Title: Lecture - Standard Model, PHYS 622

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Collection/Series: Standard Model (Elective), PHYS 622, January 6 - February 5, 2025

Subject: Particle Physics

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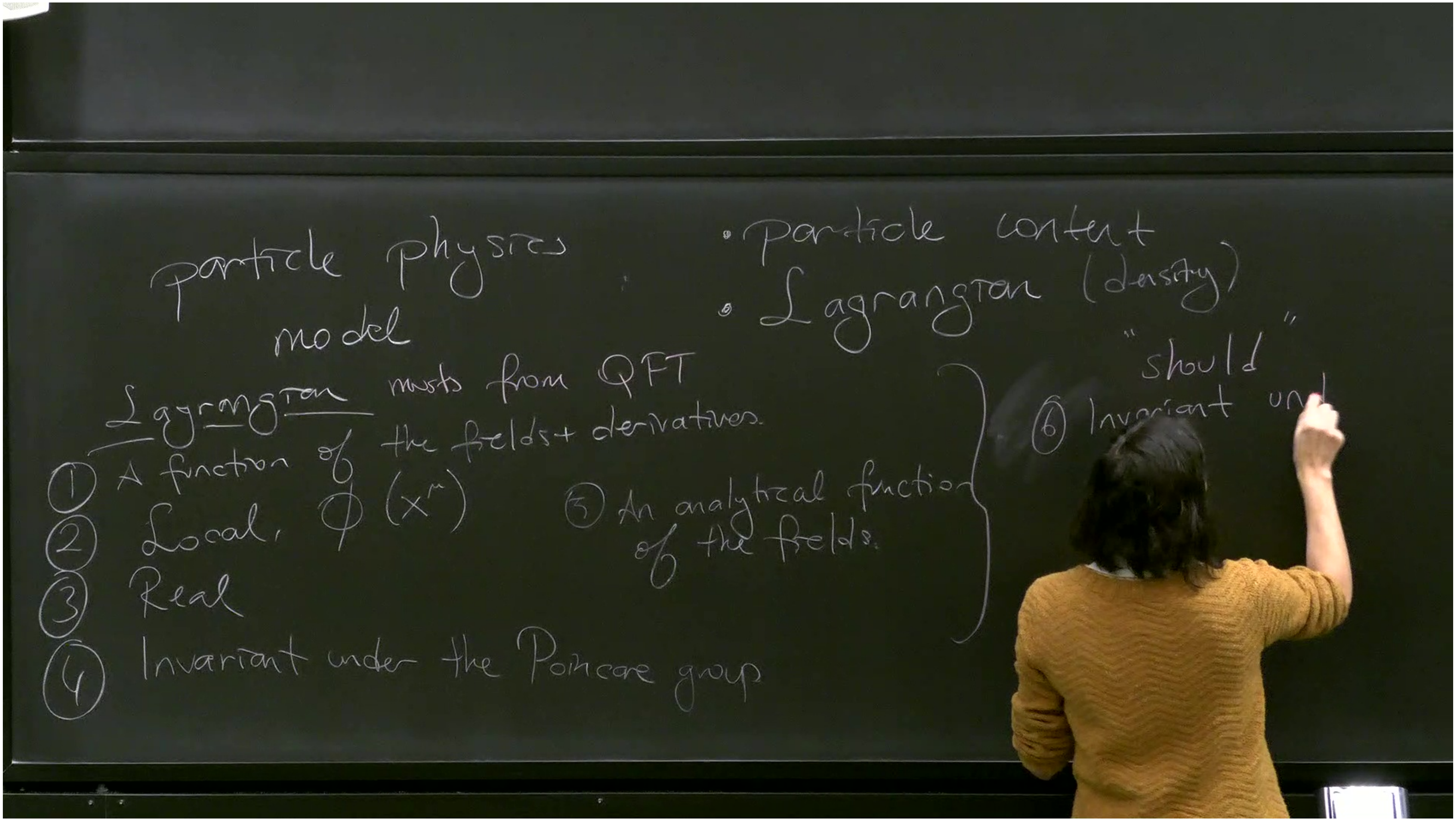
particle physics model :

- particle content
- Lagrangian


particle physics model :
• particle content
• Lagrangian (density)
Lagrangian musts from QFT
1 A function of the fields + derivatives.
2 Local, φ (x^μ)
3 Real
4 Invariant under the Poincare group
5 An analytical function of the fields.
"should"
6 Invariant und

• particle content
• $\mathcal{L}$agrangian (density)

"should"

ies.

lytical function
he fields.

