Title: Lecture - Standard Model, PHYS 622

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Collection/Series: Standard Model (Elective), PHYS 622, January 6 - February 5, 2025

Subject: Particle Physics

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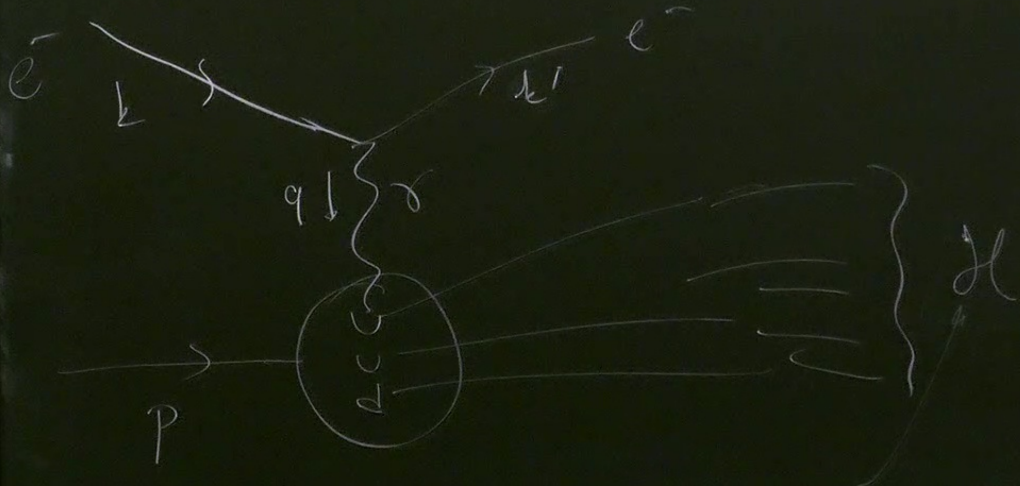
Deep Inelastic Scattering -

