Title: Lecture - Standard Model, PHYS 622

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Subject: Particle Physics

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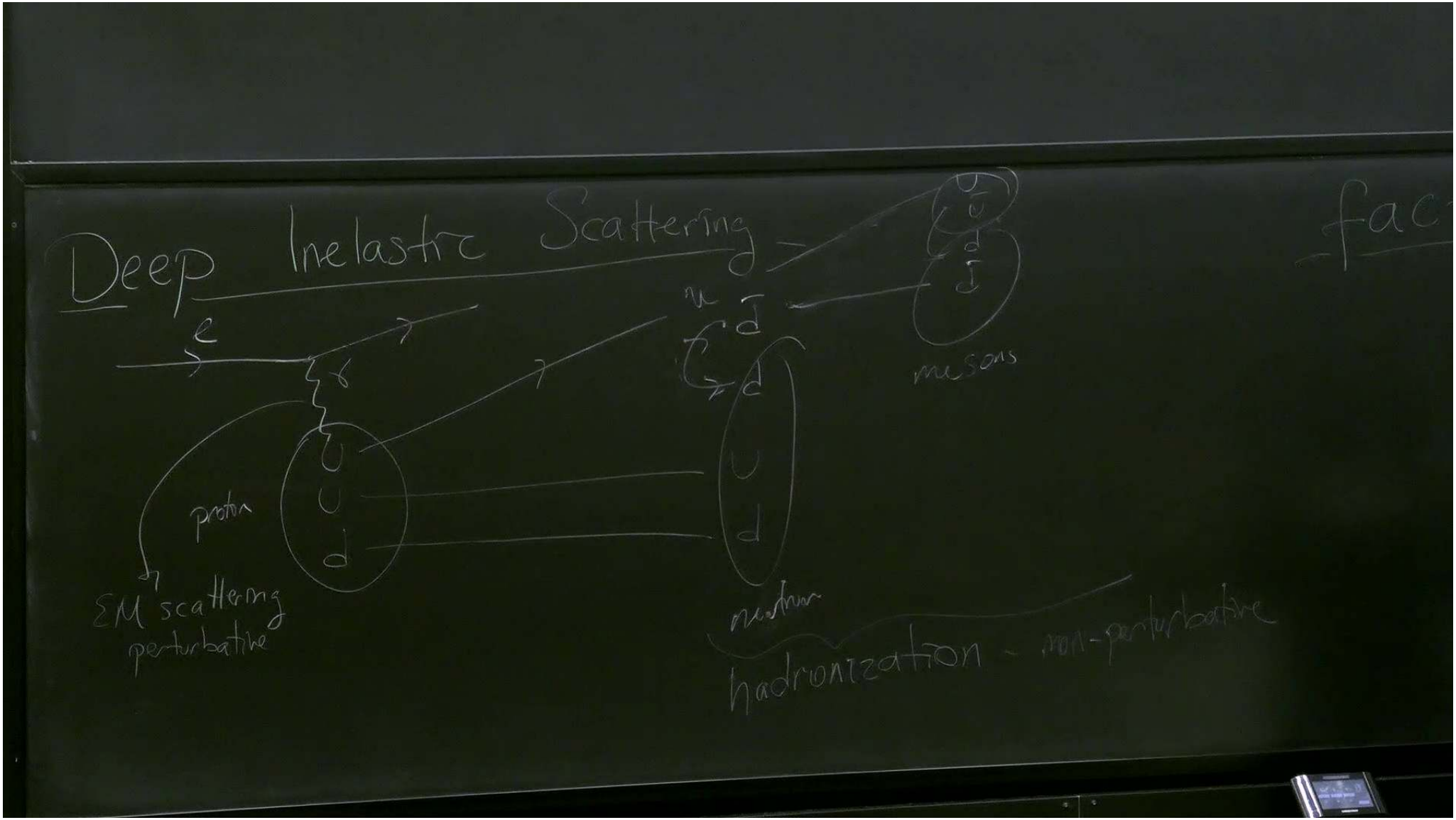
