Title: Lecture - Standard Model, PHYS 622

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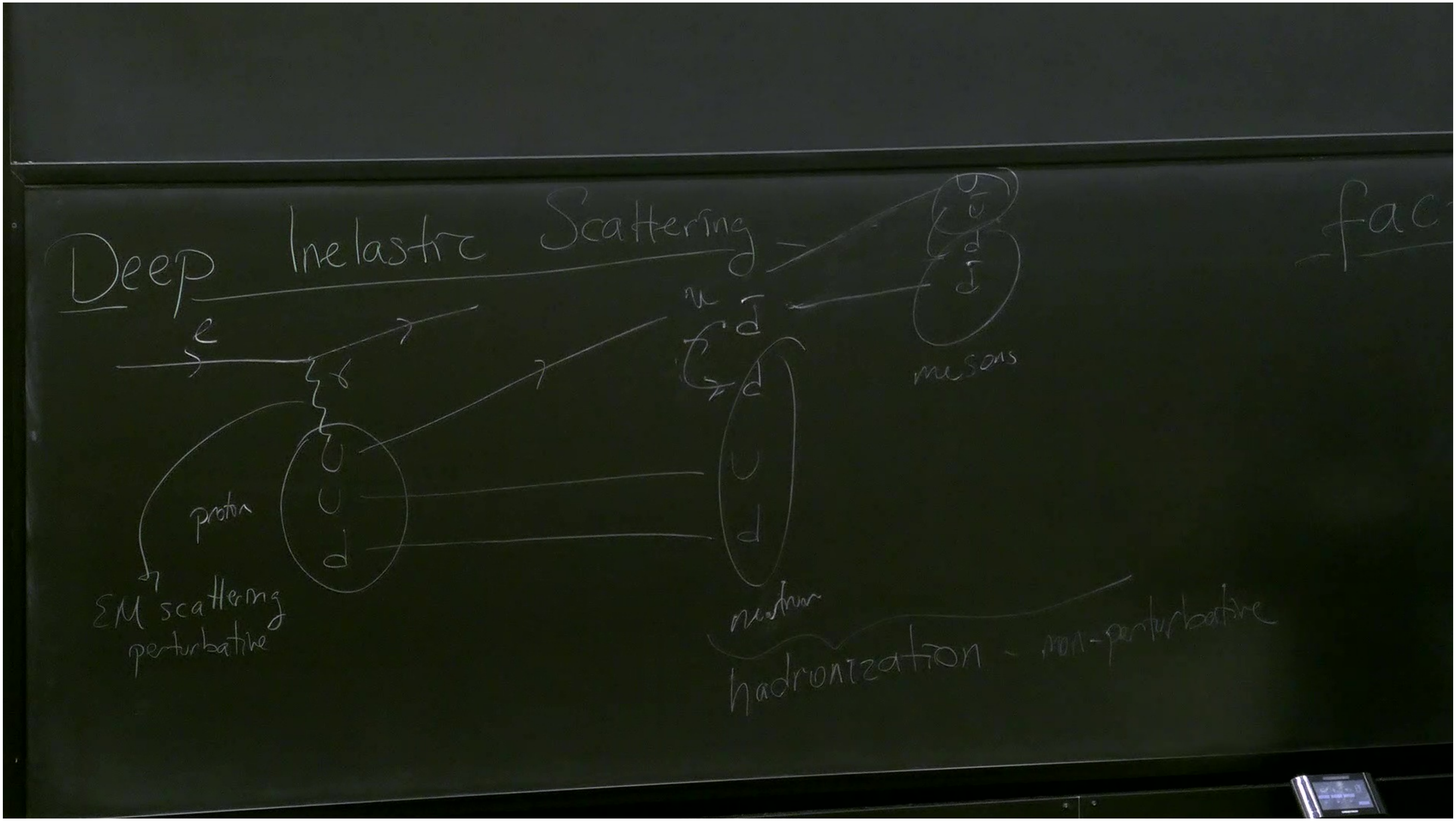
Collection/Series: Standard Model (Elective), PHYS 622, January 6 - February 5, 2025