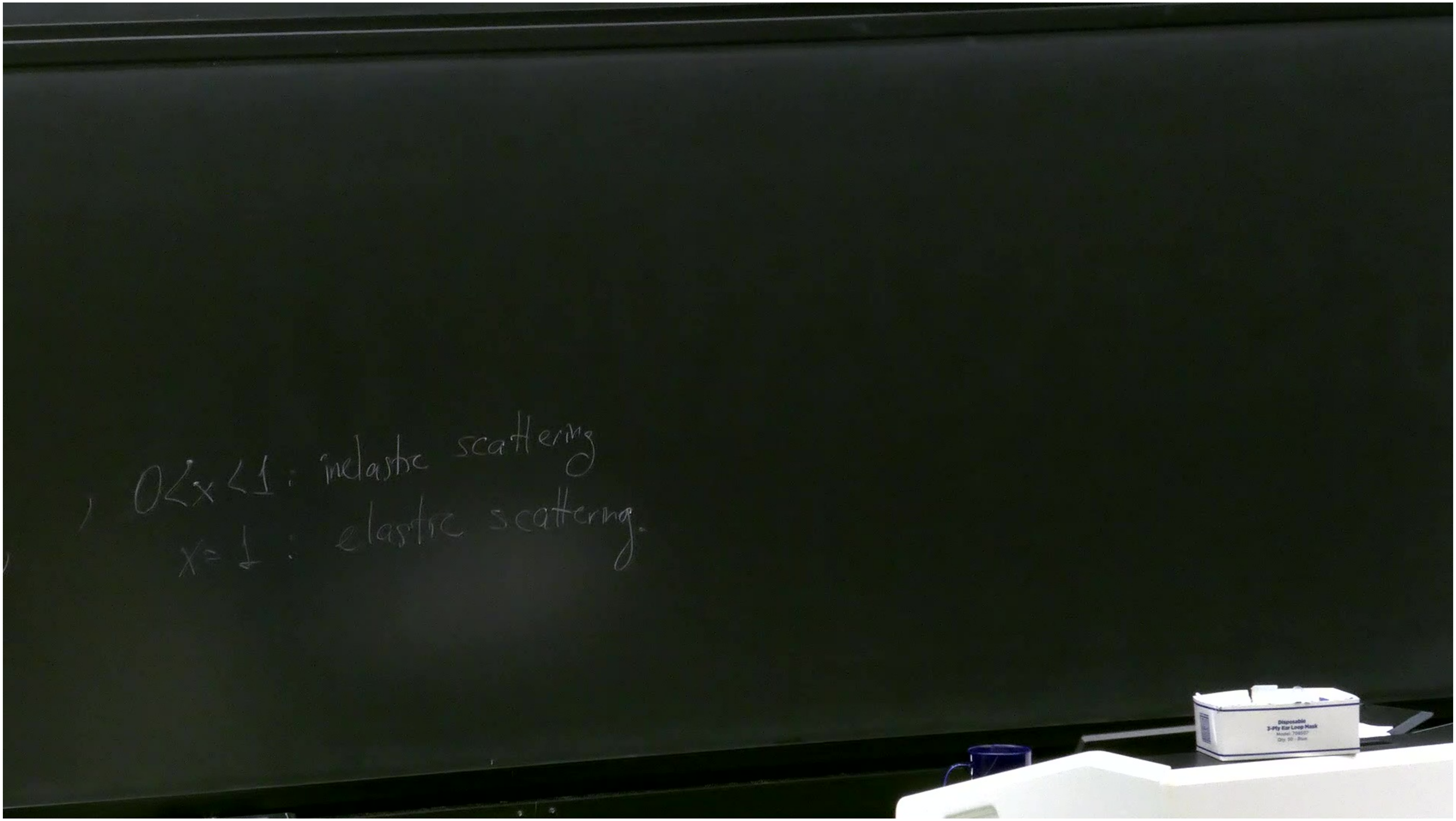


$$q^2 = (k - k')^2$$

$$\nu = \frac{-q \cdot p}{m_p}$$

$$y = \frac{q \cdot p}{k \cdot p} = \frac{\nu}{E_k}$$

$$x = \frac{q^2}{-2p \cdot q} = \frac{q^2}{2 m_p \nu}, \quad 0 < x < 1 : \text{inelas}$$


$0 < x < 1$: inelastic scattering
$x = 1$: elastic scattering.

$$\frac{d\sigma(e^-p \to e^-X)}{dx\,dy} = \frac{2\pi\alpha^2 s}{q^4}\left(x\,y^2\,\underbrace{F_1^e(x,q^2)} + 2(1-y)\,\underbrace{F_2^e(x,q^2)}\right)$$

