Title: Surface Operators and Exact Holography

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Abstract:

Non-local operators, supported on submanifolds of spacetime, often encode fascinating physical insights about a theory and can serve as order parameters for phase transitions. In this talk, we will explore various aspects of 1/2

BPS surface operators in N=4 super Yang-Mills. Specifically, I will show how supergravity computes exactly the planar limit of certain correlation functions of surface operators, even though they receive nontrivial quantum corrections. In particular, we will compute correlation functions with Chiral Primary Operators by localizing N = 4 super Yang-Mills on S^4 to a deformed version of 2d Yang-Mills on S^2. These correlation functions, which have a finite number of quantum corrections, can also be computed perturbatively in four dimensions. I will show the exact agreement between these approaches and the corresponding supergravity result. This talk is based on 2406.08541, work in collaboration with Changha Choi and Jaume Gomis.

SURFACE OPERATORS AND EXACT HOLOGRAPHY

Raquel Izquierdo García

2406.08541 with Changha Choi and Jaume Gomis

Perimeter Institute

December 9, 2024

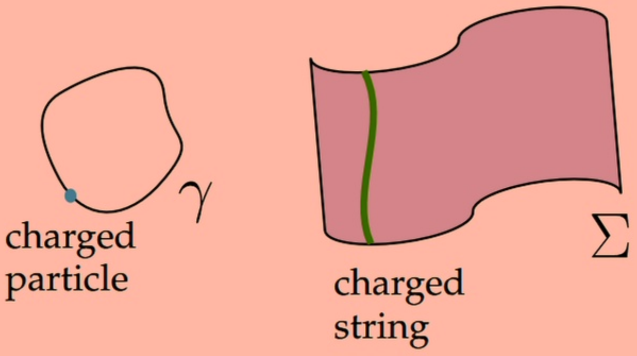
PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

"la Caixa" Foundation

Operators in a 4d theory

Characterized by their codimension

- Local operators
- Line operators
- Surface operators
- Interfaces



They introduce a probe into the theory

CORRELATION FUNCTIONS WITH SURFACE OPERATORS

$$\frac{\langle \mathcal{O}_\Sigma \, \mathcal{O} \rangle}{\langle \mathcal{O}_\Sigma \rangle} = \langle \mathcal{O} \rangle |_{\mathcal{O}_\Sigma},$$

In our case $\mathcal{O}$ will be:

- **Local operators**: Chiral Primary Operators (CPO) and stress tensor
- Wilson loops

In $\mathcal{N} = 4$ super Yang-Mills

AdS/CFT correspondence let us compute $\langle O \rangle$ in two ways:

- Perturbative QFT ($g^2 N = \lambda \ll 1$):

$$\langle \mathcal{O} \rangle |_{\mathcal{O}_\Sigma} \propto c_0 + c_1 \lambda + c_2 \lambda^2 + \dots$$

- Holographic SUGRA computation ($\lambda \gg 1$):

$$\langle \mathcal{O} \rangle |_{\mathcal{O}_\Sigma} \propto \tilde{c}_0 + \tilde{c}_1 \frac{1}{\lambda} + \tilde{c}_2 \frac{1}{\lambda^2} + \dots$$

$$\langle \mathcal{O}_{\Delta,k}\rangle|_{\mathcal{O}_\Sigma} \propto \frac{1}{\lambda^{\Delta/2}}\left(c_0 + \mathcal{O}(\lambda)\right), \qquad \lambda \ll 1$$

$$\langle \mathcal{O}_{\Delta,k}\rangle|_{\mathcal{O}_\Sigma} \propto \frac{1}{\lambda^{|k|/2}}\left(\tilde{c}_0 + \tilde{c}_1\frac{1}{\lambda} + \ldots + \tilde{c}_{\frac{\Delta-|k|}{2}}\frac{1}{\lambda^{\frac{\Delta-|k|}{2}}}\right), \quad \lambda \gg 1$$

[Drukker, Gomis, Matsuura '08] found $c_0 = \tilde{c}_{\frac{\Delta-|k|}{2}}$.

Conjecture: the correlation function of $\langle \mathcal{O}_{\Delta,k}\rangle|_{\mathcal{O}_\Sigma}$

1. has a finite expansion in λ
2. is captured by supergravity in the 't Hooft limit
3. is computable using supersymmetric localization

A 1/2 BPS $\mathcal{O}_\Sigma$ ($\Sigma = \mathbb{R}^2$ or $\Sigma = S^2$) depends on
[Gukov, Witten '06]

- A choice of Levi subgroup

$$L = \prod_{l=1}^{M} U(N_l) \subset G = U(N)$$

- $4M$ continuous parameters $(\alpha_l, \beta_l, \gamma_l, \eta_l)$

GOAL

Compute

$$\frac{\langle \mathcal{O}_\Sigma \, \mathcal{O} \rangle}{\langle \mathcal{O}_\Sigma \rangle} = \langle \mathcal{O} \rangle |_{\mathcal{O}_\Sigma},$$

as a function of g_{YM}^2, N, L and $\alpha_l, \beta_l, \gamma_l, \eta_l$ in two ways:

- 4d perturbation theory
- supersymmetric localization

GOAL

Compute

$$\frac{\langle \mathcal{O}_\Sigma \, \mathcal{O} \rangle}{\langle \mathcal{O}_\Sigma \rangle} = \langle \mathcal{O} \rangle|_{\mathcal{O}_\Sigma},$$

as a function of g^2_{YM}, N, L and $\alpha_l, \beta_l, \gamma_l, \eta_l$ in two ways:

- 4d perturbation theory
- supersymmetric localization

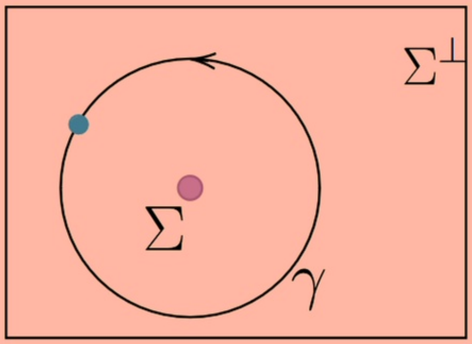
OUTLINE

1. Definition of Surface Operator
2. Perturbative calculation
3. Supersymmetric localization

SURFACE OPERATORS IN A 4D GAUGE THEORY

$A = \alpha \, d\theta.$

Induces an Aharonov-Bohm phase for a charged particle whose worldline links with the surface Σ



Locally, $\mathbb{R}^4 = \Sigma \times \Sigma^\perp$.

$$A = \alpha \, d\theta$$

The gauge group $U(N)$ is broken to $L = \prod_{l=1}^{M} U(N_l)$, the subgroup that leaves α invariant:

$$\alpha = \begin{pmatrix} \alpha_1 \cdot \mathbb{I}_{N_1} & 0 & \cdots & 0 \\ 0 & \alpha_2 \cdot \mathbb{I}_{N_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \alpha_M \cdot \mathbb{I}_{N_M} \end{pmatrix}$$

We also insert

$$\exp\left(i\sum_{l=1}^{M}\eta_l\int_{\Sigma}\operatorname{Tr}F_{(l)}\right),$$

into the $\mathcal{N}=4$ SYM path integral, where $F_{(l)}$ is the field strength of every unbroken $U(1)$ factor.

1/2 BPS $\mathcal{O}_\Sigma$ IN $\mathcal{N} = 4$ SYM

[GUKOV, WITTEN '06]

$$A_\mu \text{ and } \Phi = \tfrac{1}{\sqrt{2}}(\phi^5 + i\phi^6)$$

have singular profiles near Σ.

If $\Sigma = \mathbb{R}^2$:

$$A_0 = \alpha\, d\theta, \quad \Phi_0 = (\beta + i\gamma)\,\frac{1}{\sqrt{2}\,z},$$

where $z = re^{i\theta}$ parametrizes the normal plane to Σ.
Bosonic symmetries:

$$SO(1,3) \times SO(2) \times SO(4),$$

$$\frac{\langle \mathcal{O}_\Sigma \, \mathcal{O} \rangle}{\langle \mathcal{O}_\Sigma \rangle} = \langle \mathcal{O} \rangle |_{\mathcal{O}_\Sigma},$$

the expectation value of $\mathcal{O}$ in the $\mathcal{N} = 4$ SYM path integral prescribed by the insertion of $\mathcal{O}_\Sigma$:

$$\frac{\int_{\substack{A=A_0+a, \\ \phi^i=\phi_0+\varphi^i}} D[a] \, D[\varphi^i] \, D[\psi_\alpha] \; \mathcal{O} \; e^{i\eta \int \mathrm{Tr}\, F} \; e^{-S_{SYM}}}{\langle \mathcal{O}_\Sigma \rangle}$$

CHIRAL PRIMARY OPERATORS

$$\mathcal{O}_{\Delta,k} = \frac{(8\pi^2)^{\Delta/2}}{\lambda^{\Delta/2}\sqrt{\Delta}} C^{\Delta,k}_{i_1\ldots i_\Delta} \operatorname{Tr}\left(\phi^{\{i_1}\ldots\phi^{i_\Delta\}}\right),$$

These operators have conformal dimension Δ and $U(1)_R$ charges $k = -\Delta, -\Delta+2, \ldots, \Delta-2, \Delta$.

$$\mathcal{O}_{2,0} = \frac{4\pi^2}{\sqrt{6}\lambda} \operatorname{Tr}\left(4\Phi\bar{\Phi} - \sum_{i=1}^{4}\phi^i\phi^i\right), \quad \mathcal{O}_{2,2} = \frac{8\pi^2}{\sqrt{2}\lambda} \operatorname{Tr}(\Phi\Phi)$$

1. Expand around the background $\Phi = \Phi_0 + \varphi$
2. compute all possible Wick contractions $\langle \bar{\varphi}\, \varphi \rangle, \langle \phi^i \phi^I \rangle, I = 1, \dots, 4.$

$$\langle \mathcal{O}_{2,0} \rangle|_{\mathcal{O}_\Sigma} = \frac{16\pi^2}{\sqrt{6}\lambda}\ \mathrm{Tr}\left(\left(\Phi_0 \bar{\Phi}_0\right) + \langle \varphi\bar{\varphi} - \varphi_1\varphi_1 \rangle \right), \quad \langle \mathcal{O}_{2,2} \rangle|_{\mathcal{O}_\Sigma} = \frac{8\pi^2}{\sqrt{2}\lambda}\ \mathrm{Tr}\left(\Phi_0\Phi_0\right),$$