Subject: Particle Physics

Date: January 27, 2025 - 10:15 AM

URL: https://pirsa.org/25010022


Deep Inelastic Scattering
fac
e
γ
u
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mesons
proton
u
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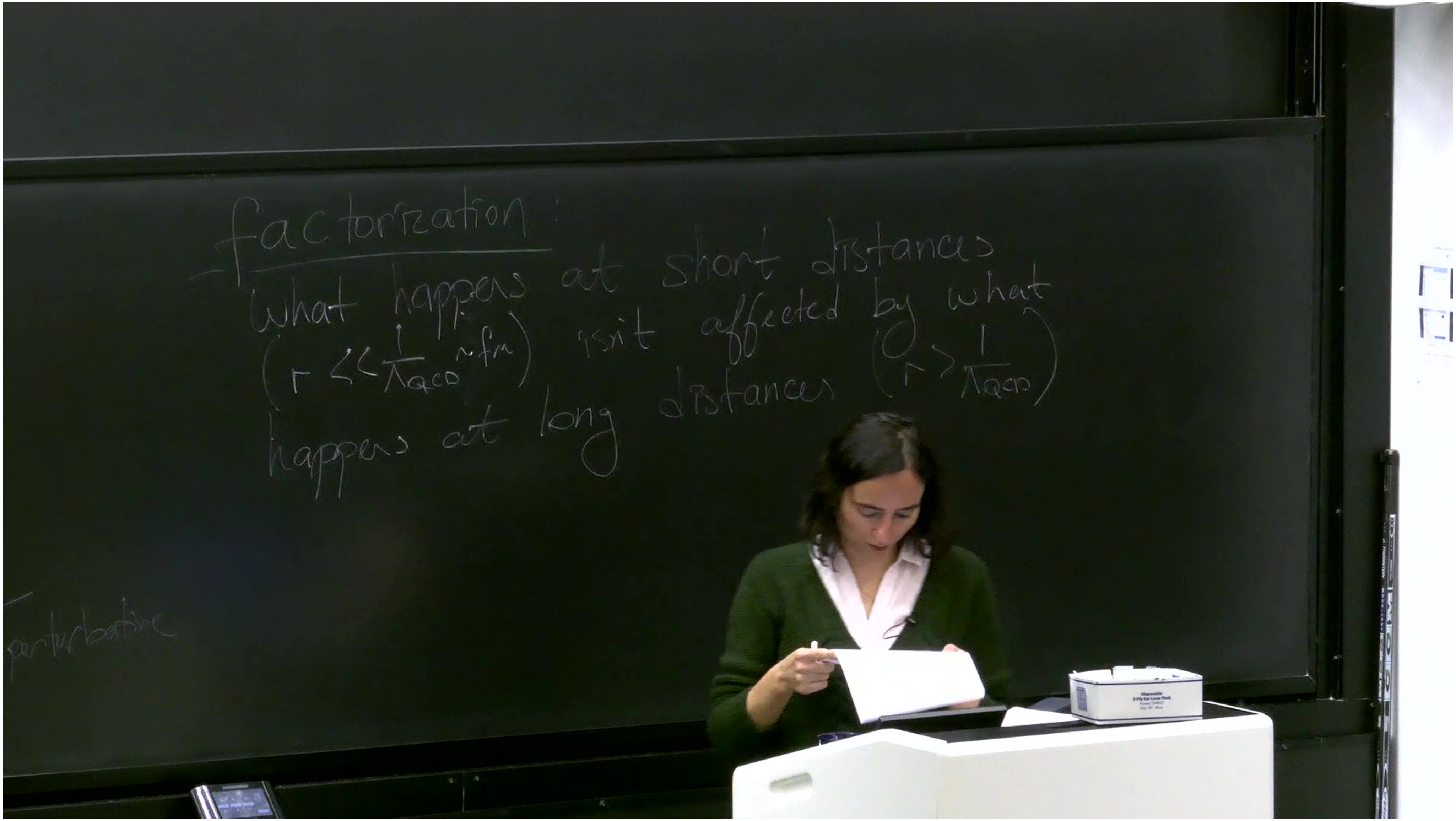
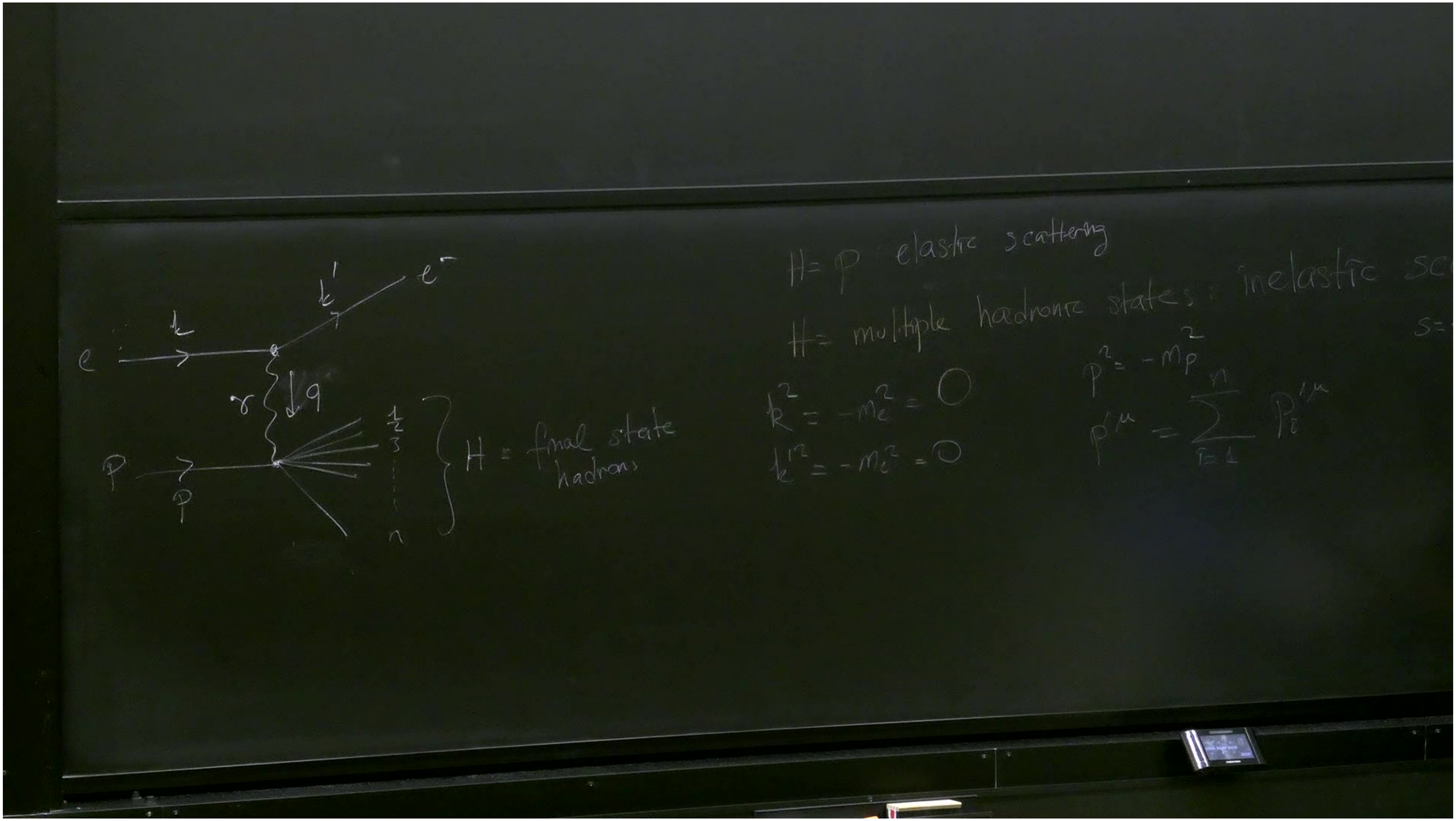
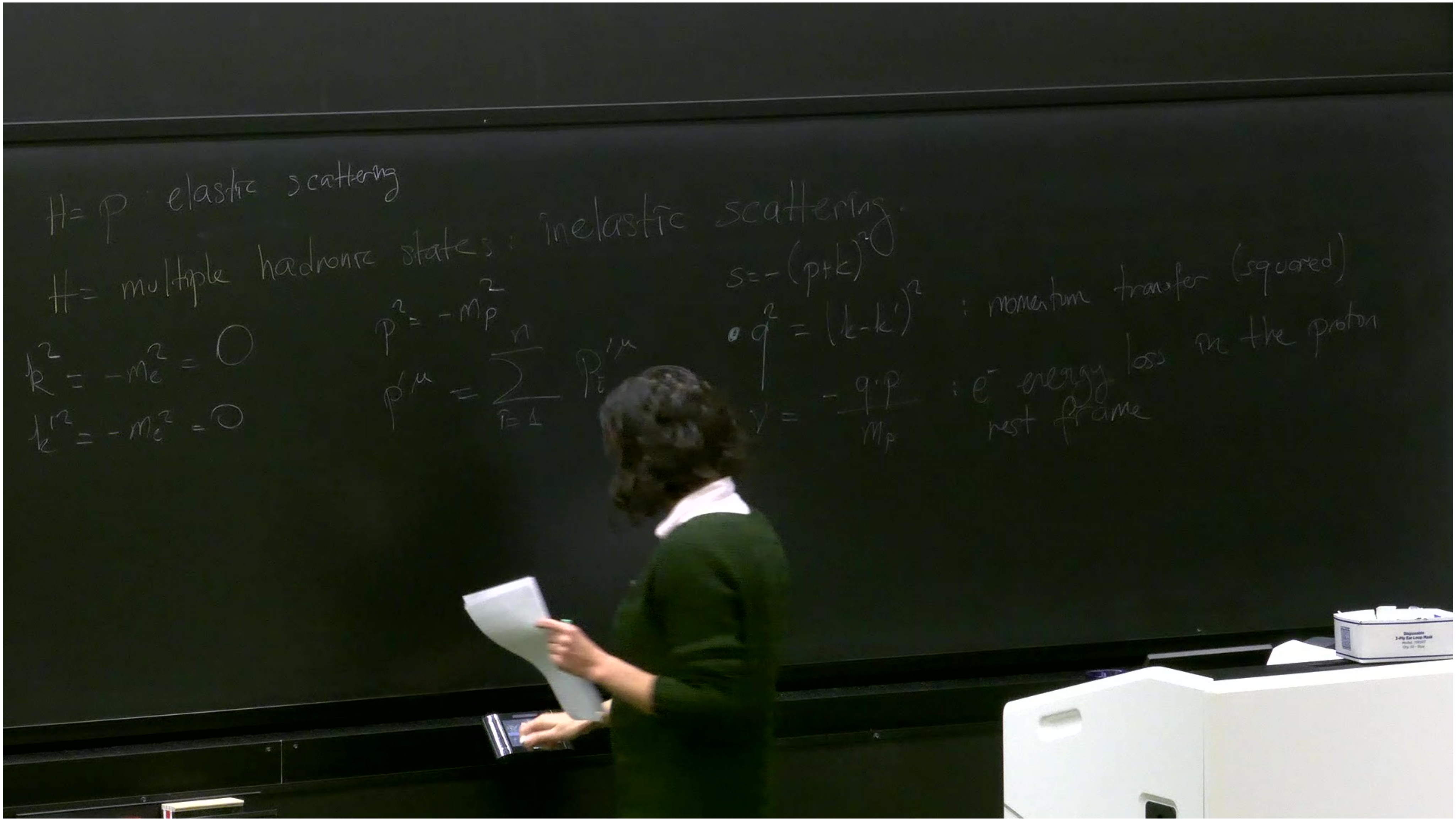
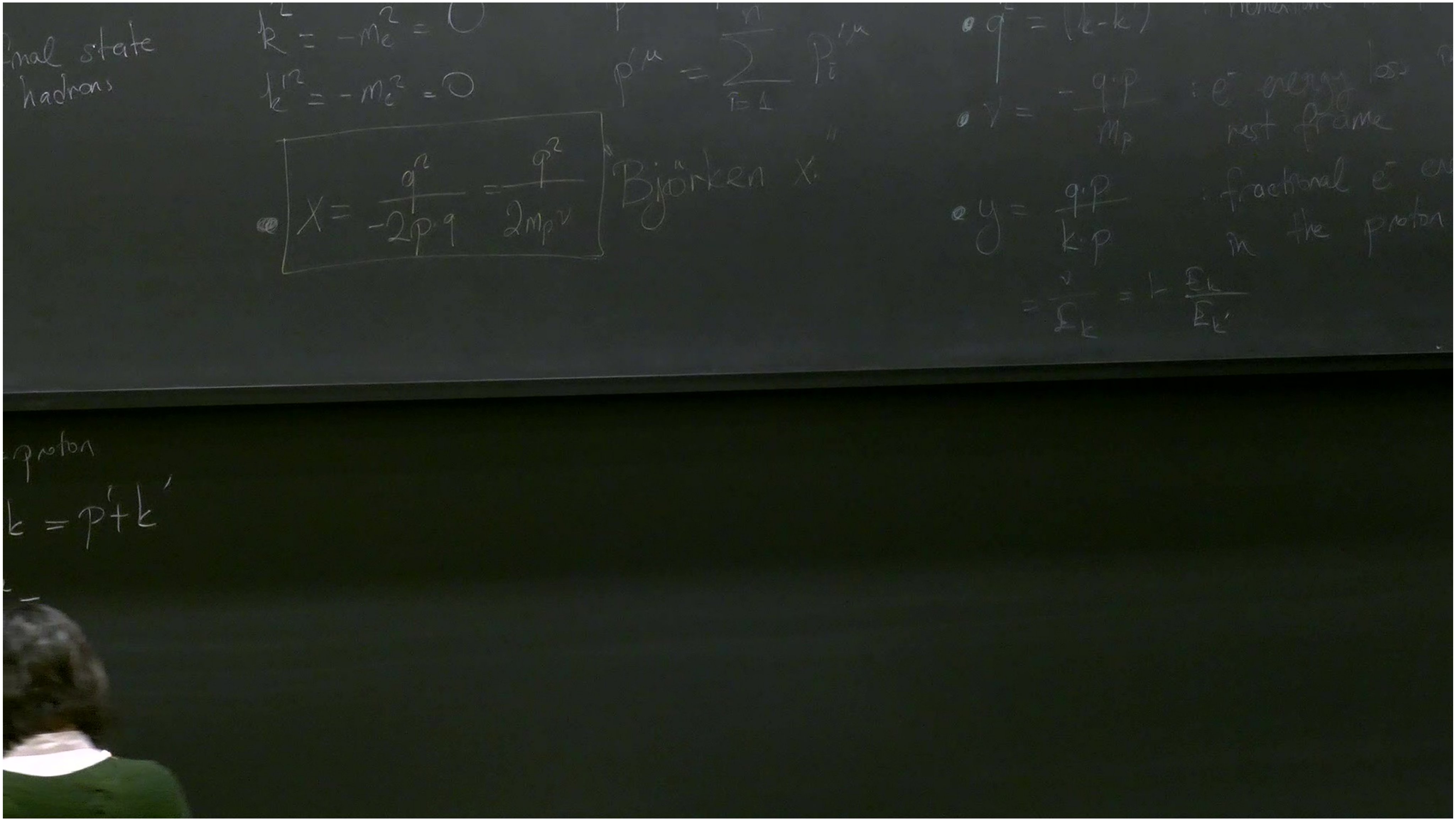
d
neutron
EM scattering
perturbative
hadronization - non-perturbative

