Title: Lecture - Standard Model, PHYS 622

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Subject: Particle Physics

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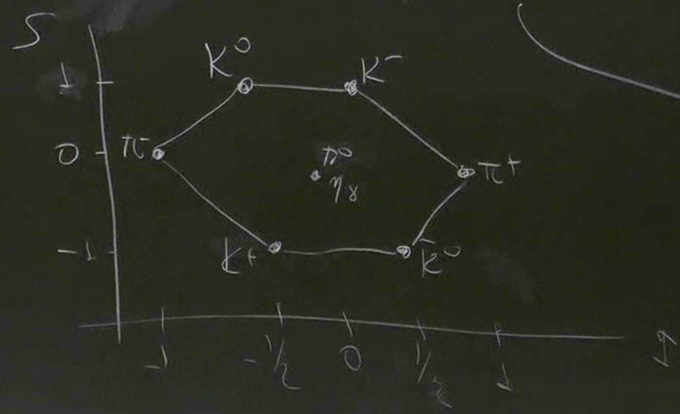
chiral perturbation theory

$$SU_L(3)\times SU_R(3)\times U_B(1) \xrightarrow[\langle \bar{q}_L q_R\rangle]{} SU_f(3)\times U_B(1)$$

$$\mathcal{M} = q\,\bar{q} \qquad q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$



$$3\otimes\bar{3} = 1\oplus 8$$

η_0 = singlet

octet

Non-linear σ model

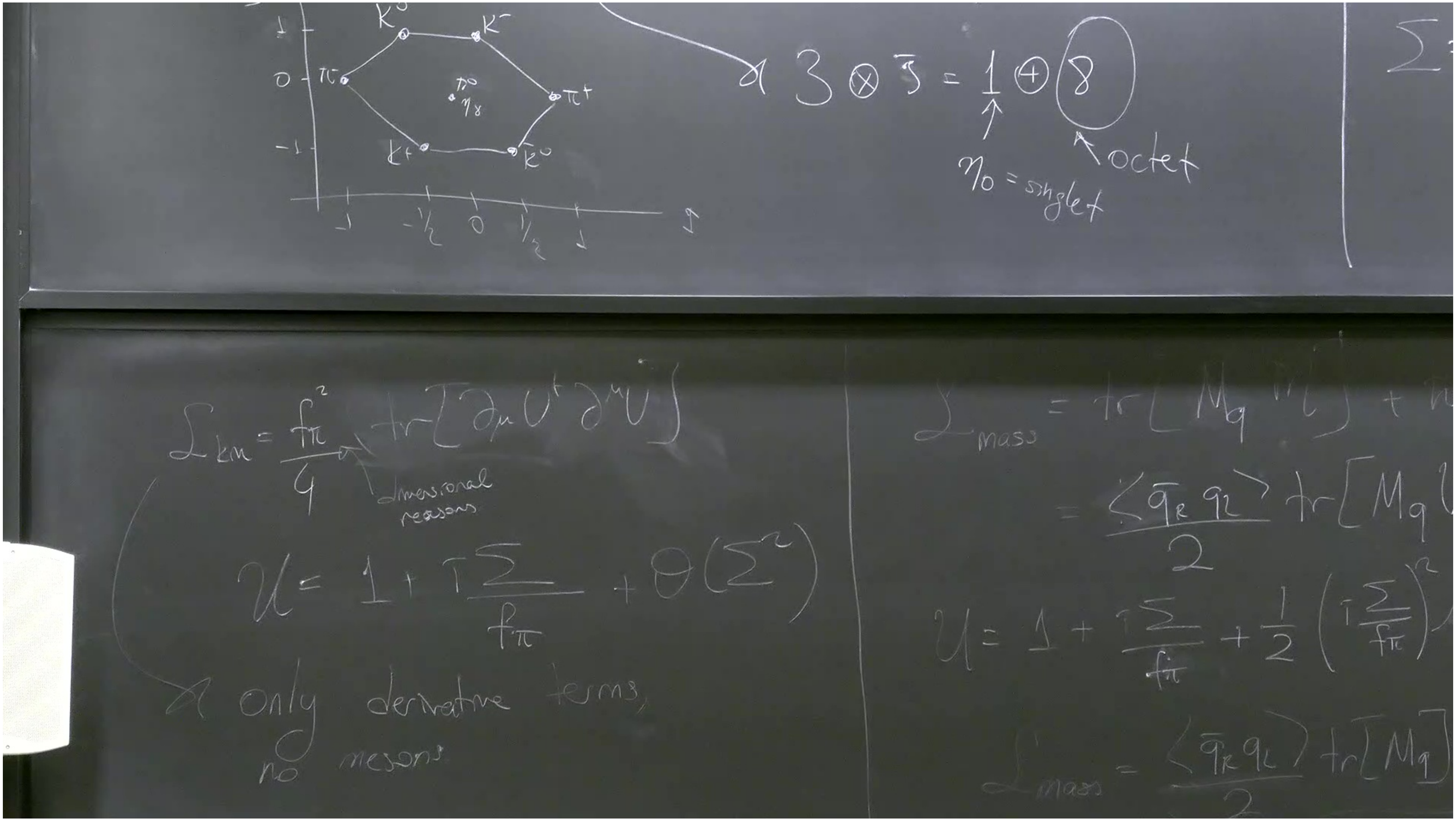
$$M = e^{i\Sigma/f_\pi}\langle M\rangle \equiv \langle \bar{q}_R q_L\rangle \underbrace{e^{i\Sigma/f_\pi}} = \langle \bar{q}_R q_L\rangle U$$

