Title: Lecture - Standard Model, PHYS 622

Speakers: Seyda Ipek

Collection/Series: Standard Model (Elective), PHYS 622, January 6 - February 5, 2025

Subject: Particle Physics

Date: January 13, 2025 - 10:15 AM

URL: https://pirsa.org/25010016

$$\boxed{SU_c(3)\times SU_L(2)\times U_Y(1)}$$

Fields	
$G_\mu^\alpha,\ \alpha=1,\dots,8$	$(8,1,0)$
$W_\mu^a,\ a=1,2,3$	$(1,3,0)$
B_μ	$(1,1,0)$

	Fields	
leptons	$l_L^m=\begin{pmatrix}\nu_L\\ e_L\end{pmatrix}_m$	$(1,2,-1/2)$
	e_R	$(1,1,-1)$
quarks	$q_L^m\begin{pmatrix}u_L\\ d_L\end{pmatrix}_m$	$(3,2,1/6)$
	u_R^m	$(3,1,2/3)$
	d_R^m	$(3,1,-1/3)$

Field

$$\Phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \qquad (1, 2, 1/2)$$

$$l_L = P_L \mathcal{L} = \begin{pmatrix} P_L \nu \\ P_L \mathcal{E} \end{pmatrix}$$

$$e_R = P_R \underline{E}$$

$$e = P_L \mathcal{E} + P_R \underline{E}$$

$$\mathcal{E} = \begin{pmatrix} e_L \\ \epsilon e_L^* \end{pmatrix}$$

$$\underline{E} = \begin{pmatrix} -\epsilon \cdot e_R^* \\ e_R \end{pmatrix}$$

Majorana fields

$$\epsilon = i\sigma_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\mathcal{L}_{SM} = -\frac{1}{4} G^{\alpha}_{\mu\nu} G_{\alpha}^{\mu\nu} - \frac{1}{4} W^{a}_{\mu\nu} W_{a}^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$- \frac{1}{2} i \bar{L}_m \not{D} L_m - \frac{1}{2} i \bar{E}_m \not{D} E_m - \frac{1}{2} i \bar{Q}_m \not{D} Q_m - \frac{1}{2} i \bar{U}_m \not{D} U_m - \frac{1}{2} i \bar{D}_m \not{D} D_m$$

$$- (D_\mu \Phi)^\dagger (D^\mu \Phi) + \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2$$

$$- \left(Y^{e}_{mn} \bar{L}_m P_R E_n \Phi + Y^{d}_{mn} \bar{Q}_m P_R D_n \Phi + Y^{u}_{mn} \bar{Q}_m P_R U_n \tilde{\Phi} + h.c. \right)$$

$$\ldots{}^{\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$- \frac{1}{2} i \bar{Q}_m \not{D} Q_m - \frac{1}{2} i \bar{U}_m \not{D} U_m - \frac{1}{2} i \bar{D}_m \not{D} D_m$$

$$\ldots E_m$$

$$\ldots{}^{+}\Phi - \lambda (\Phi^{+}\Phi)^2$$

$$\tilde{\Phi} = \epsilon \Phi^*$$

$$\ldots{}^{d}_{mn} \bar{Q}_m P_R D_n \Phi + Y^{u}_{mn} \bar{Q}_m P_R U_n \tilde{\Phi} + h.c.)$$

Covariant derivatives

$$D_\mu L_m = \partial_\mu L_m - \overset{SU(3)}{(0)} - i g_2 W_\mu^a \left(\frac{\tau_a}{2}\right)(P_L L_m) + i g_2 W_\mu^a \left(\frac{\tau_a}{2}\right)(P_R L_m) - i$$

$$D_\mu E_m = \partial_\mu E_m - (0) - \underset{SU(2)}{(0)} - i g_1 (-1) B_\mu (P_L E_m) - i g_1 (1) B_\mu$$

$$D_\mu Q_m = \partial_\mu Q_m - i g_3 G_\mu^\alpha \left(\frac{\lambda_\alpha}{2}\right)(P_L Q_m) + i g_3 G_\mu^\alpha \left(\frac{\lambda_\alpha}{2}\right)(P_R Q_m) - i g_2 W_\mu^a$$

