Title: Covariant Loop Quantum Gravity with a Cosmological Constant

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Abstract:

Covariant loop quantum gravity, commonly referred to as the spinfoam model, provides a regularization for the path integral formalism of quantum gravity. A 4-dimensional Lorentzian spinfoam model with a non-zero cosmological constant has been developed based on quantum SL(2,C) Chern-Simons theory on a graph-complement three-manifold, combined with loop quantum gravity techniques. In this talk, I will give an overview of this spinfoam model and highlight its inviting properties, namely (1) that it yields finite spinfoam amplitude for any spinfoam graph, (2) that it is consistent with general relativity with a non-zero cosmological constant at its classical regime and (3) that there exists a concrete, feasible and computable framework to calculate physical quantities and quantum corrections through stationary phase analysis. I will also discuss recent advancements in this spinfoam model and explore its potential applications.

Covariant Loop Quantum Gravity with a Cosmological Constant

Qiaoyin Pan

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Based on works with Muxin Han and Chen-Hung Hsiao

arXiv: 2310.04537, 2311.08587, 2401.14643, 2407.03242 and W.I.P.

@ Perimeter Institute —— December 2024

Why and what does covariant LQG (spinfoam model) do?

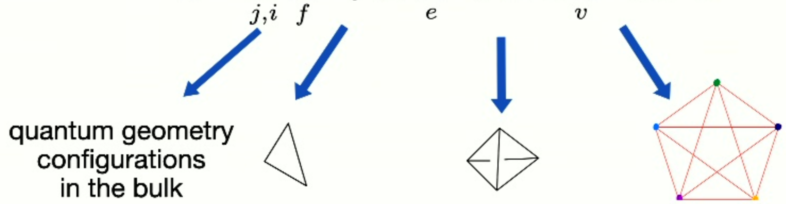
- The path integral formalism of quantum gravity is only formal and needs regularization

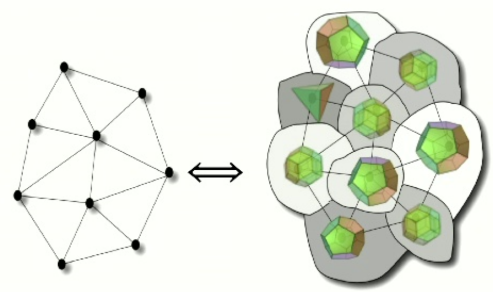
$$\int_{(h_{ab})_i}^{(h_{ab})_f} \mathcal{D}g_{\mu\nu}\, e^{i\int_{M_4} \mathrm{d}^4x\sqrt{-g}(R-2\Lambda)}$$

- A good regularized path integral formalism should
  - Given finite amplitude
  - Implement dynamics of quantum gravity $\Longrightarrow$ transition amplitude of boundary quantum states

- Spinfoam model provides a way of regularization such that
  - ☑ **Non-perturbative**: constructed directly at the quantum level, no base on perturbation theory
  - ☑ **Background-independent**: quantum geometry states from canonical LQG
  - ☑ **Triangulation-dependent**: $\mathbf{T}_4$ (cured by *Group Field Theory* (GFT))
  - ☑ **"Locally" defined**: local amplitude ansatz

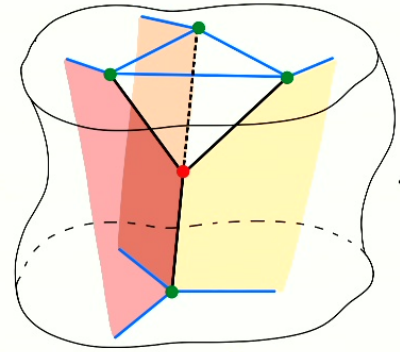
$$\mathcal{A}_{\mathbf{T}_4} = \sum_{j,i}\prod_f \mathcal{A}_f(j)\prod_e \mathcal{A}_e(j,i)\prod_v \mathcal{A}_v(j,i)$$





Why introducing a Λ?

- When the boundary $\partial\mathcal{M}_4 = \Sigma_i \cup \Sigma_f$, spinfoam amplitude = transition amplitude of LQG states
  - Spin network states (Γ, j_l, i_n) : encode quantum 3-geometries
  - Dynamics is captured by the vertex amplitude $\mathcal{A}_v$: vertex v dual to 4d event



- Spinfoam model with $\Lambda = 0$ (EPRL model) [Engle, Livine, Pereira, Rovelli '07]
  - semi-classically consistent with Einstein gravity

$$\mathcal{A}_v(\lambda j) \xrightarrow{\lambda\to\infty} \frac{1}{\lambda^{12}}\left(\mathcal{N}_+ e^{i\lambda S_{\text{Regge}}} + \mathcal{N}_- e^{-i\lambda S_{\text{Regge}}}\right)$$

spins $j \in \mathbb{N}/2$: quanta of area

$$S_{\text{Regge}} = \sum_{a<b} \mathfrak{a}_{ab}\Theta_{ab}$$

Regge action: discretized gravity

  - but has **divergence** (when area spectrum $\gg \ell_{\text{pl}}^2$) $\Leftarrow \sum_{j=0}^{\infty}$

- This motivates us to further regularize the spinfoam model — truncate spins — introduce a nonzero Λ (hinted by 3D spinfoam) [Turaev, Viro '92]

***The focus of this talk*:**

A **finite** Lorentzian 4D spinfoam model (SF) with $\Lambda \neq 0$

Plan of this talk

- **Amplitude construction in the 4D SF with $\Lambda \neq 0$**
  - The idea
  - From path integral of gravity to local topological field theory
  - From local topological field theory to SF amplitude
- **Properties of the SF model**
  - Finiteness — melonic SF amplitude
  - Consistent with GR — critical point geometry
  - Computable — critical point reconstruction program
  - Go beyond triangulation dependence
- **Outlook**

Plan of this talk

- **Amplitude construction in the 4D SF with $\Lambda \neq 0$**
  - The idea

First-order gravity action

- First-order action of Einstein gravity with Holst term $\eta = \text{diag}(-1,1,1,1)$

$$S_{\text{Holst}}[e,\mathcal{A}] = \frac{1}{2}\int_{M_4}\left[\epsilon^{IJKL} e_I \wedge e_J \wedge \mathcal{F}_{KL}(\mathcal{A}) - \frac{1}{\gamma}\epsilon^{IJKL}\left(\star(e\wedge e)\right)_{IJ} \wedge \mathcal{F}_{KL}(\mathcal{A})\right] \quad 8\pi G = 1$$

e : tetrad, $\mathfrak{sl}(2,\mathbb{C})$ one-form $\qquad$ $\mathcal{A}$: connection, $\mathfrak{sl}(2,\mathbb{C})$ one-form

$\gamma \in \mathbb{R}$: Barbero-Immirzi parameter $\qquad$ $\star$: Hodge operator