⑥ Invariant under some symmetry transformations.

⑦ Includes all terms that are not forbidden by the symmetries

⑧ Renormalizable.

up

d "
under some symmetry transformations.
all terms that are not forbidden by the symmetries
ormalizable.

the action
$$S = \int d^4x \, \mathcal{L}$$
$\mathcal{L}$agrangian density

Notation:

$\varphi(x) \equiv \varphi$ real scalar

Φ complex scalar

ψ fermion $s = 1/2$

Scalars
(real)

$$\mathcal{L} = \underbrace{\frac{1}{2}(\partial_\mu \varphi)(\partial^\mu \varphi)}_{\text{kinetic term}} - \underbrace{\frac{m^2}{2}\varphi^2}_{\text{mass term}} - \underbrace{\frac{\eta}{2\sqrt{2}}\varphi^3 - \frac{\lambda}{4}\varphi^4}_{\text{self interactions}}$$

$[\mathcal{L}] = 4$ $\quad [m] = 1$

$[\partial_\mu] = 1$ $\quad [\eta] = 1$

$[\varphi] = 1$

$[\lambda] = 0$

$$\underbrace{\frac{\lambda}{4}\varphi^4}_{\text{tions}} \qquad + \cancel{\underbrace{\frac{\varphi^5}{\Lambda}}_{\text{non-renormalizable.}}}$$

$[\lambda] = 0$

Fermions $(s = 1/2)$

"chiral" : spin vs momentum direction

$$\psi_L = P_L \psi = \frac{1-\gamma_5}{2}\psi \quad : \text{left-handed}$$

$$\underbrace{\psi_R}_{\substack{\text{Weyl spinor} \\ \text{2-component}}} = P_R \psi = \frac{1+\gamma_5}{2}\underbrace{\psi}_{\substack{\text{4-component} \\ \text{Dirac spinor}}} \quad : \text{right-handed}$$

$$\psi_R^c = C \cdot \overline{\psi_R}^T$$

$$\psi_L^c = C \cdot \overline{\psi_L}^T$$

charge conjug

$[\psi] = \frac{3}{2}$

$[\lambda] = 0$

Fermions $(s = \frac{1}{2})$

"chiral" : spin vs momentum direction

$\psi_L = P_L \psi = \frac{1-\gamma_5}{2} \psi$: left-handed

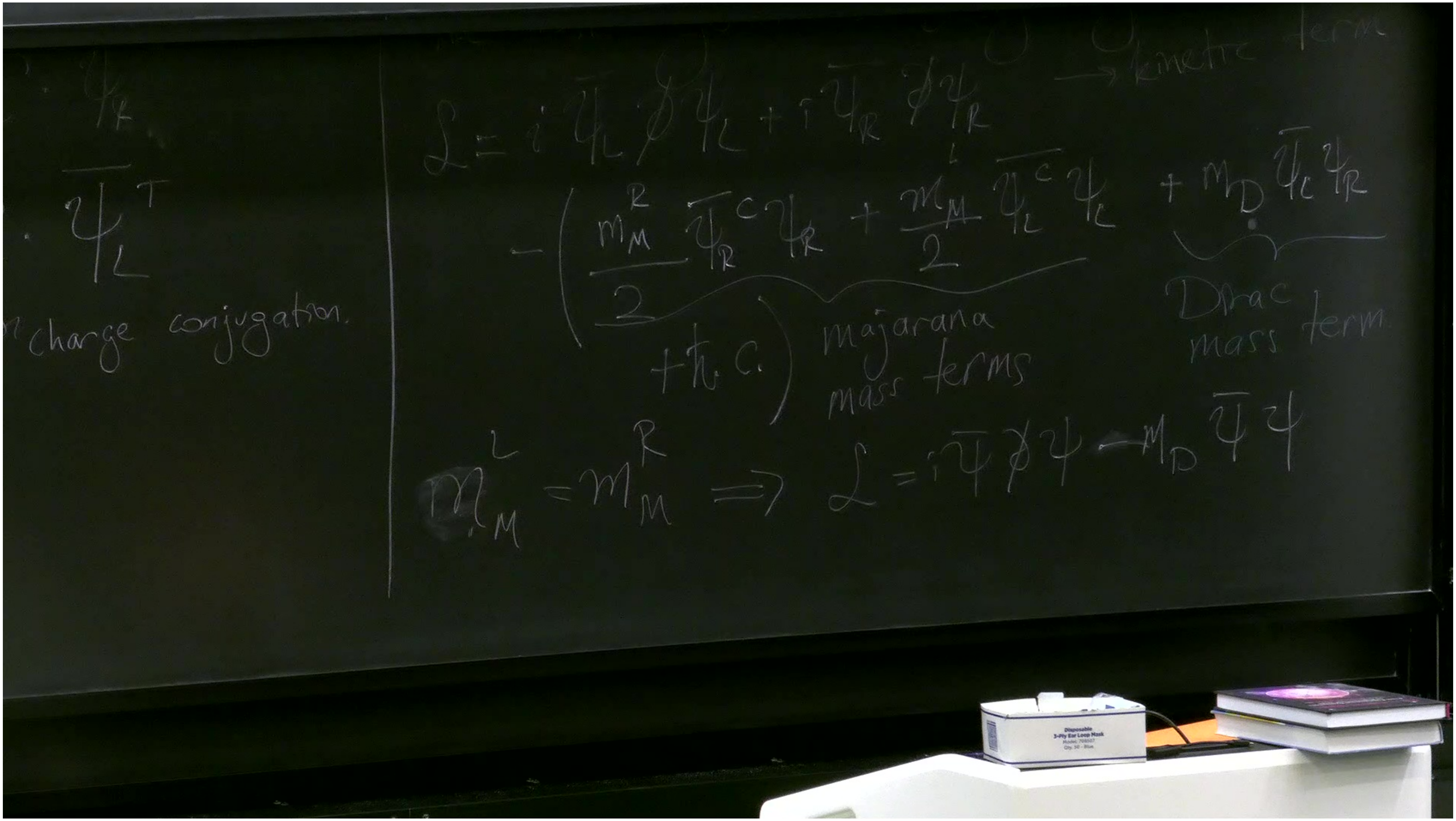
$\psi_R = P_R \psi = \frac{1+\gamma_5}{2} \psi$: right-handed