structure functions.

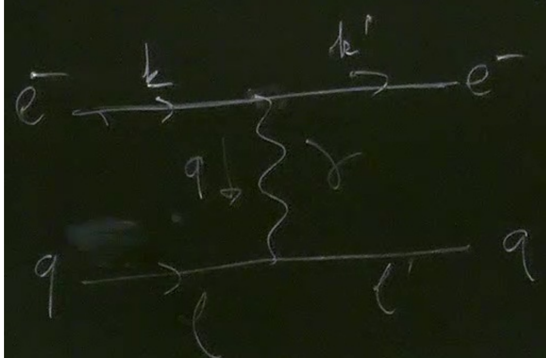
$0 < x < 1$: inelastic scattering

$x = 1$: elastic scattering.

$$\sigma(e^- p \to e^- X) = \sum_q \text{Prob}(q;\ell)\, \sigma(e^- q \to e^- q) \underbrace{\sum_H \text{Prob}(H)}_{1}$$

parton-level quantity

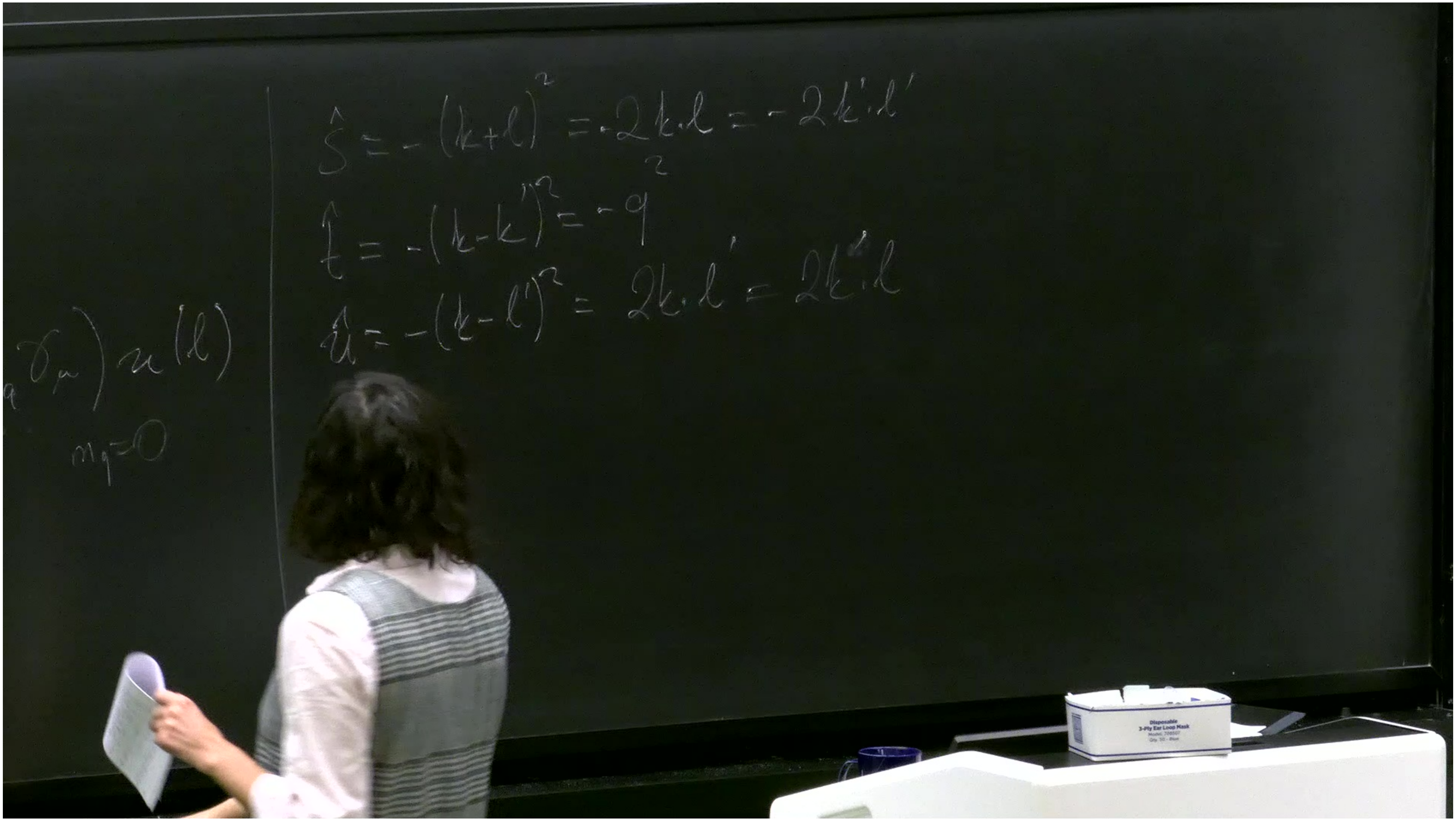
$$\sigma(e^- p \to e^- X) = \sum_q \text{Prob}(q;\ell)\, \hat{\sigma}(e^- q \to e^- q)$$