Deep Inelastic Scattering
fac
e
γ
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proton
EM scattering
perturbative
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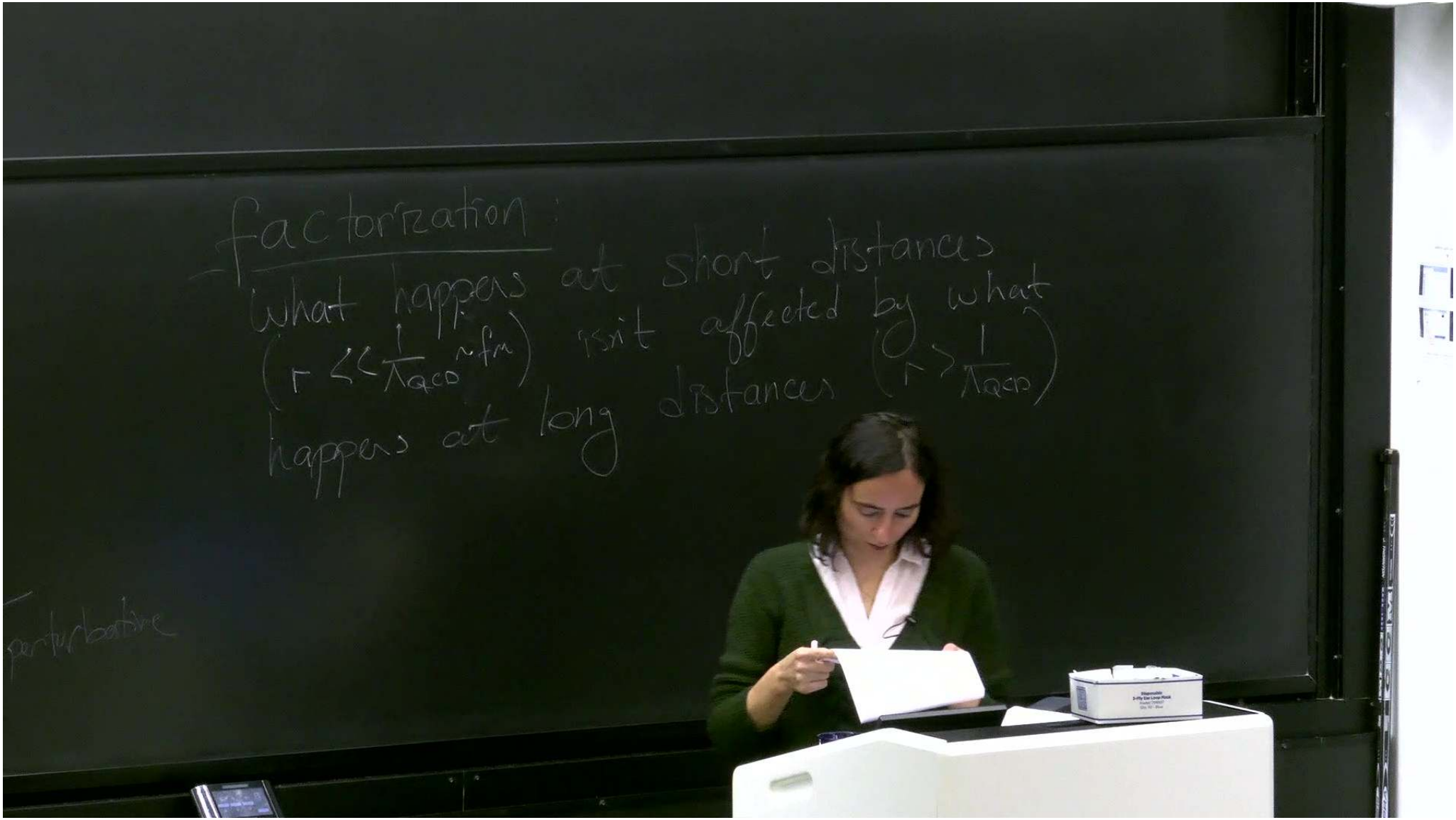
