Title: Lecture - Standard Model, PHYS 622

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Collection/Series: Standard Model (Elective), PHYS 622, January 6 - February 5, 2025

Subject: Particle Physics

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Λ_{QCD}

$\uparrow$ q, g (u, d, s) (c, b)

$\mathcal{O}(100\ \mathrm{MeV})$

$\downarrow$ $\langle \bar{q} q \rangle$

non-perturbative

$$\mathcal{L}_{QCD} = \mathcal{L}_{chiral} + \mathcal{L}_{mass}$$

$$\mathcal{L}_{chiral} = -\frac{1}{4} G^{a}_{\mu\nu} G^{a\mu\nu} - \begin{pmatrix} \bar{u} & \bar{d} & \bar{s} \end{pmatrix} \begin{pmatrix} \not{D} & 0 & 0 \\ 0 & \not{D} & 0 \\ 0 & 0 & \not{D} \end{pmatrix} \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

$$\mathcal{L}_{mass} = \begin{pmatrix} \bar{u} & \bar{d} & \bar{s} \end{pmatrix} \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

$$m_u, m_d, m_s \ll \Lambda_{QCD}$$

Symmetries of $\mathcal{L}_{chiral}$

$$U_L(3) \times U_R(3)$$

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix} \longrightarrow e^{i\theta_L^a \frac{\lambda^a}{2} P_L + i\theta_R^a \frac{\lambda_a}{2} P_R} \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

$$\lambda_0 = \sqrt{\frac{2}{3}}\,\mathbb{1} \qquad tr(\lambda_a \lambda_b) = \delta_{ab}$$

$\lambda_1, \ldots, \lambda_8$: Gell-Mann matrices

$$\theta_L = \theta_V + \theta_A$$

$$\theta_R = \theta_V - \theta_A$$

$$\longrightarrow \left[\mathbb{1} + \underbrace{\frac{i}{2}\theta_V^a \lambda_a}_{\substack{\text{anomaly-free} \\ \text{(left-right symmetric)}}} + \underbrace{\frac{i}{2}\theta_A^a \lambda_a \gamma_5}_{\substack{\text{possibly} \\ \text{anomalous}}}\right)\begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

$\lambda_0 = \sqrt{\frac{2}{3}}\,\mathbb{1}$

$\mathrm{tr}(\lambda_a \lambda_b) = \delta_{ab}$

$\lambda_{1,\dots,8}$: Gell-Mann matrices

(left-right symmetric)