- GR can be written as a constrained BF theory

Plebanski action: $S[B,\mathcal{A}] = \int_{M_4} \text{Tr}\left[\left(\star B + \frac{1}{\gamma}B\right)\wedge \mathcal{F}(\mathcal{A})\right] + \varphi_{IJKL} B^{IJ} \wedge B^{KL}$ $\qquad$ $\text{Tr}(XY) := X^{IJ}Y_{IJ}$

B : $\mathfrak{sl}(2,\mathbb{C})$ 2-form

Lagrange multiplier φ_{IJKL} satisfying $\epsilon^{IJKL}\varphi_{IJKL} = 0 \longrightarrow$ implement the **simplicity constraints**

$$\frac{\delta S}{\delta \varphi} = 0 \quad \Longrightarrow \quad \epsilon^{\mu\nu\rho\sigma} B^{IJ}_{\mu\nu} B^{KL}_{\rho\sigma} = V \epsilon^{IJKL} \quad \Longrightarrow \quad B^{IJ} = \pm e^I \wedge e^J$$

Towards 4D SF with Λ

- **Starting point — Plebanski-Holst formulation** of 4D gravity: BF theory + simplicity constraint

$$S_{\mathsf{BF}} = -\frac{1}{2}\int_{M_4} \mathrm{Tr}\left[\left(\star B + \frac{1}{\gamma}B\right)\wedge \mathcal{F}(\mathcal{A})\right] - \frac{|\Lambda|}{12}\int_{M_4} \mathrm{Tr}\left[\left(\star B + \frac{1}{\gamma}B\right)\wedge B\right]$$

$B : \mathfrak{sl}(2,\mathbb{C})$ 2-form; $\mathcal{A} : \mathfrak{sl}(2,\mathbb{C})$ connection; $\gamma \in \mathbb{R}$: Barbero-Immirzi parameter; $\star$: Hodge operator

simplicity constraint: $B = \mathrm{sgn}(\Lambda)\, e\wedge e$

- **Towards SF:** step 1 — Construct the **discretized** path integral for the BF theory $\longrightarrow$ TQFT

  step 2 — Quantize the simplicity constraint and impose it to the TQFT

- **Construct the Lorentzian path integral:** integrating out B field $\xrightarrow{\text{Gaussian integral}} \mathcal{F}[\mathcal{A}] = \frac{|\Lambda|}{3}B$

$$\int \mathrm{d}\mathcal{A}\int \mathrm{d}B\, e^{iS_{\mathsf{BF}}} = \int \mathrm{d}\mathcal{A}\, e^{\frac{3i}{4|\Lambda|}\int_{M_4}\mathrm{Tr}\left[\left(\star+\frac{1}{\gamma}\right)\mathcal{F}\wedge\mathcal{F}\right]}$$

- $\xrightarrow{M_4 \textbf{ trivial topo.}}$ $\mathrm{SL}(2,\mathbb{C})$ **Chern-Simons theory with complex coupling constant on the boundary**

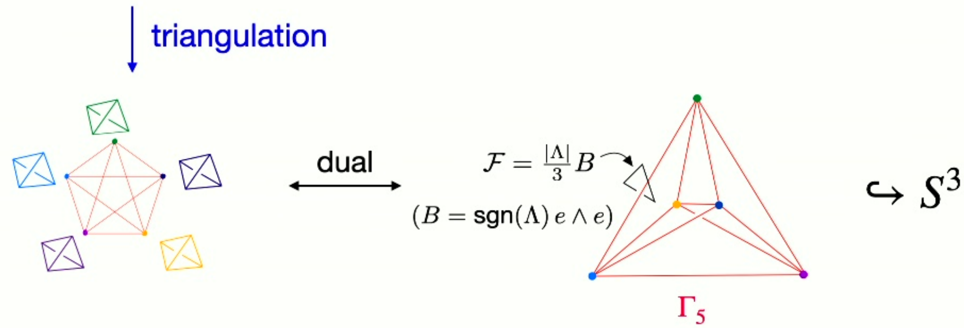
$$\frac{t}{8\pi}\int_{\partial M_4}\mathrm{Tr}\left(A\wedge \mathrm{d}A + \frac{2}{3}A\wedge A\wedge A\right) + \frac{t}{8\pi}\int_{\partial M_4}\mathrm{Tr}\left(\overline{A}\wedge \mathrm{d}\overline{A} + \frac{2}{3}\overline{A}\wedge\overline{A}\wedge\overline{A}\right)$$

$$t = k(1+i\gamma), \quad k = \frac{12\pi}{|\Lambda|\ell_{\mathrm{p}}^2\gamma} \in \mathbb{Z}_+ \implies \text{gauge invariant}$$

Towards 4D SF with Λ — cont.

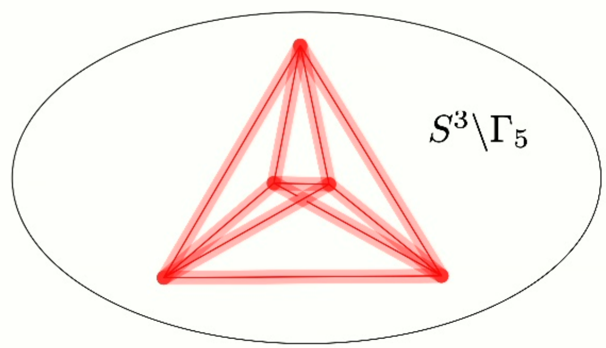
- CS theory on ∂M_4: $\frac{t}{8\pi}\int_{\partial M_4}\text{Tr}\left(A\wedge \text{d}A+\frac{2}{3}A\wedge A\wedge A\right)+\frac{t}{8\pi}\int_{\partial M_4}\text{Tr}\left(\overline{A}\wedge \text{d}\overline{A}+\frac{2}{3}\overline{A}\wedge\overline{A}\wedge\overline{A}\right)$

- Vertex amplitude: $M_4 = B^4, \partial M_4 = S^3$



- Curvatures are **line defects** on $S^3 \longrightarrow$ consider CS theory on the **graph complement**

  CS theory on $S^3\backslash\Gamma_5 \longrightarrow$ solution space: moduli space $\mathcal{M}_{\text{flat}}\left(S^3\backslash\Gamma_5, \text{SL}(2,\mathbb{C})\right)$ of $\text{SL}(2,\mathbb{C})$ flat connections



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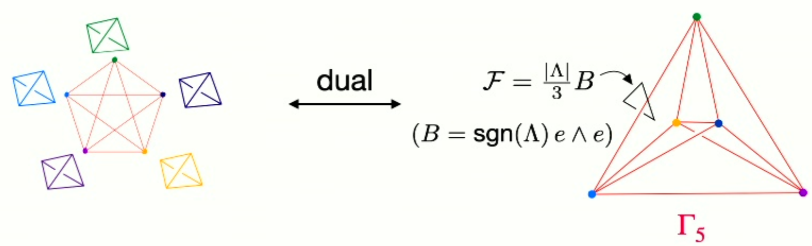
Towards 4D SF with Λ — cont.

