Title: Lecture - Standard Model, PHYS 622

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Abelian - global

$\mathcal{U}(1)$: $\Phi_1 \longrightarrow e^{iq_1\theta}\,\Phi_1$

$\Phi_2 \longrightarrow e^{iq_2\theta}\,\Phi_2$

Ex// $q_1 = 1$, $q_2 = 4$

$$\mathcal{L} = \left(\partial_\mu \Phi_i\right)^\dagger \left(\partial^\mu \Phi_i\right) - m_i^2 \left(\Phi_i^\dagger \Phi_i\right) - \lambda_{ij} \left(\Phi_i^\dagger \Phi_i\right)\left(\Phi_j^\dagger \Phi_j\right)$$

$i = 1, 2$

There is another symmetry this Lagrangian is invariant under
(accidental symmetry)

$U_a(1) \times U_b(1)$ product group.

$$\Phi_1 \longrightarrow e^{i(q_1^a \theta^a + q_1^b \theta^b)} \Phi_1 \qquad (q_1^a, q_1^b) = (0, 1)$$

$$\Phi_2 \longrightarrow e^{i(q_2^a \theta^a + q_2^b \theta^b)} \Phi_2 \qquad (q_2^a, q_2^b) = (1, 0)$$

$$\left.\begin{array}{l}\theta_{\pm} = \theta^a \pm \theta^b \\ q_i^{a} = q_i^a \pm q_i^b\end{array}\right\} U_+(1) \times U_-(1)$$

Fermions: $U(1)$

$$\psi_L \longrightarrow e^{iq_L\theta}\psi_L \qquad \overline{\psi_L^c} \longrightarrow e^{iq_L\theta}\,\overline{\psi_L^c}$$

$$\psi_R \longrightarrow e^{iq_R\theta}\psi_R \qquad \overline{\psi_R^c} \longrightarrow e^{iq_R\theta}\,\overline{\psi_R^c}$$

$$m_M^L \, \overline{\psi_L^c} \, \psi_L \longrightarrow e^{2iq_L\theta} \, m_M^L \, \overline{\psi_L^c} \, \psi_L$$

$$m_M^R \, \overline{\psi_R^c} \, \psi_R \longrightarrow e^{2iq_R\theta} \, m_M^R \, \overline{\psi_R^c} \, \psi_R$$

$q_L\theta \, \overline{\psi_L^c}$

$iq_R\theta \, \overline{\psi_R^c}$

- Majorana masses are <u>not</u> allowed for "charged" fermions

$$m_D \, \overline{\psi_L} \, \psi_R \longrightarrow e^{i(q_L - q_R)\theta} \, m_D \, \overline{\psi_L} \, \psi_R$$

- Dirac mass is allowed only for $q_L = q_R$.

$(3, 2, -1/3)$

$$m_M^R \overline{\psi_R^c} \psi_R \longrightarrow e^{2iq_R\theta} m_M^R \overline{\psi_R^c} \psi_R$$

$q_L\theta \overline{\psi_L^c}$

$iq_R\theta \overline{\psi_R^c}$

- Majorana masses are <u>not</u> allowed for "charged" fermions

$$m_D \overline{\psi_L} \psi_R \longrightarrow e^{i(q_L - q_R)\theta} m_D \overline{\psi_L} \psi_R$$

- Dirac mass is allowed only for $[q_L = q_R]$ vectorial symmetry

if $q_L \neq q_R$ "chiral"