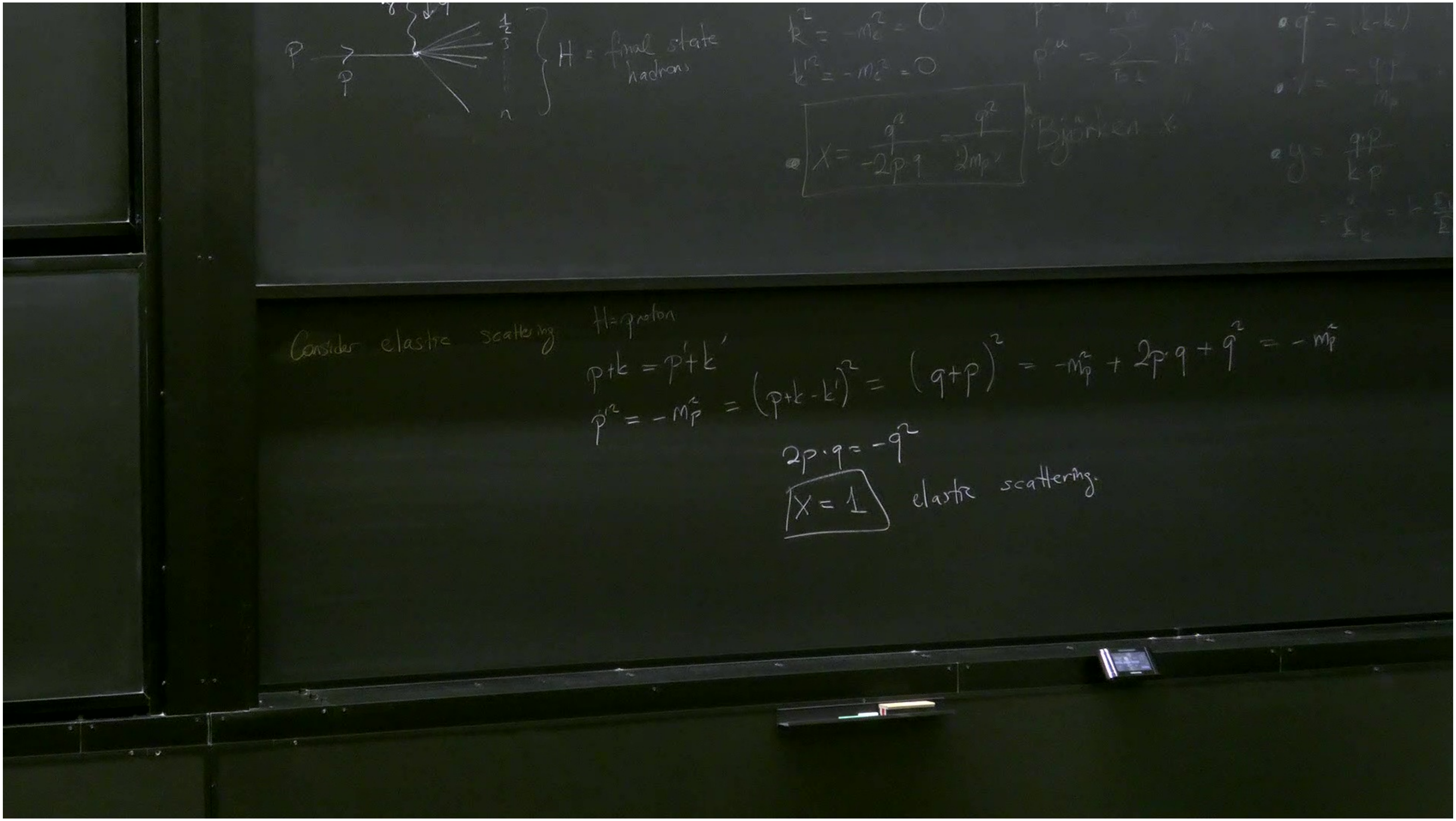

factorization:
what happens at short distances
(r << 1/Λ_QCD ~ fm) isn't affected by what
happens at long distances (r > 1/Λ_QCD)
perturbative


H = p elastic scattering
H = multiple hadronic states : inelastic sc
e
k
k'
e⁻
γ
q
p
P
1
2
3
n
H = final state hadrons
k² = -mₑ² = 0
k'² = -mₑ² = 0
P² = -mₚ²
P'^μ = Σ_{i=1}^{n} P_i'^μ
s =



H = p elastic scattering
H = multiple hadronic states : inelastic scattering
s = -(p+k)^2
p^2 = -m_p^2
q^2 = (k-k')^2 : momentum transfer (squared)
k^2 = -m_e^2 = 0
k'^2 = -m_e^2 = 0
p'^μ = Σ_{i=1}^{n} p_i'^μ
ν = - q·p / m_p : e^- energy loss in the proton rest frame


final state
hadrons
k² = −mₑ² = 0
k'² = −mₑ² = 0
p'^μ = Σ_{i=1}^{n} p_i'^μ
q² = (k−k')²
ν = −q·p / m_p : e⁻ energy loss in rest frame
X = q² / (−2p·q) = q² / (2m_p ν)
"Björken x"
y = q·p / k·p : fractional e⁻ en... in the proton
= ν / E_k = 1 − E_k' / E_k
proton
k = p' + k'


H = final state hadrons
Björken x
Consider elastic scattering H = proton
p+k = p'+k'
p'^2 = -m_p^2 = (p+k-k')^2 = (q+p)^2 = -m_p^2 + 2p·q + q^2 = -m_p^2
2p·q = -q^2
X = 1 elastic scattering.

Inelastic scattering:

$$p'^2 = (p_1' + p_2' + \dots + p_n')^2 = -\sum_{i=1}^{n} m_i^2 + 2p_1'\cdot p_2' + 2p_1'\cdot p_3' + \dots > -m_p^2$$