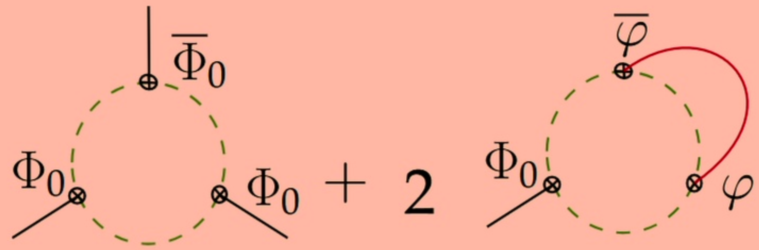
$$\langle \mathcal{O}_{3,1} \rangle|_{\mathcal{O}_\Sigma} = \frac{16\pi^3}{\lambda^{3/2}}\ \mathrm{Tr}\left(\left(\Phi_0^2 \bar{\Phi}_0\right) + 2\Phi_0 \langle \varphi\bar{\varphi} - \varphi_1\varphi_1 \rangle \right), \quad \langle \mathcal{O}_{3,3} \rangle|_{\mathcal{O}_\Sigma} = \frac{32\pi^3}{\sqrt{6}\lambda^{3/2}}\ \mathrm{Tr}\left(\Phi_0^3\right),$$

$$\langle \mathcal{O}_{4,0} \rangle|_{\mathcal{O}_\Sigma} = \frac{8\pi^4}{\sqrt{5}\lambda^2}\ \mathrm{Tr}\Big(12(\Phi_0\bar{\Phi}_0)^2 + 32\Phi_0\bar{\Phi}_0 \langle \varphi\bar{\varphi} - \varphi_1\varphi_1 \rangle + 16\Phi_0 \langle \varphi\bar{\Phi}_0\bar{\varphi} - \varphi_1\bar{\Phi}_0\varphi_1 \rangle +$$
$$+ 16 \langle \varphi\bar{\varphi} - \varphi_1\varphi_1 \rangle^2 + 8\left(\varphi_1\varphi_1\varphi_1\varphi_1 - 2\varphi\varphi_1\bar{\varphi}\varphi_1 + \varphi\varphi\bar{\varphi}\bar{\varphi} \right) \Big),$$

$$\mathcal{O}_{3,1} = \frac{8\pi^3}{\lambda^{3/2}}\ \mathrm{Tr}\left(2\Phi^2\bar{\Phi} - \Phi\sum_{i=1}^{4}\phi^i\phi^i\right)$$

Expand around the background $\Phi = \Phi_0 + \varphi$

$$\mathrm{Tr}\left(\Phi^2\overline{\Phi}\right) =$$



$\mathrm{Tr}\left(\Phi_0^2\overline{\Phi}_0\right)$ $\qquad$ $\mathrm{Tr}(\Phi_0\langle\varphi\overline{\varphi}\rangle)$

$$\langle\mathcal{O}_{3,1}\rangle|_{\mathcal{O}_\Sigma} = \frac{16\pi^3}{\lambda^{3/2}}\ \mathrm{Tr}\left(\left(\Phi_0^2\bar{\Phi}_0\right) + 2\Phi_0\langle\varphi\bar{\varphi} - \varphi_1\varphi_1\rangle\right)$$

The result of summing over all Wick contractions only depends on

$$\langle \varphi_{(ai)(bj)}\bar{\varphi}_{(ck)(dl)} - \varphi^{1}_{(ai)(bj)}\varphi^{1}_{(ck)(dl)}\rangle(x,x) = \\ = \frac{\lambda}{8\pi^2}\frac{1}{2N}\delta_{ad}\delta_{bc}\delta_{jk}\delta_{il}(1-\delta_{ab})(1-\delta_{cd})$$

$$\varphi_i, \varphi, \bar{\varphi} = \left(\begin{array}{c|c|c|c} b_{N_1,N_1}\otimes \mathbb{I}_{N_1} & \tilde{b}_{N_1,N_2} & \cdots & \tilde{b}_{N_1,N_M} \\ \hline \tilde{b}_{N_2,N_1} & b_{N_2}\otimes \mathbb{I}_{N_2,N_2} & \cdots & \tilde{b}_{N_2,N_M} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline \tilde{b}_{N_M,N_1} & \tilde{b}_{N_M,N_2} & \cdots & b_{N_M,N_M}\otimes \mathbb{I}_{N_M} \end{array}\right)$$

$$\langle \mathcal{O}_{2,0} \rangle|_{\mathcal{O}_\Sigma} = \frac{16\pi^2}{\sqrt{6}\lambda} \sum_l N_l \left(\left(\frac{\beta_l^2 + \gamma_l^2}{2} \right) + \frac{\lambda}{8\pi^2} \frac{N - N_l}{2N} \right),$$

$$\langle \mathcal{O}_{2,2} \rangle|_{\mathcal{O}_\Sigma} = \frac{4\pi^2}{\sqrt{2}\lambda} \sum_l N_l (\beta_l + i\gamma_l)^2,$$