$\underbrace{\psi_R}_{\text{Weyl spinor, 2-component}}$ $\underbrace{\psi}_{\text{4-component Dirac spinor}}$

$\psi_R^c = C \cdot \overline{\psi_R}^T$

$\psi_L^c = C \cdot \overline{\psi_L}^T$

charge conjugation


ψ_R
$\overline{\psi_L}^T$
charge conjugation.
$\mathcal{L} = i\overline{\psi_L}\not{\partial}\psi_L + i\overline{\psi_R}\not{\partial}\psi_R$ → kinetic term
$-\left(\frac{m_M^R}{2}\overline{\psi_R^c}\psi_R + \frac{m_M^L}{2}\overline{\psi_L^c}\psi_L + h.c.\right) + m_D\overline{\psi_L}\psi_R$
majarana mass terms
Dirac mass term
$m_M^L = m_M^R \Rightarrow \mathcal{L} = i\overline{\Psi}\not{\partial}\Psi - m_D\overline{\Psi}\Psi$

spin vs momentum direction

$\psi = \frac{1-\gamma_5}{2}\psi$: left-handed

$\psi = \frac{1+\gamma_5}{2}\psi$: right-handed

4-component Dirac spinor

$\psi_R^c = C \cdot \psi_L$

$\psi_L^c = C \cdot \overline{\psi}_L^T$

charge conjugation

$[\psi] = 3/2$

$\frac{\overline{\psi}\psi\overline{\psi}\psi}{\Lambda_{EW}^2}$