$$\Sigma = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta_8 & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta_8 & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & \frac{2}{\sqrt{3}}\eta_8 \end{pmatrix}$$

$$[f_\pi] = 1$$

$$[U] = 0$$

$$\varphi = v e^{ih/v}$$


K^0
K^-
π^-
π^0
η_8
π^+
K^+
$\bar{K}^0$
-1
$-\frac{1}{2}$
0
$\frac{1}{2}$
1
$3 \otimes \bar{3} = 1 \oplus 8$
η_0 = singlet
octet
$\mathcal{L}_{kin} = \frac{f_\pi^2}{4} tr[\partial_\mu U^\dagger \partial^\mu U]$
dimensional reasons
$U = 1 + i\frac{\Sigma}{f_\pi} + O(\Sigma^2)$
only derivative terms, no mesons
$\mathcal{L}_{mass} = tr[M_q$
$= \frac{\langle \bar{q}_R q_L \rangle}{2} tr[M_q$
$U = 1 + i\frac{\Sigma}{f_\pi} + \frac{1}{2}\left(i\frac{\Sigma}{f_\pi}\right)^2$
$\mathcal{L}_{mass} = \frac{\langle \bar{q}_R q_L \rangle}{2} tr[M_q]$

$\eta_0 = \text{singlet}$

$$\mathcal{L}_{mass} = \mathrm{tr}\left[M_q M^\dagger\right] + h.c.$$

$$= \frac{\langle \bar{q}_R q_L \rangle}{2} \mathrm{tr}\left[M_q U\right] + h.c.$$

$$U = 1 + \frac{i\Sigma}{f_\pi} + \frac{1}{2}\left(\frac{i\Sigma}{f_\pi}\right)^2 + \mathcal{O}(\Sigma^3)$$

$$\mathcal{L}_{mass} = \underbrace{\frac{\langle \bar{q}_R q_L \rangle}{2} \mathrm{tr}\left[M_q\right]}_{\text{vacuum energy}} - \frac{\langle \bar{q}_R q_L \rangle}{2 f_\pi^2} \mathrm{tr}\left[M_q \Sigma \Sigma\right] + \cdots$$

no mesons

$$\mathcal{L}_{mass} = \underbrace{\frac{\langle q_R q_L \rangle \, tr[M_q]}{2}}_{\text{Vacuum energy}}$$

$$\mathcal{L}_{mass} = \frac{\langle \bar{q} q \rangle}{f_\pi^2} \Big[\frac{1}{2}(m_u + m_d)\, \pi^0 \pi^0 + \frac{1}{6}(m_u + m_d + 4m_s)\, \eta_8 \eta_8 + \underbrace{\frac{m_u - m_d}{\sqrt{3}}\, \pi^0 \eta_8}_{\pi^0\text{-}\eta_8 \text{ mixing}} + (\ldots$$

$$\begin{pmatrix} \tilde{\pi}^0 \\ \tilde{\eta}_8 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} \pi^0 \\ \eta_8 \end{pmatrix}$$