$$\langle \mathcal{O}_{3,1} \rangle|_{\mathcal{O}_\Sigma} = \frac{16\pi^3}{\lambda^{3/2}} \sum_l N_l \frac{\beta_l + i\gamma_l}{\sqrt{2}} \left(\left(\frac{\beta_l^2 + \gamma_l^2}{2} \right) + 2 \frac{\lambda}{8\pi^2} \frac{N - N_l}{2N} \right),$$

$$\langle \mathcal{O}_{3,3} \rangle|_{\mathcal{O}_\Sigma} = \frac{8\pi^3}{\sqrt{3}\lambda^{3/2}} \sum_l N_l (\beta_l + i\gamma_l)^3,$$

$$\langle \mathcal{O}_{4,0} \rangle|_{\mathcal{O}_\Sigma} = \frac{8\pi^4}{\sqrt{5}\lambda^2} \left(\sum_l N_l \left(3 \left(\beta^2 + \gamma^2 \right)^2 + \frac{\lambda}{2\pi^2 N} \left(\beta^2 + \gamma^2 \right) (2N - 3N_l) \right.\right.$$
$$\left.\left. + \frac{\lambda^2}{16\pi^4 N^2} (N - N_l)^2 \right) + \frac{\lambda}{\pi^2 N} \left(\left(\sum_l N_l \frac{\beta_l + i\gamma_l}{\sqrt{2}} \right) \left(\sum_l N_l \frac{\beta_l - i\gamma_l}{\sqrt{2}} \right) \right) \right),$$

$$\langle \mathcal{O}_{4,2} \rangle|_{\mathcal{O}_\Sigma} = \frac{32\pi^4}{\sqrt{10}\lambda^2} \left(\sum_l N_l (\beta + i\gamma)^2 \left(\left(\beta^2 + \gamma^2 \right) + \frac{\lambda}{8\pi^2 N} (2N - 3N_l) \right) + \right.$$
$$\left. + \frac{\lambda}{4\pi^2 N} \left(\left(\sum_l N_l \frac{\beta_l + i\gamma_l}{\sqrt{2}} \right) \left(\sum_l N_l \frac{\beta_l + i\gamma_l}{\sqrt{2}} \right) \right) \right),$$

For $L = U(1) \times U(N-1)$:

Gauge theory

$$\langle \mathcal{O}_{2,0} \rangle|_{\mathcal{O}_\Sigma} = \frac{1}{|z|^2} \frac{8\pi^2}{\sqrt{6}\lambda} \left((\beta^2 + \gamma^2) + \frac{\lambda}{4\pi^2} \right),$$

$$\langle \mathcal{O}_{2,2} \rangle|_{\mathcal{O}_\Sigma} = \frac{1}{z^2} \frac{4\pi^2}{\sqrt{2}\lambda} (\beta + i\gamma)^2,$$

$$\langle \mathcal{O}_{3,1} \rangle|_{\mathcal{O}_\Sigma} = \frac{1}{z|z|^2} \frac{8\pi^3}{\sqrt{2}\lambda^{3/2}} (\beta + i\gamma) \left((\beta^2 + \gamma^2) + \frac{\lambda}{4\pi^2} \right),$$

$$\langle \mathcal{O}_{3,3} \rangle|_{\mathcal{O}_\Sigma} = \frac{1}{z^3} \frac{8\pi^3}{\sqrt{3}\lambda^{3/2}} (\beta + i\gamma)^3,$$

$$\langle \mathcal{O}_{4,0} \rangle|_{\mathcal{O}_\Sigma} = \frac{1}{|z|^4} \frac{24\pi^4}{\sqrt{5}\lambda^2} \left((\beta^2 + \gamma^2)^2 + \frac{\lambda (\beta^2 + \gamma^2)}{3\pi^2} + \frac{\lambda^2}{48\pi^4} \right),$$

$$\langle \mathcal{O}_{4,2} \rangle|_{\mathcal{O}_\Sigma} = \frac{1}{z^2|z|^2} \frac{32\pi^4}{\sqrt{10}\lambda^2} (\beta + i\gamma)^2 \left((\beta^2 + \gamma^2) + \frac{\lambda}{4\pi^2} \right),$$

$$\langle \mathcal{O}_{4,4} \rangle|_{\mathcal{O}_\Sigma} = \frac{1}{z^4} \frac{8\pi^4}{\lambda^2} (\beta + i\gamma)^4,$$

Probe Brane

$$\langle \mathcal{O}_{2,0} \rangle|_{\mathcal{O}_\Sigma} = \frac{1}{|z|^2} \frac{8\pi^2}{\sqrt{6}} \left(\frac{1}{4\pi^2} + (\beta^2 + \gamma^2) \frac{1}{\lambda} \right),$$

$$\langle \mathcal{O}_{2,2} \rangle|_{\mathcal{O}_\Sigma} = \frac{1}{z^2} \frac{4\pi^2}{\sqrt{2}\lambda} (\beta + i\gamma)^2,$$

$$\langle \mathcal{O}_{3,1} \rangle|_{\mathcal{O}_\Sigma} = \frac{1}{z|z|^2} \frac{8\pi^3}{\sqrt{2}\lambda^{1/2}} (\beta + i\gamma) \left(\frac{1}{4\pi^2} + (\beta^2 + \gamma^2) \frac{1}{\lambda} \right),$$