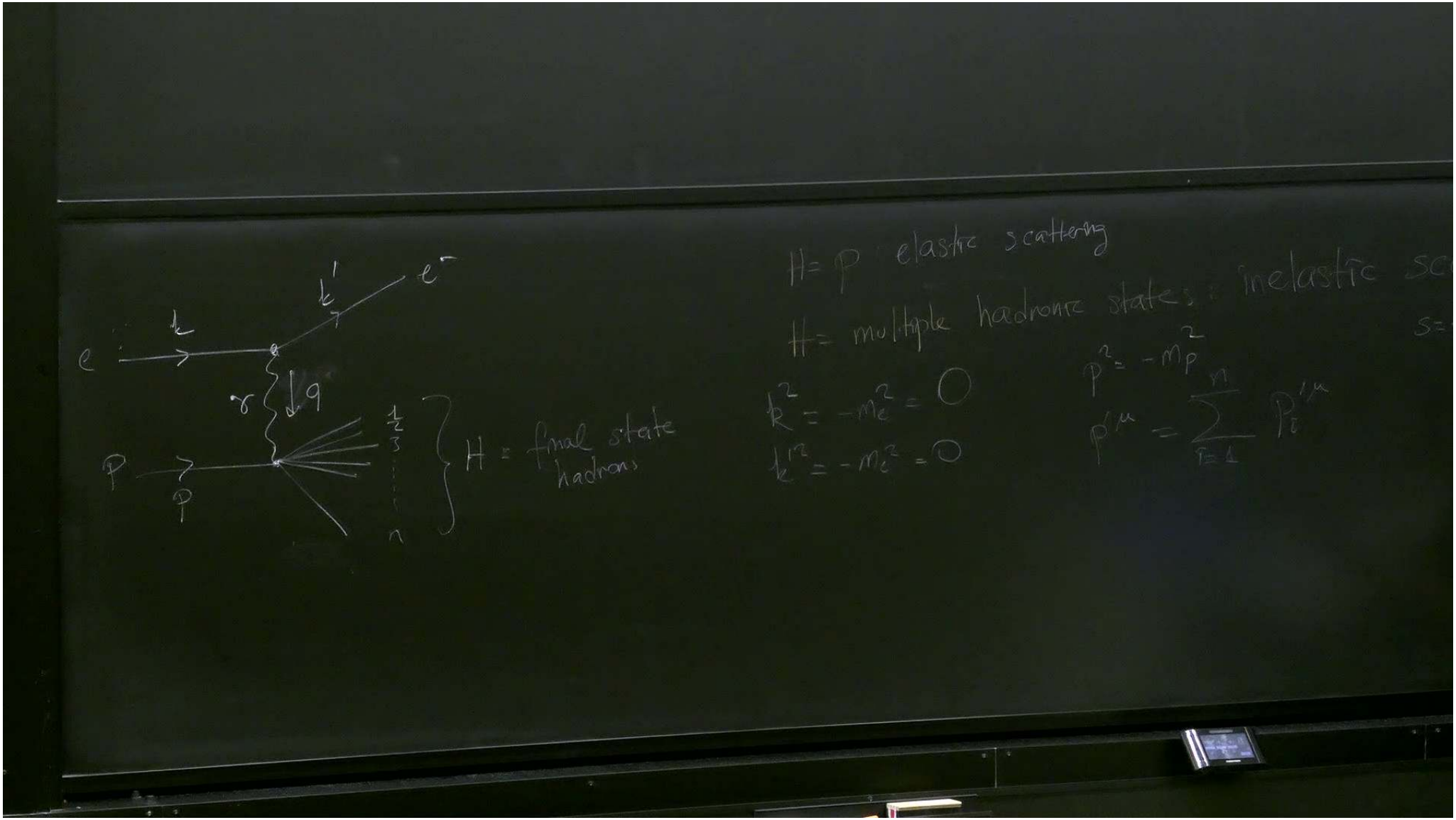
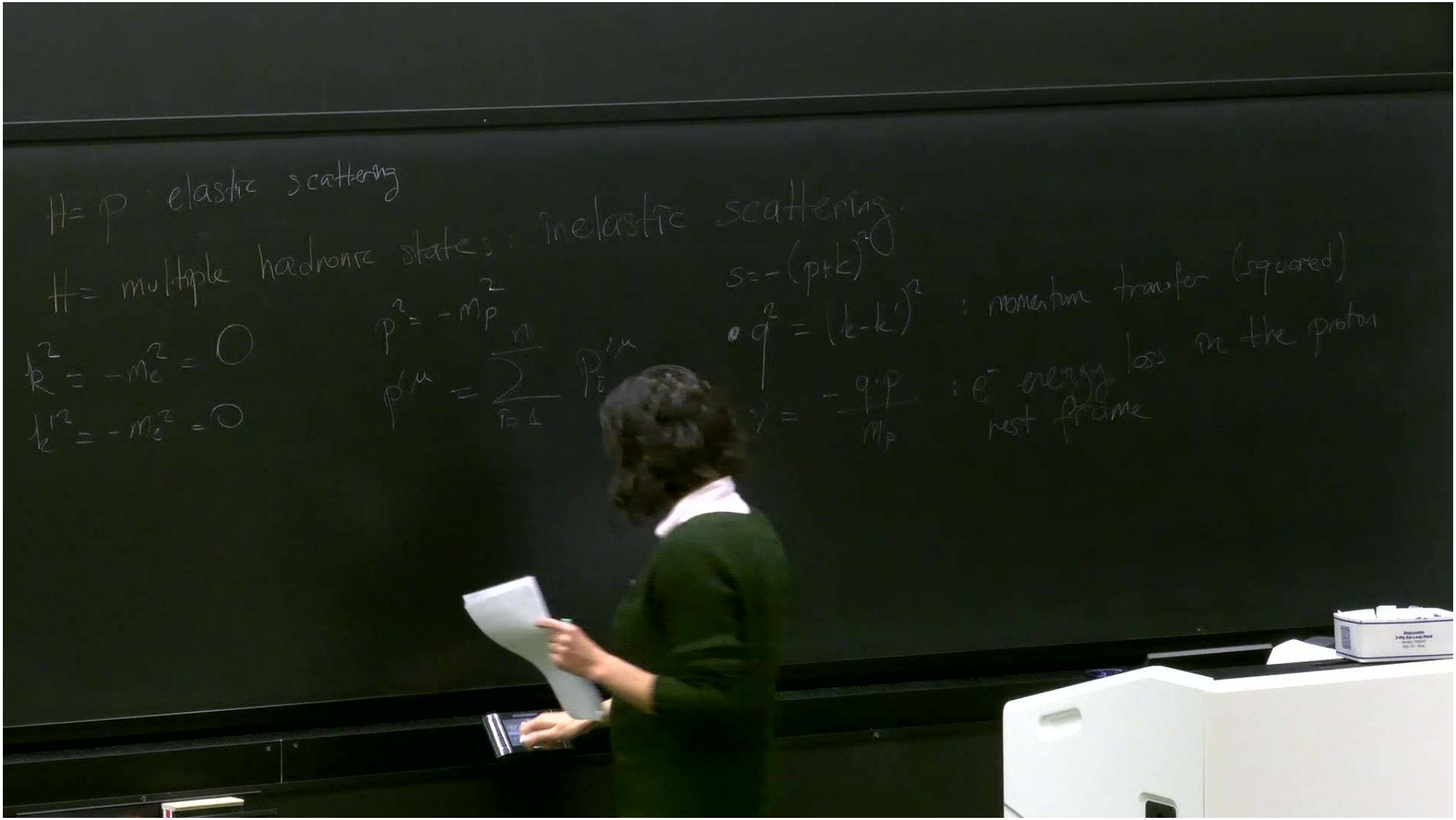
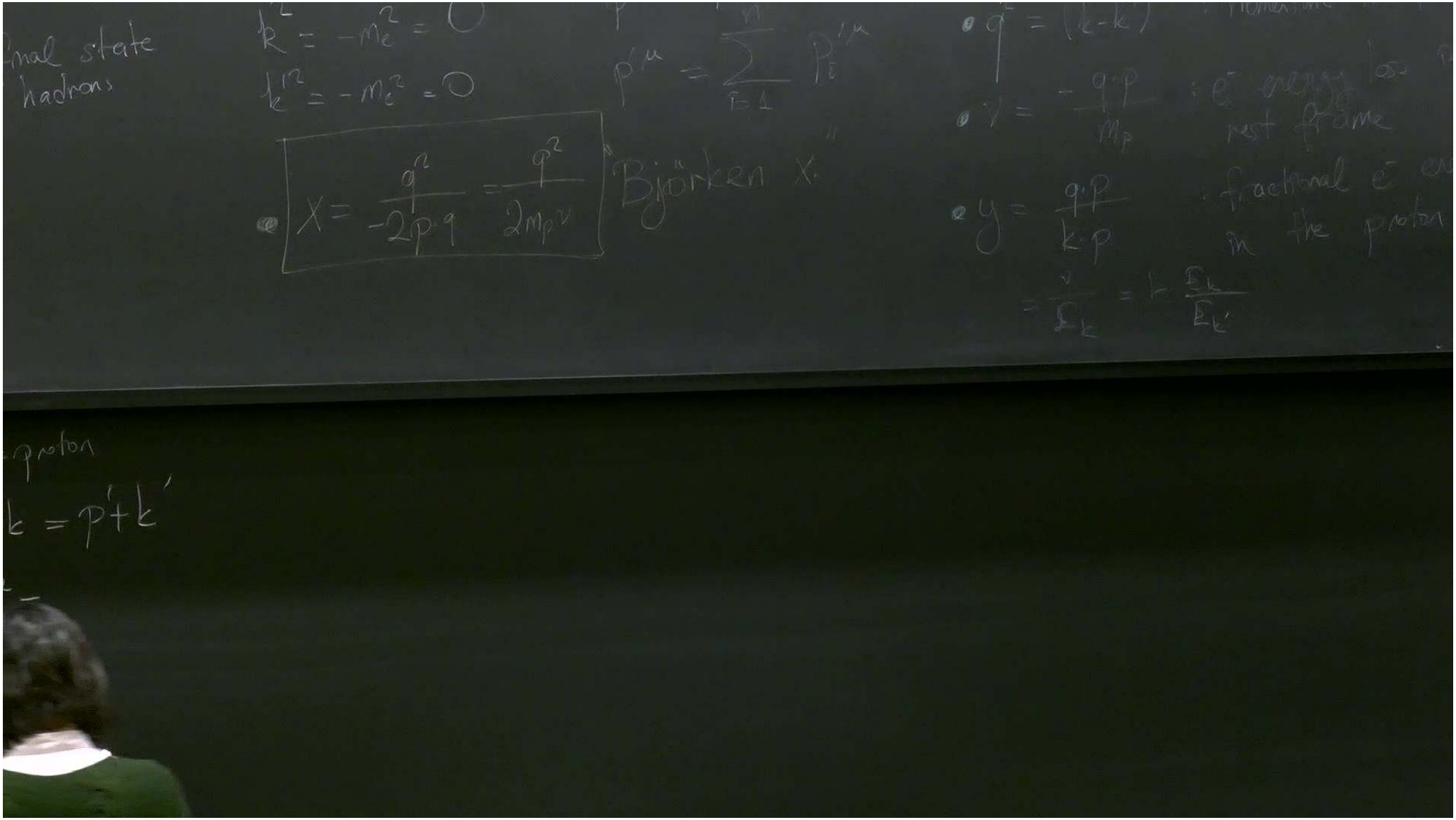
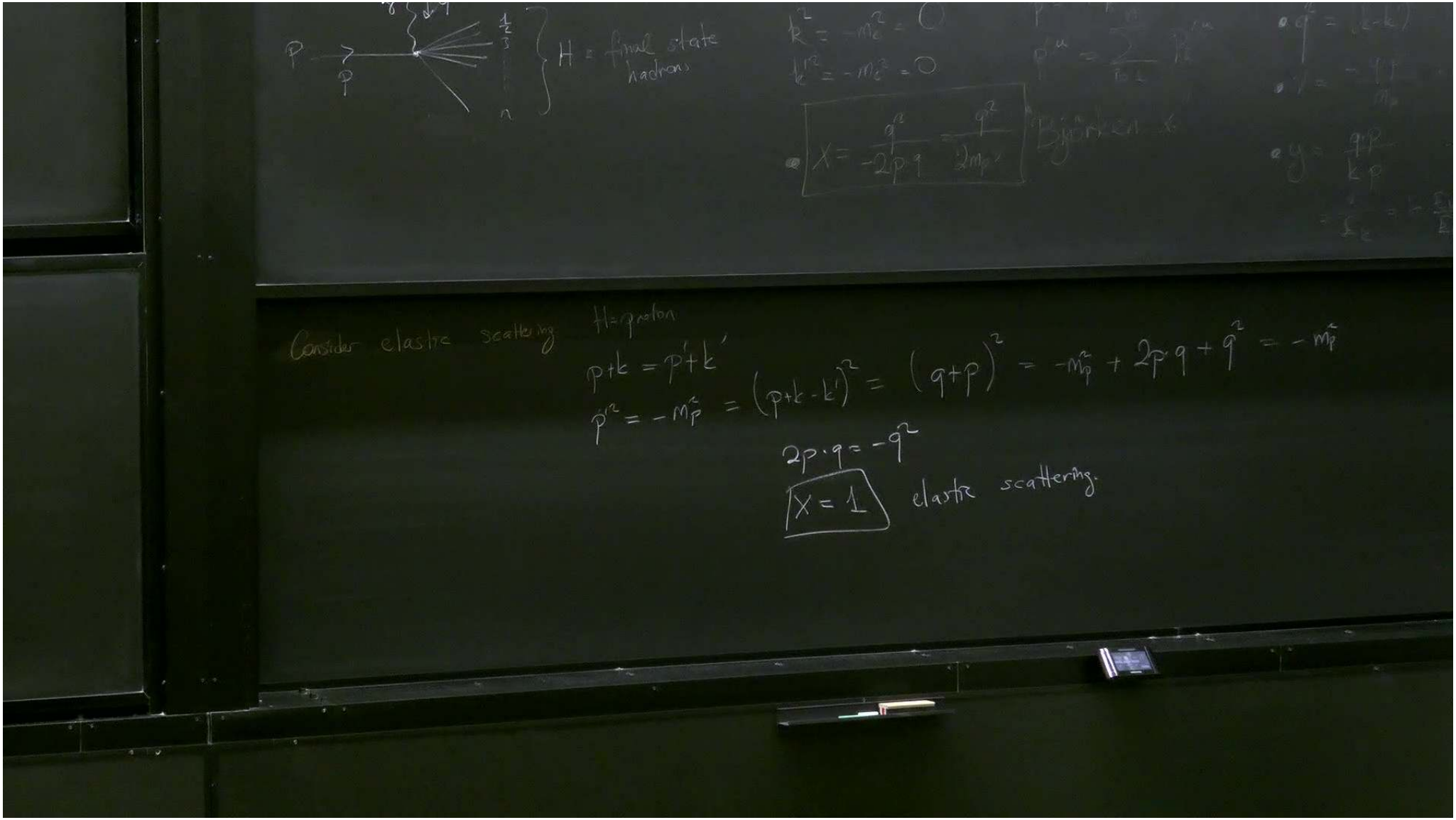
u
d
neutron
mesons
hadronization - non-perturbative