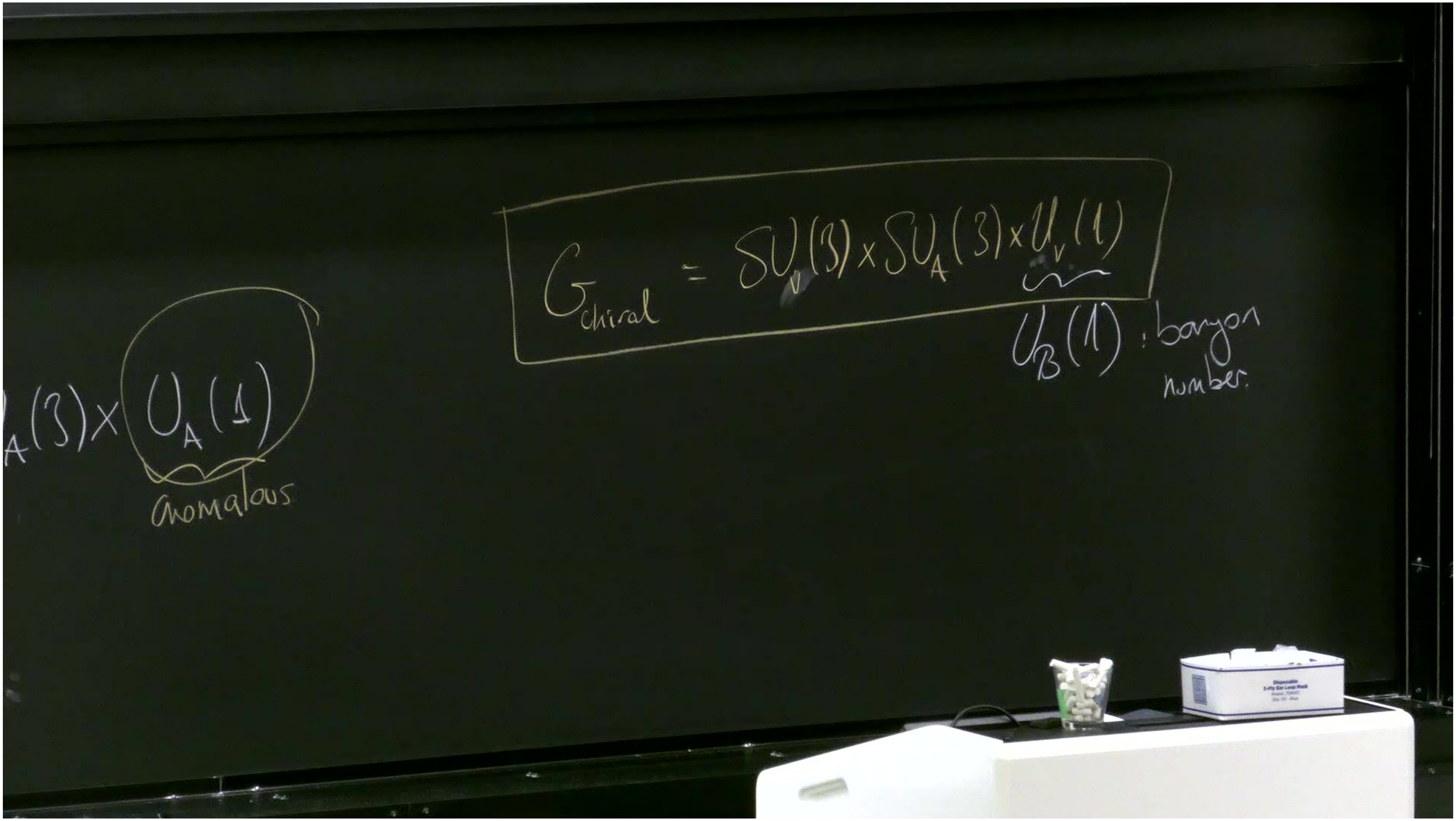
anomalous

$A(3,3,\ L-R) \propto \mathrm{Tr}\left[T^a_{L\cdot R}\right]$

$\mathcal{U}(3) = SU(3) \times \mathcal{U}(1)$

$\mathcal{U}_L(3) \times U_R(3) \equiv \mathcal{U}_V(3) \times U_A(3) = SU_V(3) \times U_V(1) \times SU_A(3) \times \underbrace{U_A(1)}_{\text{anomalous}}$

$G_{\text{chiral}} =$


G_chiral = SU_V(3) x SU_A(3) x U_V(1)
U_B(1) : baryon number
A(3) x U_A(1)
anomalous

* Mass terms will break $SU_A(3)$.

* $SU_V(3) \times U_V(1) = U_V(3)$

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix} \longrightarrow \left(1 + \frac{i}{2}\Theta_V^a \lambda_a\right)\begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

$$\begin{pmatrix} \bar{u} & \bar{d} & \bar{s} \end{pmatrix} M_q \begin{pmatrix} u \\ d \\ s \end{pmatrix} -$$

$$M_q = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

$$\delta \mathcal{L}_{mas}$$

$$M_q \begin{pmatrix} u \\ d \\ s \end{pmatrix} \longrightarrow (\bar{u}\ \bar{d}\ \bar{s})\left(1-\frac{i}{2}\theta_V^a\lambda_a\right) M_q \left(1+\frac{i}{2}\theta_V^a\lambda_a\right)\begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

$\delta\mathcal{L}_{mass} \sim \frac{i}{2}\theta_V^a\,[M_q, \lambda_a]$ invariant if $[M_q, \lambda_a] = 0$

Only λ_3, λ_8 and λ_0 commute w/ M_q

$$\lambda_0 = \sqrt{\frac{2}{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$M_q = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} = m_0 \lambda_0 + m_3 \lambda_3 + m_8 \lambda_8$$

$$m_0 = \sqrt{\frac{1}{6}} \left(m_u + m_d + m_s \right)$$

$$m_3 = \frac{1}{2} \left(m_u - m_d \right)$$

$$m_8 = \frac{1}{2\sqrt{3}} \left(m_u + m_d - 2m_s \right)$$

$\lambda_0 + m_3 \lambda_3 + m_8 \lambda_8$

$\frac{1}{6}(m_u + m_d + m_s)$

$\frac{1}{2}(m_u - m_d)$

$\frac{1}{2\sqrt{3}}(m_u + m_d - 2m_s)$

- $m_0 \lambda_0$ doesn't break any symmetries.
- $m_8 \lambda_8$ commutes w/ a subgroup:

$$\underbrace{SU_I(2)}_{\text{isospin}} \times \underbrace{U_S(1)}_{\text{strangeness}}$$