$$\mathcal{M} = \bar{u}(k')(-ie\gamma^\mu)u(k)\, \frac{-i}{q^2}\, \bar{u}(\ell')(-ieQ_q\gamma_\mu)u(\ell)$$

$m_q = 0$

$$\frac{1}{4}\sum |\mathcal{M}|^2 = \frac{2e^4 Q_q^2}{q^4}\left(\hat{s}^2 + \hat{u}^2\right)$$


$\hat{s} = -(k+l)^2 = -2k\cdot l = -2k'\cdot l'$
$\hat{t} = -(k-k')^2 = -q^2$
$\hat{u} = -(k-l')^2 = 2k\cdot l' = 2k'\cdot l$
$\gamma_\mu) u(l)$
$m_q = 0$

$q \longrightarrow \quad q \qquad q \qquad q^4$

$l \qquad l'$

$$\hat\sigma(e^- q \to e^- q) = \frac{1}{2E_k 2E_l |v_{rel}|} \int \frac{d^3k'}{(2\pi)^3} \frac{1}{2E_{k'}} \int \frac{d^3l'}{(2\pi)^3} \frac{1}{2E_{l'}} (2\pi)^4 \delta^4(k+l-k'-l') \frac{1}{4} \sum |$$

in the c.o.m frame

$$= \frac{1}{2\hat{s}} \int \frac{d^3k'}{(2\pi)^3} (2\pi)\, \delta(\sqrt{\hat s} - 2E_{k'}) \frac{1}{4} \sum |M|^2 = \frac{1}{8\pi^2 \hat{s}^2} \int$$

$$= \frac{1}{32\pi \hat{s}} \int d\cos\theta \, \frac{2e^4 Q_q^4}{q^4} (\hat{s}^2 + \hat{u}^2) \Rightarrow \frac{d\hat\sigma(e^- q \to e^- q)}{d\cos\theta} =$$

$$\frac{d^3 l'}{(2\pi)^3}\,\frac{1}{2E_{l'}}\,(2\pi)^4\,\delta^4(k+l-k'-l')\,\frac{1}{4}\sum|\mathcal{M}|^2$$

$$\sqrt{\hat{s}} - 2E_{k'}\Big)\,\frac{1}{4}\sum|\mathcal{M}|^2 = \frac{1}{8\pi^2\hat{s}^2}\int E_{k'}^2\,dE_{k'}\int d\Omega\,\frac{1}{2}\,\delta\Big(E_{k'}-\frac{\sqrt{\hat{s}}}{2}\Big)\,\frac{1}{4}\sum|\mathcal{M}|^2$$

$$(\hat{s}^2+\hat{u}^2) \Rightarrow \frac{d\hat{\sigma}(e^-q\to e^-q)}{d\cos\theta} = \frac{\pi\alpha^2 Q_q^2}{\hat{s}\,\hat{t}^2}(\hat{s}^2+\hat{u}^2)$$

$$\hat{t} = -(k-k')^2 = -2E_k E_{k'} + 2|\vec{k}||\vec{k}'|\cos\theta = -\frac{1}{2}\hat{s}(1-\cos\theta)$$

$$d\hat{t} = \frac{1}{2}\hat{s}\, d\cos\theta$$

$$\frac{d\hat{\sigma}}{d\hat{t}} = \frac{2\pi\alpha^2 Q_q^2}{\hat{s}^2\hat{t}^2}\left(\hat{s}^2+\hat{u}^2\right)$$