$$p'^2 + m_p^2 = 2p\cdot q + q^2 = 2p\cdot q(1-x) > 0$$

since $\nu > 0 \Rightarrow p\cdot q > 0 \Rightarrow x <$

$$\Rightarrow \boxed{0 < x < 1}$$

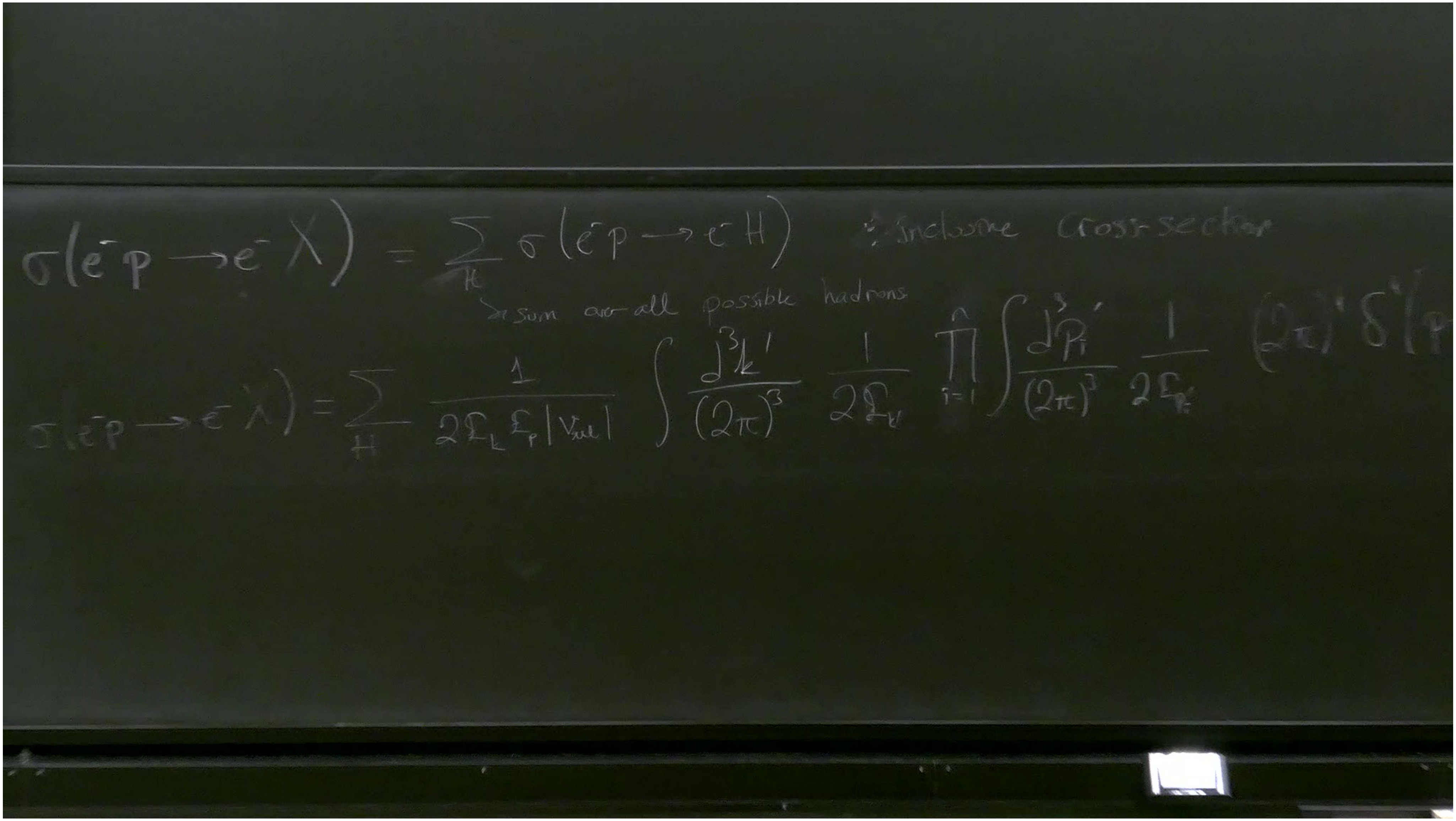
$> - m_p^2$

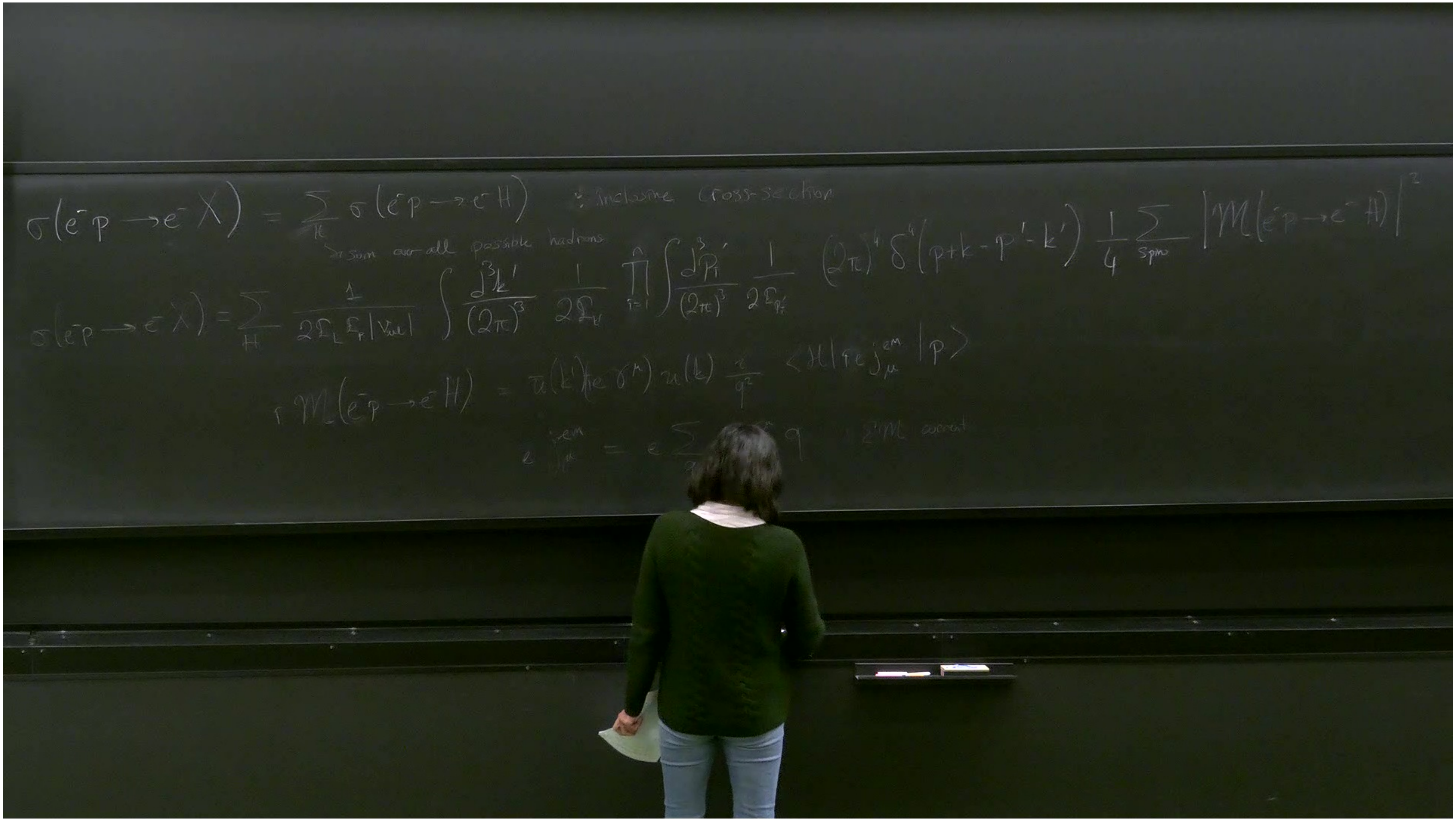
0

$) \Rightarrow x < 1$

$< x < 1$ inelastic scattering

$\equiv$ fraction of the proton's momentum that is carried by the quark that the e^- is scattering off (calculated in the infinite momentum frame, $\vec{p}$ is very large.)
(Breit frame)


$\sigma(e^- p \to e^- X) = \sum_H \sigma(e^- p \to e^- H)$: inclusive cross-section
sum over all possible hadrons
$\sigma(e^- p \to e^- X) = \sum_H \frac{1}{2E_k E_p |v_{rel}|} \int \frac{d^3k'}{(2\pi)^3} \frac{1}{2E_{k'}} \prod_{i=1}^{n} \int \frac{d^3p_i'}{(2\pi)^3} \frac{1}{2E_{p_i'}} (2\pi)^4 \delta^4(p$