$\mathcal{L} = i\,\overline{\psi}_L \ldots$

$-\left(\frac{m_M^R}{2} \ldots\right.$

$m_M^L = m_M^R$

Weyl spinor
2-component

Dirac spinor

Scalars + Fermions

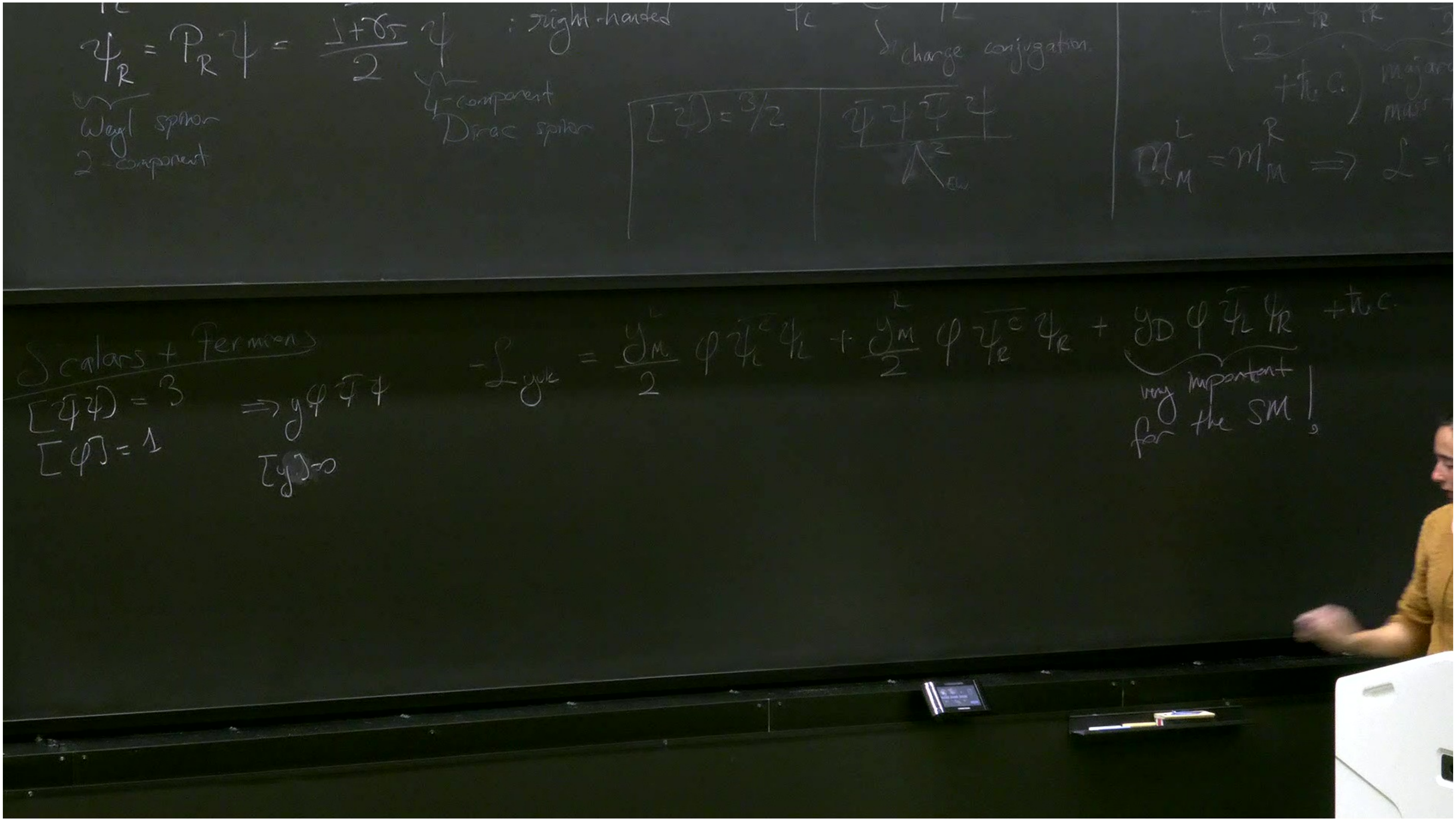
$[\bar{\psi}\psi] = 3$

$[\varphi] = 1$

$\Rightarrow y \varphi \bar{\psi} \psi$

$[y] = 0$

$$-\mathcal{L}_{yuk} = \frac{y_M^L}{2} \varphi \overline{\psi_L^c} \psi_L + \frac{y_M^R}{2} \varphi \overline{\psi_R^c}$$


: right-handed
Weyl spinor
2-component
4-component
Dirac spinor
charge conjugation
+h.c.
Scalars + Fermions
very important
for the SM!
+h.c.

Symmetries

- Abelian (commuting)
  - global
  - local
- non-Abelian (non-commuting)
  - global
  - local

Abelian - Global

$\varphi \to -\varphi$ if $\mathcal{L} \xrightarrow{\varphi \to -\varphi} \mathcal{L}$

Z_2

this will be a symmetry of your model.

$$\mathcal{L}(\varphi) = \frac{1}{2}(\partial_\mu \varphi)(\partial^\mu \varphi)$$

$(\partial_\mu \varphi)(\partial^\mu \varphi) - \frac{m^2}{2}\varphi^2 - \frac{\lambda}{4}\varphi^4$