Go f

Go from parton level to hadron level:

$$l = x \cdot p$$

$$\hat{s} = -(k+l)^2 = -2k \cdot l = -2x\, k \cdot p = x \cdot s = \hat{s}$$

$$\hat{t} = -y \cdot x \cdot s$$

$$\hat{u} = -s \cdot x(1-y)$$

$$\frac{d\hat{\sigma}}{dy} = \frac{d\hat{\sigma}}{d\hat{t}} \left| \frac{d\hat{t}}{dy} \right| = \frac{2\pi\alpha^2 Q_q^2}{q^4} \left(y^2 + 2(1-y) \right) x \cdot s$$

Define: $f_q(x)\,dx$: prob. of finding a quark w/ momentum fraction

$$\sigma(e^-p \to e^-X) = \sum_q \int_0^1 dx\, f_q(x)\, \hat{\sigma}(e^-q \to e^-q)$$

$$\frac{d\sigma(e^-p \to e^-X)}{dx} = \sum_q f_q(x)\, \hat{\sigma}(e^-q \to e^-q)$$

...tum fraction between x and x+dx inside the proton.

$$\frac{d\sigma(e^-p \to e^-X)}{dx\,dy} = \sum_q f_q(x) \frac{d\hat{\sigma}(e^-q \to e^-q)}{dy}$$

$$\frac{d\sigma(e^-p \to e^-X)}{dx\,dy} = \frac{2\pi\alpha^2 s}{q^4}\left(\sum_q x y^2 Q_q^2 f_q(x) + \sum_q 2(1-y) x f_q(x)\right)$$

...tum fraction between x and $x+dx$ inside the proton.

$$(\ldots e^- q)$$

$$\frac{d\sigma(e^-p\to e^-X)}{dx\,dy} = \sum_q f_q(x)\,\frac{d\hat{\sigma}(e^-q\to e^-q)}{dy}$$

$$\frac{d\sigma(e^-p\to e^-X)}{dx\,dy} = \frac{2\pi\alpha^2 s}{q^4}\left(\sum_q x y^2 Q_q^2 f_q(x) + \sum_q 2(1-y)\,x f_q(x)\right)$$

$$= \frac{2\pi\alpha^2 s}{q^4}\left(x y^2 F_1^e(x,q^2) + 2(1-y)\,F_2^e(x,q^2)\right)$$

$$\left.\begin{aligned} F_1^e(x) &= \sum_q Q_q^2 f_q(x) \\ F_2^e(x) &= x \sum_q Q_q^2 f_q(x) \end{aligned}\right\} \quad \boxed{F_2^e = x F_1^e}$$

Callan-Gross relation.

$\underbrace{F_1^e, F_2^e}$ don't depend on q^2 : Björken scaling.

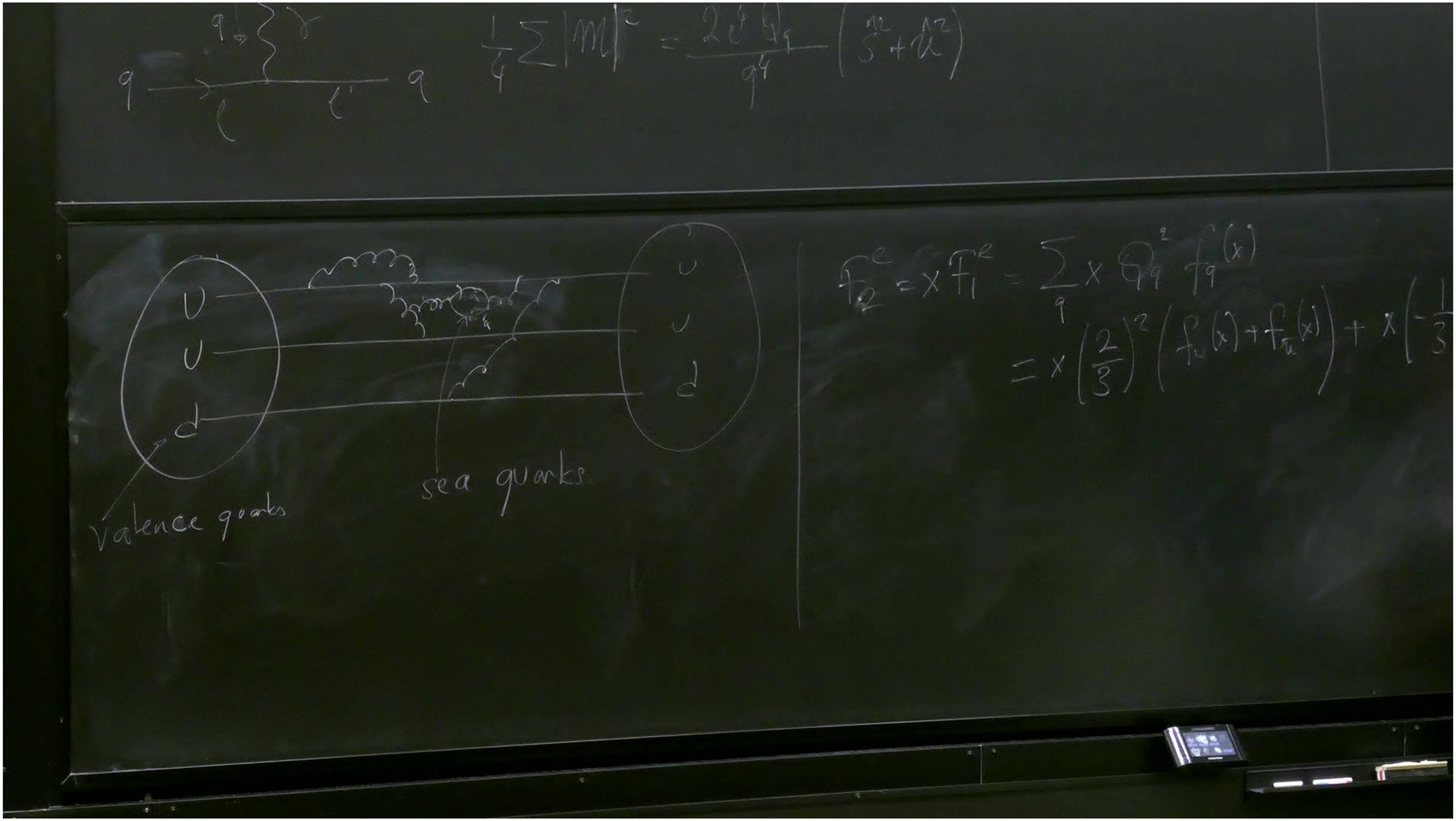
(violated at $\mathcal{O}(\alpha_s)$)