+ antiparticles

$(3, 2, 1/6)$

$(3, 1, 2/3)$

$(3, 1, -1/3)$

quarks

$s = 1/2$

Abelian - Local

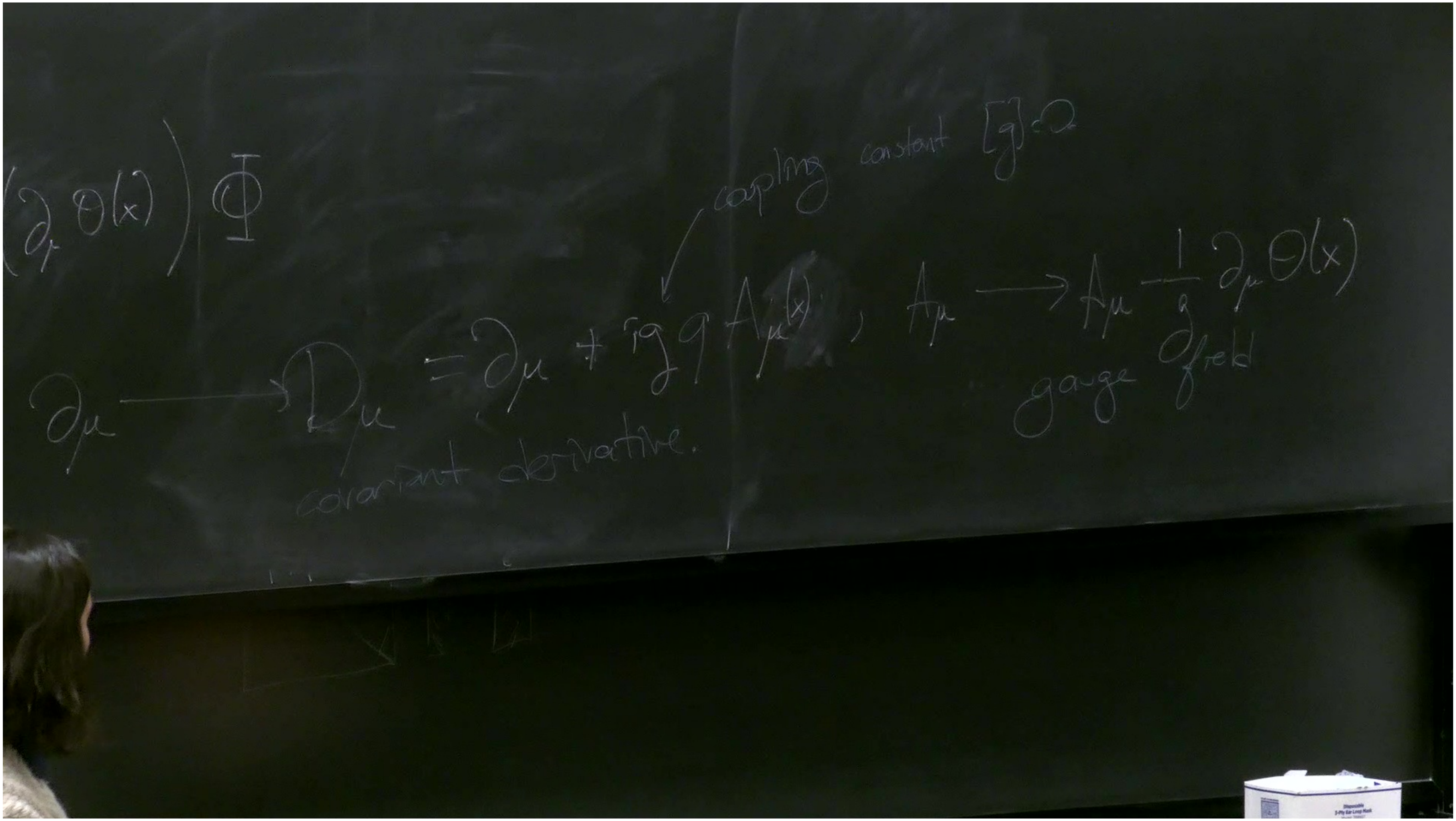
$\theta \to \theta(x)$ "U(1) is gauged"

$$\Phi(x) \longrightarrow e^{iq\theta(x)}\,\Phi(x)$$

$\Phi^\dagger \Phi$ is still invariant under gauged $U(1)$

$$\partial_\mu \Phi \longrightarrow \partial_\mu\left(e^{iq\theta(x)}\,\Phi(x)\right) = e^{iq\theta(x)}\,\partial_\mu \Phi + iq\,e^{iq\theta(x)}\left(\partial_\mu \theta(x)\right)\Phi$$

$$\partial_\mu \Phi^\dagger \partial^\mu \Phi \not\longrightarrow (\partial_\mu \Phi)^\dagger (\partial^\mu \Phi) \Longrightarrow \partial_\mu \longrightarrow D_\mu$$


$(\partial_\mu \theta(x))\Phi$
coupling constant $[g]=0$
$\partial_\mu \longrightarrow D_\mu = \partial_\mu + igA_\mu(x)$; $A_\mu \longrightarrow A_\mu - \frac{1}{g}\partial_\mu \theta(x)$
covariant derivative
gauge field

$$[D_\mu, D_\nu] = i g q \underbrace{F_{\mu\nu}}_{\text{field strength}}$$

$$\cdot\, M^2 A_\mu A^\mu \quad \text{is } \underline{\text{not}} \text{ allowed by}$$

for $U(1)$: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

• $m^2 A_\mu A^\mu$ is not allowed by gauged $U(1)$. (photon for EM)

• $i\bar{\psi} \not{D} \psi = i\bar{\psi} \not{\partial} \psi - g q \bar{\psi} \not{A} \psi$

interaction te

If you have $U_a(1) \times U_b(1)$

$$\begin{array}{cc} \downarrow & \downarrow \\ A^a_\mu & A^b_\mu \\ | & | \\ g_a & g_b \end{array}$$

Non-Abelian Symmetries - Global

$SU(N)$: • consider a field in a representation R of dim $M > 1$.