factorization:
what happens at short distances
($r \ll \frac{1}{\Lambda_{QCD}} \sim$ fm) isn't affected by what
happens at long distances ($r \gg \frac{1}{\Lambda_{QCD}}$)
perturbative


e
k
k'
e⁻
γ
q
P
P
H = final state hadrons
H = P elastic scattering
H = multiple hadronic states: inelastic sc
$k^2 = -m_e^2 = 0$
$k'^2 = -m_e^2 = 0$
$P^2 = -m_P^2$
$P'^{\mu} = \sum_{i=1}^{n} P_i^{\mu}$


H=p elastic scattering
H= multiple hadronic states: inelastic scattering
$k^2 = -m_e^2 = 0$
$k'^2 = -m_e^2 = 0$
$p^2 = -m_p^2$
$p'^\mu = \sum_{i=1}^{n} p_i'^\mu$
$s = -(p+k)^2$
$q^2 = (k-k')^2$: momentum transfer (squared)
$\gamma = \frac{-q \cdot p}{m_p}$: e^- energy loss in the proton rest frame


final state
hadrons
$k^2 = -m_e^2 = 0$
$k'^2 = -m_e^2 = 0$
$p'^\mu = \sum_{i=1}^{n} p_i'^\mu$
$q = (k - k')$
$\nu = \frac{-q \cdot P}{m_p}$: e⁻ energy loss
rest frame
$X = \frac{q^2}{-2p \cdot q} = \frac{q^2}{2m_p \nu}$ Björken X.
$y = \frac{q \cdot P}{k \cdot P}$: fractional e⁻ en
in the proton
$= \frac{\nu}{E_k} = 1 - \frac{E_{k'}}{E_k}$
proton
$k = p' + k'$


p
p
H = final state hadrons
Consider elastic scattering H=proton
p+k = p'+k'
p'^2 = -m_p^2 = (p+k-k')^2 = (q+p)^2 = -m_p^2 + 2p·q + q^2 = -m_p^2
2p·q = -q^2
X = 1 elastic scattering.