- $m_{\pi^0}^2 \simeq m_{\pi^+}^2 = \frac{\langle \bar{q} q \rangle}{f_\pi^2}(m_u + m_d)$
- $m_{K^+}^2 = \frac{\langle \bar{q} q \rangle}{f_\pi^2}(m_u + m_s)$
- $m_\eta^2 = \frac{\langle \bar{q} q \rangle}{f_\pi^2}(m_u + m_d + 4m_s)$
- $m_{K^0}^2 = \frac{\langle \bar{q} q \rangle}{f_\pi^2}(m_d + m_s)$

$$\frac{\langle \bar{q}_R q_L \rangle}{2} \mathrm{tr}[M_q] - \frac{\langle \bar{q}_R q_L \rangle}{2 f_\pi^2} \mathrm{tr}[M_q \Sigma \Sigma^\dagger]$$

vacuum energy

$$\underbrace{\frac{m_u - m_d}{\sqrt{3}} \pi^0 \eta_8}_{\pi^0-\eta_8 \text{ mixing}} + (m_u + m_d)\, \pi^+ \pi^- + (m_u + m_s)\, K^+ K^- + (m_d + m_s)\, \bar{K}^0 K^0$$

$$\boxed{m_\eta^2 \simeq \frac{1}{3}\left(2\left(m_{K^\pm}^2 + m_{K^0}^2\right) - m_\pi^2\right)}$$

Gell-Mann - Okubo formula

$$\frac{\langle \bar{q} q \rangle}{f_\pi^2} (m_u + m_s)$$

$$\frac{\langle \bar{q} q \rangle}{f_\pi^2} (m_d + m_s)$$

$$m_\pi \simeq 137 \text{ MeV}$$

$$m_K \simeq \sqrt{\frac{1}{2}\left(m_{K^+}^2 + m_{K^0}^2\right)} = 496 \text{ MeV}$$

$$m_\eta = 549 \text{ MeV}$$

$$\begin{pmatrix}\pi^+\\ \pi^0\\ \pi^-\end{pmatrix}$$

$SU_I(2)$ - triplet

higher degeneracy.

$$\underbrace{\underset{S=1}{\begin{pmatrix}K^0\\ K^+\end{pmatrix}}, \quad \underset{S=-1}{\begin{pmatrix}K^-\\ \bar{K}^0\end{pmatrix}}}_{\text{higher degeneracy.}}$$

$SU_I(2)$ - doublet

η_8

$SU_I(2)$ singlet

$U_A(1)$ – anomalous

$$j_5^\mu = \bar{q}\gamma^\mu\gamma_5 \frac{\lambda_0}{2} q = \frac{1}{\sqrt{6}}\left(\bar{u}\gamma^\mu\gamma_5 u + \bar{d}\gamma^\mu\gamma_5 d + \bar{s}\gamma^\mu\gamma_5 s\right)$$

anomalous w.r.t. $SU_c(3)^2$

$$\partial_\mu j_5^\mu = \sqrt{\frac{1}{6}}\, 3 \cdot \frac{g_3^2}{16\pi^2} G^\alpha_{\mu\nu} \tilde{G}^{\alpha\mu\nu}$$