σ(e⁻p → e⁻X) = Σ_H σ(e⁻p → e⁻H) ← inclusive cross-section
sum over all possible hadrons
σ(e⁻p → e⁻X) = Σ_H 1/(2E_k E_p |v_rel|) ∫ d³k′/(2π)³ 1/(2E_k′) Π_{i=1}^n ∫ d³p_i′/(2π)³ 1/(2E_{p_i′}) (2π)⁴ δ⁴(p+k−p′−k′) 1/4 Σ_spin |M(e⁻p → e⁻H)|²
iM(e⁻p → e⁻H) = ū(k′)(ieγ^μ) u(k) i/q² ⟨H| ie j_μ^em |p⟩
EM current

$$\sigma(e^- p \to e^- X) = \sum_H \frac{1}{2E_L E_p |v_{rel}|} \int \frac{d^3k'}{(2\pi)^3} \frac{1}{2E_{k'}} \prod_{i=1} \int \frac{d^3p_i}{(2\pi)^3} \frac{1}{2E_{p_i'}}$$

$$i\mathcal{M}(e^- p \to e^- H) = \bar{u}(k')(ie\gamma^\mu) u(k) \frac{i}{q^2} \langle H | ie j^{em}_\mu | p \rangle$$

How does j^{em}_μ connect $|p\rangle$ to $\langle H|$? non-perturbative "hadronization"

$$e j^{em}_\mu = e \sum_q \bar{q} \gamma^\mu q \quad \text{EM current}$$

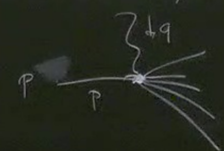
$$\frac{1}{4}\sum |\mathcal{M}|^2 = \frac{e^4}{q^4} \frac{1}{4} \mathrm{Tr}[\not{k}' \gamma^\mu \not{k} \gamma^\nu] \langle H | j^{em}_\mu | p \rangle \langle p | j^{em}_\nu | H \rangle$$

$$\sigma(e^- p \to e^- X) = \frac{(2\pi)}{2E_L |v_{rel}|} \int \frac{d^3k'}{(2\pi)^3} \frac{1}{2E_{k'}} \frac{e^4}{q^4} \underbrace{\frac{1}{2} \mathrm{Tr}[\not{k}' \gamma^\mu \not{k} \gamma^\nu]}_{L^{\mu\nu}} \underbrace{\frac{1}{2E_p} \sum_H \prod_{i=1}^n \int \frac{d^3p_i'}{(2\pi)^3} \frac{1}{2E_{p_i'}} (2\pi)^3 \delta^4(p+k-p'-k') \frac{1}{2} \langle H | j^{em}_\mu | p \rangle \langle p | j^{em}_\nu | H \rangle}_{W_{\mu\nu}}$$

$$W_{\mu\nu} \supset \eta_{\mu\nu},\ P_\mu P_\nu,\ \underbrace{q_\mu q_\nu,\ P_\mu q_\nu,\ P_\nu q_\mu}_{0 \text{ when w/ } L^{\mu\nu}},\ \underbrace{\epsilon_{\mu\nu\alpha\beta} P^\alpha q^\beta}_{\text{parity odd}}$$

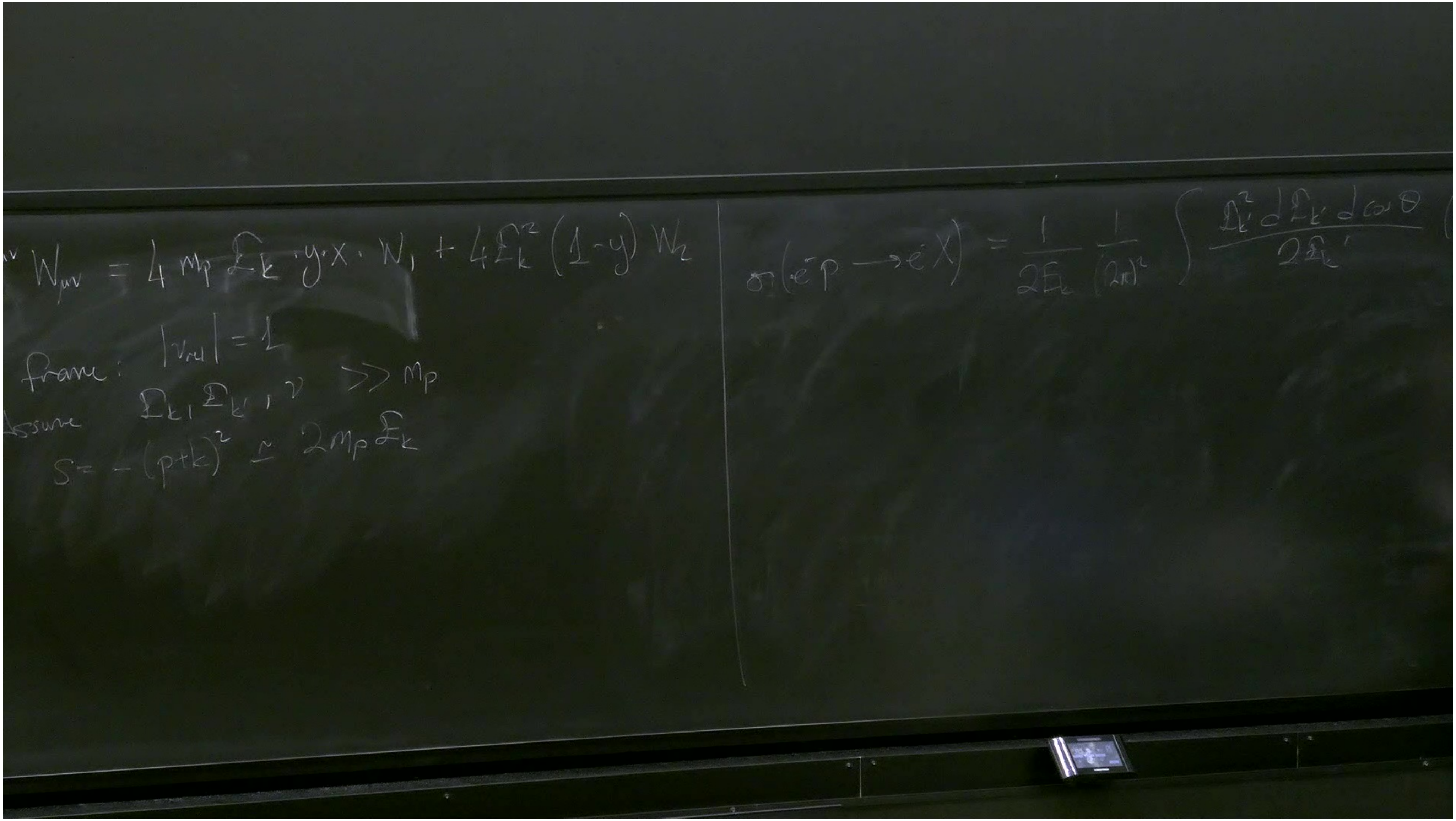
parity odd, EM is not (would be allowed by weak int.)