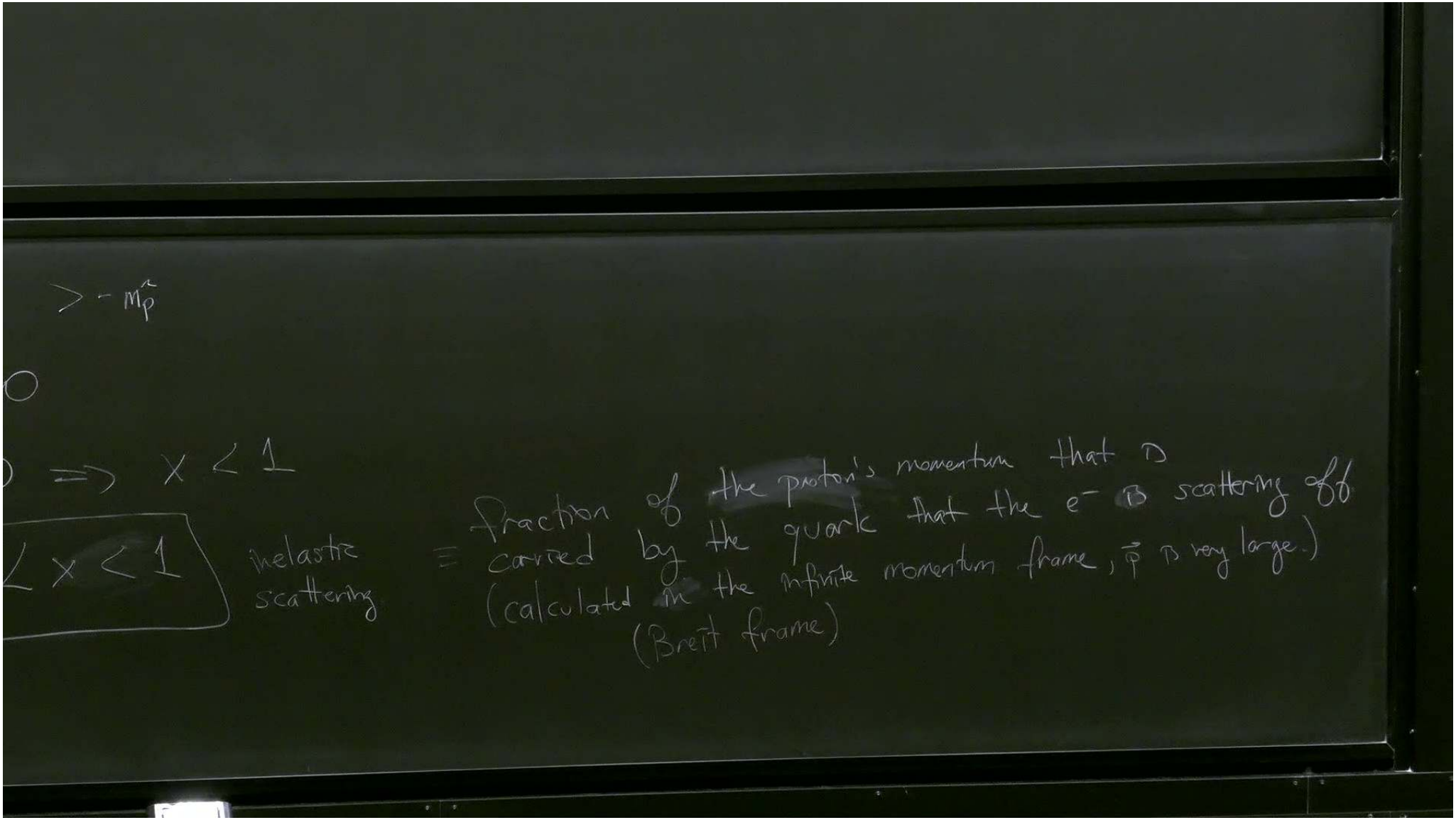
Inelastic scattering:

$$p'^2 = (p_1' + p_2' + \cdots + p_n')^2 = \sum_{i=1}^{n} m_i^2 + 2p_1' \cdot p_2' + 2p_1' \cdot p_3' + \cdots > m_p^2$$