$$= \frac{2\pi\alpha^2 s}{q^4}\left(x y^2 \underbrace{F_1^e(x,q^2)} + 2(1-y) \underbrace{F_2^e(x,q^2)} \right)$$

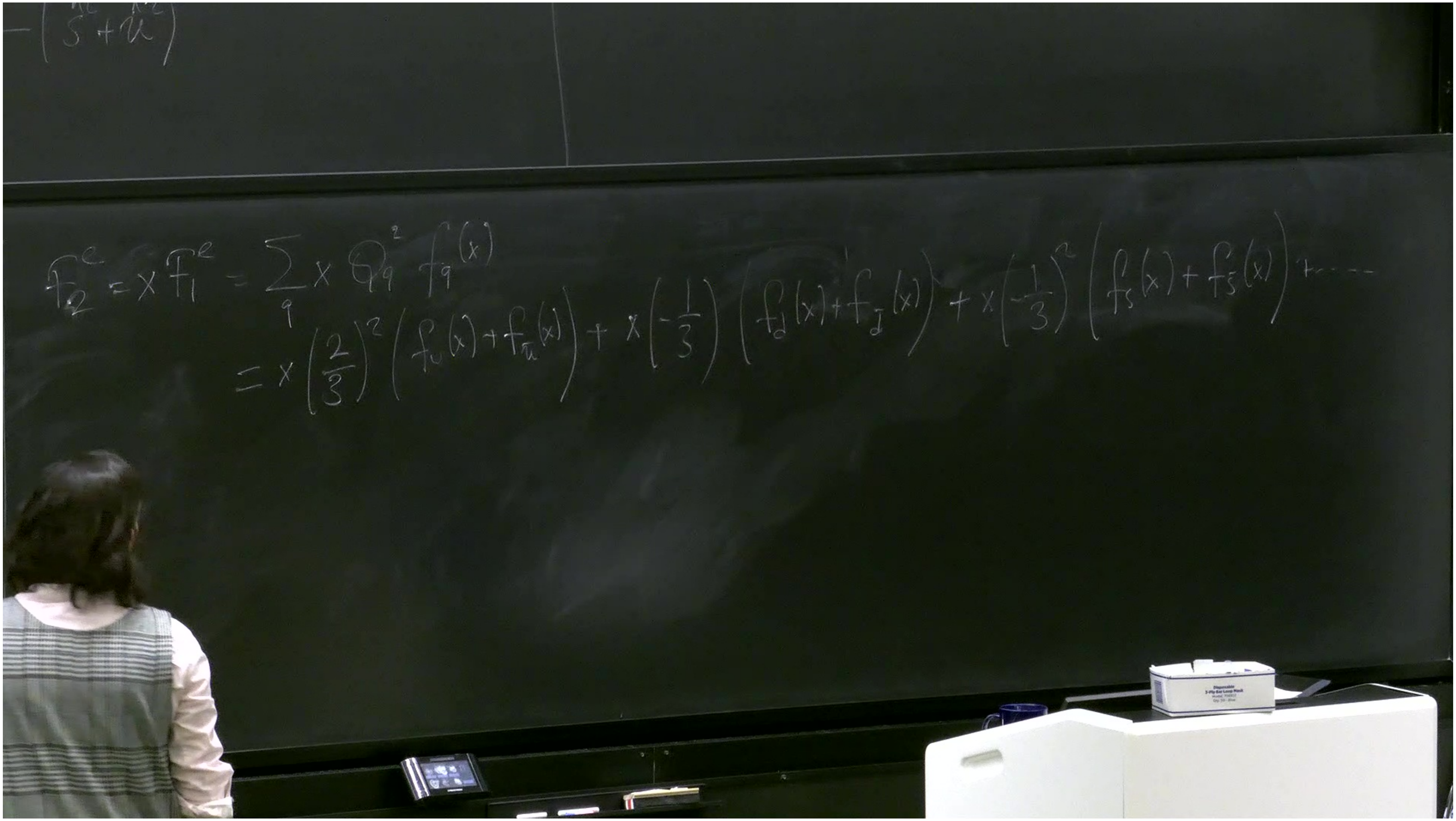
Gross

ation.

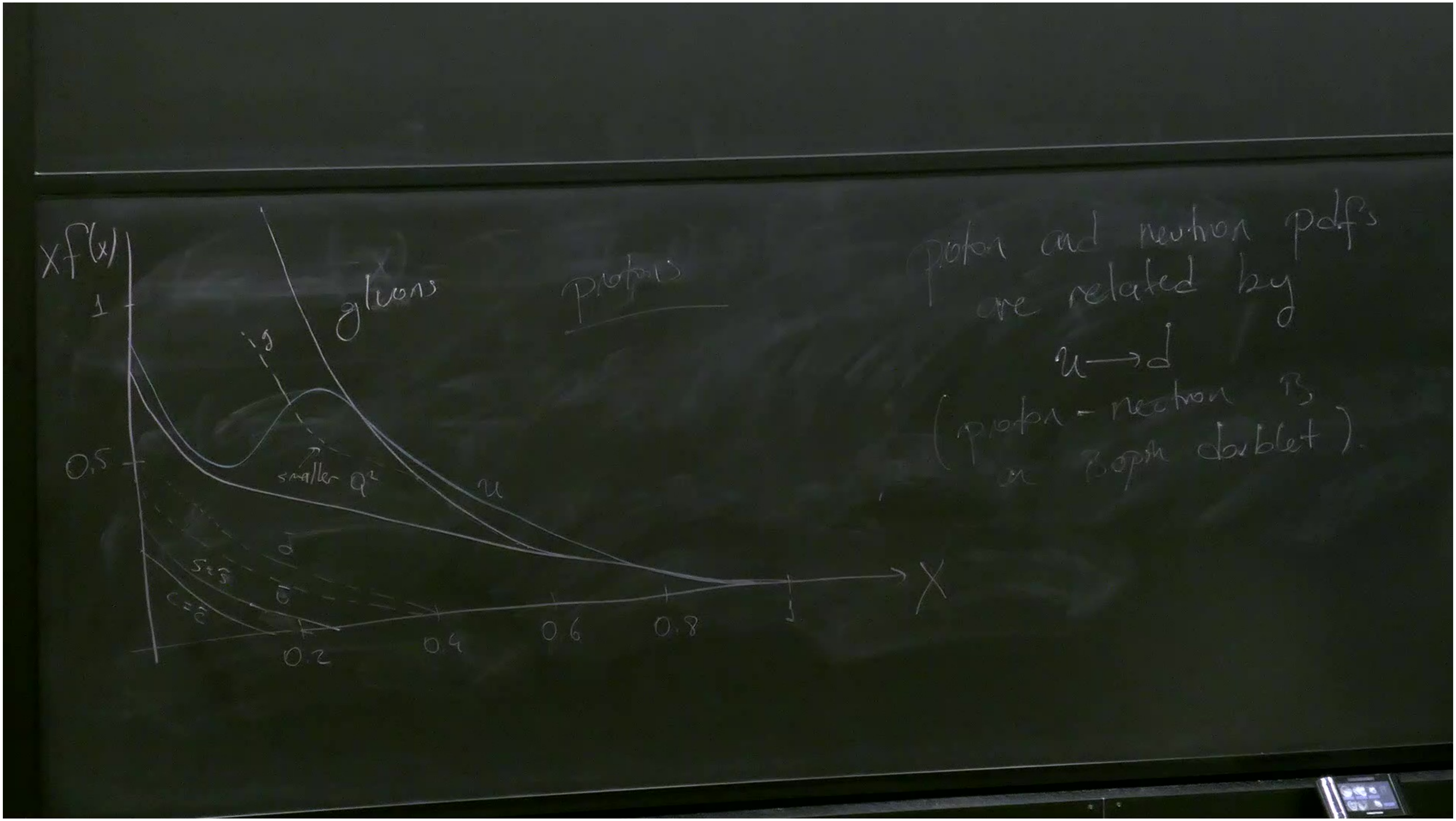
$f_q(x)$: parton distribution function (p.d.f)



