Title: Lecture - Standard Model, PHYS 622

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Subject: Particle Physics

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$$SU_L(2) \times U_Y(1) \xrightarrow{\langle \Phi \rangle = v} U_{EM}(1)$$

$$SU_L(2) \to W^a_\mu,\ a=1,2,3 \qquad U_Y(1) \to B_\mu$$

$$V(\Phi) = \lambda\left(\Phi^\dagger \Phi - \frac{\mu^2}{2\lambda}\right)$$

$$W^\pm_\mu = W^1_\mu \pm i\, W^2_\mu$$

$$M_W^2 = \frac{g_2^2 v^2}{4}$$

$$Z_\mu = \frac{-g_1 B_\mu + g_2 W^3_\mu}{\sqrt{g_1^2 + g_2^2}} \equiv \cos\theta_W W^3_\mu -$$

$$\boxed{\sin\theta_W = \ldots}$$

$\to U_{EM}(1)$

$\Phi - \frac{\mu^2}{2\lambda}\Big)^2$

$W_\mu^3 = \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu$

$$\boxed{\sin\theta_W = \frac{g_1}{\sqrt{g_1^2+g_2^2}}}$$

Weinberg angle

$$SU_L(2) \times U_Y(1) \xrightarrow{\langle \Phi \rangle = v} U_{EM}(1)$$

$\downarrow$ $\qquad\qquad\qquad\qquad$ $\downarrow$

W^a_μ, $a = 1,2,3$ $\qquad\qquad$ B_μ

$$V(\Phi) = \lambda\left(\Phi^\dagger \Phi - \frac{\mu^2}{2\lambda}\right)^2$$

$\pm\, i\, W^2_\mu$

$$Z_\mu = \frac{-g_1 B_\mu + g_2 W^3_\mu}{\sqrt{g_1^2 + g_2^2}} \equiv \cos\theta_W W^3_\mu - \sin\theta_W B_\mu$$

$$\frac{g_2^2 v^2}{4}$$

$$M_Z^2 = \frac{(g_1^2 + g_2^2)\, v^2}{4}$$

$$\boxed{\sin\theta_W = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}}$$ Weinberg angle

$$A_\mu = -\sin\theta_W W^3_\mu + \cos\theta_W B_\mu$$

$$M_A = 0$$

$$\ldots\theta_W W^3_\mu + \sin\theta_W B_\mu$$

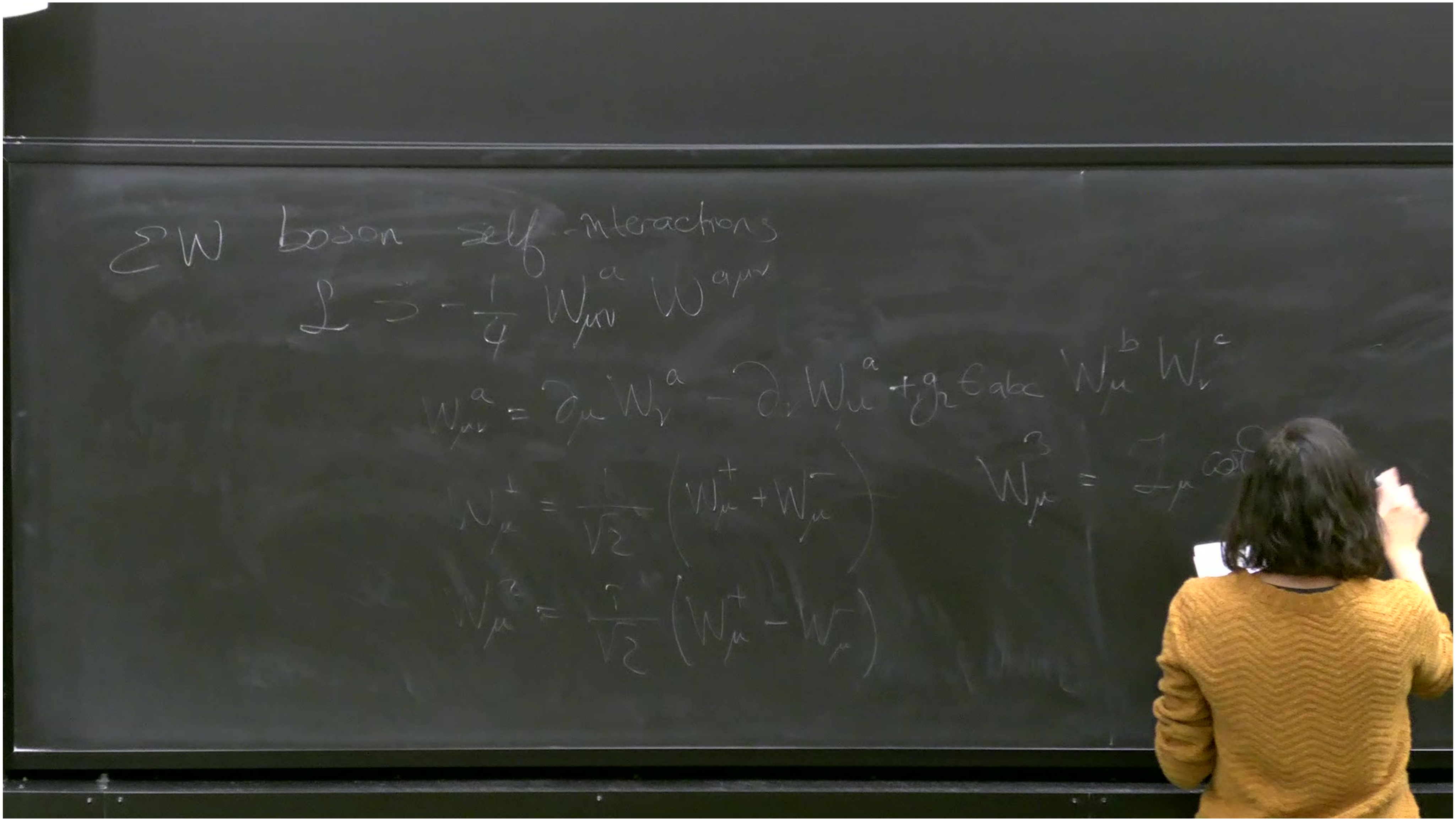
$$\boxed{\ldots\theta_W = \frac{g_1}{\sqrt{g_1^2+g_2^2}}}$$ Weinberg angle

