Title: Bound state corrections and high-energy scattering

Speakers: Ryan Plestid

Collection/Series: Particle Physics

Subject: Particle Physics

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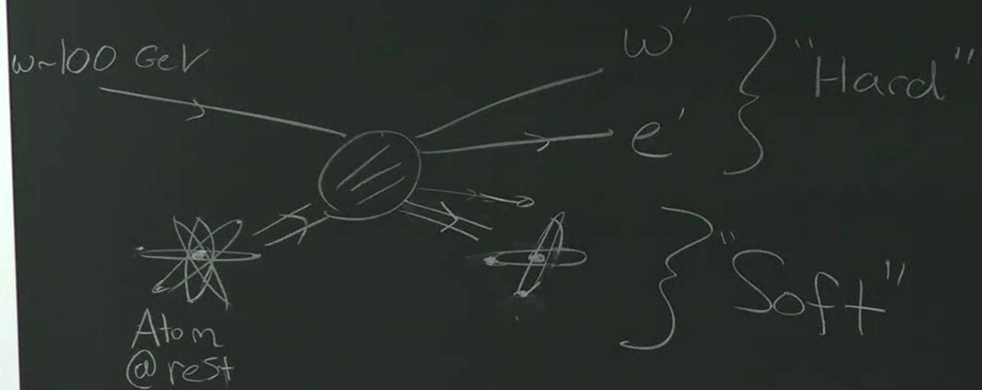
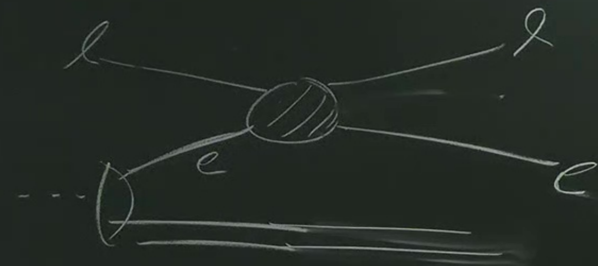
Abstract:

Many fundamental-physics experiments scatter high energy beams off of fixed targets composed of ordinary matter i.e., atoms. When considering the scattering off of atomic electrons we often make the approximation that the electron is free and at rest, however one can ask how good this approximation really is? This becomes especially important in the face of demanding precision goals of certain experiments. For example the planned MuonE experiment will attempt to measure the shape of $\mu e \rightarrow \mu e$ scattering as a function of angle with a precision of 10 ppm. In this talk I will explain how to systematically include bound-state corrections arising from the difference between a free-and-at-rest electron and those bound in atomic orbitals. When the final state of the atom is not measured, a surprisingly simple and elegant formula can be obtained that reduces the leading order corrections to a single atomic matrix element. New developments related to Coulomb corrections for inelastic systems will also be discussed. Based on (arXiv:2403.12184, 2407.21752).

Sketch of Problem



Microscopically



- Expand around $\vec{p}_e = 0$

Ex//

2 GeV $\nu\,{}^{40}\mathrm{Ar} \to \nu e\, I^+$

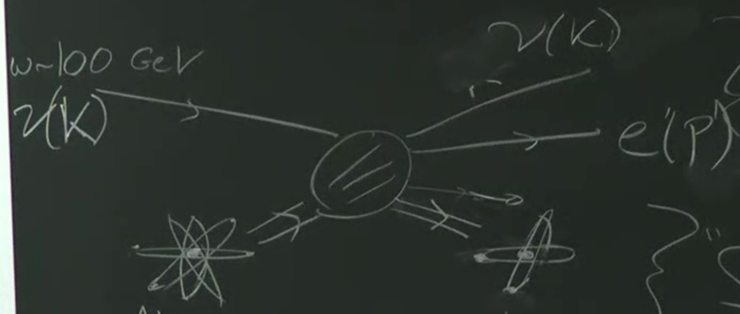
150 GeV $\mu\,{}^{12}\mathrm{C} \to \mu e\, I^*$