$$p'^2 + m_p^2 = 2p \cdot q + q^2 = 2p \cdot q(1-x) > 0$$

since $\nu > 0 \Rightarrow p \cdot q > 0 \Rightarrow x < 1$

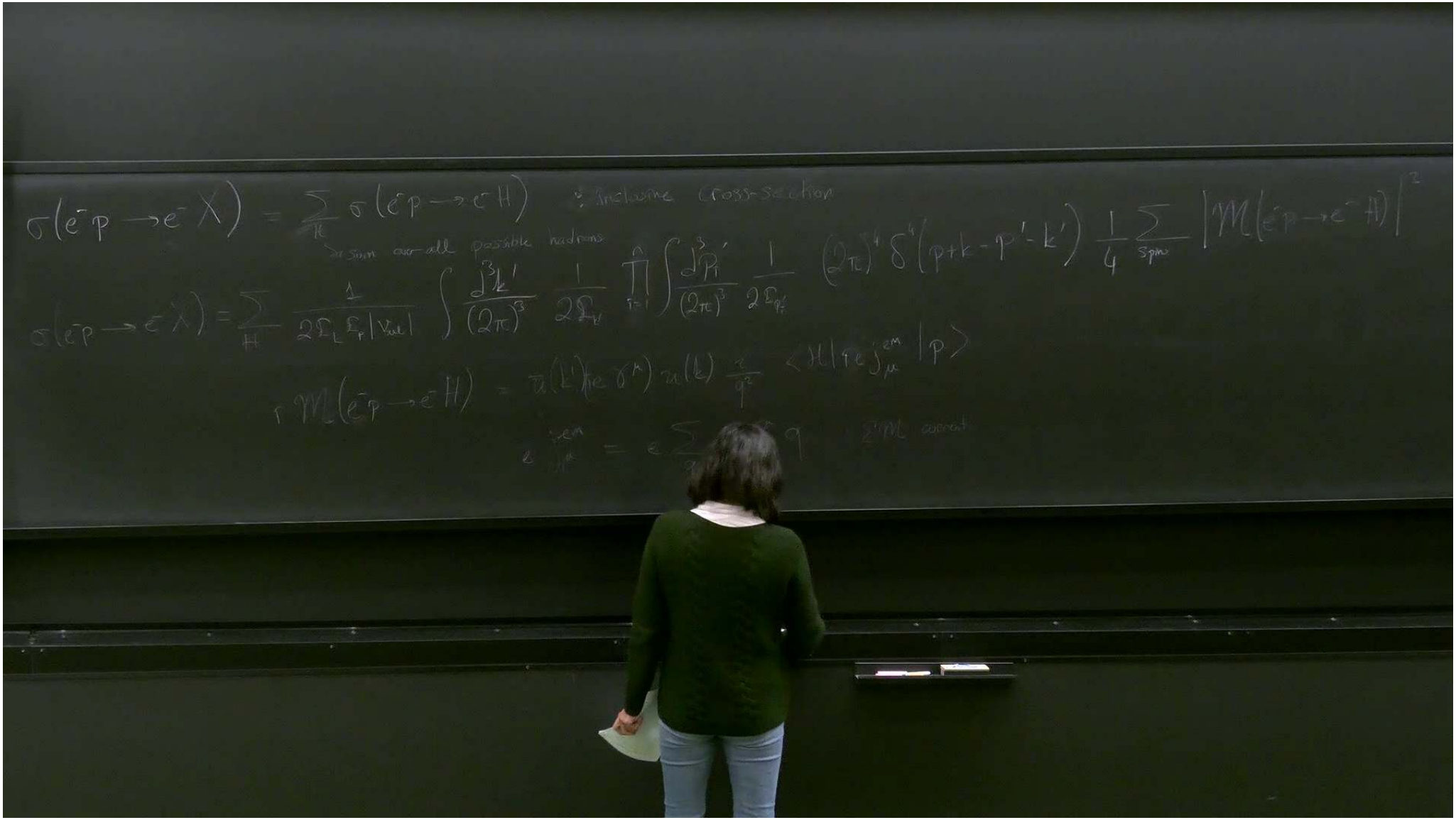
$$\Rightarrow \boxed{0 < x < 1}$$


$> -m_p^2$
0
$\Rightarrow x < 1$
$< x < 1$
inelastic scattering
= fraction of the proton's momentum that ▷ carried by the quark that the e^- ▷ scattering off
(calculated in the infinite momentum frame, $\vec{p}$ ▷ very large.)
(Breit frame)

$$\sigma(e^- p \to e^- X) = \sum_H \sigma(e^- p \to e^- H)$$: inclusive cross section

> sum over all possible hadrons

$$\sigma(e^- p \to e^- X) = \sum_H \frac{1}{2\mathcal{L}_k \mathcal{L}_p |V_{rel}|} \int \frac{d^3k'}{(2\pi)^3} \frac{1}{2\mathcal{L}_{k'}} \prod_{i=1} \int \frac{d^3p_i'}{(2\pi)^3} \frac{1}{2\mathcal{L}_{p_i'}} (\quad)^4 \delta^4(p$$


σ(e⁻p → e⁻X) = Σ_H σ(e⁻p → e⁻H) : inclusive cross-section
sum over all possible hadrons
σ(e⁻p → e⁻X) = Σ_H 1/(2E_L E_p |v_rel|) ∫ d³k'/(2π)³ 1/2E_k' ∏_{i=1}^{n} ∫ d³p_i'/(2π)³ 1/2E_{p_i'} (2π)⁴ δ⁴(p+k−p'−k') 1/4 Σ_spin |M(e⁻p → e⁻H)|²
iM(e⁻p → e⁻H) = ū(k')(ie γ^μ) u(k) i/q² ⟨H| ie j_μ^em |p⟩
e j_μ^em = e Σ q
EM current

$\sigma(e^- p \to e^- \lambda) = \frac{\sum_H}{} \frac{1}{2E_L E_T |v_{rel}|} \int \frac{d^3k'}{(2\pi)^3} \frac{1}{2E_{k'}} \prod_{i=1}^{n} \int \frac{d^3p_i'}{(2\pi)^3 2E_{p_i'}}$

$i\mathcal{M}(e^- p \to e^- H) = \bar{u}(k') ie\gamma^\mu u(k) \frac{i}{q^2} \langle H | ie J_\mu^{em} | p \rangle$

$J_\mu^{em} = e \sum_q q \bar{q} \gamma^\mu q$ EM current

How does J_μ^{em} connect $|p\rangle$ to $\langle H|$?
non-perturbative
"hadronization"

$\frac{1}{4}\sum |\mathcal{M}|^2 = \frac{e^4}{q^4} \frac{1}{4} \mathrm{Tr}[k' \gamma^\mu k \gamma^\nu] \langle H | J_\mu^{em} | p \rangle \langle p | J_\nu^{em} | H \rangle$

$\sigma(e^- p \to e^- X) = \frac{(2\pi)}{2E_L |v_{rel}|} \int \frac{d^3k'}{(2\pi)^3} \frac{1}{2E_{k'}} \frac{e^4}{q^4} \underbrace{\frac{1}{2} \mathrm{Tr}[k' \gamma^\mu k \gamma^\nu]}_{L^{\mu\nu}} \underbrace{\frac{1}{2E_p} \frac{\sum}{H} \prod_{i=1}^{n} \int \frac{d^3p_i'}{(2\pi)^3 2E_{p_i'}} (2\pi)^3 \delta^4(p+k-p'-k') \frac{1}{2} \langle H | J_\mu^{em} | p \rangle \langle p | J_\nu^{em} | H \rangle}_{W_{\mu\nu}}$