$$\frac{M_W}{M_Z} = \frac{g_2}{\sqrt{g_1^2+g_2^2}} = \cos\theta_W$$

"custodial" SU(2)

$$\Phi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \end{pmatrix} \xrightarrow{\text{vacuum}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ v \end{pmatrix}$$

invariant under $O(3)$


EW boson self-interactions
$\mathcal{L} \supset -\frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu}$
$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2 \epsilon_{abc} W^b_\mu W^c_\nu$
$W^1_\mu = \frac{1}{\sqrt{2}} (W^+_\mu + W^-_\mu)$
$W^2_\mu = \frac{i}{\sqrt{2}} (W^+_\mu - W^-_\mu)$
$W^3_\mu = Z_\mu \cos$

EW boson self-interactions

$$\mathcal{L} \supset -\frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu}$$

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2 \epsilon_{abc} W^b_\mu W^c_\nu$$

$$W^1_\mu = \frac{1}{\sqrt{2}}\left(W^+_\mu + W^-_\mu\right)$$

$$W^3_\mu = Z_\mu \cos\theta_W + A_\mu \sin$$

$$W^2_\mu = \frac{i}{\sqrt{2}}\left(W^+_\mu - W^-_\mu\right)$$

$\ldots \theta_W + A_\mu \sin\theta_W$

$$\mathcal{L}_{WWZ} = i g_2 \cos\theta_W \Big[W^+_{\mu\nu} W^{-\mu} Z^\nu - W^-_{\mu\nu} W^{+\mu} Z^\nu + W^+_\mu W^-_\nu Z^{\mu\nu} \Big]$$

$$\mathcal{L}_{WW\gamma} = i g_2 \sin\theta_W \Big[W^+_{\mu\nu} W^{-\mu} A^\nu - W^-_{\mu\nu} W^{+\mu} A^\nu + W^+_\mu W^-_\nu F^{\mu\nu} \Big]$$

$$W^+_{\mu\nu} = \partial_\mu W^+_\nu - \partial_\nu W^+_\mu$$

$$Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$W_\mu^2 = \frac{i}{\sqrt{2}}\left(W_\mu^+ - W_\mu^-\right)$$

$$\mathcal{L}_{WWWW} = -\frac{1}{2} g_2^2 \left[\left(W_\mu^+ W^{-\mu}\right)^2 - \left(W_\mu^+ W^{+\mu}\right)\left(W_\nu^- W^{-\nu}\right) \right]$$