- CS theory on ∂M_4: $\frac{t}{8\pi}\int_{\partial M_4} \mathrm{Tr}\left(A \wedge \mathrm{d}A + \frac{2}{3} A \wedge A \wedge A\right) + \frac{t}{8\pi}\int_{\partial M_4} \mathrm{Tr}\left(\overline{A} \wedge \mathrm{d}\overline{A} + \frac{2}{3}\overline{A} \wedge \overline{A} \wedge \overline{A}\right)$
- Vertex amplitude: $M_4 = B^4, \partial M_4 = S^3$

triangulation



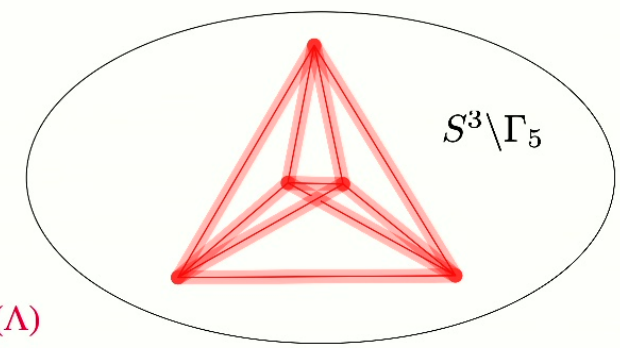
$\hookrightarrow S^3$

- Curvatures are **line defects** on $S^3 \longrightarrow$ consider CS theory on the **graph complement**

CS theory on $S^3 \backslash \Gamma_5 \longrightarrow$ solution space: moduli space $\mathcal{M}_{\text{flat}}\left(S^3 \backslash \Gamma_5, \mathrm{SL}(2,\mathbb{C})\right)$ of $\mathrm{SL}(2,\mathbb{C})$ flat connections

> 4D quantum gravity with $\Lambda \neq 0$
>
> = complex CS theory on boundary\graph
>
> + **simplicity constraints** on the graph $\mathcal{F} = \frac{\Lambda}{3} e \wedge e$

encode $\mathrm{sgn}(\Lambda)$



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Plan of this talk

- **Amplitude construction in the 4D SF with $\Lambda \neq 0$**
  - From path integral of gravity to local topological field theory

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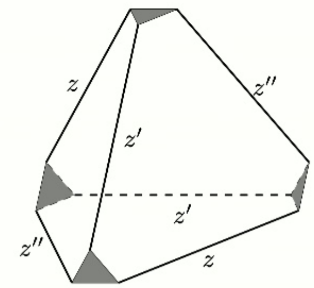
CS partition function on $S^3\backslash\Gamma_5$

- **Step 1**: CS partition function on $S^3\backslash\Gamma_5$
  - Discretization of $S^3\backslash\Gamma_5$ is composed of 20 ideal tetrahedra $\triangle$'s
  - CS partition function on one $\triangle$: **quantum dilogarithm function**



ideal tetrahedron
(vertices truncated)

$$\Psi_\triangle(z,\tilde{z}) = \prod_{j=0}^{\infty} \frac{1-\tilde{q}^{j+1}\tilde{z}^{-1}}{1-q^{-j}z^{-1}} \quad \text{(meromorphic)} \quad \left| \begin{array}{l} q = \exp\left(\frac{4\pi i}{t}\right) \\ \tilde{q} = \exp\left(\frac{4\pi i}{\bar{t}}\right) \end{array} \right.$$

CS phase space coordinates on $\triangle$:

$$z(\mu,j) = \exp\left[\tfrac{2\pi i}{k}(-ib\mu - 2j)\right] \qquad b = \sqrt{\tfrac{1-i\gamma}{1+i\gamma}}$$

$$\tilde{z}(\mu,j) = \exp\left[\tfrac{2\pi i}{k}\left(-ib^{-1}\mu + 2j\right)\right] \qquad k = \tfrac{12\pi}{|\Lambda|\ell_{\rm P}^2\gamma}$$

$$\left(\mu \in \mathbb{R}\ ,\ \text{spin}\ j \in \mathbb{Z}/2\right)$$

The coordinates are periodic in j: $z(j) = z(j+\mathbb{Z}k/2)$, $\tilde{z}(j) = \tilde{z}(j+\mathbb{Z}k/2)$

$$j = 0, \tfrac{1}{2}, \cdots, \tfrac{k-1}{2}$$

Truncation in spin **by construction**, implied by Λ

[Faddeev '95, Kashaev '96, Dimofte, Gaiotto, Gukov '14-15]

CS partition function on $S^3\backslash\Gamma_5$ — cont.

- Gluing ideal tetrahedra $\longrightarrow$ gluing constraints + unitary transformations

- **Result:** CS partition function $\mathcal{Z}_{S^3\backslash\Gamma_5}$

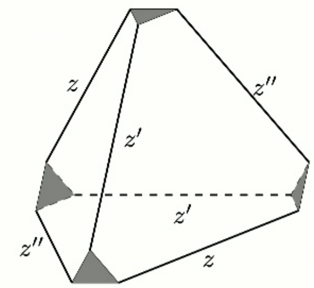
$$= \text{Unitary transformations} \cdot \text{gluing constraints} \cdot \prod_{i=1}^{20} \Psi_\triangle(i)$$

$$= \text{finite sum of convergent state integral}$$

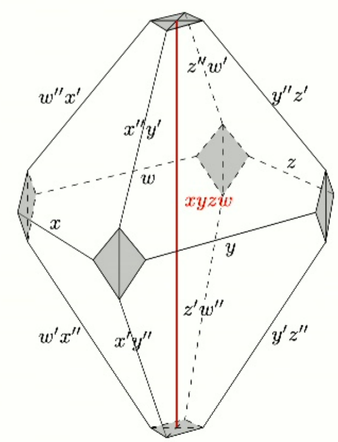
Bounded!

$$\mathcal{Z}_{S^3\backslash\Gamma_5}(\vec{\mu} \mid \vec{j}) = \frac{4i}{k^{15}} \sum_{2\vec{l}\in(\mathbb{Z}/k\mathbb{Z})^{15}} \int_{\mathcal{C}} \mathrm{d}^{15}\vec{\nu}\, e^{S_0} \prod_{i=1}^{20} \Psi_\triangle(i)$$

$$S_0 = \frac{\pi i}{k}\left[-2\left(\vec{\mu} - \frac{iQ}{2}\vec{t}\right)\cdot\vec{\nu} + 8\vec{j}\cdot\vec{l} - \vec{\nu}\cdot\mathbf{AB}^\top\cdot\vec{\nu} + 4(k+1)\vec{l}\cdot\mathbf{AB}^\top\cdot\vec{l}\right]$$



ideal tetrahedron (vertices truncated)



ideal octahedron

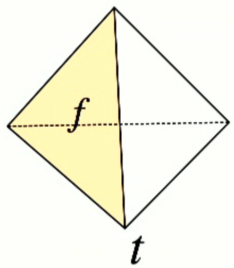
simplicity constraints and vertex amplitude