$$\langle \mathcal{O}_{3,3} \rangle|_{\mathcal{O}_\Sigma} = \frac{1}{z^3} \frac{8\pi^3}{\sqrt{3}\lambda^{3/2}} (\beta + i\gamma)^3,$$

$$\langle \mathcal{O}_{4,0} \rangle|_{\mathcal{O}_\Sigma} = \frac{1}{|z|^4} \frac{24\pi^4}{\sqrt{5}} \left(\frac{1}{48\pi^4} + \frac{(\beta^2 + \gamma^2)}{3\pi^2} \frac{1}{\lambda} + \frac{(\beta^2 + \gamma^2)^2}{\lambda^2} \right),$$

$$\langle \mathcal{O}_{4,2} \rangle|_{\mathcal{O}_\Sigma} = \frac{1}{z^2|z|^2} \frac{32\pi^4}{\sqrt{10}\lambda} (\beta + i\gamma)^2 \left(\frac{1}{4\pi^2} + \frac{\beta^2 + \gamma^2}{\lambda} \right),$$

$$\langle \mathcal{O}_{4,4} \rangle|_{\mathcal{O}_\Sigma} = \frac{1}{z^4} \frac{8\pi^4}{\lambda^2} (\beta + i\gamma)^4,$$

Supersymmetric localization

We consider a SQFT in a curved background. Given a supercharge $\mathcal{Q}$:

$$\mathcal{Q}^2 = B = \mathcal{L}_V + R,$$

We want to compute

$$Z = \int [D\phi]\,[D\psi]\; e^{-S}, \quad \langle \mathcal{O} \rangle = \int [D\phi]\,[D\psi]\; \mathcal{O}\; e^{-S}$$

invariant under some supersymmetry $\mathcal{Q}$:

$$\mathcal{Q}\,S = 0, \quad \mathcal{Q}\,\mathcal{O} = 0$$

Supersymmetric localization

- Deform the action $S(t) = S + t\,\mathcal{Q}\,V$ $(\mathcal{Q}^2\,V = 0)$

$$Z[t] = \int [D\phi]\,[D\psi]\,e^{-S-t\mathcal{Q}V}$$

- Saddle point *approximation* at $t \to \infty$[1]
- Z localizes to supersymmetric configurations obeying

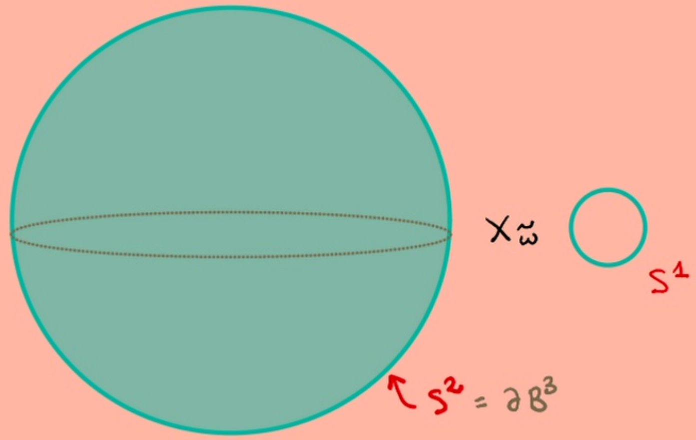
$$\mathcal{Q}V = 0$$

[Witten '91]

[1] We prove that $\partial_t Z[t] = 0$, the result is semiclassical wrt to t, **exact with respect to all other parameters**.

Localization of $\mathcal{N} = 4$ SYM on S^4 to $2d$ YM on S^2 [Pestun, '09]

$$S^4 \sim_{conf} B^3 \times_{\tilde{w}} S^1$$



$$\mathcal{Q}^2 = \frac{1}{r}\left(R_{05} - R_\tau\right)$$

$$ds^2 = d\tilde{x}_i^2 + r^2\left(1 - \frac{\tilde{x}^2}{4r^2}\right)^2 d\tau^2$$

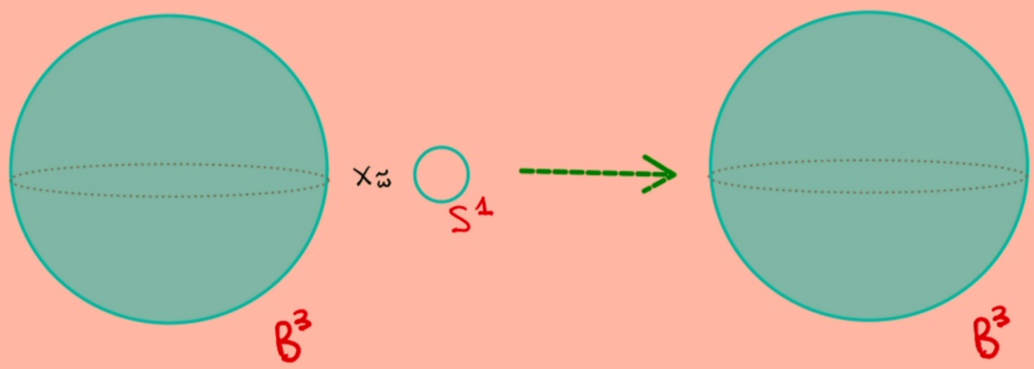
Supersymmetry equations: $V = (\Psi, \overline{\mathcal{Q}\,\Psi})$
$\mathcal{Q}V = 0 \implies \mathcal{Q}\Psi = 0$ (16 complex equations)