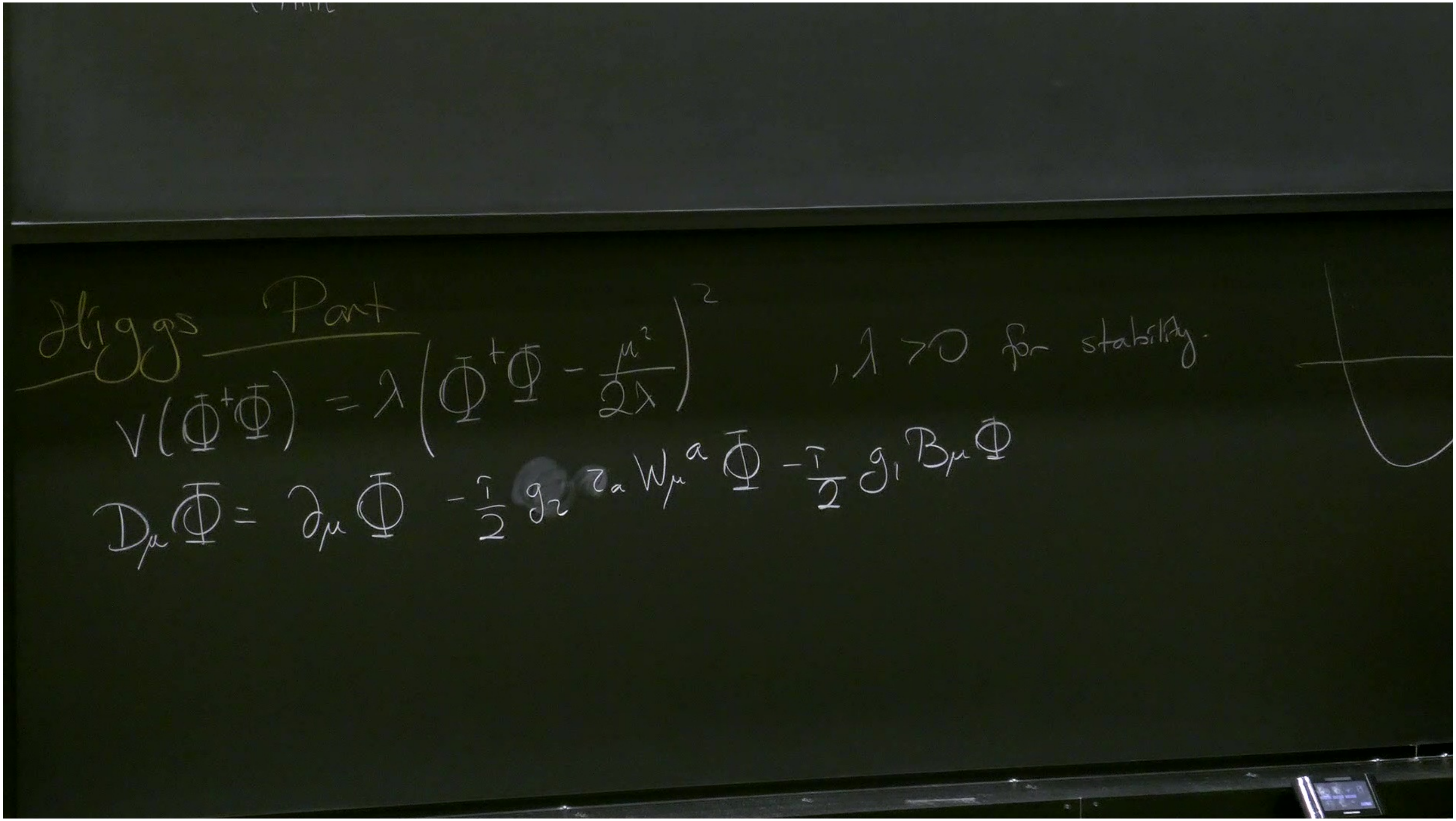
$$D_\mu U_m = \partial_\mu U_m - i g_3 G_\mu^\alpha \left(\frac{\lambda_\alpha}{2}\right)(P_L U_m) + i g_3 G_\mu^\alpha \left(\frac{\lambda_\alpha}{2}\right)(P_R U_m) - \underset{SU(2)}{(0)}$$