- **Step 2** towards vertex amplitude: impose the **simplicity constraints** on Γ_5

2-form constraint: $B = \text{sgn}(\Lambda)\, e \wedge e \xrightarrow{\text{discretize}} B_f^{IJ}(t) = \text{sgn}(\Lambda)\, e^I(t) \wedge e^J(t)$

discrete B-field on f: $B_f^{IJ}(t) := \int_f B^{IJ}(t)$

$e^I(t) \in$ Cartesian coordinate patch covering t



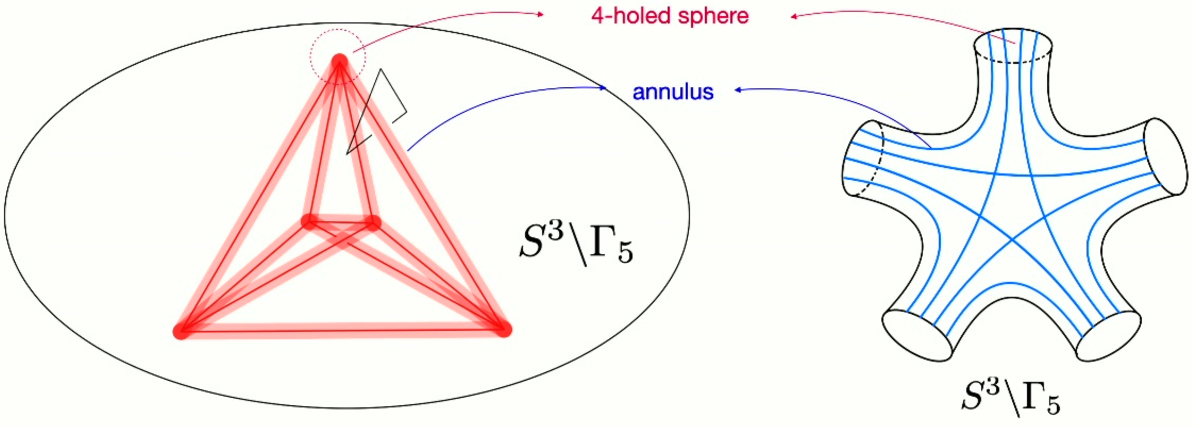
- Improve this constraint: observe that a tetrahedron is a 3D object

$$\exists\, N^I \in \mathbb{R}^{1,3} \quad \text{such that } N^I \perp f \in t\,, \quad \forall\, f \in t$$

Linear constraint: $\exists\, N^I \in \mathbb{R}^{1,3}$ such that $N^I\, B_{IJ} = 0\,, \;\forall\, f \in t$

$\Longleftrightarrow$ restrict the gauge group SL$(2,\mathbb{C})$ to SU(2) locally at each tetrahedron

simplicity constraints and vertex amplitude — cont.



$\mathcal{Z}_{S^3\backslash\Gamma_5}(\vec{\mu} \mid \vec{j}) \Longrightarrow (\vec{\mu}, \vec{j})$ are **localized** coordinates

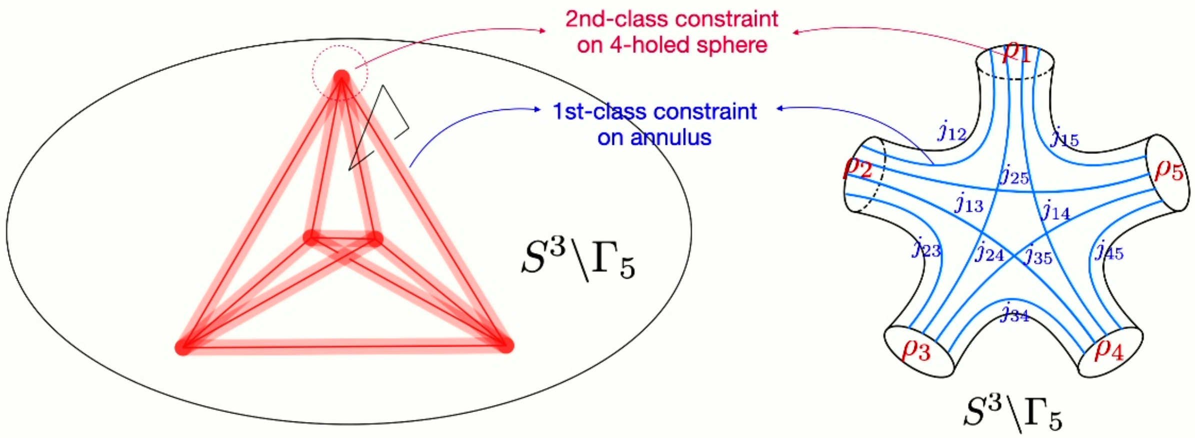
Linear constraint: $\exists\, N^I \in \mathbb{R}^{1,3}$ such that $N^I B_{IJ} = 0\,,\quad \forall$ tetra

$$\xrightarrow{\mathcal{F}=\frac{|\Lambda|}{3}B} N^I \mathcal{F}_{IJ} = 0, \quad \forall \text{ 4-holed sphere } \Sigma_{0,4}$$

$$\mathcal{M}_{\text{flat}}(\Sigma_{0,4}, \mathsf{SL}(2,\mathbb{C})) \xrightarrow{\text{simplicity constraint}} \mathcal{M}_{\text{flat}}(\Sigma_{0,4}, \mathsf{SU}(2))$$

[Han '21, Han, QP '23]

simplicity constraints and vertex amplitude — cont.



$$\mathcal{M}_{\text{flat}}(\Sigma_{0,4},\text{SL}(2,\mathbb{C})) \xrightarrow{\text{simplicity constraint}} \mathcal{M}_{\text{flat}}(\Sigma_{0,4},\text{SU}(2))$$

First-class constraints

Impose strongly

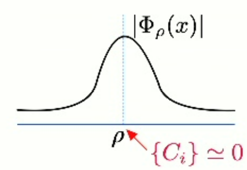
$$\begin{aligned} \mathcal{Z}_{S^3\backslash\Gamma_5} &= \mathcal{Z}_{S^3\backslash\Gamma_5}\left(\{\lambda_{ab}\}_{a<b};\cdots\right), \\ \lambda_{ab} &= \exp\left(\frac{2\pi i}{k}j_{ab}\right) = \exp\left(\frac{i|\Lambda|}{6}\mathfrak{a}_{ab}\right) \in \text{U}(1) \end{aligned}$$

