

**Title:** Lecture - Standard Model, PHYS 622

**Speakers:** Seyda Ipek

**Collection/Series:** Standard Model (Elective), PHYS 622, January 6 - February 5, 2025

**Subject:** Particle Physics

**Date:** January 29, 2025 - 10:15 AM

**URL:** <https://pirsa.org/25010023>

# Deep Inelastic Scattering -



$$q^2 = (k - k')^2$$

$$y = \frac{-q \cdot p}{m_p v}$$

$$y = \frac{q \cdot p}{k \cdot p} = \frac{v}{E_k}$$

$$x = \frac{q^2}{-2p \cdot q} = \frac{q^2}{2m_p v}$$

$0 < x < 1$  : inelas

$0 < x < 1$ : inelastic scattering  
 $x = 1$ : elastic scattering.

$$\frac{d\sigma(e^-p \rightarrow e^-X)}{dx dy} = \frac{2\pi\alpha^2 s}{q^4} \left( xy^2 \underbrace{F_1^e(x, q^2)} + 2(1-y) \underbrace{F_2^e(x, q^2)} \right)$$

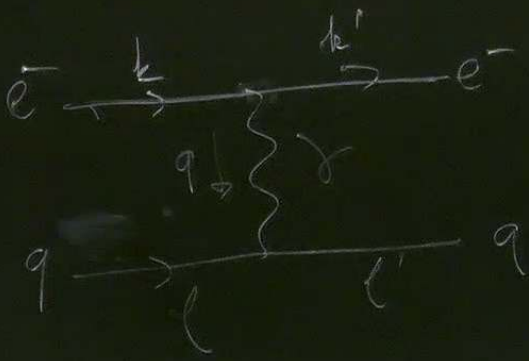
structure functions.

$0 < x < 1$ : inelastic scattering  
 $x = 1$ : elastic scattering.

$$\sigma(e^- p \rightarrow e^- X) = \sum_q \text{Prob}(q; l) \sigma(e^- q \rightarrow e^- q) \underbrace{\sum_H \text{Prob}(H)}_1$$

parton-level quantity

$$\sigma(e^- p \rightarrow e^- X) = \sum_q \text{Prob}(q; l) \hat{\sigma}(e^- q \rightarrow e^- q)$$



$$iM = \bar{u}(k') (-ie\gamma^\mu) u(k) \frac{-i}{q^2} \bar{u}(l') (-ieQ_q\gamma_\mu) u(l)$$

$m_q=0$

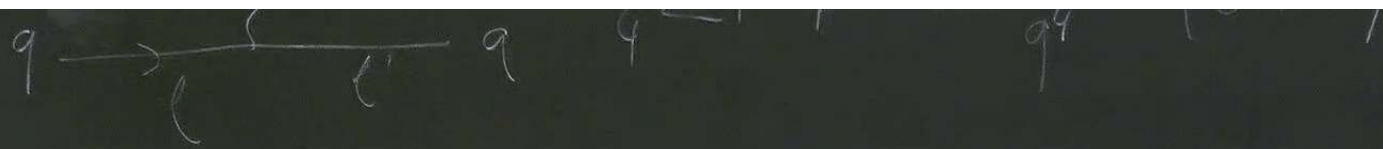
$$\frac{1}{4} \sum |M|^2 = \frac{2e^4 Q_q^2}{q^4} (s^2 + u^2)$$

