Title: Quantum Gravity and Black Hole Evaporation

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Abstract:

Hawking's seminal result, that black holes behave as black bodies with a non-vanishing temperature, suggests that black holes should evaporate. However, Hawking's derivation is incomplete, as it neglects the backreaction between radiation and geometry. In this talk, we will present a novel approach to black hole perturbation theory that incorporates backreaction and is valid to arbitrary order. The applications to the physics of evaporating black holes is discussed, and we explore potential experimental implications. The intention is to eventually derive corrections to semi-classical computations in the literature and to determine the fate of evaporating black holes.

Quantum Gravity and Black Hole Evaporation

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Content

1. Motivation
2. Perturbation theory in reduced phase space formulation
3. Application to black holes and black hole evaporation

Motivation

Quantum gravity aims to address fundamental questions about gravity:

- Breakdown of general relativity (GR) at singularities (physical quantities become infinite)
- Dynamical spacetime of GR incompatible with fixed spacetime in quantum field theory (QFT)
- Quantisation of GR using standard techniques of QFT unpredictive (non-renormalisable)

Challenges for Quantum Gravity:

- Lack of experimental guidance: So far no experimental observation of quantum gravity
- Development of suitable experiments to test quantum gravity theories
- Mathematical Complexity: Quantisation of dynamical spacetimes in full generality very complicated

Therefore

- Study consequences of quantum gravity for astrophysical objects, e.g. black holes
- Exploit symmetries and use perturbation theory to simplify mathematics

Classical Theory of Black Holes

- Black holes form in the universe through gravitational collapse of matter
- Completely characterised by mass, charge and angular momentum
- Simplest case: Schwarzschild black hole (non-rotating, uncharged)

Important theorem in classical general relativity:

Singularity Theorem [Hawking & Penrose]

Assume:

- Globally hyperbolic spacetime with connected Cauchy surface
- Matter satisfies strong energy condition
- Spacetime contains a trapped surface

Then, spacetime is null geodesic incomplete

- Singularities inside black holes consequence of GR (failure of classical GR)

- Furthermore: Area Theorem [Hawking], area of the event horizon never decreasing
- Black holes stable objects

Quantum Field Theory on Black Hole Spacetimes

Idea: Study the properties of quantum fields on classical spacetimes

- **Hawking effect**: Emission of radiation with a black body spectrum (temperature $T_H \propto M^{-1}$)
- Decrease of black hole mass (**backreaction**) $\rightarrow$ Violation of classical area theorem

- Backreaction not fully understood (no first-principle derivation)
- Simple heuristic argument based on Stefan Boltzmann law: $P \propto AT^4 \propto M^{-2}$
- Change of black hole mass given by emitted power: $\dot{M} = -P \propto -M^{-2}$
- Black hole lifetime $t_{\text{evap}} \propto M^3$

More detailed calculation including backscattering effects [Page 1976]:

- Modifications to thermal spectrum at small energies of the particles (**graybody factors**)
- Neutrinos and photons important messengers for the study of Hawking evaporation
- Secondary interactions of emitted particles important for observations

However no first-principle derivation using minimal assumptions

Challenges for the Study of Black Holes in Quantum Gravity

Symmetries:

- Solutions of Einstein equations only available using symmetry assumptions
- Schwarzschild black hole: rotational symmetry; Kerr black hole: axial symmetry
- Perturbation theory to treat non-symmetric degrees of freedom analytically

Gauge Invariance:

- GR is a gauge theory $\rightarrow$ redundancies in theoretical description
- Hamiltonian formulation of GR fully constrained
- Determine observables, i.e. gauge-invariant variables to study the physics

Backreaction:

- Non-symmetric variables influence the dynamics of the symmetric variables
- Example: Hawking radiation as part of non-symmetric variables and black hole mass as part of symmetric variables

Black Holes in Canonical Quantum Gravity

In Loop Quantum Gravity (LQG) black holes are extensively studied: [Agullo, Ashtekar, Bodendorfer, Corichi, Gambini, Giesel, Han, Haggard, Husain, Liu, Mena Marugán, Olmedo, Pullin, Rovelli, Singh, Vidotto, Weigl, Wilson-Ewing, ...]

