$(GM \ll \ell)$
$\cos 3b = \frac{3\sqrt{3}GM}{\ell}$
T
T_h
T_c
GM
ℓ/3√3

Pure dS : $$ds^2 = \left(1-\frac{r^2}{\ell^2}\right)dt^2 - \frac{dr^2}{1-\frac{r^2}{\ell^2}} - r^2 d\Omega_{\mathrm{II}}^2$$

STATIC PATCH



the horizon is $\quad r \to r_+ \quad \frac{g_{t\phi}}{g_{\phi\phi}} = \frac{a}{r_+^2 + a^2}$

S-AdS $\quad \exists$ 1 horizon $\quad 1 - \frac{2GM}{r} + \frac{r^2}{L^2}$

$$r_h = \frac{2}{\sqrt{3}} L \sinh\beta \qquad \sinh 3\beta = \frac{3\sqrt{3}GM}{L}$$

$\sim 2GM$ if $GM \ll L$

$3\sqrt[3]{2GML^2}$ $\quad GM \gg L$

$$T = \frac{f'(r_+)}{4\pi} = \frac{1}{4\pi r_+}\left[1 + \frac{3r_+^2}{L^2}\right]$$