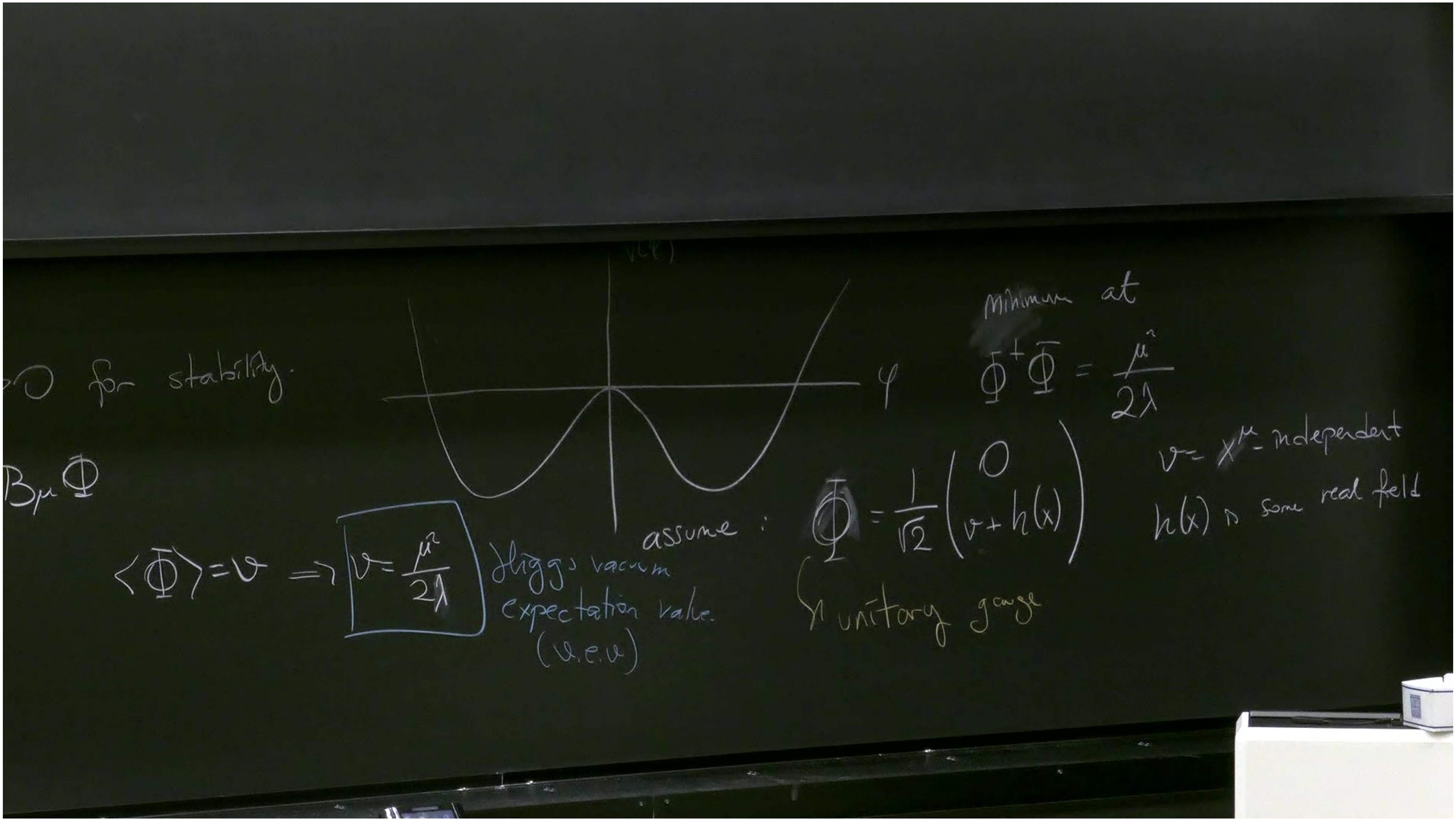
$$\ldots \tfrac{\tau_a}{2}\Big)(P_R L_m) - i\left(-\tfrac{1}{2}\right) g_1 B_\mu (P_L L_m) - i\left(\tfrac{1}{2}\right) g_1 B_\mu (P_R L_m)$$