Split on bottom and top equations with respect to $-i\Gamma^0\Gamma^1$:

$$\mathcal{Q}\Psi^{top} = 0, \quad \mathcal{Q}\Psi^{bottom} = 0.$$

FROM S^4 TO B^3

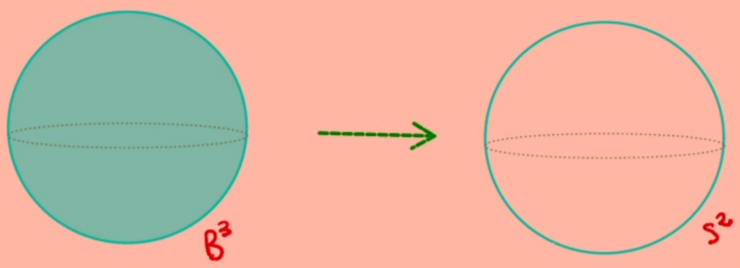
1. $\mathcal{Q}\Psi^{bottom} = 0$ (8 complex equations)
   $\nabla_\tau A_\mu = \nabla_\tau \phi^I = 0$



*Natural since $R_\tau \subset \mathcal{Q}^2$

FROM B^3 TO S^2

$\mathcal{Q}\Psi^{top} = 0$ determines the 3d theory on B^3.



We can write the $\mathcal{Q}^2$ invariant 3d action as

2. $S_{B^3} = \int (\mathcal{Q}\Psi^{top})^2 + \partial(\ldots)$, localization to $S^2 = \partial B^3$

$$S_{2d} = \frac{2\pi}{g^2} \int_{S^2} d\Omega \, \Phi_n^2, \quad \Phi_n = \frac{x_i}{|x|} \phi_{i+4}$$

2d YM

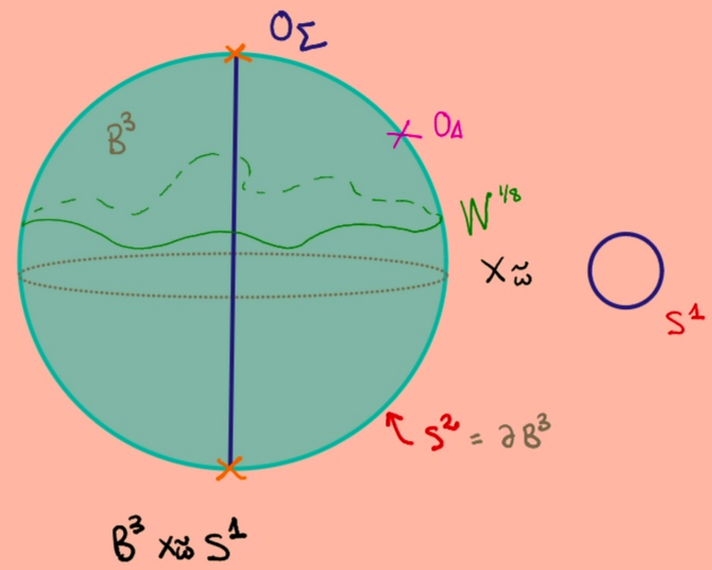
Defining a complexified connection on S^2
$A_{\mathbb{C}} = A_t - i *_{2d} \phi_t$ and using the susy equations

$$S_{2d} = \frac{1}{g^2_{2d^2}} \int_{S^2} d\Omega \, (*_{2d} F_{A_{\mathbb{C}}})^2, \quad g^2_{2d} = -\frac{g^2}{4\pi r^2},$$

Q is a symmetry of

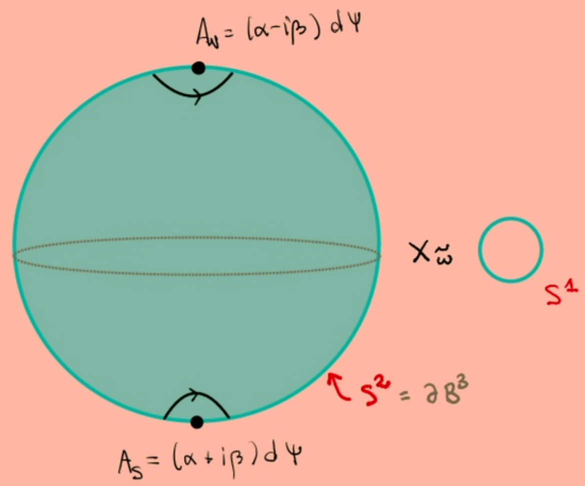
- $\mathcal{O}|_{\Sigma}$
- $\mathcal{O}_{\Delta}$ **[Drukker, Plefka; '09]**
- $W^{1/8}_{\theta_0,\psi_0}$

$$Q^2 = \frac{1}{r}\left(R_{05} - R_{\tau}\right)$$



$$\Sigma = I \times_{\tilde{w}} S^1 \subset B^3 \times_{\tilde{w}} S^1$$

ADDING $\mathcal{O}_\Sigma$ TO THIS STORY



From $2d$ YM pov: non trivial background for $A_{\mathbb{C}}$

$$A_{\mathbb{C}} = \alpha d\psi - i\gamma \csc\theta d\theta - i\beta\cos\theta d\psi, \quad F_{\mathbb{C}} = i\beta d\mathrm{Vol}_{S^2}$$

$$\mathrm{Hol}_N(\nabla_A) = e^{2\pi(-\alpha+i\beta)}, \quad \mathrm{Hol}_S(\nabla_A) = e^{2\pi(\alpha+i\beta)}$$

LOCALIZABLE CPOs

We cannot use $\mathcal{O}_{\Delta,k}$ because they are not invariant under $\mathcal{Q}^2$.