Φ_ν DUNE

$(g-2) \leftrightarrow$ HVP $\}$ 10^{-5}

MuonE $\}$ 10 ppm

Sketch of Problem



"Hard"

"Soft"

$(\sim p)$

Atom
@ rest

Ex//

2 GeV $\nu \; {}^{40}Ar \rightarrow \nu \, e \, I^{+}$

150 GeV $\mu \; {}^{12}C \Rightarrow \mu \, e \, I^{*}$

Φ_ν DUNE

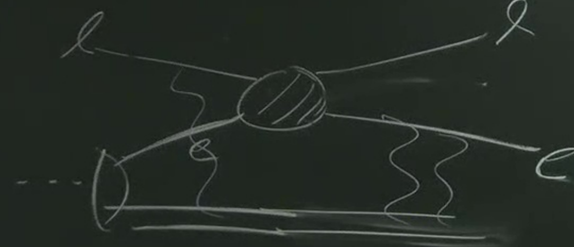
$(g-2) \Leftrightarrow$ HVP

MuonE

10^{-5}

10 ppm

Microscopically



• Expand around $\vec{p}_e = 0$

Questions

↳ What controls corr.?

$\sim \cancel{\frac{Z\alpha\, m_e}{\omega}}$ $\quad \left(\frac{p_e}{m_e}\right)^2$

$S \simeq 2p\cdot k$

↳ Do they ever Matter? ✓

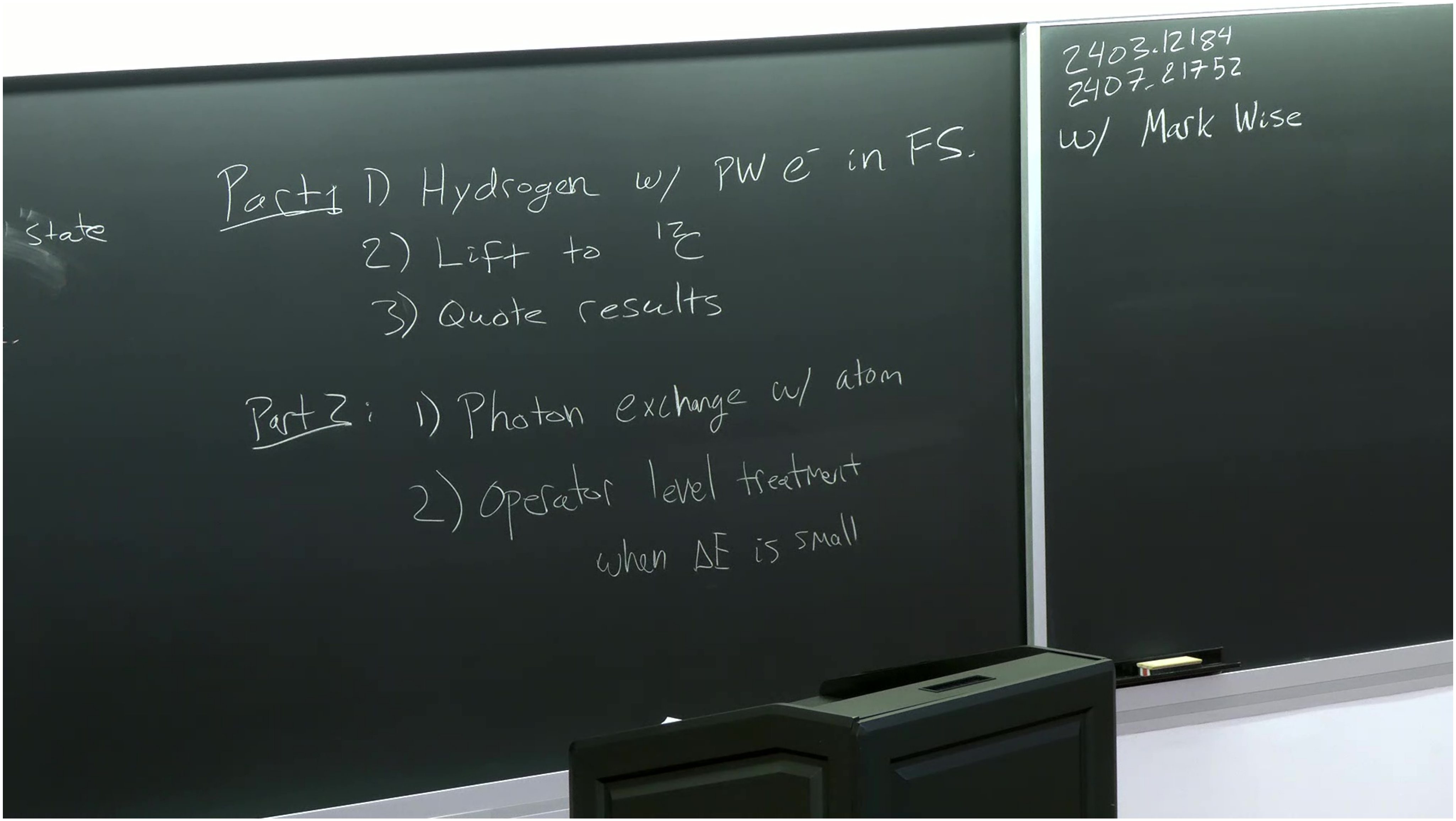
↳ How to control RCs?