Partially Conserved Axial Cur

$$\ldots + \bar{d}\gamma^\mu\gamma_5 d + \bar{s}\gamma^\mu\gamma_5 s\Big)$$

Partially Conserved Axial Current Relationship (PCAC)

$$j^\mu_{5i} = \bar{q}\gamma^\mu\gamma_5 \frac{\lambda_i}{2} q$$

$$\langle 0 | j^\mu_{5i} | \pi_j(k) \rangle = -i f_\pi k^\mu e^{-ikx} \delta_{ij}$$

$\eta_0 \equiv \eta'$

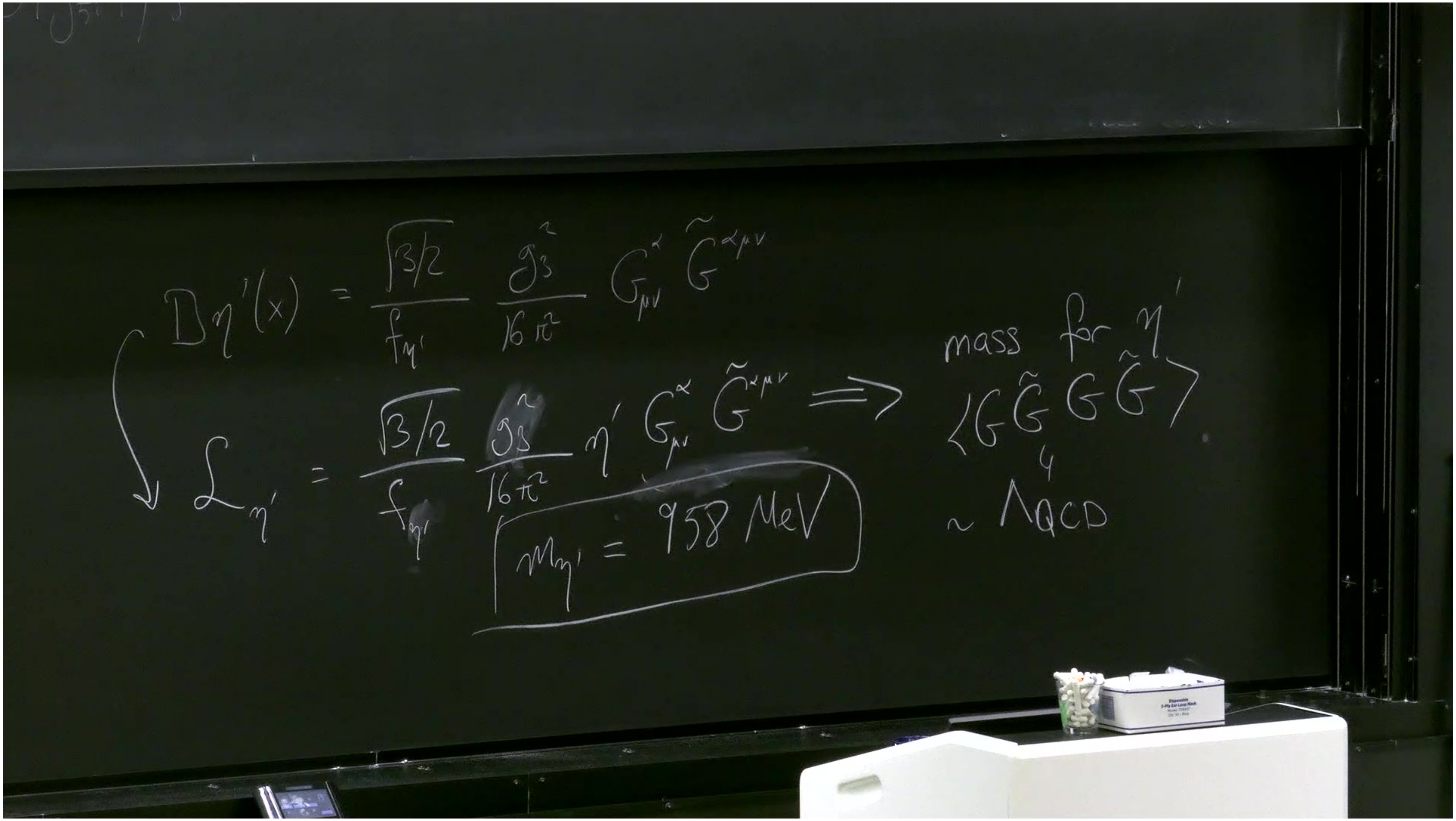
$\boxed{i=0}$

$$\langle 0 | j_5^\mu | \eta'(k) \rangle = -i f_{\eta'} k^\mu e^{-ikx}$$

$$\Rightarrow \quad j_5^\mu(x) = f_{\eta'} \partial^\mu \eta'(x)$$

$$\partial_\mu j_5^\mu = \frac{\sqrt{3/2}\, g_3^2}{16\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu} = f_{\eta'} \partial_\mu \partial^\mu \eta'(x)$$