$\sigma_{\mu} u(l)$   
 $m_g = 0$

$$\hat{J} = -(k+l)^2 = -2kl = -2k'l'$$

$$\hat{J} = -(k-k')^2 = -q^2$$

$$\hat{J} = -(k-l')^2 = 2kl' = 2k'l$$



$$\begin{aligned}
 \delta(\bar{e} q \rightarrow \bar{e} q) &= \frac{1}{2E_k 2E_{k'} |v_{rel}|} \int \frac{d^3 k'}{(2\pi)^3} \frac{1}{2E_{k'}} \int \frac{d^3 l}{(2\pi)^3} \frac{1}{2E_l} (2\pi)^4 \delta^4(k+l-k'-l') \frac{1}{4} \sum_{\text{spins}} |M|^2 \\
 &= \frac{1}{2\hat{s}} \int \frac{d^3 k'}{(2\pi)^3} (2\pi) \delta(\sqrt{s} - 2E_{k'}) \frac{1}{4} \sum_{\text{spins}} |M|^2 = \frac{1}{8\pi^2 \hat{s}^2} \int \frac{d^3 k'}{(2\pi)^3} \\
 &= \frac{1}{32\pi \hat{s}} \int d\cos\theta \frac{2e^4 Q_q^4}{q^4} (\hat{s}^2 + \hat{u}^2) \Rightarrow \frac{d\hat{\sigma}(\bar{e} q \rightarrow \bar{e} q)}{d\cos\theta} =
 \end{aligned}$$

in the c.m.  
frame

$$\begin{aligned}
 & \frac{1}{(2\pi)^3} \frac{1}{2E_k} (2\pi)^4 \delta^4(k+l-k'-l') \frac{1}{4} \sum |M|^2 \\
 & \frac{1}{(s - 2E_k)} \frac{1}{4} \sum |M|^2 = \frac{1}{8\pi^2 s} \int \Omega_k' d\Omega_k' \int d\Omega \frac{1}{2} \delta(\Omega_k' - \frac{\sqrt{s}}{2}) \frac{1}{4} \sum |M|^2 \\
 & (s + u) \Rightarrow \frac{d\sigma(e^-q \rightarrow e^-q)}{d\cos\theta} = \frac{\pi^2 Q_q^2}{s^2} (s^2 + u^2)
 \end{aligned}$$

$$\hat{t} = -(\mathbf{k} - \mathbf{k}')^2 = -2E_k E_{k'} + 2|\mathbf{k}||\mathbf{k}'|\cos\theta = -\frac{1}{2}\hat{s}(1 - \cos\theta)$$

$$d\hat{t} = \frac{1}{2}\hat{s} d\cos\theta$$

$$\frac{d\hat{s}}{d\hat{t}} = \frac{2\pi\alpha^2 Q^2 q^2}{\hat{s}^2 \hat{t}} \left( \frac{\hat{s}^2}{\hat{s}} + \hat{u}^2 \right)$$

Go f

So from parton level to hadron level:

$$l = x \cdot p$$

$$\hat{s} = -(k+l)^2 = -2k \cdot l = -2x k \cdot p = x \cdot s = \hat{s}$$

$$\hat{t} = -y \cdot x \cdot s$$

$$\hat{u} = -s \cdot x(1-y)$$

$$\frac{d\hat{\sigma}}{dy} = \frac{d\hat{\sigma}}{d\hat{t}} \left| \frac{d\hat{t}}{dy} \right| = \frac{2\pi\alpha^2 Q_q^2}{q^4} (y^2 + 2(1-y)) x \cdot s$$

Define:  $f_q(x) dx$  = prob. of finding a quark w/ momentum fraction

$$\sigma(e p \rightarrow e X) = \sum_q \int_0^1 dx f_q(x) \hat{\sigma}(e q \rightarrow e q)$$

$$\frac{d\sigma(e p \rightarrow e X)}{dx} = \sum_q f_q(x) \hat{\sigma}(e q \rightarrow e q)$$

fraction between  $x$  and  $x+dx$  inside the proton.

$$\frac{d\sigma(e p \rightarrow e X)}{dx dy} = \sum_q f_q(x) \frac{d\hat{\sigma}(e q \rightarrow e q)}{dy}$$

$$\frac{d\sigma(e p \rightarrow e X)}{dx dy} = \frac{2\pi^2 s}{q^4} \left( \sum_q x y^2 Q_q^2 f_q(x) + \sum_q 2(1-y)x f_q(x) \right)$$

un fraction between  $x$  and  $x+dx$  inside the proton.