valence quarks
sea quarks
u
u
d



$F_2^e = x F_1^e = \sum_q x Q_q^2 f_q(x)$
$= x\left(\frac{2}{3}\right)^2 \left(f_u(x) + f_{\bar{u}}(x)\right) + x\left(-\frac{1}{3}\right)^2 \left(f_d(x) + f_{\bar{d}}(x)\right) + x\left(-\frac{1}{3}\right)^2 \left(f_s(x) + f_{\bar{s}}(x)\right) + \cdots$


xf(x)
gluons
protons
proton and neutron pdf's
are related by
u ⟶ d
(proton - neutron is
an isospin doublet).
smaller Q²
u
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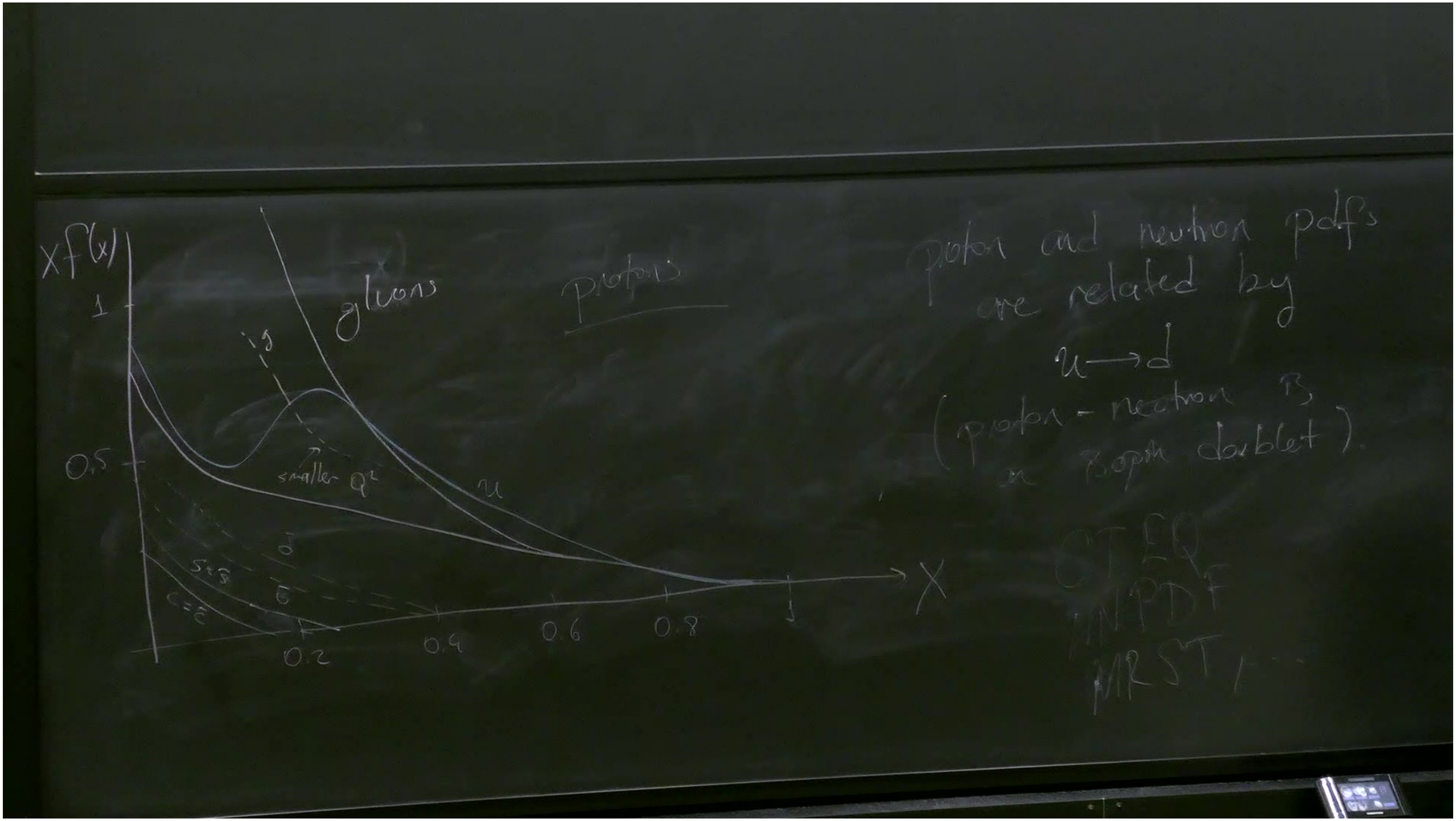
1
0.5
0.2
0.4
0.6
0.8
1
X