$$\mathcal{L}_{WWZZ} = -\left(g_2 \cos\theta_w\right)^2 \left[\left(W_\mu^+ W^{-\mu}\right) Z^\nu Z_\nu - \left(W_\mu^+ Z^\mu\right)\left(W_\nu^- Z^\nu\right) \right]$$

$$\mathcal{L}_{WW\gamma\gamma} = -\Big(\underbrace{g_2 \sin\theta_w}_{\equiv e}\Big)^2 \left[\left(W_\mu^+ W^{-\mu}\right) A_\nu A^\nu - \left(W_\mu^+ A^\mu\right)\left(W_\nu^- A^\nu\right) \right]$$

$$Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\mathcal{L}_{WWZ\gamma} = -(g_2 \sin\theta_W)(g_2 \cos\theta_W)\Big[2\,(W^+_\mu W^{-\mu})\, Z_\nu A^\nu - (W^+_\mu Z^\mu)(W^-_\nu A^\nu) - (W^+_\mu A^\mu)(W^-_\nu Z^\nu)\Big]$$

Generalized Stokes thm:

EW: boson-fermion interactions

$\bar{f}\, \not{D} f$

Charged-current interactions

$$\mathcal{L}_{cc} = \frac{i\, g_2}{2\sqrt{2}} \Big[W_\mu^+ \bar{\nu}_m' \gamma^\mu (1+\gamma_5) e_m' + V_{mn} W_\mu^+ \bar{u}_m' \gamma^\mu (1+\gamma_5) d_n' + $$

$$V_{mn} = \left(U^{u\dagger} U^d \right)_{mn}$$

Generalized Stokes thm:

EW boson-fermion interactions

$\bar{f} \not{D} f$

Charged-current interactions

$$\mathcal{L}_{cc} = \frac{i\, g_2}{2\sqrt{2}} \Big[\overbrace{W^+_\mu \, \bar{\nu}'_m \, \gamma^\mu (1+\gamma_5)\, e'_m}^{\text{lepton flavor universality}} + V_{mn} W^+_\mu \, \bar{u}'_m \, \gamma^\mu (1+\gamma_5)\, d'_n + h.c. \Big)$$

$V_{mn} = \left(U^{u\dagger} U^{d}\right)_{mn}$: CKM matrix

Neutral-current interactions

$$\mathcal{L}_{nc} = -\sum \left[\bar{f} \gamma^\mu P_L \left(-i g_2 W_\mu^3 T_3 - i g_1 B_\mu Y_L \right) f + \bar{f} \gamma^\mu P_R \left(-i g_1 B_\mu Y_R \right) f \right] \equiv i \sum \bar{f} M_\mu \gamma^\mu f$$

$$Q = T_3 + Y_L = Y_R$$

$$M_\mu = P_L g_2 W_\mu^3 T_3 + P_L g_1 B_\mu Y_L + g_1 P_R B_\mu Y_R$$
$$= P_L g_2 W_\mu^3 T_3 + P_L g_1 B_\mu (Q - T_3) + g_1 P_R B_\mu Y_R$$

$$l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}_{e/\mu} \quad T_3 = +1/2,\ T_3 = -1/2 \qquad q_L = \begin{pmatrix} u \\ d \end{pmatrix}$$

$$M_\mu = T_3 P_L \left(g_2 W_\mu^3 - g_1 B_\mu \right) + g_1 B_\mu Q$$

$$= \frac{e}{\sin\theta_W \cos\theta_W} \left[T_3 P_L - Q \sin^2\theta_W \right] Z_\mu + e Q A_\mu$$

$$\mathcal{L}_{em} = \sum_f e Q A_\mu \bar{f} \gamma^\mu f$$

$\mathcal{L}_{nc}$

$l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad T_3 = -1/2 \qquad q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \quad T_3 = -1/2$

A_μ

$$\mathcal{L}_{NC} = \frac{ie}{\sin\theta_W \cos\theta_W} \sum_f Z_\mu \bar{f} \gamma^\mu \left(P_L T_3 - Q \sin^2\theta_W \right) f$$

$$= \frac{ie}{\sin\theta_W \cos\theta_W} \sum_f Z_\mu \bar{f} \gamma^\mu \left(g_V + \gamma_5 g_A \right) f$$

$$g_V = \frac{1}{2} T_3 - Q \sin^2\theta_W \qquad g_A = \frac{1}{2} T_3$$

$$f_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\mathcal{L}_{h\text{-}f} = -\sum_f \frac{m_f}{v} h \bar{f} f$$