- $\Phi = \frac{1}{\sqrt{2}}(\varphi_R + i\varphi_I)$

the most general Lagrangian

$(\partial_\mu \varphi)(\partial^\mu \varphi) - \frac{m^2}{2}\varphi^2 - \frac{\lambda}{4}\varphi^4$

- $\Phi = \frac{1}{\sqrt{2}}(\varphi_R + i\varphi_I)$

the most general Lagrangian before imposing any symmetries:

$$\mathcal{L} = \frac{1}{2}\delta_{ij}(\partial^\mu \varphi_i)(\partial_\mu \varphi_j) - \frac{m_{ij}^2}{2}\varphi_i\varphi_j - \frac{\eta_{ijk}}{6}\varphi_i\varphi_j\varphi_k - \frac{\lambda_{ijkl}}{4!}\varphi_i\varphi_j\varphi_k\varphi_l$$

$i,j,k,l = R, I$

$$\left(\begin{aligned} &\mathcal{L} = K - V \\ &V = \frac{m^2}{2}\varphi^2 + \frac{\eta}{?}\varphi^3 \\ &\quad + \lambda\varphi^4 \end{aligned}\right)$$

$\mathbb{Z}_2$

(this will be a symmetry of your model.)

Consider the cont. rotation:

$$\begin{pmatrix} \varphi_R \\ \varphi_I \end{pmatrix} \longrightarrow \mathcal{O} \begin{pmatrix} \varphi_R \\ \varphi_I \end{pmatrix} \qquad \mathcal{O} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$\rightsquigarrow SO(2)$

$$- \frac{\lambda_{ijk}}{6}\varphi_i\varphi_j\varphi_k - \frac{\lambda_{ijkl}}{4!}\varphi_i\varphi_j\varphi_k\varphi_l$$

$i, j, k, l = R, I$

$$\left(\begin{aligned} &\mathcal{L} = K - V \\ &V = \frac{m^2}{2}\varphi^2 + \frac{\eta}{3!}\varphi^3 + \lambda\varphi^4 \end{aligned} \right)$$

$$\mathcal{L}(\varphi_R, \varphi_I) \longrightarrow \mathcal{L}(\varphi_R, \varphi_I)$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi_R)(\partial^\mu \varphi_R) + \frac{1}{2}(\partial_\mu \varphi_I)(\partial^\mu \varphi_I) - \frac{m^2}{2}(\varphi_R\varphi_R + \varphi_I\varphi_I)$$

$$- \frac{\lambda}{4}\left(\varphi_R^4 + \varphi_I^4 + 2\varphi_R\varphi_R\varphi_I\varphi_I\right)$$

$\lambda_{ijkl} \equiv \lambda$

$m_{LL} = m_{RR} = m$

$\mathbb{Z}_2$

this will be a symmetry of your model.

Consider the cont. rotation: $SO(2)$

$$\begin{pmatrix} \varphi_R \\ \varphi_I \end{pmatrix} \longrightarrow \mathcal{O} \begin{pmatrix} \varphi_R \\ \varphi_I \end{pmatrix} \qquad \mathcal{O} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$\boxed{\mathcal{L} = (\partial_\mu \Phi)^\dagger (\partial^\mu \Phi) - m^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2}$$

$\Phi \to e^{i\theta} \Phi$

$\Phi^\dagger \to e^{-i\theta} \Phi^\dagger$

$U(1)$

$$-\frac{\mu_{ijk}}{6}\varphi_i\varphi_j\varphi_k - \frac{\lambda_{ijkl}}{4!}\varphi_i\varphi_j\varphi_k\varphi_l$$

$i, j, k, l = R, I$

$$\left(\begin{aligned}&\mathcal{L} = K - V\\ &V = \frac{m^2}{2}\varphi^2 + \frac{\mu}{3}\varphi^3 + \lambda\varphi^4\end{aligned}\right)$$

$\to e^{i\vartheta}\Phi$

$\to e^{-i\vartheta}\Phi^\dagger$

$\mathcal{U}(1)$

$$\mathcal{L}(\varphi_R, \varphi_I) \longrightarrow \mathcal{L}(\varphi_R, \varphi_I)$$