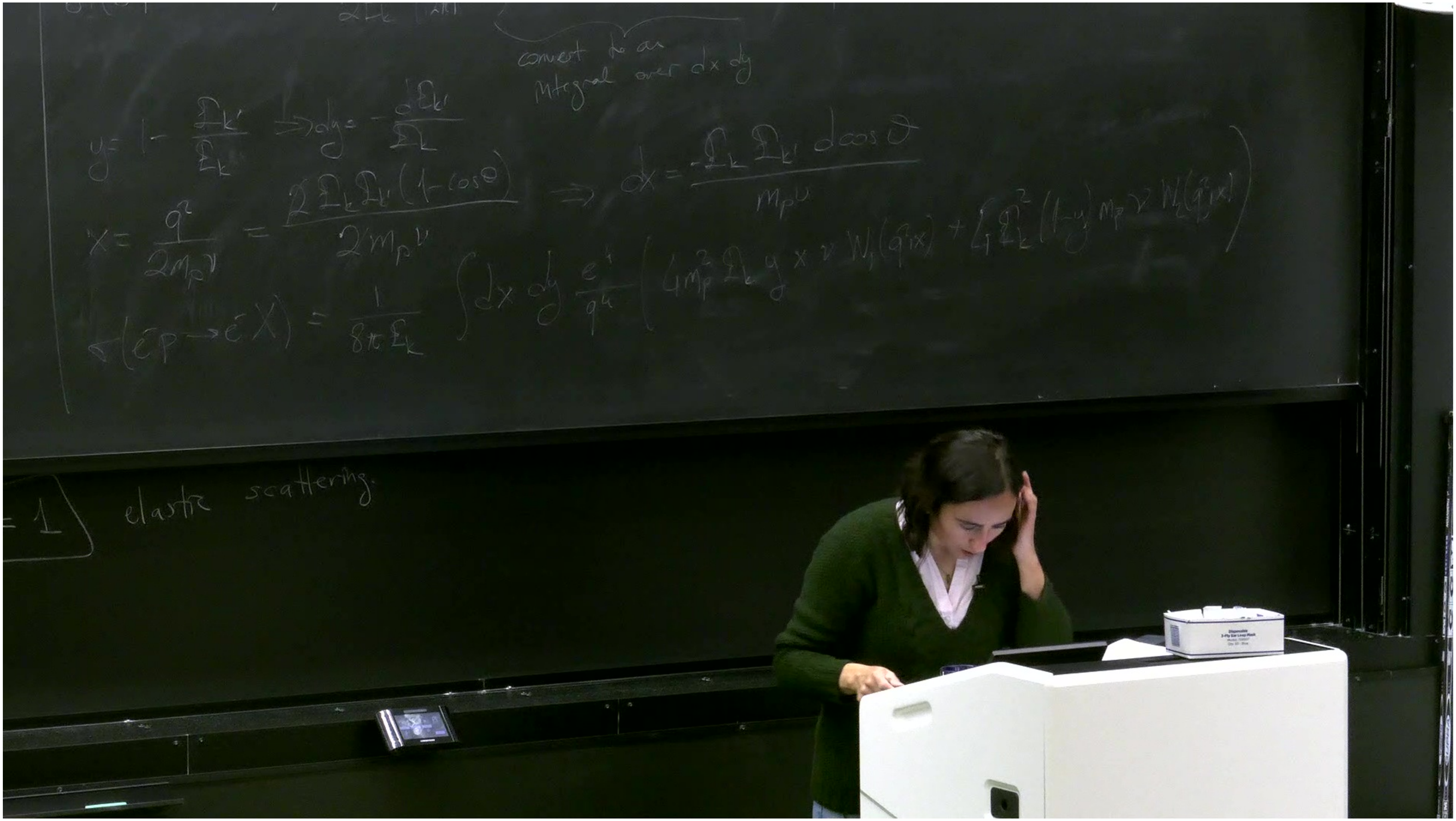
$$L^{\mu\nu} = 2(k_\mu k'_\nu + k_\nu k'_\mu - \eta_{\mu\nu} k \cdot k')$$



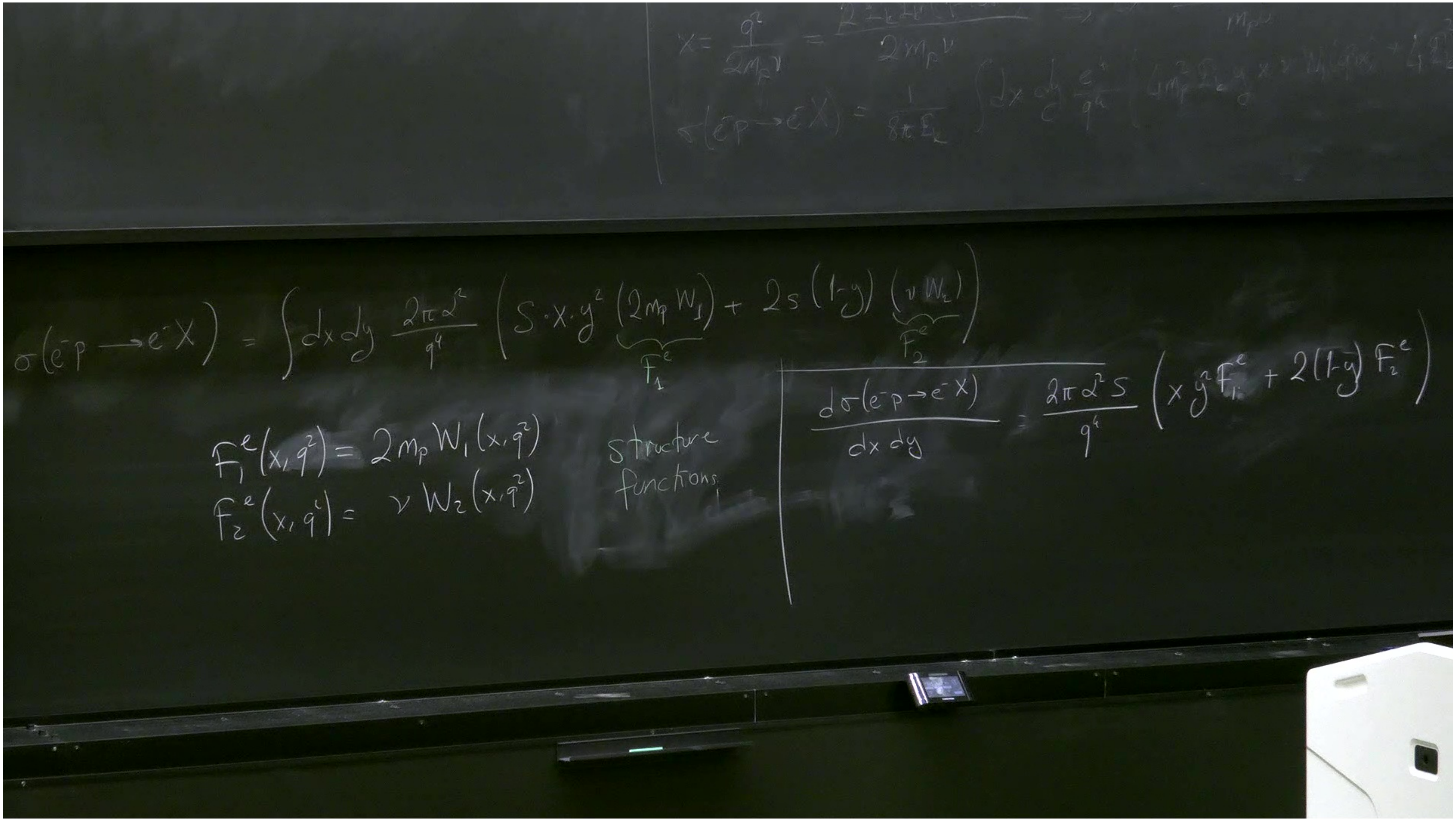
$$W_{\mu\nu} = -W_1 \eta_{\mu\nu} + W_2 \frac{P_\mu P_\nu}{m_p^2}$$