$I(u) = 1/2$

$I(d) = -1/2$

$S = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ $\quad S(s) = -1$

$\sqrt{3}\begin{pmatrix} 0 & 0 & -2 \end{pmatrix}$ $\qquad m_8 \ldots \frac{\;}{2\sqrt{3}}$

• $m_3 \lambda_3$ term breaks everything to: $U_u(1) \times U_d(1) \times U_s(1)$

$$U_L(3) \times U_R(3) \xrightarrow{U_A(1) \text{ is anomalous}} U_V(3) \times SU_A(3) \xrightarrow{\text{mass terms}} U_V($$

(1)

$$\xrightarrow{m)} U_V(3) \longrightarrow SU_I(2)\times U_S(1) \longrightarrow U_u(1)\times U_d(1)\times U_s(1)$$

$\lambda_1, \ldots, 8$: Gell-Mann matrices

Mesons : $q\bar{q}$

3 quark $\times$ 3 antiquark = 9 combinations.

$$\mathcal{M} = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \begin{pmatrix} \bar{u} & \bar{d} & \bar{s} \end{pmatrix} = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix}$$

et's look at how $\mathcal{M}$ transforms under

$$SU_V(3) \equiv SU_f(3)$$

flavor

$$\mu_1 - i\mu_2 \qquad\qquad \mu_4 - i\mu_5$$

$$-\mu_3 + \sqrt{\frac{1}{3}}\,\mu_8 + \sqrt{\frac{2}{3}}\,\mu_0 \qquad\qquad \mu_6 - i\mu_7$$

$$\mu_6 + i\mu_7 \qquad\qquad -\frac{2}{\sqrt{3}}\,\mu_8 + \sqrt{\frac{2}{3}}\,\mu_0$$