Define position dependant CPOs **[Drukker, Plefka, 09']** compatible with $\mathcal{Q}$ **[Giombi, Pestun, '10]**:

$$\Phi_n = \frac{1}{2R}\left(x^2\phi_6 + \frac{1}{\sqrt{2}}\left(z\bar{\Phi} + \bar{z}\Phi\right)\right).$$

$$O_\Delta = \mathrm{Tr}\left(\Phi_n - i\phi_9\right)^\Delta$$

$\mathcal{O}_\Delta$ IN THE $2d$ YANG-MILLS THEORY

Using susy equations ($\mathcal{Q}\Psi = 0$) the $2d$ action can be written in different ways:

$$S_{2d} = \frac{\pi}{g^2}\int_{S^2} d\Omega \Phi_n^2 = \frac{\pi}{g^2}\int_{S^2} d\Omega (d_A^{*2d}\phi_t)^2 = \frac{1}{g_{2d}^2}\int_{S^2} d\Omega \left(*_{2d}F_{A_{\mathbb{C}}}\right)^2$$

In the localization locus:

$$\Phi_n \Longleftrightarrow -i * F_{2d}$$

and

$$\langle O_\Delta \mathcal{O}|_\Sigma\rangle_{4d} \quad \Longleftrightarrow \quad \left\langle \mathrm{Tr}\left[(-i * F_{2d})^\Delta\right]\right\rangle_{2d}$$

Gaussian action for F, only nontrivial coincident points propagator for modes outside of the Levi subgroup:

$$F_0 = i\beta, \quad \langle F_{(ai)(bj)} F_{(ck)(dl)} \rangle = \frac{\lambda}{16\pi^2 N} \delta_{ad}\delta_{bc}\delta_{il}\delta_{jk}(1 - \delta_{ab}) \tag{1}$$

Example:

$$\langle \mathcal{O}_2 \rangle |_{\mathcal{O}_\Sigma} = \sum_{l=1}^{M} N_l \beta_l^2 + \frac{g_{2d}^2}{8\pi} N_l (N - N_l) \tag{2}$$

In order to compare with $\langle \mathcal{O}_{\Delta,k} \mathcal{O}|_{\Sigma} \rangle$:

$$P[\mathcal{O}_\Delta] = \sum_{k=-N}^{N} z^{\Delta+k/2} \bar{z}^{\Delta-k/2} c_k \mathcal{O}_{\Delta,k} + \mathcal{R}[\mathcal{O}_\Delta]$$

Example:

$$\mathcal{P}[\mathcal{O}_2] = \frac{z^2}{2} \frac{\sqrt{2}\lambda}{8\pi^2} \mathcal{O}_{2,2} + \frac{\bar{z}^2}{2} \frac{\sqrt{2}\lambda}{8\pi^2} \mathcal{O}_{2,-2} + \frac{\sqrt{6}\lambda}{16\pi^2} \mathcal{O}_{2,0}$$

Comparison

4*d* calculation

$$\langle \mathcal{O}_{2,0} \rangle |_{\mathcal{O}_\Sigma} = \frac{16\pi^2}{\sqrt{6}\lambda} \sum_l N_l \left(\left(\frac{\beta_l^2 + \gamma_l^2}{2} \right) + \frac{\lambda}{8\pi^2} \frac{N - N_l}{2N} \right),$$

$$\langle \mathcal{O}_{2,2} \rangle |_{\mathcal{O}_\Sigma} = \frac{4\pi^2}{\sqrt{2}\lambda} \sum_l N_l (\beta_l + i\gamma_l)^2,$$

$$\langle \mathcal{O}_{3,1} \rangle |_{\mathcal{O}_\Sigma} = \frac{16\pi^3}{\lambda^{3/2}} \sum_l N_l \frac{\beta_l + i\gamma_l}{\sqrt{2}} \left(\left(\frac{\beta_l^2 + \gamma_l^2}{2} \right) + 2 \frac{\lambda}{8\pi^2} \frac{N - N_l}{2N} \right),$$

$$\langle \mathcal{O}_{3,3} \rangle |_{\mathcal{O}_\Sigma} = \frac{8\pi^3}{\sqrt{3}\lambda^{3/2}} \sum_l N_l (\beta_l + i\gamma_l)^3,$$