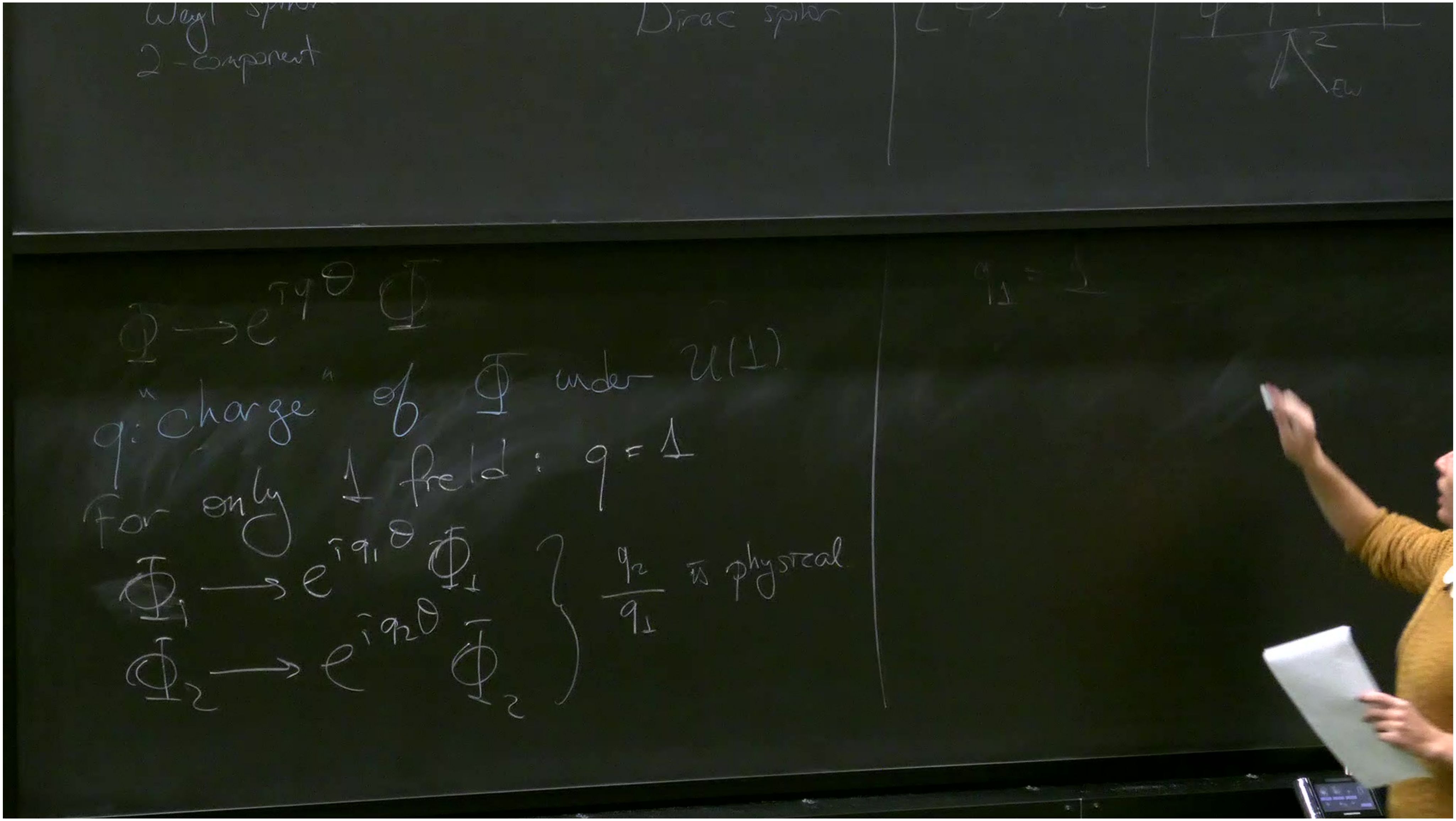
$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi_R)(\partial^\mu \varphi_R) + \frac{1}{2}(\partial_\mu \varphi_I)(\partial^\mu \varphi_I) - \frac{m^2}{2}(\varphi_R\varphi_R + \varphi_I\varphi_I)$$