$$\sqrt{2}\,\pi^+ \qquad\qquad \sqrt{2}\,K^+$$

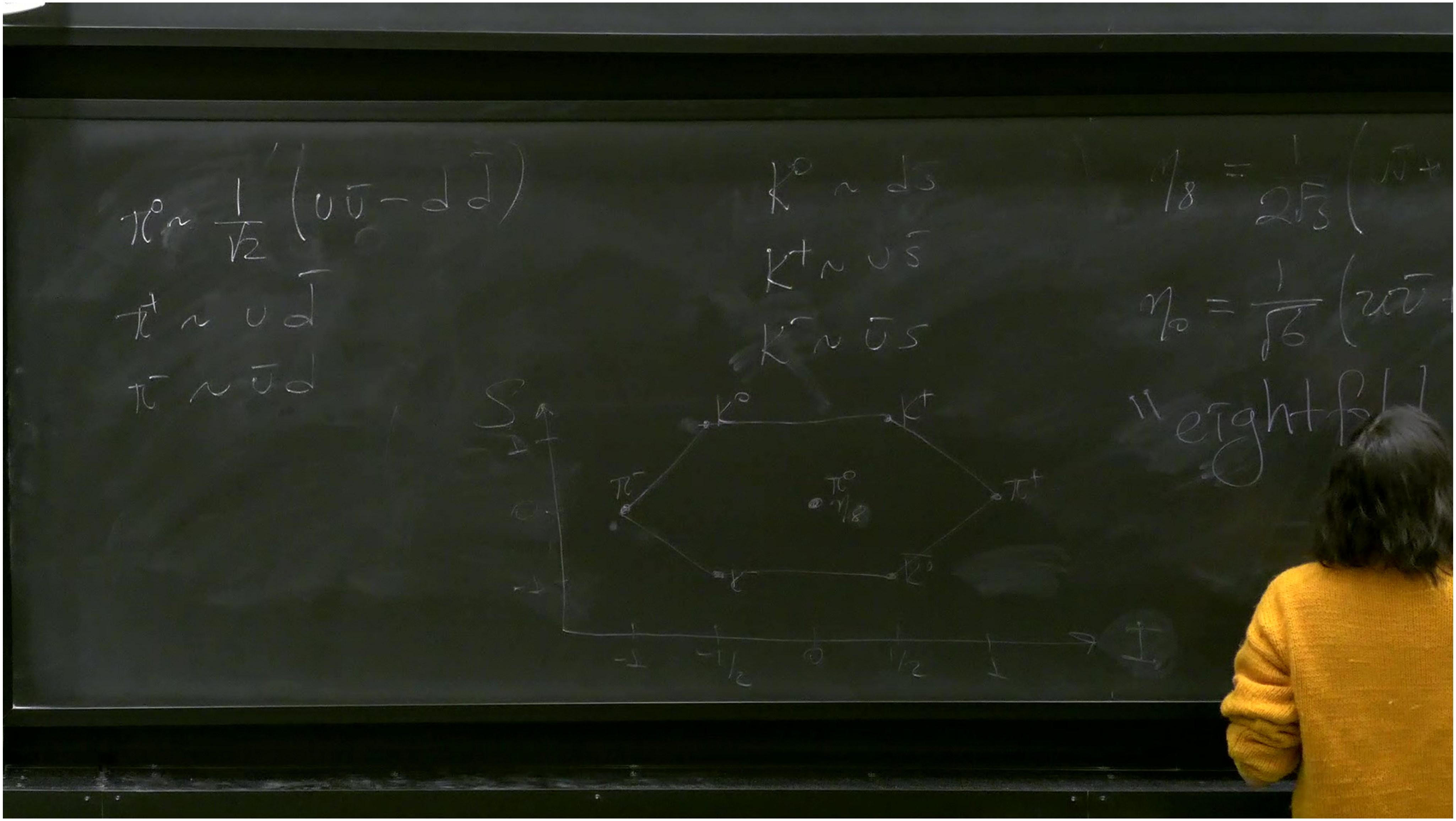
$$-\pi^0 + \frac{1}{\sqrt{3}}\,\eta_8 + \sqrt{\frac{2}{3}}\,\eta_0 \qquad\qquad \sqrt{2}\,K^0$$

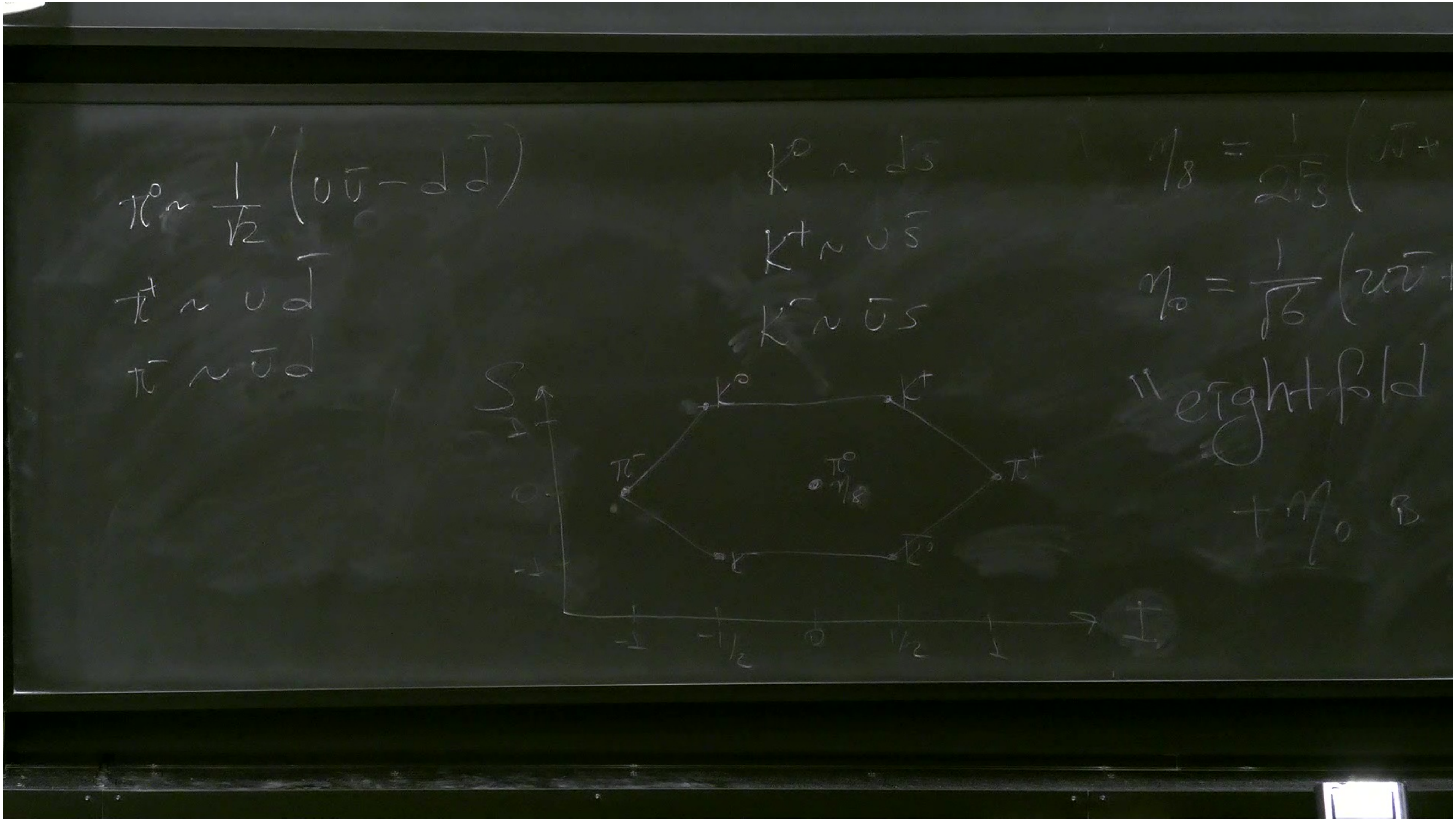
$$\sqrt{2}\,K^0 \qquad\qquad -\frac{2}{\sqrt{3}}\,\eta_8 + \sqrt{\frac{2}{3}}\,\eta_0$$