$$\ldots i\, g_1 (1) B_\mu (P_R E_m)$$

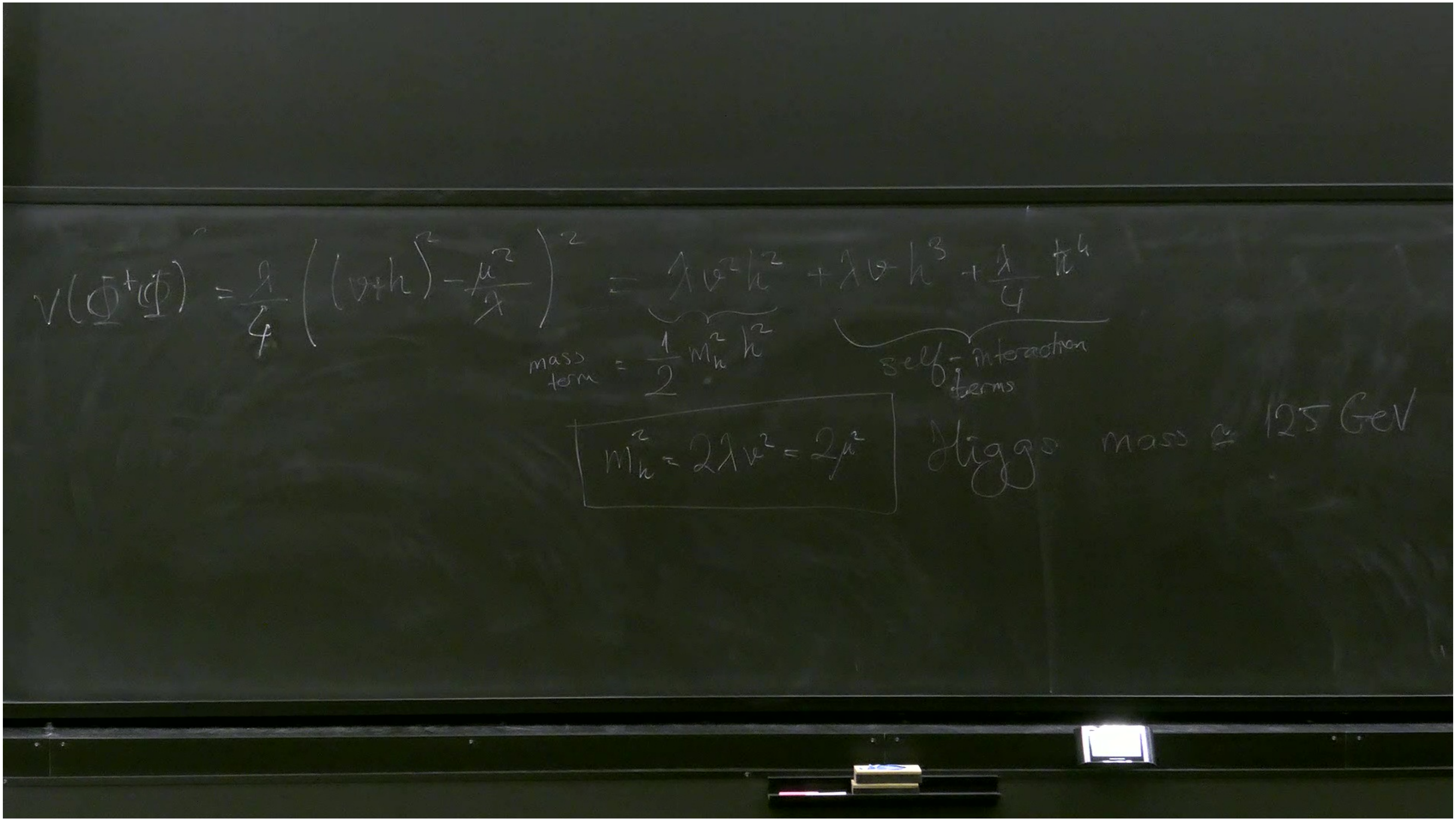
$$\ldots m\Big) - i g_2 W_\mu^a \left(\frac{\tau_a}{2}\right)(P_L Q_m) + i g_2 W_\mu^a \left(\frac{\tau_a}{2}\right)(P_R Q_m) - i g_1 \left(\frac{1}{6}\right) B_\mu (P_L Q_m) - i g_1 \left(-\frac{1}{6}\right) B_\mu (P_R Q_m)$$

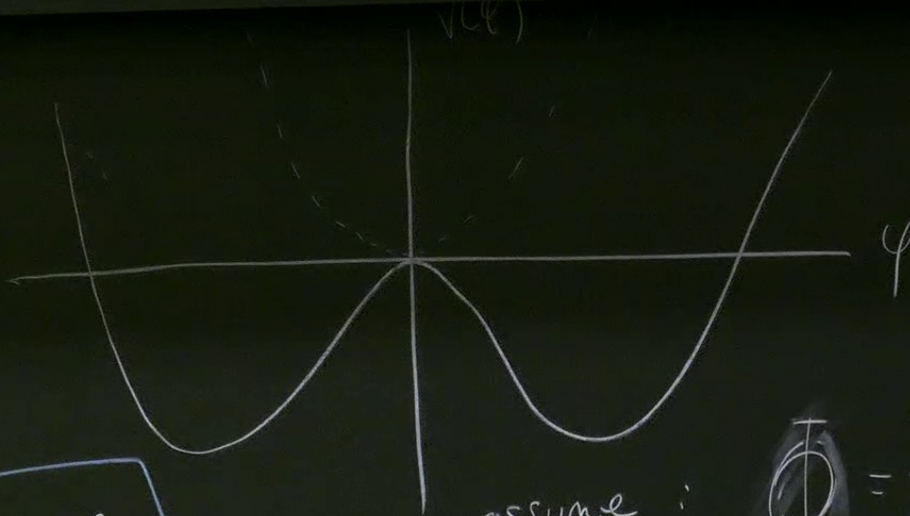
$$\ldots\Big) - \underbrace{(0)}_{SU(2)} - i\left(\frac{2}{3}\right) g_1 B_\mu (P_L U_m) - i\left(-\frac{2}{3}\right) g_1 B_\mu (P_R U_m)$$