Fix the triangle **area** by spin

Second-class constraints

Impose weakly

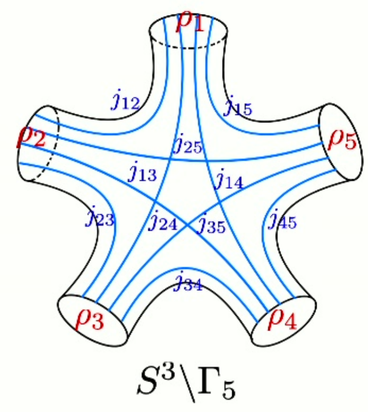
Couple with coherent state Ψ_ρ



Restrict ρ to label the tetrahedron **shape**

[Engle, Livine, Pereira, Rovelli, Freidel, Krasnov, Asante, Dittrich, Haggard, Padua-Arguelles, Han, QP, *etc.*]

Vertex amplitude — Result



- The **vertex amplitude** is defined by the inner product of the CS partition function with 5 coherent states

$$\mathcal{A}_v(j,\rho) = \langle \Psi_\rho \mid \mathcal{Z}^j_{S^3\backslash\Gamma_5}\rangle$$

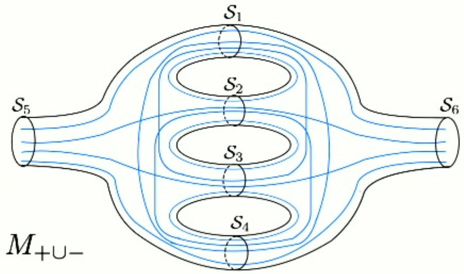
  - **Finite** by construction
  - Large k-limit: oscillatory action $\mathcal{A}_v \overset{k\to\infty}{\sim} \int [\mathrm{d}\vec{X}] e^{ikS(\vec{X})}$ $k = \frac{12\pi}{|\Lambda|\ell_{\rm p}^2\gamma}$
  - Stationary phase analysis $\Longrightarrow$ reproduce **4D Regge action with constant curvature** [Haggard, Han, Kaminski, Riello '14-15; Han '21]

$$\mathcal{A}_v = \left(\mathcal{N}_+ e^{iS_{\text{Regge},\Lambda}} + \mathcal{N}_- e^{-iS_{\text{Regge},\Lambda}}\right)[1 + O(1/k)]$$

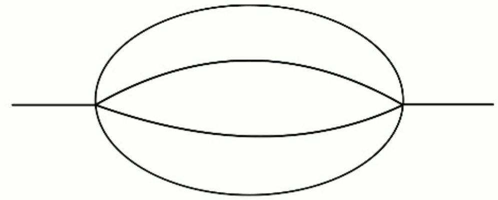
Plan of this talk

- **Properties of the SF model**
  - Finiteness — melonic SF amplitude

SF amplitude for a melon graph

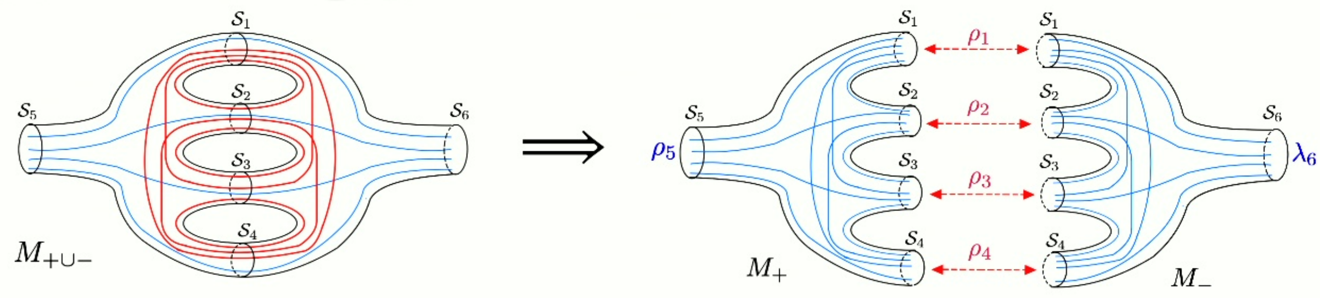


$\Longleftrightarrow$



melonic spinfoam graph

SF amplitude for a melon graph



finite sum — integral over compact space — bounded amplitude

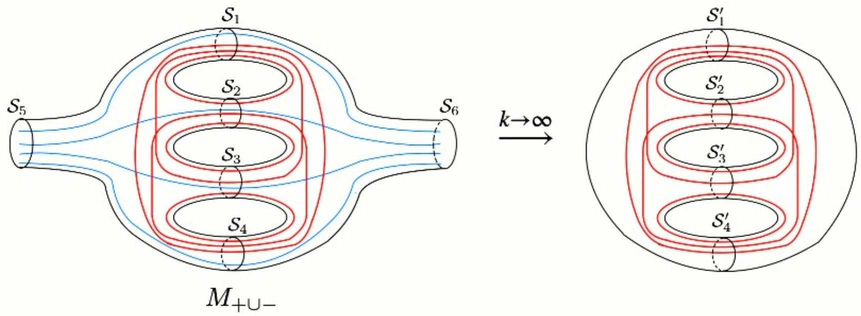
$$\mathcal{A}_{\text{melon}}\left(\{j_b\},\rho_5,\lambda_6\right)=\sum_{\{j_f\}=0}^{(k-1)/2}\prod_{f=1}^{6}[2j_f+1]_{\mathfrak{q}}\int[\mathbf{d}\rho_e]^{\times 4}\,\mathcal{A}_{v,+}\left(\{j_f\},\{j_b\},\{\rho_e\},\rho_5\right)\mathcal{A}_{v,-}\left(\{j_f\},\{j_b\},\{\rho_e\},\lambda_6\right)$$

Finite!

quantum dimension with $\mathfrak{q}=e^{\frac{2\pi i}{k}}$ $\quad k=\frac{12\pi}{|\Lambda|\ell_{\mathrm{P}}^2\gamma}$

- Face amplitude — consistent with EPRL face amplitude $[2j_f+1]_{\mathfrak{q}}\xrightarrow{\Lambda\to 0}2j_f+1$
- The **finiteness** can be generalized to **any** spinfoam graph using similar mechanism [Han, QP '23]

SF amplitude for a melon graph — cont.