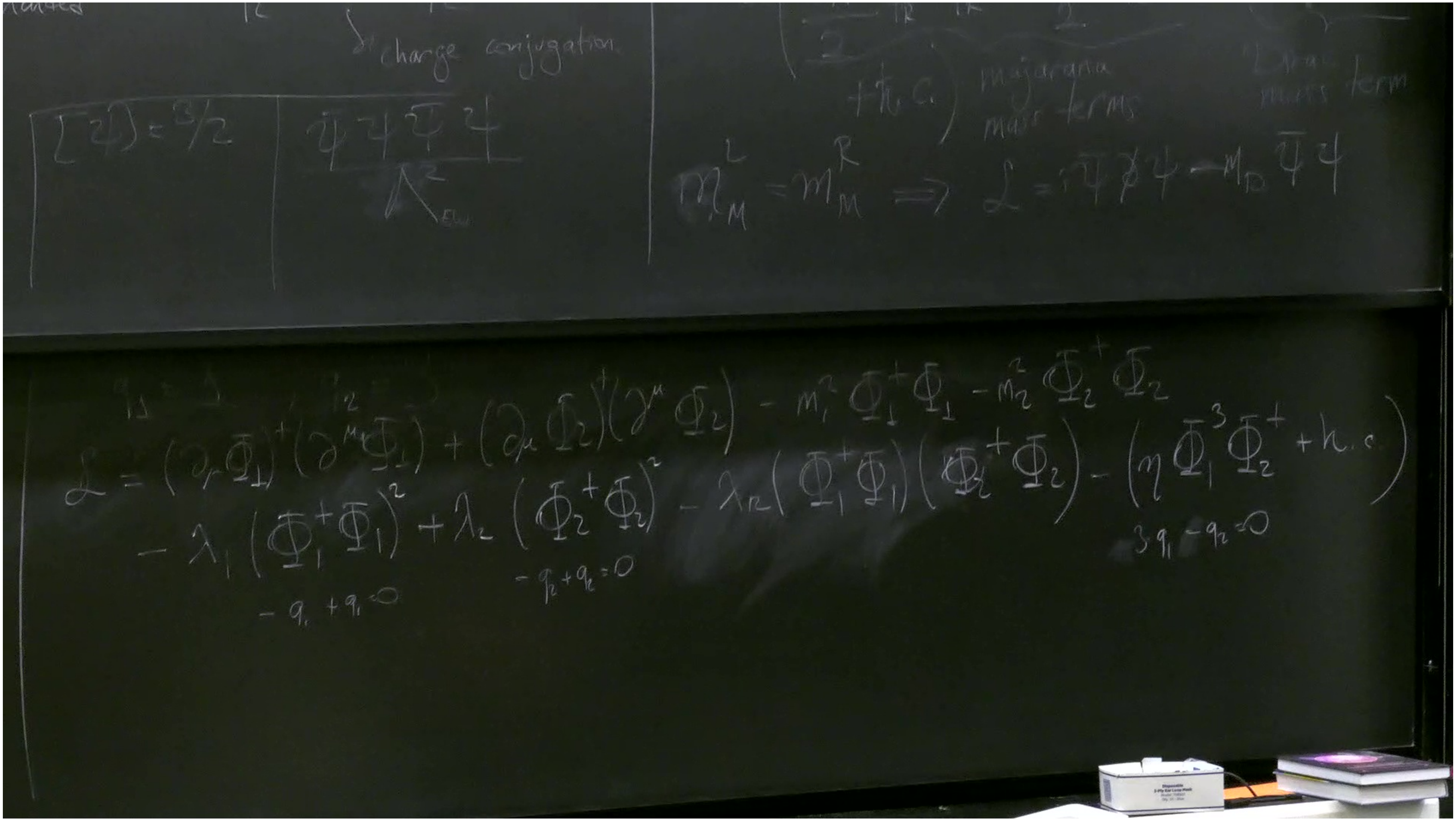
$$-\frac{\lambda}{4}\left(\varphi_R^4 + \varphi_I^4 + 2\varphi_R\varphi_R\varphi_I\varphi_I\right)$$