Electron is in a bound state

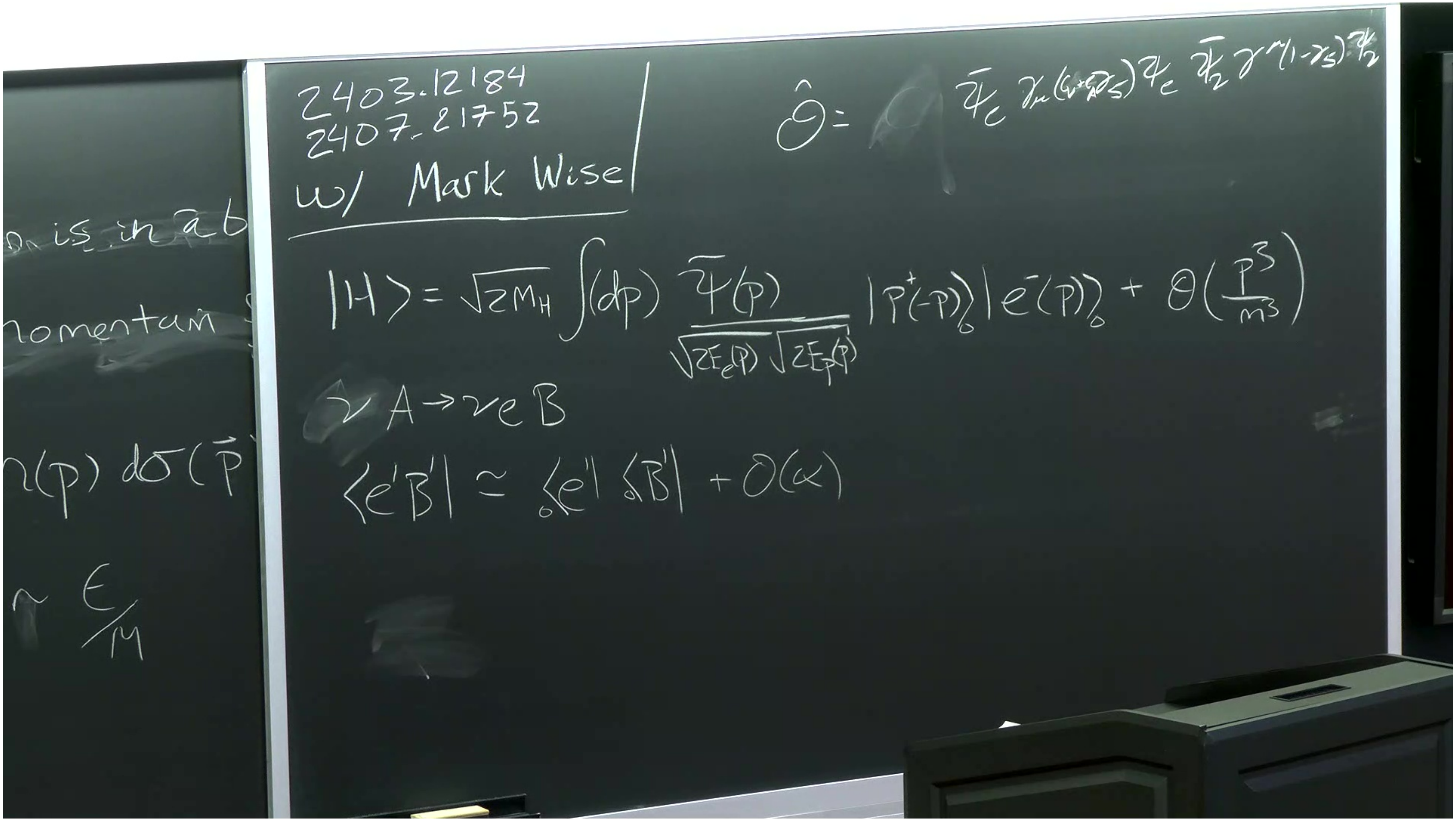
$\exists$ momentum fluc.