$$\Box \eta'(x) = \frac{\sqrt{3}}{f_{\eta}}$$


$\partial \eta'(x) = \frac{\sqrt{3/2}}{f_{\eta'}} \frac{g_s^2}{16\pi^2} G^{\alpha}_{\mu\nu} \tilde{G}^{\alpha\mu\nu}$
$\mathcal{L}_{\eta'} = \frac{\sqrt{3/2}}{f_{\eta'}} \frac{g_s^2}{16\pi^2} \eta' G^{\alpha}_{\mu\nu} \tilde{G}^{\alpha\mu\nu} \Longrightarrow$
mass for η'
$\langle G\tilde{G}\, G\tilde{G} \rangle$
$\sim \Lambda_{QCD}^4$
$\boxed{m_{\eta'} = 958 \text{ MeV}}$

π^0 decay

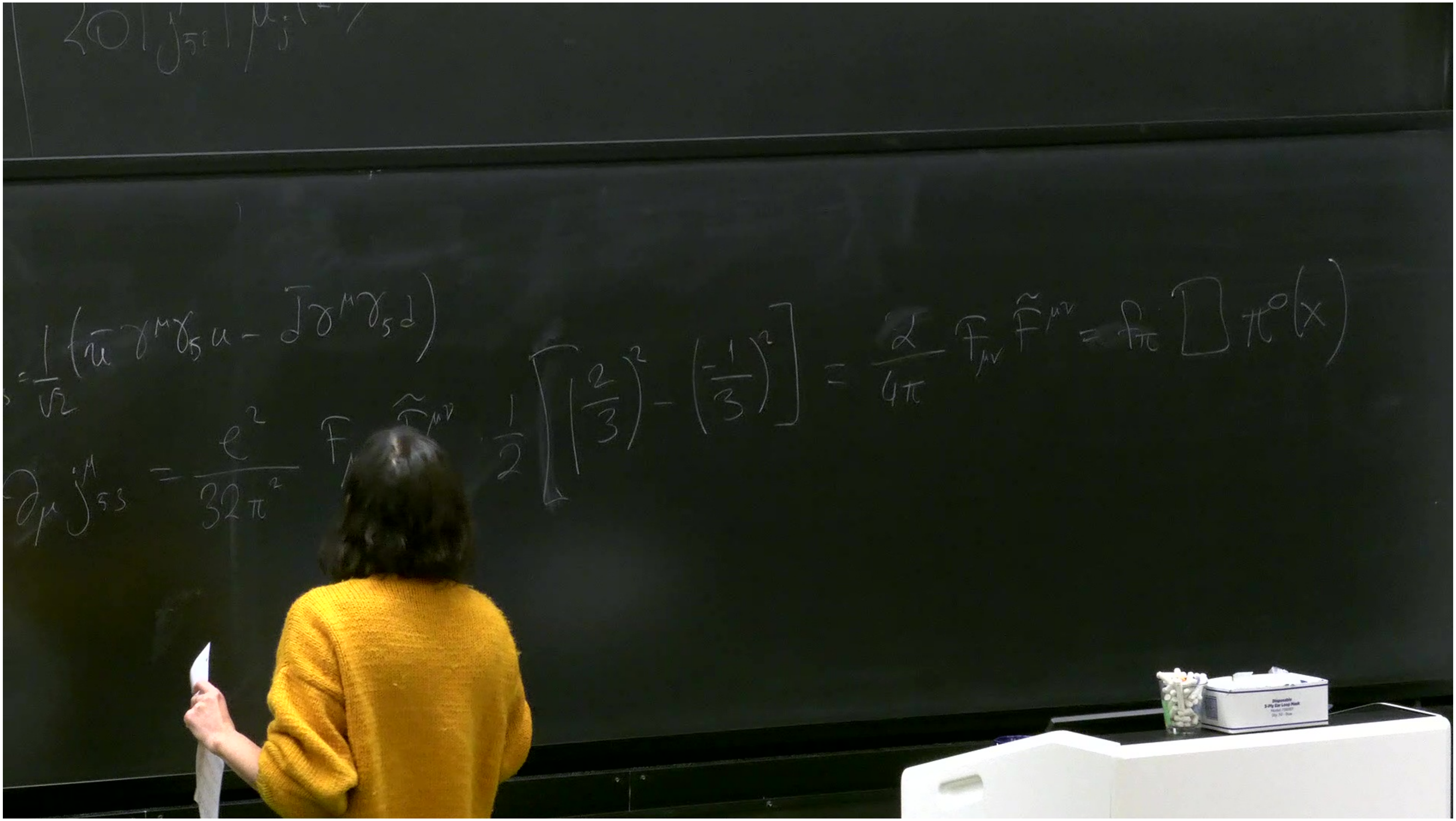
$i=3$: $\langle 0 | j^\mu_{53} | \pi^0(x) \rangle = -i f_\pi k^\mu e^{-ikx}$