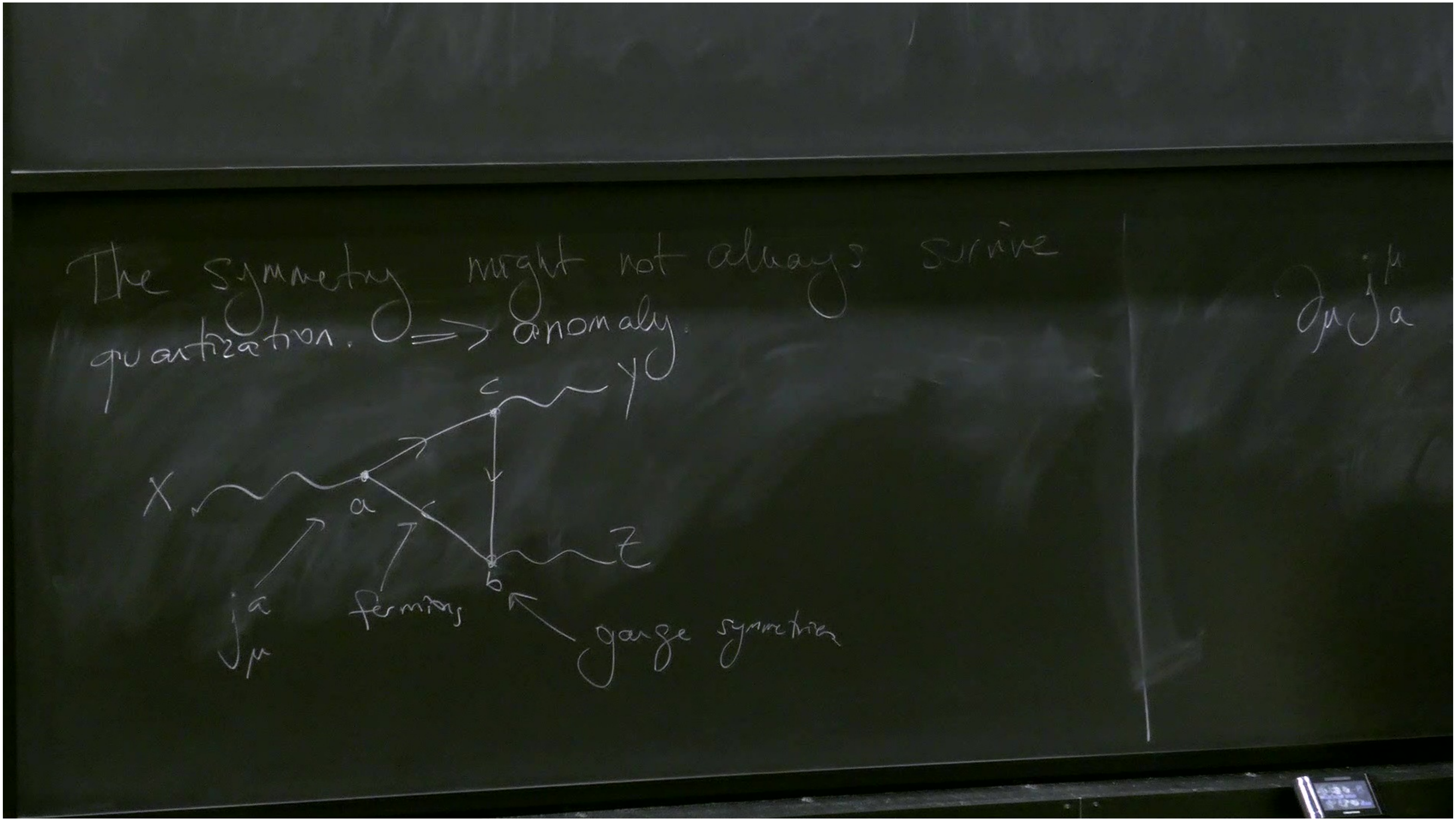
No flavor changing neutral currents (FCNCs)