$$\frac{d\sigma(\bar{e}p \rightarrow e^+X)}{dx dy} = \sum_q f_q(x) \frac{d\hat{\sigma}(\bar{e}q \rightarrow e^+q)}{dy}$$

$$\begin{aligned} \frac{d\sigma(\bar{e}p \rightarrow e^+X)}{dx dy} &= \frac{2\pi\alpha^2 S}{q^4} \left( \sum_q xy^2 \underbrace{Q_q^2}_{\text{charge}} f_q(x) + \sum_q 2(1-y)x \underbrace{f_q(x)}_{\text{parton}} \right) \\ &= \frac{2\pi\alpha^2 S}{q^4} \left( xy^2 \underbrace{F_1^e(x, q^2)}_{\text{structure}} + 2(1-y) \underbrace{F_2^e(x, q^2)}_{\text{structure}} \right) \end{aligned}$$

$$\begin{aligned}
 F_1^e(x) &= \sum_q Q_q^2 f_q(x) \\
 F_2^e(x) &= x \sum_q Q_q^2 f_q(x)
 \end{aligned}$$

don't depend on  $q^2$  : Björken scaling

$$F_2^e = x F_1^e$$

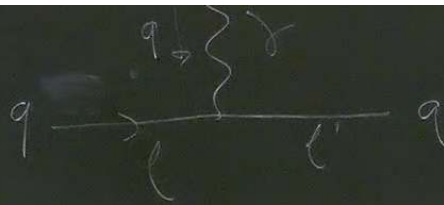
Callan-Gross relation.

(violated at  $\mathcal{O}(\alpha_s)$ )

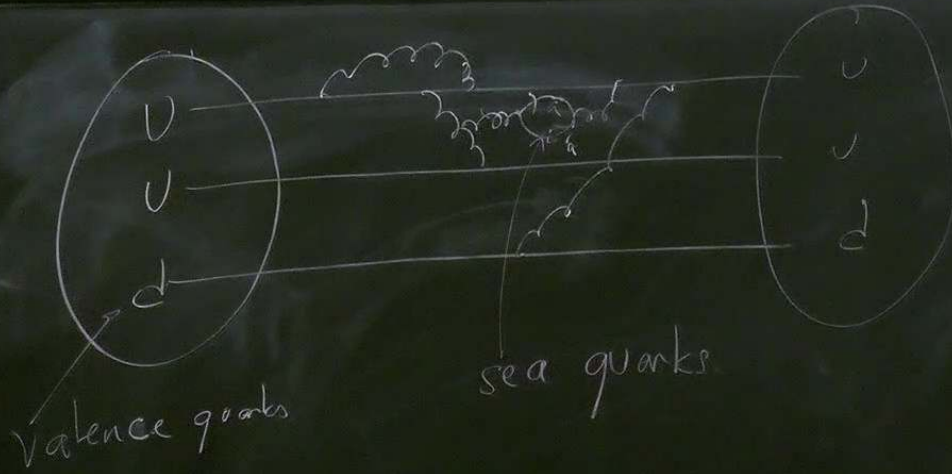
$$= \frac{2\pi\alpha^2 S}{q^4} \left( x y^2 \underbrace{F_1^e(x, q^2)} + 2(1-y) \underbrace{F_2^e(x, q^2)} \right)$$

$f_q(x)$  : parton distribution function (p.d.f)

Gross  
function.