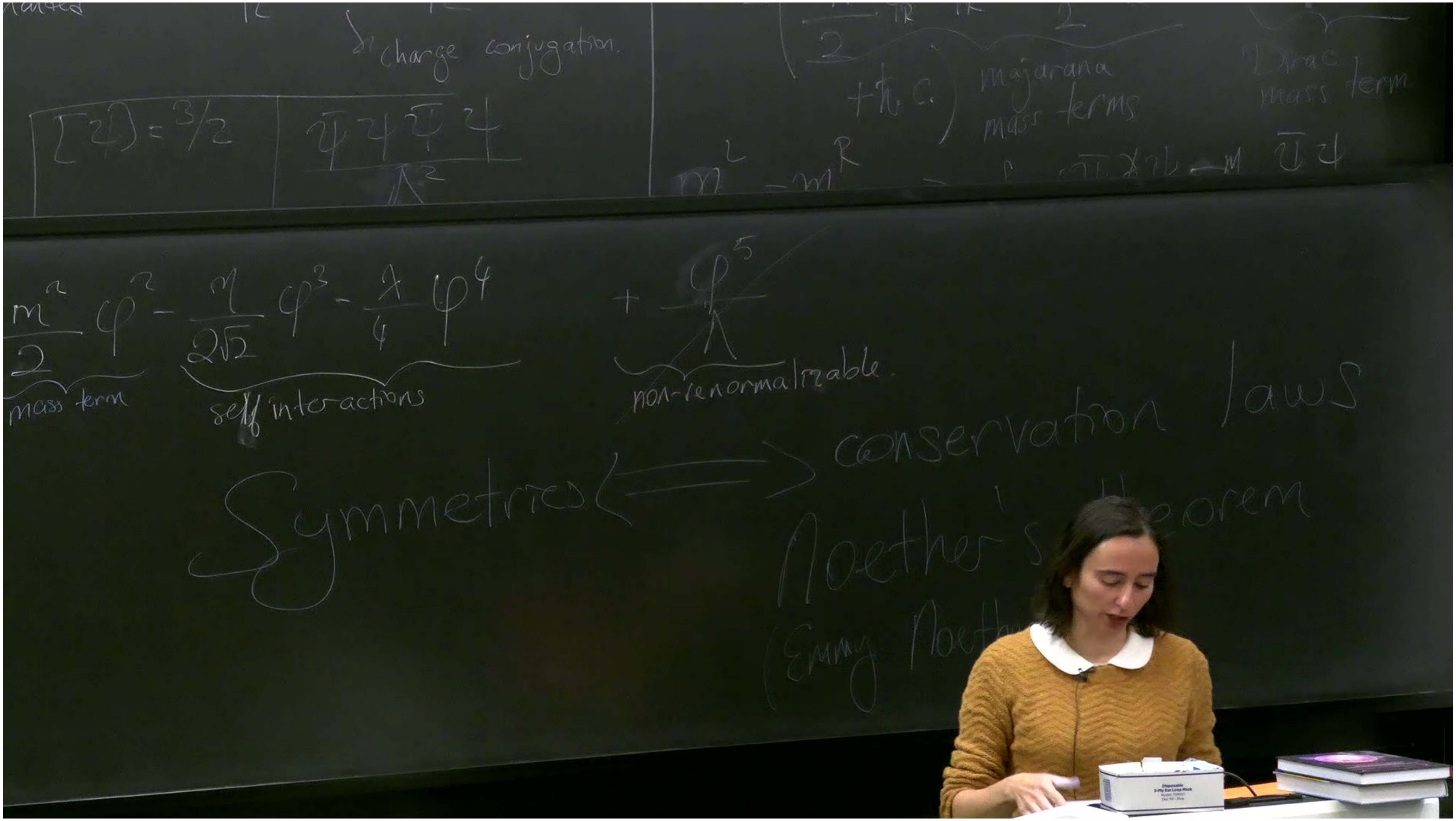
$\to \lambda_{ijkl} \equiv \lambda$

λ

$m_{LL} = m_{RR} = m$


2-component
Dirac spinor
$\Phi \to e^{iq\theta}\,\Phi$
q: "charge" of Φ under $U(1)$
For only 1 field: $q = 1$
$\Phi_1 \to e^{iq_1\theta}\,\Phi_1$
$\Phi_2 \to e^{iq_2\theta}\,\Phi_2$
$\frac{q_2}{q_1}$ is physical
$q_1 = 1$


charge conjugation
$[\Psi] = 3/2$
$\bar\Psi\Psi\bar\Psi\Psi$
Λ^2_{EW}
$+h.c.$
majorana mass terms
Dirac mass term
$m_M^L = m_M^R \Rightarrow \mathcal{L} = i\bar\Psi \partial\!\!\!/ \Psi - m_D \bar\Psi\Psi$
$\mathcal{L} = (\partial_\mu\Phi_1)^\dagger(\partial^\mu\Phi_1) + (\partial_\mu\Phi_2)^\dagger(\partial^\mu\Phi_2) - m_1^2\Phi_1^\dagger\Phi_1 - m_2^2\Phi_2^\dagger\Phi_2$
$- \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 - \lambda_{12}(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) - (\eta\Phi_1^3\Phi_2^\dagger + h.c.)$
$-q_1 + q_1 = 0$
$-q_2 + q_2 = 0$
$3q_1 - q_2 = 0$


charge conjugation
+h.c.
majarana mass terms
Dirac mass term
mass term
self interactions
non-renormalizable
Symmetries
conservation laws
Noether's theorem
(Emmy Noether