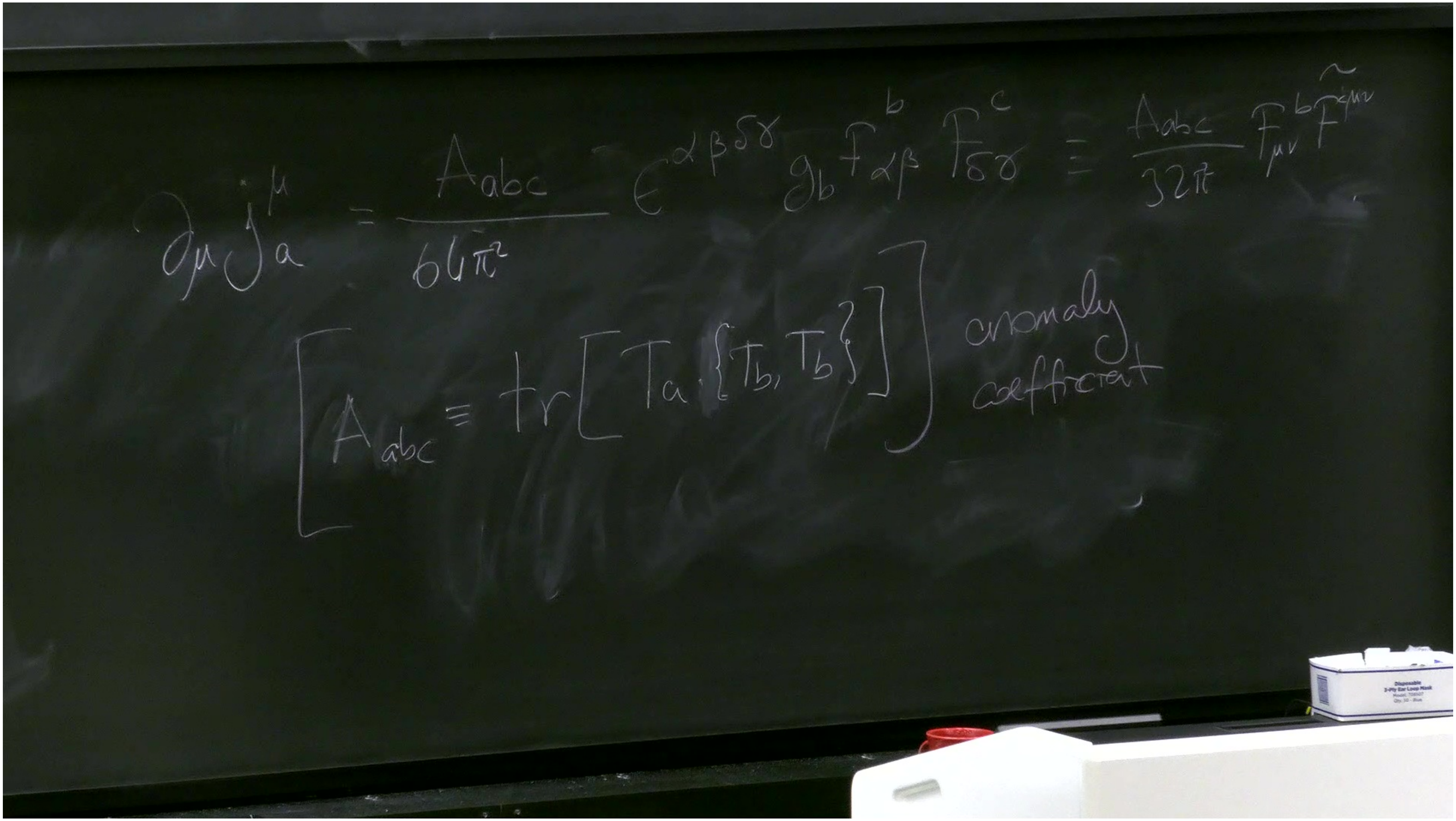
Anomalies

Take a Lagrangian invariant under some symmetry.

Symmetries $\Longleftrightarrow$ conserved current

$$\partial_\mu j^\mu_a = 0 \quad \Longleftrightarrow \quad Q_a = \int d^3x\, j^0_a$$


The symmetry might not always survive quantization. ⟹ anomaly.
$\partial_\mu j^\mu_a$
X
Y
Z
a
b
c
j^a_μ
fermions
gauge symmetries


$\partial_\mu j_a^\mu = \frac{A_{abc}}{64\pi^2} \epsilon^{\alpha\beta\delta\gamma} g_b F_{\alpha\beta}^{b} F_{\delta\gamma}^{c} \equiv \frac{A_{abc}}{32\pi} F_{\mu\nu}^{b} \tilde{F}^{c\mu\nu}$
$\left[A_{abc} \equiv \mathrm{tr}\left[T_a \{ T_b, T_b \} \right] \right]$
anomaly coefficient