$$\frac{1}{4} \sum |m_i|^2 = \frac{2 \cdot 3 \cdot 4}{9^4} (5 + 12)$$

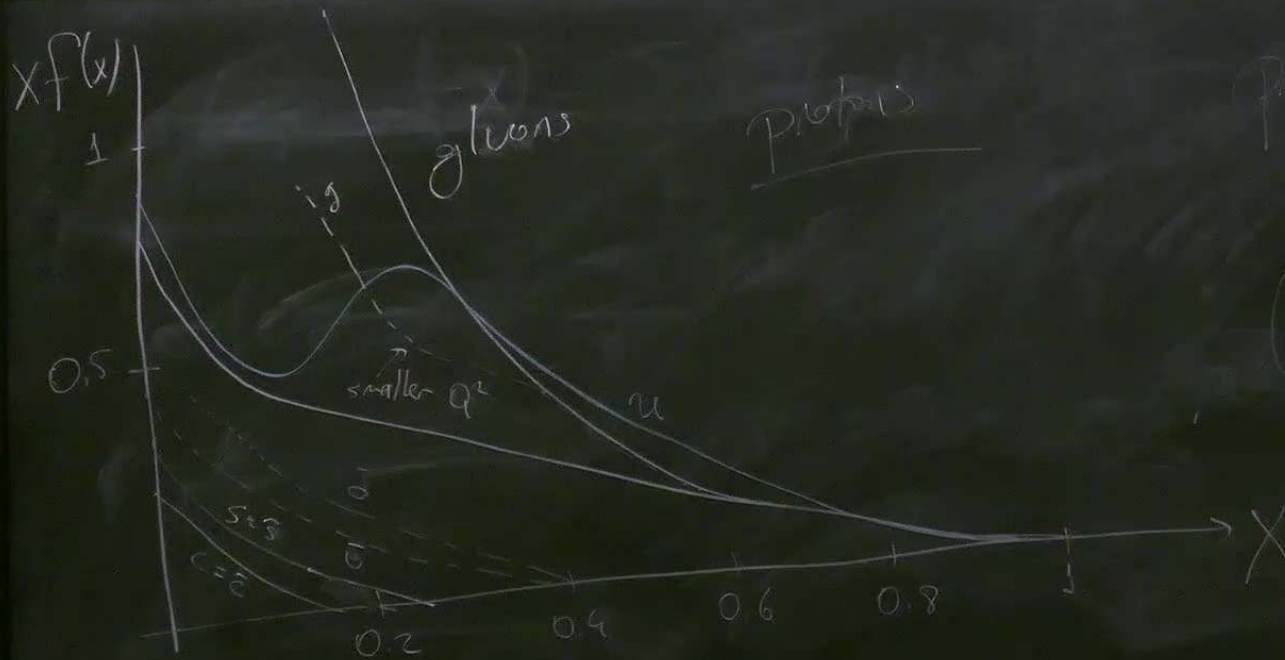


$$F_2^e = x F_1^e = \sum_q x Q_q^2 f_q(x)$$

$$= x \left(\frac{2}{3}\right)^2 (f_u(x) + f_{\bar{u}}(x)) + x \left(-\frac{1}{3}\right)^2 (f_d(x) + f_{\bar{d}}(x))$$

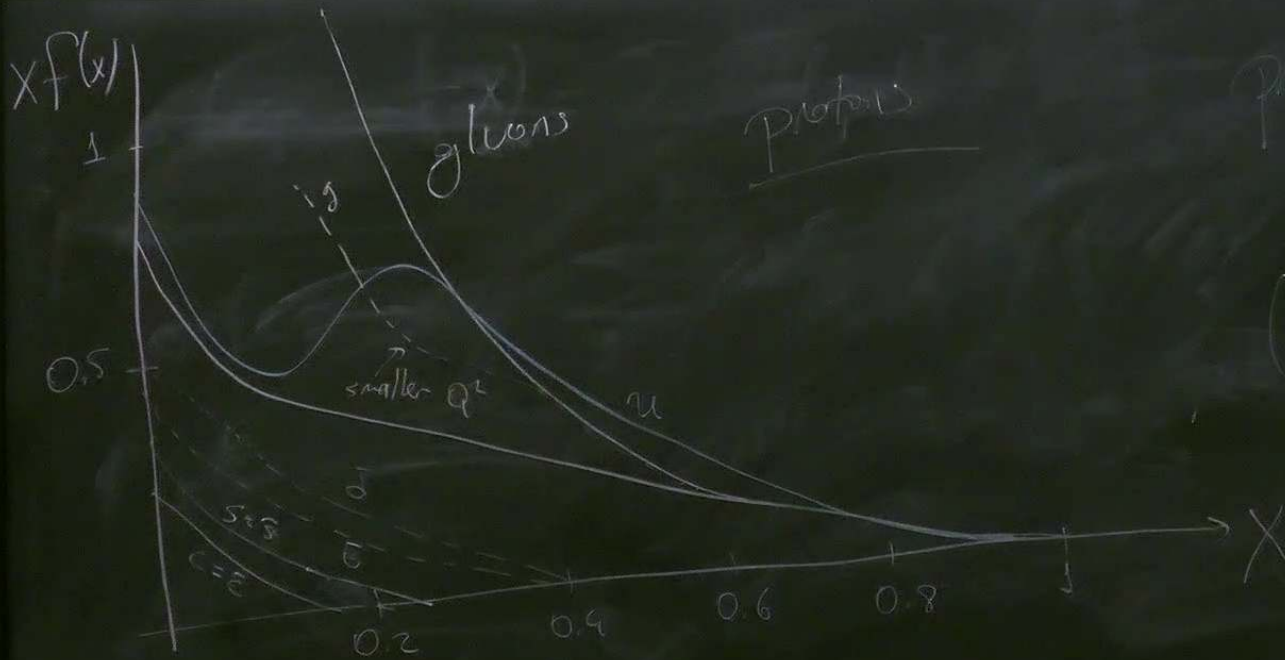
$$-\left(\frac{1}{5} + \frac{1}{10}\right)$$