$$\int (dp)\, n(p)\, d\sigma(\vec{p})$$

$$\frac{p^2}{M^2} \sim \frac{\epsilon}{M}$$



State
Part 1 1) Hydrogen w/ PW e^- in FS.
2) Lift to ^{12}C
3) Quote results
Part 2: 1) Photon exchange w/ atom
2) Operator level treatment
when ΔE is small
2403.12184
2407.21752
w/ Mark Wise



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w/ Mark Wise
$\hat{\mathcal{O}} = \bar{\psi}_e \gamma_\mu (g_V + g_A \gamma_5) \psi_e \, \bar{\psi}_\nu \gamma^\mu (1-\gamma_5) \psi_\nu$
$|H\rangle = \sqrt{2M_H} \int (dp) \frac{\Psi(p)}{\sqrt{2E_e(p)}\sqrt{2E_p(p)}} |p^+(-p)\rangle_0 |e^-(p)\rangle_0 + \mathcal{O}\left(\frac{p^3}{m^3}\right)$
$\nu A \to \nu e B$
$\langle e' B'| \simeq {}_0\langle e'| \, {}_0\langle B'| + \mathcal{O}(\alpha)$
is in a b
momentum
$n(p)\, d\sigma(\vec{p}$
$\sim \epsilon / M$

Sketch of Problem

$$\mathcal{M} = {}_0\langle e'\nu | \hat{O} | e\nu\rangle_0 \qquad |\mathcal{M}|^2 \sim |\tilde{\psi}(p)|^2 \, |M|^2 \, \frac{2M_H \, 2E_p}{2E_e}$$

Function of 3-momentum

$$p_\mu = \left(\sqrt{m_e^2+\vec{p}^{\,2}}, \vec{p}\right)$$

$$\boxed{p\cdot k,\ p\cdot k',\ p\cdot p'} \sim \mathcal{O}(m_e\omega)$$

$$p'\cdot k,\ p'\cdot k',\ k\cdot k'$$

$$|\mathcal{M}|^2 \frac{2M_H 2E_p}{2E_e}$$

Questions

$$\int d\Pi_e \, d\Pi_p \, \frac{1}{2\omega'} (2\pi)\delta(\Sigma E)$$

$$u_\mu = (1,0,0,0) \qquad \Sigma E = (\omega + m_e - \epsilon) - (E_e' + \omega')$$

$$m_H = m_e + m_p - \epsilon$$

$$\frac{p^2}{m^2} \sim \frac{\epsilon}{M}$$

$$k + p + \langle V \rangle u = k' + p'$$

$$\langle V \rangle \simeq -\epsilon - \left(\sqrt{m_e^2 + \vec{p}^{\,2}} - m_e\right)$$