$$= \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix}$$

$$M = \sum \mu_\alpha \lambda_\alpha = \begin{pmatrix} \mu_3 + \frac{1}{\sqrt{3}}\mu_8 + \sqrt{\frac{2}{3}}\mu_0 & \mu_1 - i\mu_2 \\ \mu_1 + i\mu_2 & -\mu_3 + \sqrt{\frac{1}{3}}\mu_8 + \sqrt{\frac{2}{3}}\mu_0 \\ \mu_4 + i\mu_5 & \mu_6 + i\mu_7 \end{pmatrix}$$

$$= \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt{2}\,\pi^+ \\ \sqrt{2}\,\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 \\ \sqrt{2}\,K^- & \sqrt{2}\,K^0 \end{pmatrix}$$


π⁰ ~ 1/√2 (uū − dd̄)
π⁺ ~ ud̄
π⁻ ~ ūd
K⁰ ~ ds̄
K⁺ ~ us̄
K⁻ ~ ūs
η₈ = 1/(2√3) (
η₀ = 1/√6 (
"eight fold
S
I
π⁰
η₈
K⁰
K⁺
π⁻
π⁺
K̄⁰
-1
-1/2
0
1/2
1


$\pi^0 \sim \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$
$\pi^+ \sim u\bar{d}$
$\pi^- \sim \bar{u}d$
$K^0 \sim d\bar{s}$
$K^+ \sim u\bar{s}$
$K^- \sim \bar{u}s$
$\eta_8 = \frac{1}{2\sqrt{3}}($
$\eta_0 = \frac{1}{\sqrt{6}}($
"eightfold
$+\ \eta_0$
S
I
K^0
K^+
π^-
π^0
η_8
π^+
K^-
$\bar{K}^0$
1
0
-1
-1
-1/2
0
1/2
1

$= \frac{1}{2\sqrt{3}}\left(u\bar{u} + d\bar{d} - 2s\bar{s}\right)$

$\frac{1}{\sqrt{6}}\left(u\bar{u} + d\bar{d} + s\bar{s}\right)$

ghtfold way"

$+ \eta_0$ is singlet under $SU_f(3)$

λ_0 doesn't break any symmetries.