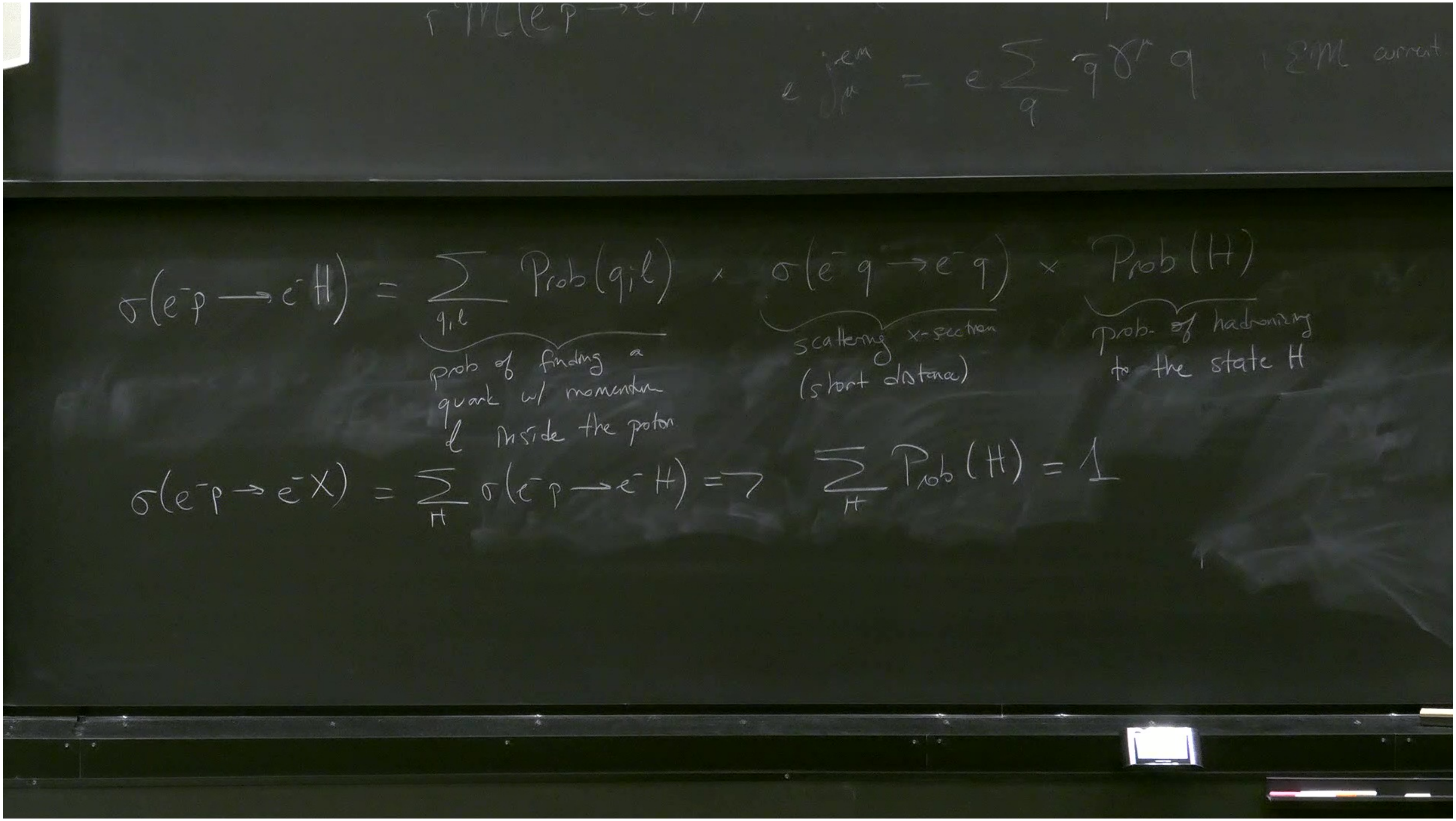
$W_1(x, q^2),\ W_2(x, q^2)$: Lorentz scalars


W_μν = 4 m_p E_k · y · x · W_1 + 4 E_k^2 (1 - y) W_2
frame: |v_rel| = 1
Assume E_k, E_k', ν >> m_p
s = -(p+k)^2 ≃ 2 m_p E_k
σ(e⁻ p → e⁻ X) = 1/(2E_k) 1/(2π)^2 ∫ E_k'^2 dE_k' d cos θ / 2E_k'



convert to an
integral over dx dy
elastic scattering


$\sigma(e^- p \to e^- X) = \int dx\, dy\, \frac{2\pi\alpha^2}{q^4}\left(S \cdot x \cdot y^2 (2 m_p W_1) + 2s(1-y)(\nu W_2) \right)$
F_1^e
F_2^e
$\frac{d\sigma(e^- p \to e^- X)}{dx\, dy} = \frac{2\pi\alpha^2 S}{q^4}\left(x y^2 F_1^e + 2(1-y) F_2^e \right)$
$F_1^e(x, q^2) = 2 m_p W_1(x, q^2)$
$F_2^e(x, q^2) = \nu W_2(x, q^2)$
structure functions
$x = \frac{q^2}{2 m_p \nu}$
$\sigma(e^- p \to e^- X) = \frac{1}{8\pi E_L} \int dx\, dy\, \frac{e^4}{q^4}$


e j_μ^em = e Σ_q q̄ γ^μ q
EM current
σ(e⁻p → e⁻H) = Σ_{q,ℓ} Prob(q,ℓ) × σ(e⁻q → e⁻q) × Prob(H)
prob of finding a quark w/ momentum ℓ inside the proton
scattering x-section (short distance)
prob. of hadronizing to the state H
σ(e⁻p → e⁻X) = Σ_H σ(e⁻p → e⁻H) ⇒ Σ_H Prob(H) = 1