$\nu e \to \nu e$

$$\frac{1}{2m}\langle |M_0|^2\rangle\left(1 + \frac{1}{3}\frac{p^2}{m_e^2} - \frac{\epsilon}{m_e}\right)$$

$\mu e \to \mu e$

$$\frac{1}{2m}\left(\langle |M_0|^2\rangle\left(1 + \frac{\epsilon}{m_e}\right) - \frac{2e^4 m_\mu^2}{(p\cdot p')_\mu}\left[\frac{\epsilon}{m_e} + \frac{1}{3}\frac{p_e^2}{m_e^2}\right]\right)$$

$\omega')$

$$\frac{p^2}{m^2} \sim \epsilon/M$$

Sketch of Problem

$$\int (dp)\, |\tilde{\psi}(p)|^2 \left(\ldots \right)$$

$$\int d\epsilon \int (dp)\; S_A(\epsilon, p) \left(\ldots\ \ldots \right)$$

$$M_A - M_B \equiv m_e - \epsilon_{AB}$$

$$S_A(\epsilon, p) = \int \frac{d^3 q}{(2\pi)^3} \langle A | a_p^{\dagger}\, \delta(\epsilon_A + \hat{H} - \epsilon)\, a_q | A \rangle$$

$$\int d\epsilon \int (dp)\, S_A(\epsilon,p) = Z$$

$$\int d\epsilon \int (dp)\, S_A(\epsilon,p)\, \frac{p^2}{2m} = \langle \hat{T} \rangle_A$$

$$\int d\epsilon \int (dp)\, (-\epsilon)\, S_A(\epsilon,p) = \langle \hat{T} \rangle_A + \langle \hat{V}_{1B} \rangle_A + 2\langle \hat{V}_{2B} \rangle_A$$

$$\left\langle \sum_i \frac{1}{\hat{r}_i} \right\rangle$$

$$\langle T \rangle_A = \epsilon_A$$

$$\langle \hat{V}_{1B} + \hat{V}_{2B} \rangle = -2\epsilon_A$$

$\nu e \to \nu e$

$\frac{1}{2m}\langle |M_0|^2 \rangle (1 +$

$\mu e \to \mu e$

$\frac{1}{2m}(\langle |M_0|^2 \rangle ($

-2

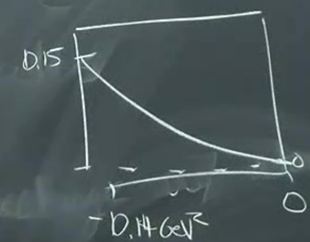
νe

$$\ldots \rangle \left(1 + \frac{1}{3}\frac{p^2}{m_e^2} - \frac{\epsilon}{m_e} \right)$$

μe

$$\langle |M_0|^2 \rangle \left(1 + \frac{\epsilon}{m_e} - \frac{2 e^4 m_\mu^2}{(p\cdot p')_\mu}\left[\frac{\epsilon}{m_e} + \frac{1}{3}\frac{p_e^2}{m_e^2} \right] \right)$$

$$\frac{1}{\sigma}\frac{d\sigma}{dt} = \frac{1}{\sigma_0}\frac{d\sigma_0}{dt}\bigg|_{\mu e \to \mu e} + (45 \times 10^{-5})\, \underline{f(t)}$$



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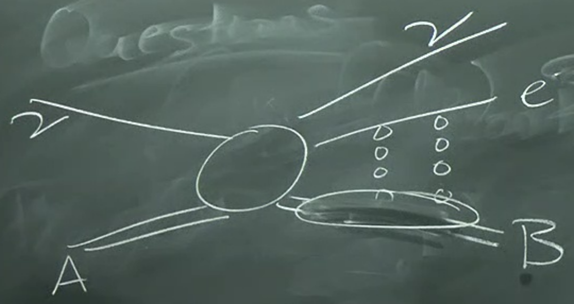
w/ Mark Wise