$$j^\mu_{53} = \frac{1}{\sqrt{2}} \left(\bar{u} \gamma^\mu \gamma_5 u - \bar{d} \gamma^\mu \gamma_5 d \right)$$

$$j^\mu_{53} \leftrightarrow f_\pi \partial^\mu \pi^0(x)$$

$$\partial_\mu j^\mu_{53} = -\frac{e^2}{32\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

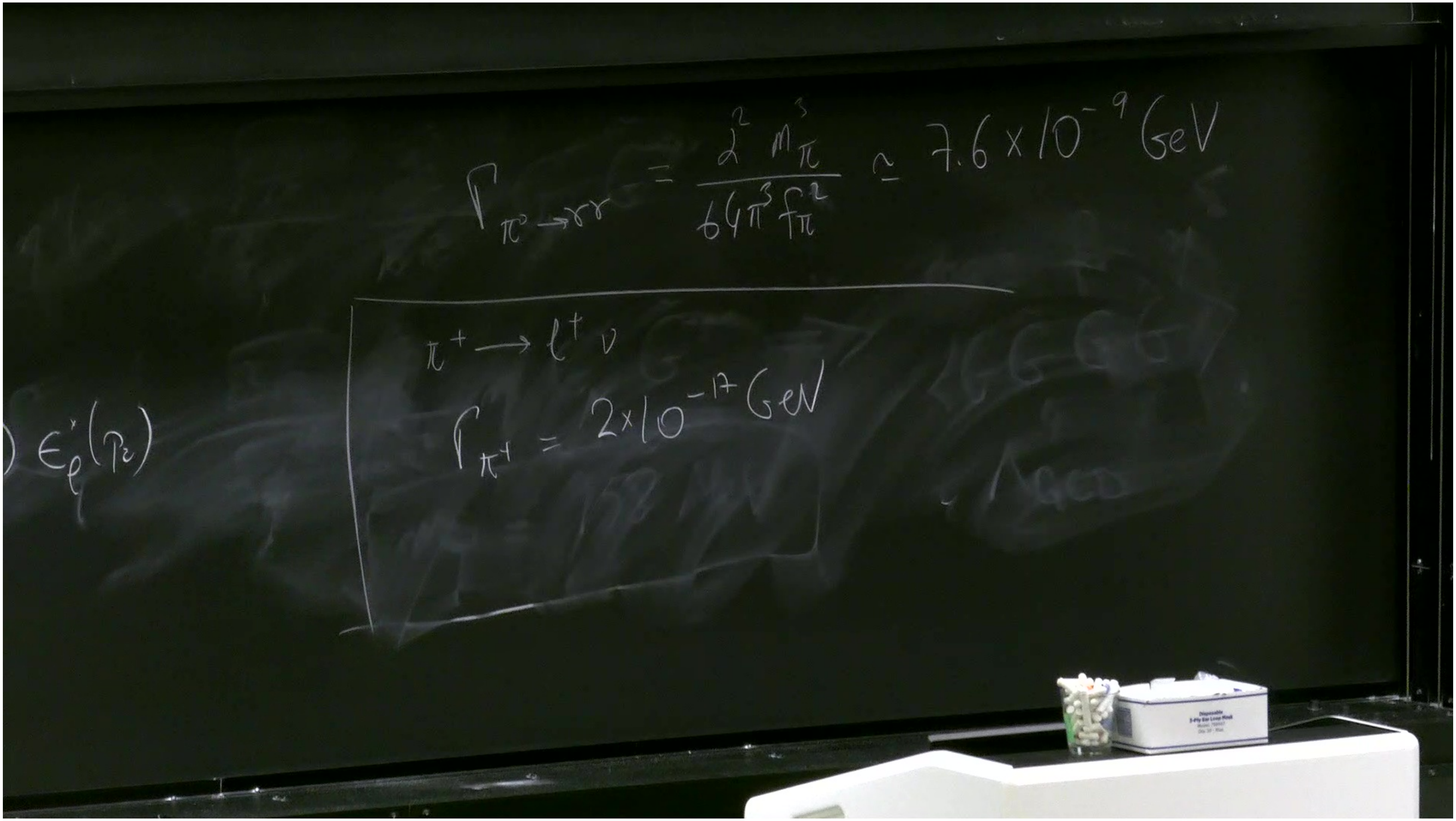
- <u>not</u> anomalous w.r.t. $SU_c(3)^2$
- anomalous w.r.t. $U_{em}(1)^2$


$= \frac{1}{\sqrt{2}}\left(\bar{u}\gamma^\mu\gamma_5 u - \bar{d}\gamma^\mu\gamma_5 d\right)$
$\partial_\mu j^\mu_{53} = \frac{e^2}{32\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu} \cdot \frac{1}{2}\left[\left(\frac{2}{3}\right)^2 - \left(-\frac{1}{3}\right)^2\right] = \frac{\alpha}{4\pi} F_{\mu\nu}\tilde{F}^{\mu\nu} = f_\pi \Box \pi^0(x)$