λ_8 commutes w/ a subgroup:

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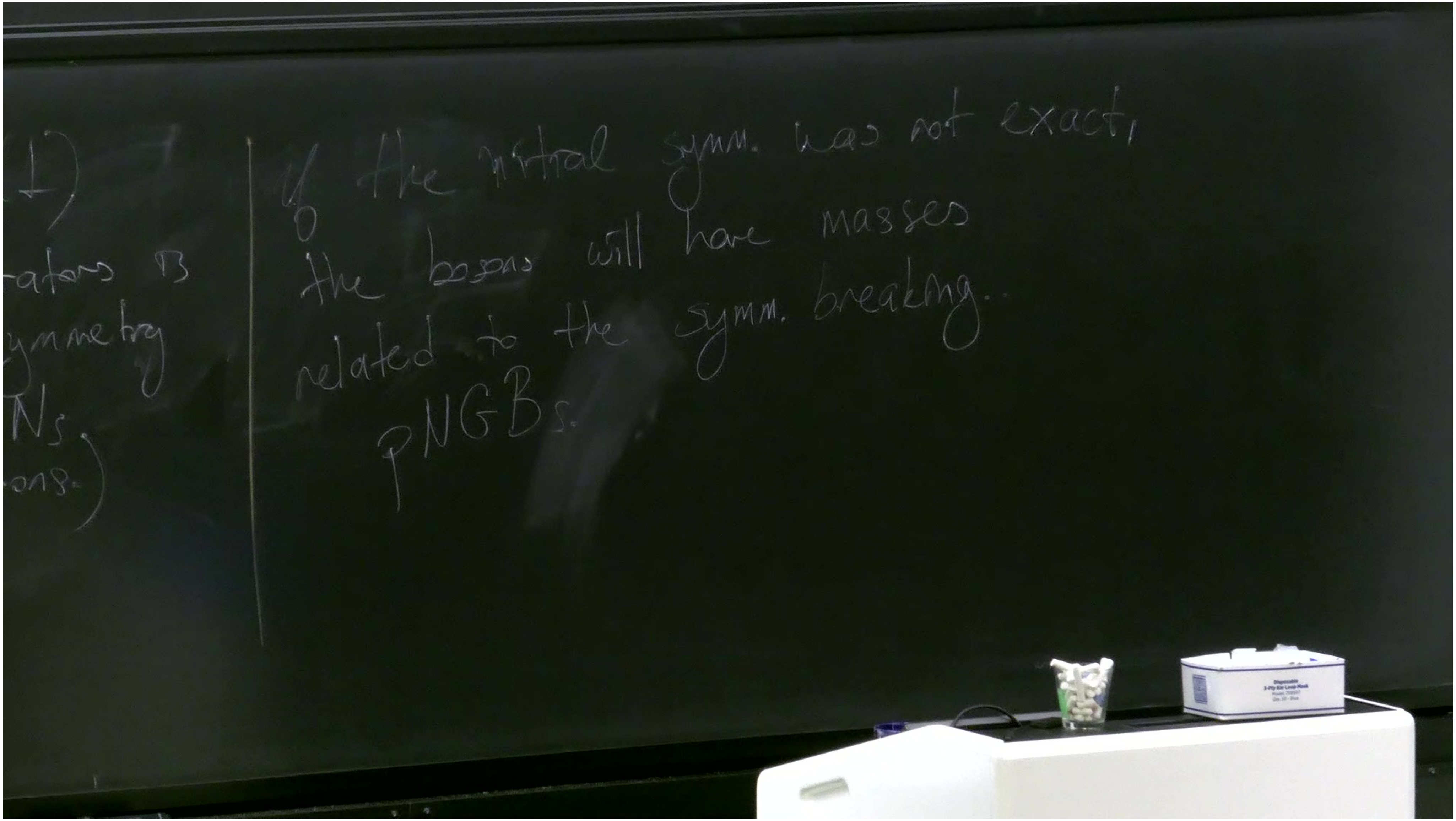
Let's look

$$SU_V(3) \times SU_A(3) \times U_B(1) \xrightarrow[\langle \bar{q}_R q_L \rangle \sim \Lambda^3_{QCD}]{} SU_f(3) \times U_B(1)$$

Goldstone's theorem

When a global symmetry w/ N_G generators is broken spontaneously down to a smaller symmetry w/ N_S generators, you will get $N_G - N_S$ massless bosons. (Nambu-Goldstone bosons.)


If the initial symm. was not exact,
the bosons will have masses
related to the symm. breaking..
PNGBs.