$L^{\mu\nu} = 2(k_\mu k'_\nu + k_\nu k'_\mu - \eta_{\mu\nu} k \cdot k')$

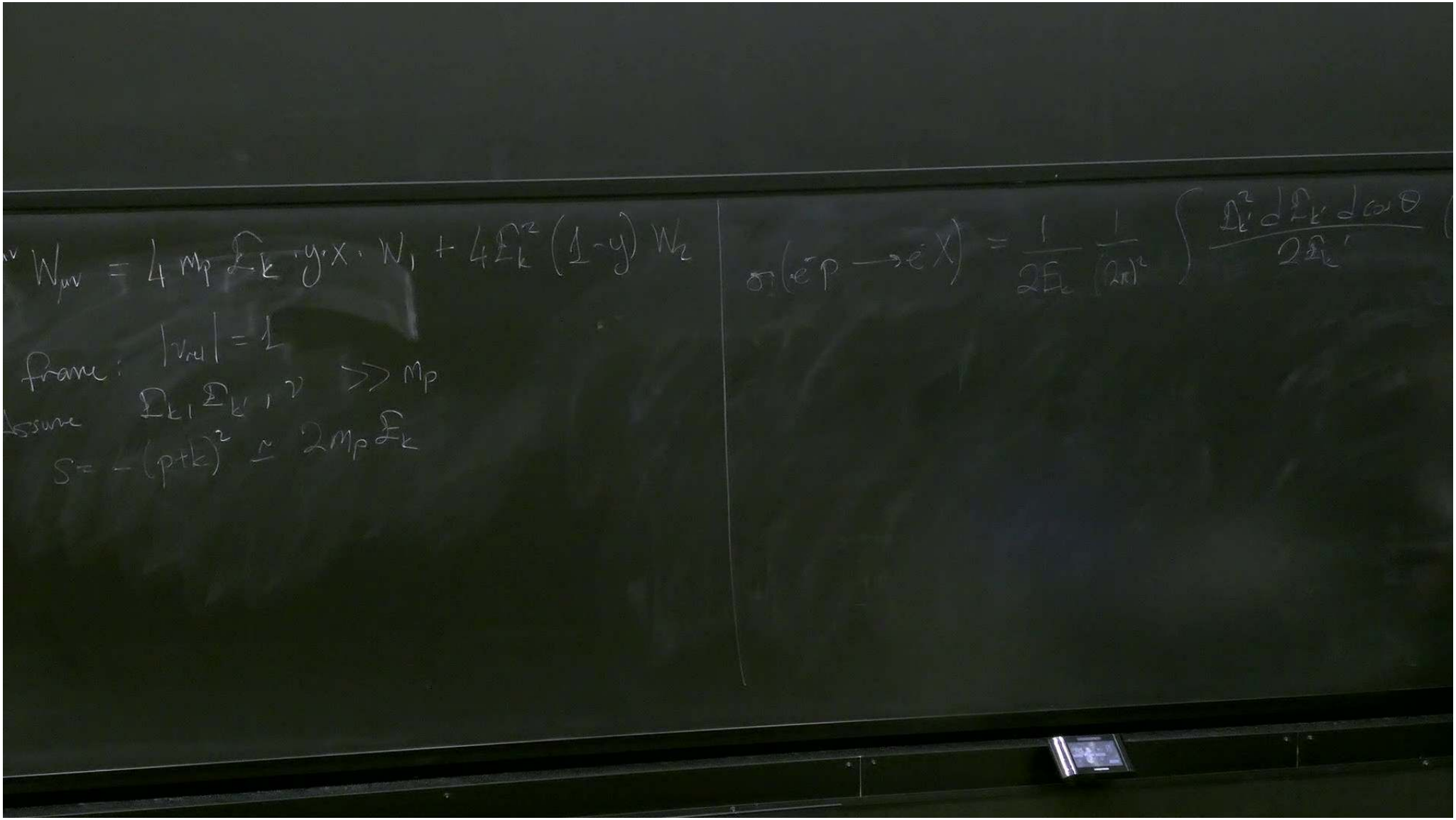
$W_{\mu\nu} \supset \eta_{\mu\nu}, P_\mu P_\nu, q_\mu q_\nu, P_\mu q_\nu, P_\nu q_\mu, \epsilon_{\mu\nu\alpha\beta} P^\alpha q^\beta$