$\to \Phi$ is a vector of dim (M).

φ_i, $i = 1, 2, \ldots, M$.

e.g. $s = 1/2$ fields in a $M=2$ rep. of $SU(2)$.

$g_a \qquad g^b$

R of dim $M > 1$.

$$\phi_i \longrightarrow \left(e^{i T_a \theta_a}\right)_{ij} \phi_j \qquad i,j = 1,2,\dots,M \qquad a = 1,\dots N^2-1$$

T_a : generators of SU(N) algebra.

$$[T_a, T_b] = i f_{abc} T_c$$

structure constants.

$S_1 = \frac{1}{2}$ $\quad S_2 = \frac{1}{2}$ $\quad 2 \otimes 2 = 1 \oplus 3$

$$\underbrace{S_1 = \tfrac{1}{2} \quad S_2 = \tfrac{1}{2}}_{S=0 \text{ or } S=1}$$

- You need to combine multiple fields to make a singlet under $SU(N)$.

How do you do that?

$$\boxed{R \otimes \bar{R} = 1 \oplus (N^2-1)}$$

M ... m ... M ... m_I ... D ...

Non-Abelian Symmetries - Global

Consider $U(N) = SU(N) \times U(1)$

$\psi_L \; : \; N \; , \; 1$

$\psi_R \; : \; N \; , \; 1$

$\mathcal{L} = i \bar{\psi}_L \partial \psi_L + i \bar{}$

Non-Abelian Local $\quad \theta \to \theta(x) \quad$ "$SU(N)$ is gauged"

$$\Phi(x) \longrightarrow e^{iq\theta(x)}\,\Phi(x)$$

$\Phi^\dagger \Phi$ is still invariant under gauged $U(1)$

$$\partial_\mu \Phi \longrightarrow \partial_\mu\left(e^{iq\theta(x)}\,\Phi(x)\right) = e^{iq\theta(x)}\,\partial_\mu \Phi + iq\,e^{iq\theta(x)}\left(\partial_\mu \theta(x)\right)\Phi$$

$$\partial_\mu \Phi^\dagger \partial^\mu \Phi \not\longrightarrow \left(\partial_\mu \Phi\right)^\dagger \left(\partial_\mu \Phi\right) \Longrightarrow \partial_\mu \longrightarrow$$

$$\partial_\mu \longrightarrow \underbrace{D_\mu}_{\text{covariant derivative}} = \partial_\mu + i\overset{\text{coupling constant } [g]=0}{g}\, \underbrace{T_a}_{\substack{M\times M \text{ matrix for an } M\text{-dim}\\ \text{representation of } SU(N)}}\, G^a_\mu$$

Non-Abelian Local $\theta \to \theta(x)$ "SU(N) is gauged"

$$\to e^{iq\theta(x)}\,\Phi(x)$$

...ill invariant under gauged U(1)

$$G_a^{\mu} \to G_a^{\mu} - f_{abc}\,\theta_b(x)\,G_c^{\mu} - \frac{1}{g}\partial^{\mu}\theta_a$$

$$G_a^{\mu\nu} = \partial^{\mu} G_a^{\nu} - \partial^{\nu} G_a^{\mu} - g f_{abc} G_b^{\mu} G_c^{\nu} \quad \Rightarrow \quad \partial_{\mu}$$

$$\boxed{SU_c(3) \times SU_L(2) \times U_Y(1)}$$

weak hypercharge

gauge bosons ($s=1$)

Fields	Charges
G^{α}_{μ}, $\alpha = 1,2,\dots,8$	$(8,1,0)$ gluon
W^{a}_{μ}, $a = 1,2,3$	$(1,3,0)$
B_μ	$(1,1,0)$

will combine to $W^{\pm}, Z, \gamma$

$m = e, \mu, \tau$

Fields	Charges	
$l^m_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}_m$	$(1,2,-1/2)$	leptons $s=1/2$
e^m_R	$(1,1,-1)$	
$q^m_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}_m$	$(3,2,1/6)$	quarks $s=1/2$
u^m_R	$(3,1,2/3)$	
d^m_R	$(3,1,-1/3)$	

$m = 1,2,3$

u, c, t

d, s, b

Charges	
$(1, \underline{2}, -1/2)$	leptons $s = 1/2$
$(1, 1, -1)$	
$(3, \underline{2}, 1/6)$	quarks $s = 1/2$
$(3, 1, 2/3)$	
$(3, 1, -1/3)$	

Field	Charges	
$\Phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$	$(1, 2, -1/2)$	Higgs boson $s=0$

$+$ antiparticles (opposite charges)