xf(x)
1
0.5
gluons
protons
smaller Q²
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0.6
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1
x
proton and neutron pdf's
are related by
u ⟶ d
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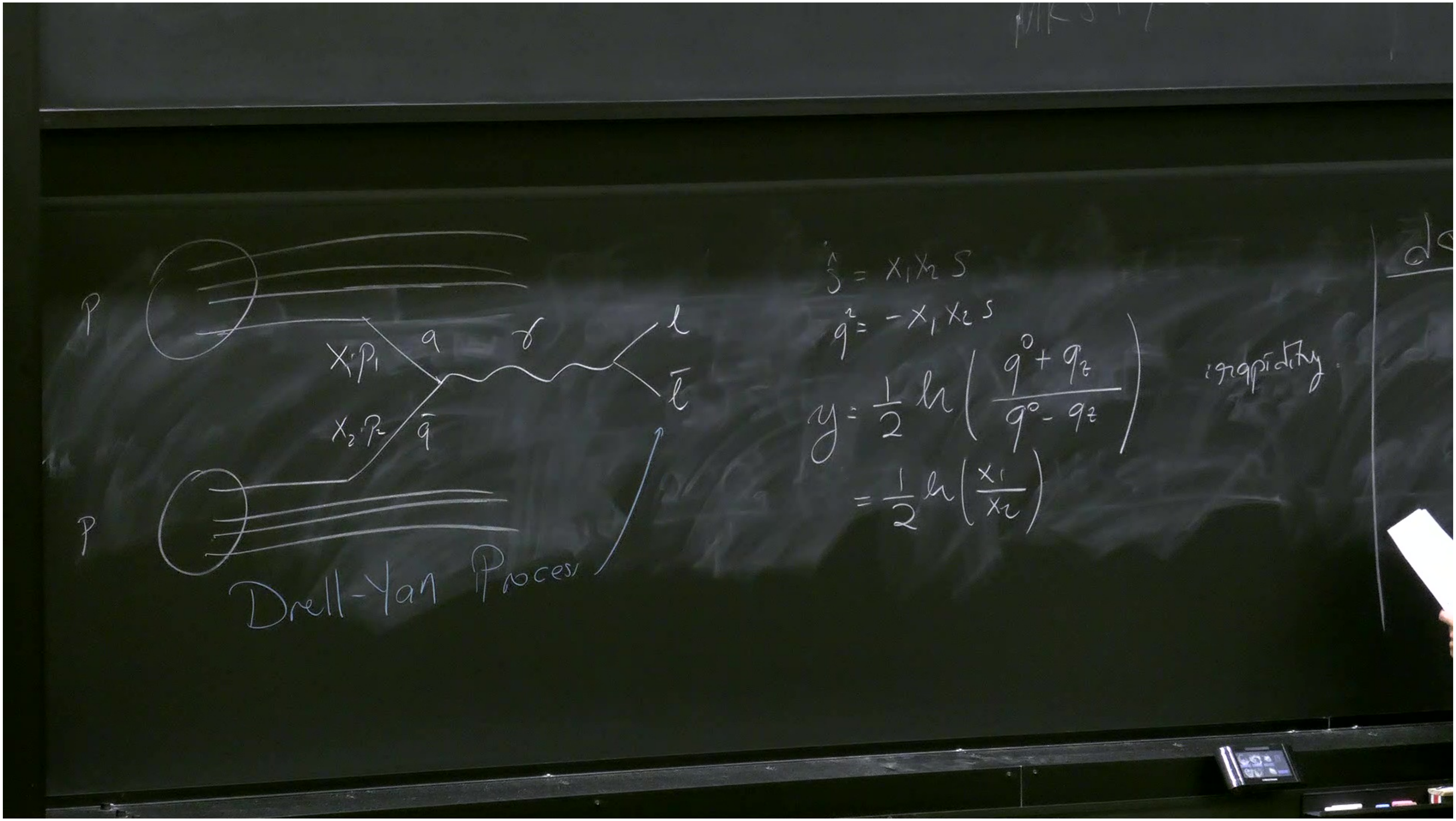
CTEQ
NNPDF
MRST, ...


p
p
$x_1 p_1$
$x_2 p_2$
q
$\bar{q}$
γ
l
$\bar{l}$
Drell-Yan Process
$\hat{s} = x_1 x_2 s$
$q^2 = -x_1 x_2 s$
$y = \frac{1}{2} \ln\left(\frac{q^0 + q_z}{q^0 - q_z}\right)$
$= \frac{1}{2} \ln\left(\frac{x_1}{x_2}\right)$
rapidity

rapidity.

$$\frac{d\sigma(pp\to \ell\bar{\ell})}{dq^2\,dy} = \int_0^1 dx_1 \int_0^1 dx_2 \sum_q \left(f_q(x_1)\, f_{\bar{q}}(x_2) + f_{\bar{q}}(x_1)\, f_q(x_2) \right)$$

$$\times\, \hat{\sigma}(q\bar{q}\to \ell\bar{\ell}) \cdot \delta\left(|q^2| - x_1 x_2 s\right) \cdot \delta\left(y - \tfrac{1}{2}\ln(x_1/x_2)\right)$$

$$\sigma(q\bar{q}\to \ell\bar{\ell}) = \frac{4\pi\alpha^2 Q_q^2}{9 x_1 x_2 s}$$