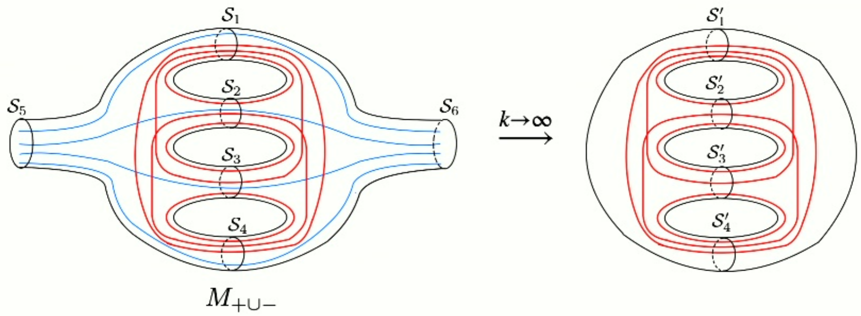
- We fix the boundary data $(\{j_b\}, \rho_5, \lambda_6)$ and consider the $\Lambda \to 0\,(k \to \infty)$ for the melonic amplitude $k = \frac{12\pi}{|\Lambda|\ell_{\mathrm{P}}^2\gamma}$
- Oscillatory action $\mathcal{A}_{\text{melon}} \overset{k\to\infty}{\sim} \int [\mathrm{d}\vec{X}] e^{ikS(\vec{X})} \Longrightarrow$ Stationary phase analysis

  $\Longrightarrow$Scaling behaviour (lower bound): $\mathcal{A}_{\text{melon}} \sim k^{21}$

Finite!

[Han, QP '23]

SF amplitude for a melon graph — cont.



- We fix the boundary data $(\{j_b\}, \rho_5, \lambda_6)$ and consider the $\Lambda \to 0\,(k \to \infty)$ for the melonic amplitude $k = \frac{12\pi}{|\Lambda|\ell_{\rm p}^2\gamma}$
- Oscillatory action $\mathcal{A}_{\text{melon}} \overset{k\to\infty}{\sim} \int [\mathrm{d}\vec{X}] e^{ikS(\vec{X})} \Longrightarrow$ Stationary phase analysis

  $\Longrightarrow$Scaling behaviour (lower bound): $\mathcal{A}_{\text{melon}} \sim k^{21}$ **Finite!**

- Comparison with the melonic radiative correction in the EPRL model
  - Introduce a cut-off for representation label **by hand** $\sum_{j=0}^{\infty} \to \sum_{j=0}^{j_{\text{max}}}$ and consider large j_{max} **Infinite!**
  - Divergent behaviour (numerical result): $\mathcal{A}_{\text{melon}} \sim j_{\text{max}}$ [Frisoni, Gozzini, Vidotto '22]

[Han, QP '23]

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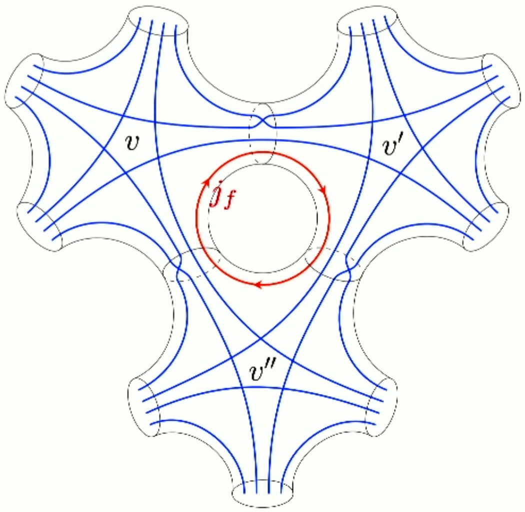
Critical point geometry

- **Critical point geometry:**

Vertex amplitude: **constantly curved 4-simplex** geometry

gluing 4-simplices by identifying boundary **constantly curved tetrahedra**

What if internal triangles form?



Δ_3 three-manifold

[Han, QP '24]

Critical point geometry

- **Critical point geometry:**

Vertex amplitude: **constantly curved 4-simplex** geometry

gluing 4-simplices by identifying boundary **constantly curved tetrahedra**

What if internal triangles form?

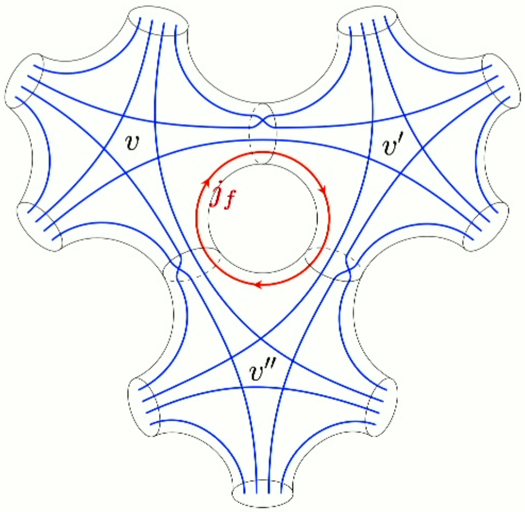
Stationary phase analysis for internal spin j_f:

$\Longrightarrow$ critical deficit angle

$$\varepsilon_f \equiv \sum_{v \in f} \Theta_f = 4\pi N_f/\gamma, \quad N_f \in \mathbb{Z} \quad \xrightarrow{N_f=0} \quad 0$$

$\Longrightarrow$ 4D bulk is smoothly dS/AdS $\begin{cases} \text{every 4-simplex is constantly curved} \\ \text{zero deficit angle} \end{cases}$

Valid for any 4-complex with internal triangle(s)



Δ_3 three-manifold

[Han, QP '24]

Critical point geometry — cont.

- **Critical point geometry:**
  - Every **4-simplex** is **constantly curved**
  - 4-simplices are glued by identifying boundary **constantly curved tetrahedra**
  - **Vanishing deficit angle** hinged by each internal triangle $(\text{mod } 4\pi\mathbb{Z}/\gamma)$

- in the **semiclassical** regime $\Longrightarrow$ the critical point of the spinfoam amplitude describes a **4D dS spacetime** (when $\Lambda > 0$) or a **4D AdS spacetime** (when $\Lambda < 0$) — **"(A)dS-ness property"**

[Han, QP '24]

Critical point geometry — cont.

- **Critical point geometry:** $\longrightarrow$ real critical point

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  - 4-simplices are glued by identifying boundary **constantly curved tetrahedra**
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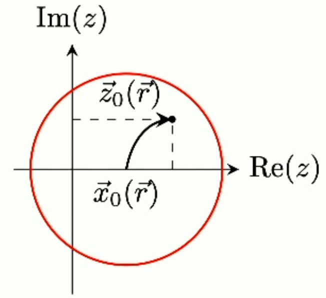


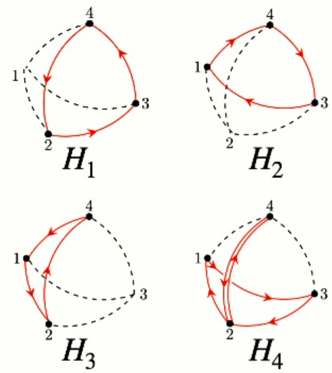
- **But NOT (A)dS-ness problem!**
- Consider complex critical point — Hörmander's theorem
  - **Non-(A)dS geometries** are captured by the complex critical points
  - Similar situation happens in the EPRL model — "flatness property" and the effective spinfoam model [Asante, Dittrich, Haggard '20, Han, Huang, Liu, Qu '21]

[Han, QP '24]

Plan of this talk

- **Properties of the SF model**
  - Consistent with GR — critical point geometry
  - Computable — critical point reconstruction program

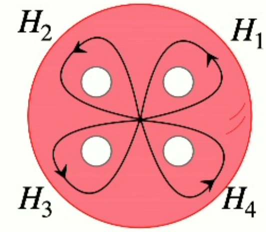
How does a 3D theory describe 4D geometry?