$$\mathcal{L}_{eff}(\pi^0) = \frac{1}{2}\partial_\mu \pi^0 \partial^\mu \pi^0 + \frac{1}{2} m_\pi^2 \pi^0 \pi^0 + \frac{\alpha}{4\pi f_\pi} \pi^0 F_{\mu\nu} \tilde{F}^{\mu\nu}$$

$$\mathcal{L}_{int} = \frac{\alpha}{2\pi f_\pi} \pi^0 \epsilon^{\mu\nu\sigma\rho} \partial_\mu A_\nu \partial_\sigma A_\rho$$

$$i\mathcal{M}(\pi^0 \to \gamma\gamma) = 2 \cdot \left(-\frac{\alpha}{2\pi f_\pi}\right) \epsilon^{\mu\nu\sigma\rho} (ip_1)_\mu (ip_2)_\sigma \; \epsilon^*_\nu(p_1)\, \epsilon^*_\rho(p_2)$$


$\Gamma_{\pi^0 \to \gamma\gamma} = \frac{\alpha^2 m_\pi^3}{64\pi^3 f_\pi^2} \simeq 7.6 \times 10^{-9}$ GeV
$\pi^+ \longrightarrow \ell^+ \nu$
$\Gamma_{\pi^+} = 2 \times 10^{-17}$ GeV
$\epsilon^*_\rho(p_2)$