2d calculation

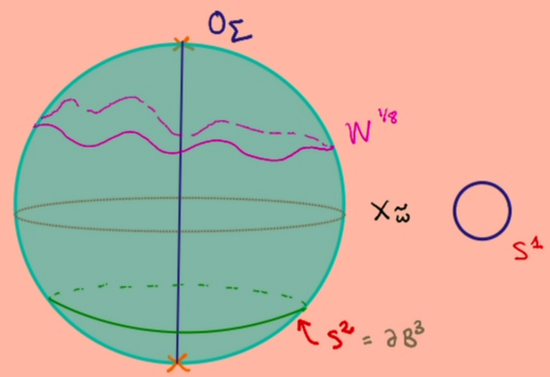
$$\langle \mathcal{O}_2 \rangle \, |_{\mathcal{O}_\Sigma} = \sum_{l=1}^{M} N_l \beta_l^2 + \frac{g_{2d}^2}{8\pi} N_l (N - N_l)$$

$$\langle \mathcal{O}_2 \rangle \, |_{\mathcal{O}_\Sigma} = \sum_{l=1}^{M} N_l \beta_l^3 + \frac{3 g_{2d}^2}{8\pi} \beta_l N_l (N - N_l)$$

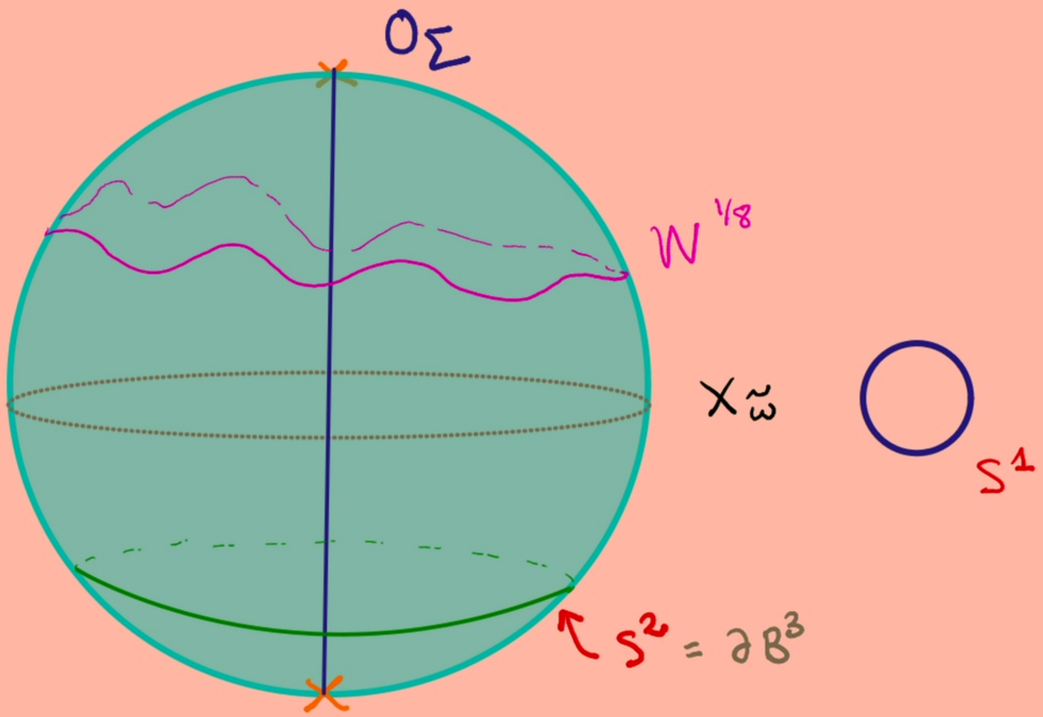
More correlation functions

Other interesting correlators

- $\left\langle \mathcal{O}_\Sigma W^{1/8}_{\theta_0,\psi_0} \right\rangle$
- $\langle \mathcal{O}_\Sigma T_{\mu\nu} \rangle$
- $\langle \mathcal{O}_\Sigma \rangle$



CORRELATION FUNCTION WITH WILSON LOOP



$$W_{\theta_0,\psi_0} = \frac{1}{N}\ \mathrm{Tr}\exp\int d\psi\left(iA_\psi + |z|\cos\theta_0\,\phi^1 + \sqrt{2}\sin\theta_0\ \mathrm{Re}\left(z\,\Phi e^{-i\psi_0}\right)\right),$$

First quantum correction:

$$\frac{\langle W_{\theta_0,\psi_0}\,\mathcal{O}_\Sigma\rangle}{\langle\mathcal{O}_\Sigma\rangle} = W_{\theta_0,\psi_0}|_{\Phi_0,A_0}\left(1 + \frac{\sum_{l=1}^{M} N_l^2}{N}\frac{g^2}{8}\ \cos^2\theta_0 + \ldots\right).$$

Only rainbow diagrams contribute to the full correlation function.

SUMMARY, CONCLUSIONS, ONGOING WORK...

- We proved the conjectures:
  1. $\langle \mathcal{O}_{\Delta,k} \mathcal{O}_{\Sigma} \rangle$ receives a finite number of quantum corrections (using susy localization
  2. **they are captured by supergravity** in the planar limit
- Surface operators have conformal anomalies:

$$\delta_\sigma \log \langle \mathcal{O}_\Sigma \rangle = \frac{1}{24\pi} \int d^2x \sqrt{h} \delta\sigma \left(b\, R_\Sigma + c_1\, g_{mn} h^{\mu\sigma} h^{\nu\rho} \hat{K}^m_{\mu\nu} \hat{K}^n_{\rho\sigma} - c_2\, W_{\mu\nu\rho\sigma} h^{\mu\rho} h^{\nu\sigma} \right)$$

we want to compute them using the 10d supergravity solutions