$\pi_1(\text{sk}_1(\text{3-simplex})) \xrightarrow{\text{isomorphism}} \pi_1(S^2\backslash 4 \text{ points})$

$\omega_{\text{spin}} \searrow \qquad \swarrow \omega_{\text{flat}}$

$$\{H_1, H_2, H_3, H_4 \in \text{SU}(2) | H_4 H_3 H_2 H_1 = \mathbb{I}_{\text{SU}(2)}\}/\text{SU}(2)$$



3D (constantly curved) geometries are encoded in holonomies on 2D surface: $H_f(\mathfrak{a}_f, \hat{\mathfrak{n}}_f) = e^{\frac{\Lambda}{3}\mathfrak{a}_f \hat{\mathfrak{n}}_f \cdot \vec{\tau}}, \quad \forall f = 1, \cdots, 4$ [Haggard, Han, Riello '16]

One dimension higher:

4-simplex geometry $\longleftarrow$ $\pi_1(\text{sk}_1(\text{4-simplex})) \xrightarrow{\text{isomorphism}} \pi_1(S^3\backslash\Gamma_5) \longrightarrow$ coordinates of

triangle areas & normals $\qquad \mathcal{M}_{\text{flat}}(S^3\backslash\Gamma_5, \text{SL}(2,\mathbb{C}))$

$\{\mathfrak{a}_f, \hat{\mathfrak{n}}_f\}$ $\qquad \omega_{\text{spin}} \searrow \qquad \swarrow \omega_{\text{flat}} \qquad \{z, \tilde{z}, \cdots\}$

$$\{\{\widetilde{H}_{ab}\} \in \text{SL}(2,\mathbb{C}) | \text{closure conditions}\}/\text{SL}(2,\mathbb{C})$$

$$\widetilde{H}_{ab}(\mathfrak{a}_{ab}, \hat{\mathfrak{n}}_{ab}) = \widetilde{H}_{ab}(\{z, \tilde{z}, \cdots\})$$

Stationary phase approximation of SF amplitude

1. 4-simplex geometry

triangle areas & normals

$\{\mathfrak{a}_f, \hat{\mathfrak{n}}_f\}$

↓ [Haggard, Han, Kaminski, Riello '15, Han '15, Haggard, Han, Riello '16]

$\mathcal{M}_{\text{flat}}\left(S^3\backslash\Gamma_5, \mathrm{SL}(2,\mathbb{C})\right)$

coordinates $\{z, \tilde{z}, \cdots\}$

2. finite-dimensional integral with oscillatory action

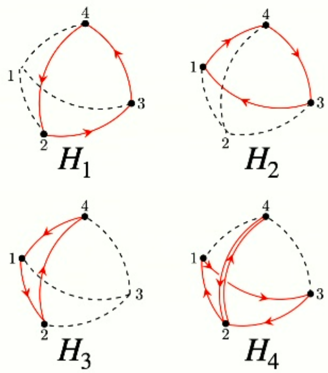
e.g. one $\mathcal{A}_v$: 40-dim integral

$\triangle_3$: 114-dim integral

↓

stationary phase approximation

How does a 3D theory describe 4D geometry?



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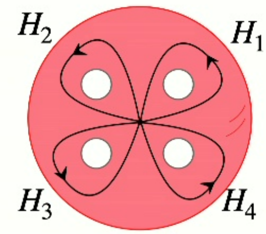

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4D (constantly curved) geometries are encoded in holonomies on 3D manifold

[Haggard, Han, Kaminski, Riello '15, Han '15]

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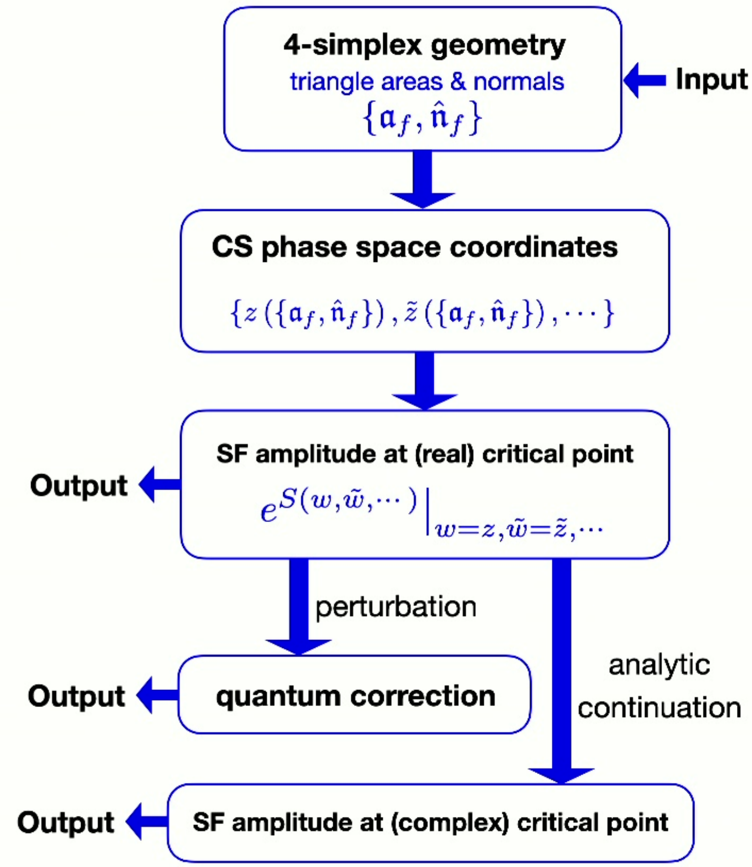
stationary phase approximation



[Han, QP, W.I.P.]

Plan of this talk

- **Properties of the SF model**
  - Consistent with GR — critical point geometry
  - Go beyond triangulation dependence

Toward a triangulation-independent formalism

"Spinfoam model is a regularization of the path-integral formalism of quantum gravity"

$$\int \mathcal{D}g_{\mu\nu}\, e^{iS_{\mathsf{EH}}}$$

The SF model with Λ is good 4D QG formalism because
it is finite and semi-classically consistent with the Einstein gravity

A better regularization should be triangulation independent!