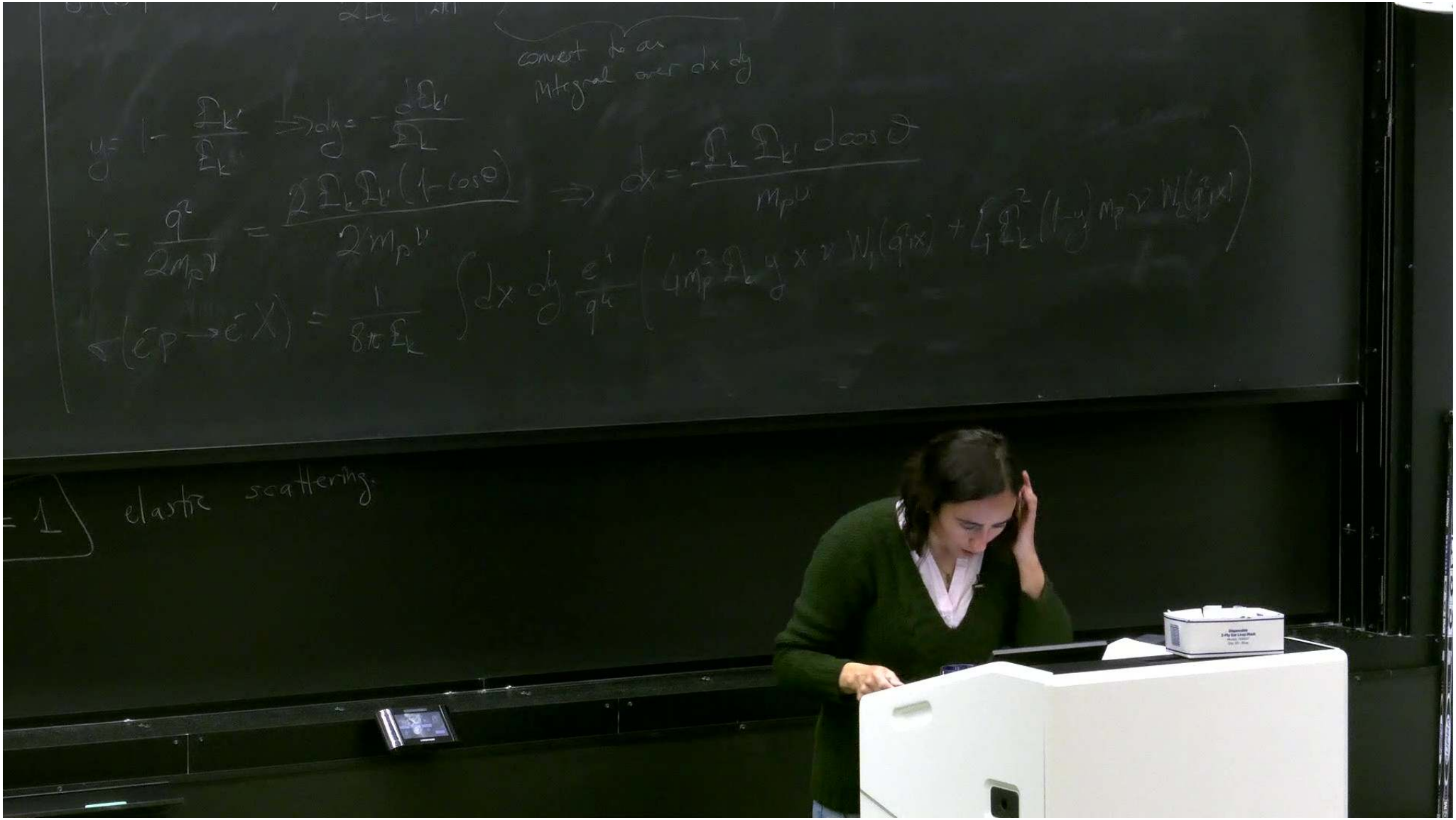
0 when $\times L^{\mu\nu}$

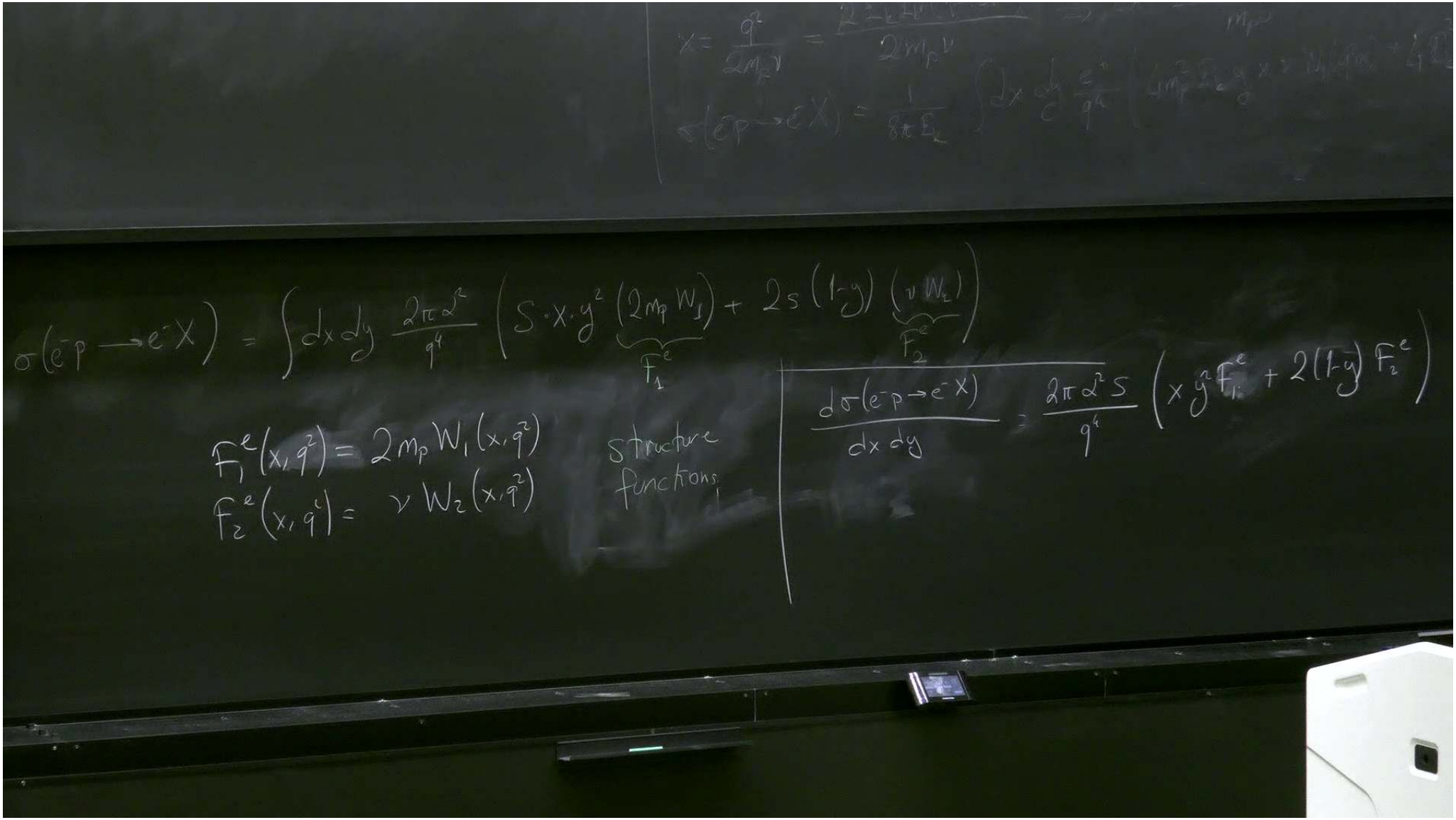
parity violating EM $\Rightarrow$ no! (could be allowed by weak int.)

$W_{\mu\nu} = -W_1 \eta_{\mu\nu} + W_2 \frac{P_\mu P_\nu}{m_p^2}$

$W_1(x, q^2), W_2(x, q^2)$ Lorentz scalars


$W_{\mu\nu} = 4 m_p E_k \cdot y \cdot x \cdot W_1 + 4 E_k^2 (1-y) W_2$
frame: $|\vec{v}_{rel}| = 1$
Assume $\Omega_k, E_k, \nu \gg m_p$
$S = -(p+k)^2 \simeq 2 m_p E_k$
$\sigma(e^- p \to e^- X) = \frac{1}{2E_e} \frac{1}{(2\pi)^2} \int \frac{\Omega_{k'}^2 \, dE_{k'} \, d\omega\theta}{2E_{k'}}$


convert to an integral over dx dy
elastic scattering.


$x = \frac{q^2}{2m_p \nu}$
$\sigma(ep \to eX) = \frac{1}{8\pi E_k}$
$\sigma(e^- p \to e^- X) = \int dx\, dy\, \frac{2\pi\alpha^2}{q^4} \left(S \cdot x \cdot y^2 \underbrace{(2m_p W_1)}_{F_1^e} + 2S(1-y) \underbrace{(\nu W_2)}_{F_2^e} \right)$
$F_1^e(x, q^2) = 2m_p W_1(x, q^2)$
$F_2^e(x, q^2) = \nu W_2(x, q^2)$
structure functions
$\frac{d\sigma(e^- p \to e^- X)}{dx\, dy} = \frac{2\pi\alpha^2 S}{q^4} \left(x y^2 F_1^e + 2(1-y) F_2^e \right)$

$\sigma(e^- p \to e^- H) = \sum_{q,\ell} \text{Prob}(q,\ell) \times \sigma(e^- q \to e^- q) \times \text{Prob}(H)$

Prob of finding a quark w/ momentum ℓ inside the proton

scattering x-section (short distance)

Prob. of hadronizing to the state H

$\sigma(e^- p \to e^- X) = \sum_H \sigma(e^- p \to e^- H) \Rightarrow \sum_H \text{Prob}(H) = 1$