$\hat{\mathcal{O}} = \ldots \tilde{\Psi}_c$

$$|H\rangle = \sqrt{2M_H}\int (dp)\, \frac{\tilde{\Psi}(p)}{\sqrt{2E_e(p)}\sqrt{2E_p(p)}}\, |p^+(-p)\rangle_0 |e \ldots$$

$$\nu\, {}^1H \to \nu e p^+$$

$$\langle e' p^{+\prime}| \simeq {}_0\langle e'|\, {}_0\langle p^{+\prime}| + \mathcal{O}(\alpha)$$



$\hookrightarrow$ Dominant region

$\vec{q} \sim Z^{1/3}\alpha\, m_e$

$$\frac{1}{2P\cdot q + q^2} \sim \frac{1}{2P\cdot q}$$

$$\langle \alpha\, \beta^2 \rangle \sim \underline{Z^{2/3}\alpha^3}$$

$\beta \sim Z^{1/3}\alpha$

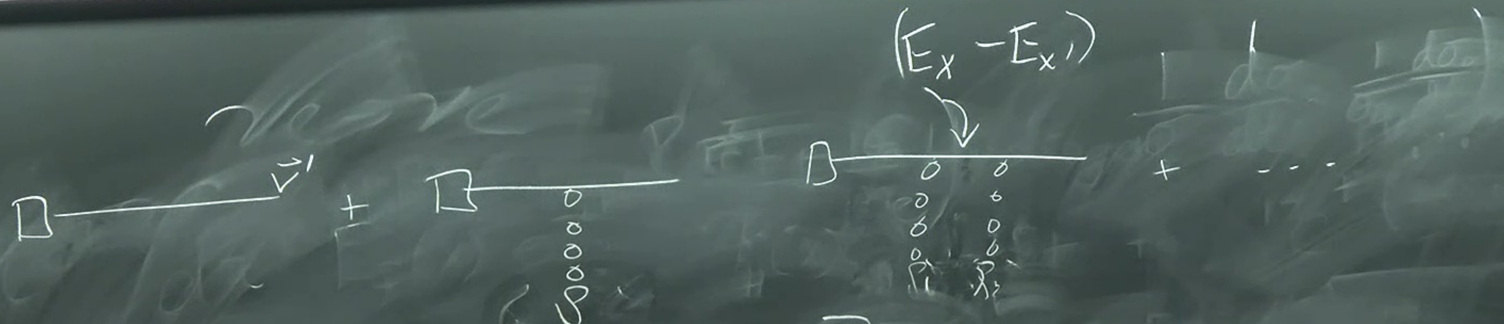
$$\mathcal{M} \sim \int d^3x \;\; E_f(x)$$

$$\int (d\Pi)\, |\mathcal{M}|^2 = \int dy\, dx \;\; E_f^*(y)$$

$$\mathcal{M} \sim \int d^3x \; E_f(x)\, e^{-i\vec{k}\cdot x}\, \psi_H(x)$$

$$\sim \underline{Z^{2/3}\,\alpha^3}$$

$$\int (d\vec{k}) \, |\mathcal{M}|^2 = \int dy\, dx \; E_f^*(y)\, E_f(x)\, \psi_H^*(x)\, \psi_H(x) \underline{\int d\vec{k}\; e^{-i\vec{k}\cdot(\vec{x}-\vec{y})}}$$

$W_{\vec{v}'}$ 

$\varepsilon^{2/3} \alpha^3$

$$W_{\vec{v}'} = \exp\left[-ie^2 \int_0^\infty ds\, \hat{V}(s\vec{v}')\right]$$

Sketch of Problem

$S_A \to \tilde{S}_A$

$\int (dp)\, |\tilde{\psi}(p)|^2 (\ldots)$ $\quad a^\dagger, a \to a^\dagger W^\dagger, \quad W a$

$= \int d\epsilon \int (dp)\, S_A(\epsilon, p) (\ldots\ \ldots)$

$S_A(\epsilon, p) = \int \frac{d^3q}{(2\pi)^3} \langle A | a_p^\dagger\, \delta(\epsilon_A + \hat{H} - \epsilon)\, \hat{a}_q | A \rangle$