$$\begin{aligned} F_2^e &= x F_1^e = \sum_9 x Q_9^2 f_9(x) \\ &= x \left(\frac{2}{3}\right)^2 (f_0(x) + f_a(x)) + x \left(-\frac{1}{3}\right) (f_d(x) + f_j(x)) + x \left(-\frac{1}{3}\right)^2 (f_s(x) + f_5(x)) + \dots \end{aligned}$$



protons

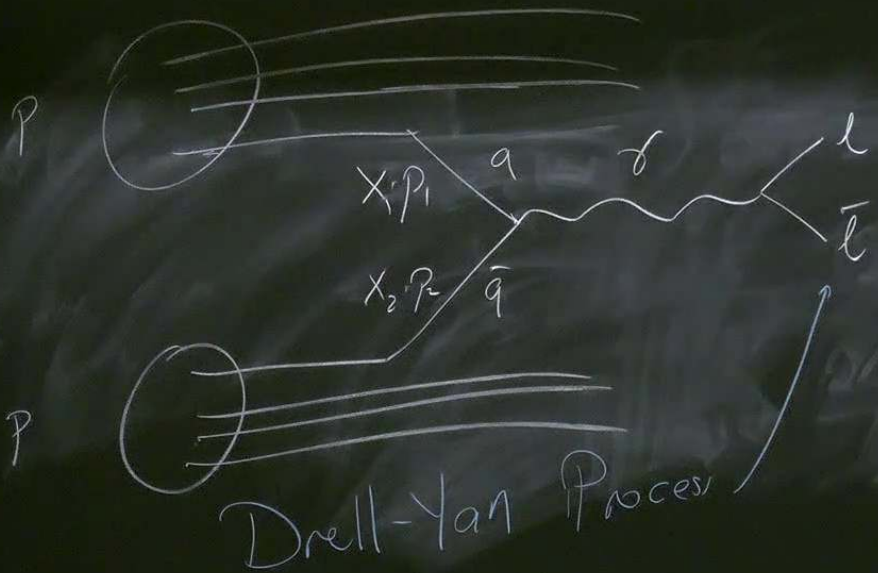
proton and neutron pdfs  
are related by  
 $u \rightarrow d$   
(proton - neutron  $B_3$   
or  $B_8$  doublet).



proton and neutron pdfs  
 are related by  
 $u \rightarrow d$   
 (proton - neutron is  
 an isospin doublet)

CTEQ  
 NNPDF  
 MRST

PHYS 117



$$\hat{S} = x_1 x_2 S$$

$$q^2 = -x_1 x_2 s$$

$$y = \frac{1}{2} \ln \left( \frac{q^0 + q_z}{q^0 - q_z} \right)$$

$$= \frac{1}{2} \ln \left( \frac{x_1}{x_2} \right)$$

rapidity

ds

isospin

$$\frac{d\sigma(pp \rightarrow e\bar{e})}{dq^2 dy} = \int_0^1 dx_1 \int_0^1 dx_2 \sum_q \left( f_q(x_1) f_{\bar{q}}(x_2) + f_{\bar{q}}(x_1) f_q(x_2) \right) \times \delta(q\bar{q} \rightarrow e\bar{e}) \cdot \delta(|q^2 - x_1 x_2 s|) \cdot \delta\left(y - \frac{1}{2} \ln\left(\frac{x_1}{x_2}\right)\right)$$

$$\sigma(q\bar{q} \rightarrow e\bar{e}) = \frac{4\pi\alpha^2 Q_q^2}{9x_1 x_2 s}$$