- A "**moduli space field theory**" formalism of the SF model with Λ, in an analogous way to the GFT
  - Consider a field $\Psi(j,\iota) : \mathcal{M}_{\mathsf{flat}}(\Sigma_{0,4}, \mathsf{SU}(2)) \to \mathbb{C}$ (j,ι) : configuration of a tetrahedron
  - Consider a generalized moduli-space field action

$$S[\Psi] = K[\Psi] + V[\Psi] + c.c.$$

kinetic: $K[\Psi] = \sum_{\{j\}\in(\mathbb{Z}/k\mathbb{Z})^4} \int [\mathsf{d}\iota]\ \Psi(j,\iota^*)\Psi(j,\iota)$

potential: $V[\Psi] = \frac{g}{5!} \sum_{\{j_{ab}\}_{a<b}} \prod_{a=1}^{5} \int [\mathsf{d}\iota_a]\ \mathcal{A}_v(\{j_{ab},\iota_a\}) \prod_{a=1}^{5} \Psi(\{j_{ab}\},\iota_a)$

[Han, QP, W.I.P.]

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  - Expectation: (1) $\int \mathcal{D}\Psi\, e^{iS[\Psi]} = \sum_{\Gamma} \frac{g^{N_\Gamma}}{\text{Sym}(\Gamma)} \mathcal{A}_\Gamma$ gives **finite** amplitude order-by-order

    (2) triangulation-independent

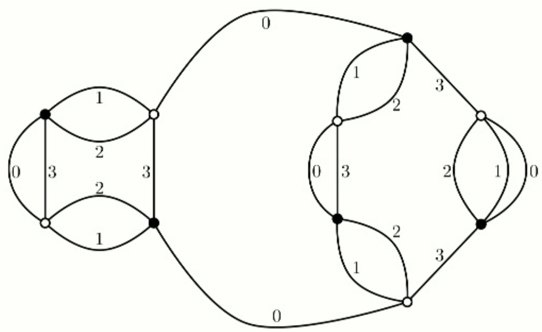
[Han, QP, W.I.P.]

Relation to colored tensor model

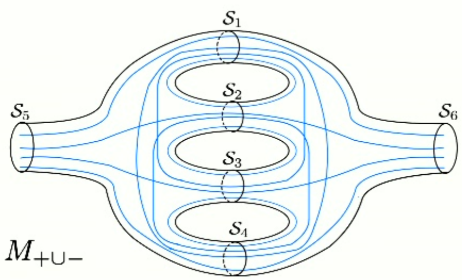
- For a **colored graph** (as in colored tensor model), the spinfoam amplitude takes a **simple form**

- A (D+1)-colored graph Γ_c $\begin{cases} \text{colored white or black on (D+1)-valent **nodes**} \\ \text{colored } \{0,1,\cdots,D\} \text{ on incident **links**} \\ \text{each link connects a white node and a black node} \end{cases}$

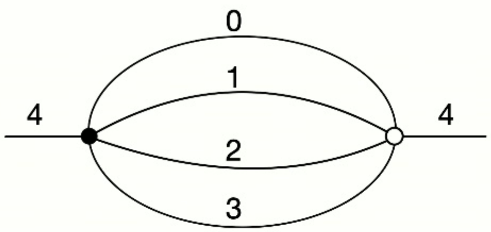
- It restricts the gluing to be **"simple"**, *e.g.* no crossing strands
  - Gluing constraints at the CS level
  - Impose simplicity constraints **all at once** on CS partition function $\mathcal{Z}_{M_{+\cup-}}$



an example of Γ_c at D=3



$\Longleftrightarrow$



colored melonic spinfoam graph

[Han, QP, W.I.P.]

Relation to colored tensor model — cont.

- A "**colored tensorial moduli space field theory**" formalism
  - Consider a tensor field $\Psi^{a}_{\vec{n}_a}(j,\iota) : \mathcal{M}_{\text{flat}}\,(\Sigma_{0,4}, \mathrm{SU}(2)) \to \mathbb{C}$
  - Consider a generalized moduli-space field action

$\vec{n}_a = (n_{a\,a-1}, \cdots, n_{a0}, n_{a4}, \cdots, n_{a\,a+1})$

$n_{ab} = n_{ba}$

$$S[\Psi] = K[\Psi] + V[\Psi] + c.c.$$

kinetic: $$K[\Psi] = \sum_{a=0}^{4} \sum_{\vec{n}_a} \sum_{\{j\}\in(\mathbb{Z}/k\mathbb{Z})^4} \int [\mathrm{d}\iota]\, \overline{\Psi^{a}_{\vec{n}_a}}\,(j,\iota)\, \Psi^{a}_{\vec{n}_a}(j,\iota)$$

potential: $$V[\Psi] = \frac{g}{5!k^3} \sum_{a=0}^{4} \sum_{\vec{n}_a} \sum_{\{j_{ab}\}_{a<b}} \prod_{a=1}^{5} \int [\mathrm{d}\iota_a]\, \mathcal{A}_v(\{j_{ab}, \iota_a\}) \prod_{a=1}^{5} \Psi^{a}_{\vec{n}_a}(\{j_{ab}\}, \iota_a)$$

  - Expectation: (1) $\int \mathcal{D}\Psi\, e^{iS[\Psi]} = \sum_{\Gamma_c} \frac{g^{V_{\Gamma_c}} k^{-\omega(\Gamma_c)}}{\mathrm{Sym}(\Gamma_c)} \mathcal{A}_{\Gamma_c}$ gives **finite** amplitude order-by-order for **closed colored graph**

    (2) dominate at 4-sphere triangulation [Gurau '11, Bonzom, Gurau, Riello, Rivasseau 11, Gurau, Ryan '12]

[Han, QP, W.I.P.]

Further developments and applications

- Generalize the model to include **timelike tetrahedra**: $\mathrm{SU}(2) \to \mathrm{SL}(2,\mathbb{R})$ [Conrady, Hnybida '11, Han, Liu '18]

- A **quantum group** representation of the SF model?
  - Clue 1: combinatorial quant. of CS theory $\longrightarrow$ quantum group rep. [Alekseev, Grosse, Schomerus '94-95, Buffenoir, Noui, Roche '02]
  - Clue 2: Turaev-Viro model
  - Clue 3: quantum state of constantly curved tetrahedron = $\mathfrak{q}$-deformed intertwiner [Han, Hsiao, QP '23]

- **Applications** of this SF model to physical systems (presumably rely on *numerical development*)

  Cosmology
  - Investigate the bouncing probability in the SF scenario
  - Expect to obtain a modified Friedman equation with a bare Λ

  (Asymptotically dS/AdS) black hole
  - Study the transition amplitude of boundary BH states
  - Study the black-to-while hole tunneling using SF method

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Thank you for your attention!