Higgs Part
$V(\Phi^\dagger\Phi) = \lambda\left(\Phi^\dagger\Phi - \frac{\mu^2}{2\lambda}\right)^2$, $\lambda > 0$ for stability.
$D_\mu \Phi = \partial_\mu \Phi - \frac{i}{2} g_2 \tau_a W_\mu^a \Phi - \frac{i}{2} g_1 B_\mu \Phi$



>0 for stability.
$B_\mu \Phi$
φ
minimum at
$\Phi^\dagger \Phi = \frac{\mu^2}{2\lambda}$
v = x^μ- independent
$h(x)$ is some real field
assume: $\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$
unitary gauge
$\langle \Phi \rangle = v \Rightarrow \boxed{v = \frac{\mu^2}{2\lambda}}$
Higgs vacuum expectation value. (v.e.v)


$V(\Phi^\dagger\Phi) = \frac{\lambda}{4}\left((v+h)^2 - \frac{\mu^2}{\lambda}\right)^2 = \underbrace{\lambda v^2 h^2}_{\text{mass term} = \frac{1}{2} m_h^2 h^2} + \underbrace{\lambda v h^3 + \frac{\lambda}{4} h^4}_{\text{self-interaction terms}}$
$\boxed{m_h^2 = 2\lambda v^2 = 2\mu^2}$
Higgs mass ≃ 125 GeV



Minimum at

$$\Phi^\dagger \Phi = \frac{\mu^2}{2\lambda}$$

$v = x^\mu$-independent

$h(x)$ is some real field

assume : $\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$

unitary gauge.

$$\boxed{v = \frac{\mu^2}{2\lambda}}$$

Higgs vacuum expectation value. (v.e.v)

$\frac{\lambda}{4} \, h^4$

...eraction

...s

...ggs mass $\simeq$ 125 GeV

$$y^e_{mn} \bar{L}_m \, \mathcal{E}_n \, \Phi = y^e_{mn} \begin{pmatrix} \bar{\nu}_m \\ \bar{\mathcal{E}}_m \end{pmatrix}^T P_R \, \mathcal{E}_n \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$= \underbrace{y^e_{mn} \frac{v}{\sqrt{2}} \, \bar{\mathcal{E}}_m P_R \, \mathcal{E}_n}_{\text{fermion mass term}} + \underbrace{\frac{y^e_{mn}}{\sqrt{2}} h \, \bar{\mathcal{E}}_m P_R \, \mathcal{E}_n}_{h-f-\bar{f} \text{ interaction}}$$

$$\mathcal{L}_{\substack{\text{fermion}\\ \text{masses}}} = -\frac{y^e_m v}{\sqrt{2}}\,\bar{e}_m e_m - \frac{y^u_m v}{\sqrt{2}}\,\bar{u}_m u_m - \frac{y^d_m v}{\sqrt{2}}\,\bar{d}_m d_m$$

Ex: Find the Yukawa couplings.

$m_e = 511\ \text{keV}$

$m_\mu = 105\ \text{MeV}$

$m_\tau = 1.7\ \text{GeV}$

$m_u = 2\ \text{MeV}$

$m_d = 4\ \text{MeV}$

$m_s \simeq 87\ \text{MeV}$

$m_c \simeq 1.31\ \text{GeV}$

$m_b = 4.24\ \text{GeV}$

$m_t = 173\ \text{GeV}$

inter-action.

$$\frac{y^d_m v}{\sqrt{2}} \bar{d}_m d_m$$

Ex: Find the Yukawa couplings.

Ex: Write the dim-5 term that will give you ν-masses:

$$m_\nu \overline{\nu_L^c} \nu_L$$