- **Symmetry reduction**: Classical restriction to spherically symmetric degrees of freedom
- Take advantage of description of black hole interior as Kantowski-Sachs cosmology
- Using tools from Loop Quantum Cosmology (LQC): Singularity is resolved

Singularity resolution also using other methods:

- Modified classical equations of motion and analysis of dust collapse scenarios [Fazzini, Giesel, Husain, Kelly, Liu, Santacruz, Weigl, Wilson-Ewing, ...]
- Study of black hole to white hole transitions using spin foam numerics [Dona, Vidotto, Rovelli, ...]

LQG Models:

- Neglecting non-symmetric degrees of freedom
- No backreaction is considered: Cannot study evaporating black holes
- Usually treat gauge invariance by solving the quantum constraint equations

Our approach:

- Inclusion of non-symmetric degrees of freedom
- Interaction between non-symmetric and symmetric degrees of freedom
- Reduced phase space formulation to solve constraints classically

Recent development: Perturbations in Kantowski Sachs interior of black hole [Mena Marugán, Mínguez Sánchez 2024]

Our Approach to Perturbation Theory

The procedure works as follows

1. Fix symmetry group of class of exact solutions
   - Action of symmetry group defines invariant (symmetric) degrees of freedom: background (q, p, Q, P)
   - Non-invariant (non-symmetric) degrees of freedom: perturbations (x, y, X, Y)
   - Split test functions: symmetric test functions f and non-symmetric test functions g
   - Split constraints: symmetric constraints C and non-symmetric constraints Z
2. Split symmetric and non-symmetric variables into observable (true) and non-observable (gauge) degrees of freedom. Notation:

	Observable	**Non-Observable**	
Symmetric	(Q, P)	(q, p)	"background"
Non-Symmetric	(X, Y)	(x, y)	"perturbations"

   - Choose one pair of gauge degrees of freedom for every constraint
   - Choice arbitrary; often motivated by physical situation
   - Physics completely described by observable degrees of freedom
3. Apply reduced phase space formulation
   - Select gauge fixing conditions, $q = q_*$ and $x = x_*$
   - Solve symmetric constraints C for p ($p = p_*$) and non-symmetric constraints Z for y ($y = y_*$)
   - Analytic solution of constraints not available $\rightarrow$ Perturbative solution
   - Determine f_*, g_* through stability condition of the gauge fixing
   - Reduced phase space coordinatised by Q, P, X, Y

Our Approach to Perturbation Theory

4. Boundary term analysis
   - Spacetime usually includes boundaries (e.g. boundary at infinity for black holes)
   - Define decay properties of fields (e.g. asymptotic flatness)
   - For well-defined variational principle, need to add counter boundary term $B(f,g)$ to constraints
   - Distinguish between gauge transformations ($B(f,g)=0$) and symmetry transformations ($B(f,g)\neq 0$)

5. Calculate the physical Hamiltonian H (in the presence of boundaries):
   - For any function $F(Q,P,X,Y)$ of the true degrees of freedom define H as

$$\{H,F\} = \{C(f)+Z(g)+B(f,g),F\}_{p=p_*,q=q_*,f=f_*,y=y_*,x=x_*,g=g_*}$$

   - Assume boundary term of the form $B(f,g) = f\cdot j_f + g\cdot j_g$ with boundary currents j_f, j_g
   - Rewrite variation as total differential on reduced phase space: $f_*\cdot\delta j_{f*} + g_*\cdot\delta j_{g*} = \delta H$

Remarks:

- Knowledge of boundary term crucial for derivation of physical Hamiltonian
- GR fully constrained: On constraint surface only boundary terms remain
- Note that $H\neq B(f_*,g_*)$ in general (f_*, g_* could contain canonical variables)

Our Approach to Perturbation Theory

Key Advantage:

- Formalism applicable to many scenarios: cosmology, Schwarzschild and Kerr black holes
- Disentangle the definition of observables from perturbation theory
- No need to discuss gauge invariance at every order (no consensus in the literature)
- Hamiltonian H computable in X, Y to any order: $H = H_0 + H_1 + H_2 + \ldots$
- Full reduction and derivation of a physical Hamiltonian $\rightarrow$ No constraints in quantum theory
- No issues with closure of the constraint algebra in the quantum theory (no anomalies)

Application to Schwarzschild Black Holes – Step 1

Step 1 – The symmetry group:

- Page showed: Angular momentum radiated away faster than mass
- Restrict to spherical symmetry (rotation group $SO(3)$): Schwarzschild black hole
- Work in ADM formalism (induced metric $m_{\mu\nu}$ and conjugate momentum $W^{\mu\nu}$, $\mu = 1, 2, 3$)
- Spherical symmetry → spherical harmonics L_{lm}
- Expand $m_{\mu\nu}$, $W^{\mu\nu}$ in terms of spherical scalar, vector and tensor harmonics:

[Freeden, Gervens, Gutting, Schreiner]

$$m_{33} = q_v + \sum_{l\geq 1,m} x^v_{lm} L_{lm} \qquad \frac{W^{33}}{\sqrt{\Omega}} = p_v + \sum_{l\geq 1,m} y^v_{lm} L_{lm}$$

$$m_{3A} = \sum_{l\geq 1,m,I} x^I_{lm} [L_{I,lm}]_A \qquad \frac{W^{3A}}{\sqrt{\Omega}} = \frac{1}{2} \sum_{l\geq 1,m,I} y^I_{lm} L^A_{I,lm}$$

$$m_{AB} = q_h \Omega_{AB} + \sum_{l\geq 1,m} x^h_{lm} [L_{h,lm}]_{AB} + \sum_{l\geq 2,m,I} X^I_{lm} [L_{I,lm}]_{AB} \qquad \frac{W^{AB}}{\sqrt{\Omega}} = \frac{p_h}{2} \Omega^{AB} + \frac{1}{2} \sum_{l\geq 1,m} y^h_{lm} L^{AB}_{h,lm} + \sum_{l\geq 2,m,I} Y^I_{lm} L^{AB}_{I,lm}$$

where Ω_{AB} the metric on the sphere and $\sqrt{\Omega} = \sqrt{\det \Omega}$

- Background degrees of freedom spherically symmetric: (q, p)
- Perturbation degrees of freedom: (x, y) and (X, Y)
- Symmetric constraints: symmetric Hamiltonian constraint C_v and symmetric radial diffeomorphism constraint C_h
- Non-symmetric constraints: non-symmetric Hamiltonian constraint Z_v, non-symmetric radial diffeomorphism constraint Z_h and angular diffeomorphism constraints $Z_{e/o}$

Application to Schwarzschild Black Hole – Steps 2 & 3

Step 2 – Selection of gauge degrees of freedom:

	True	**Gauge**
Symmetric	M	(q, p)
Non-Symmetric	(X, Y)	(x, y)

with M the Schwarzschild mass

Step 3 – The reduced phase space formulation:

- Choose Gullstrand Painlevé (GP) gauge

$$m_{33} = 1, \qquad m_{3A} = 0, \qquad \Omega^{AB} m_{AB} = 2r^2 \,,$$

  where $\Omega_{AB} = \mathrm{diag}(1, \sin^2\theta)$ is the metric on the sphere
- GP gauge implies $q_v = 1$, $q_h = r^2$ and $x = 0$
- Advantage of GP coordinates: non-singular at black hole horizon; Explore interior and exterior of black hole simultaneously
- Can work with 2 asymptotic ends (black to white hole transition)
- Solution of constraints: Solve C_v and C_h for q_v, q_h and the constraints Z for the y's
- In this step: Iterative solution $p_v = p_v^{(0)} + p_v^{(1)} + \dots$ and similarly for p_h and y ($p_v^{(i)}$ is a polynomial of degree i in X, Y)

Application to Schwarzschild Black Hole – Step 3

Solution of the constraints order by order for the non-rotating black hole:

Zeroth Order:

- Only symmetric constraints C_v and C_h (Integral of non-symmetric quantity over the sphere vanishes)
- The solution depends on an integration constant M:

$$p_v^{(0)} = 2\sqrt{2Mr}$$

$$p_h^{(0)} = \frac{1}{r^2}\sqrt{2Mr}$$

- Above expressions agree with Schwarzschild solution in GP coordinates

First Order:

- Only non-symmetric constraints non-vanishing
- Determine $y_v^{(1)}, y_h^{(1)}, y_{e/o}^{(1)}$ as linear functions of X, Y

Second Order:

- For 2nd order physical Hamiltonian: Only need to consider the second order symmetric constraints
- We obtain a solution for $p_v^{(2)}, p_h^{(2)}$ quadratic in X, Y

Application to Schwarzschild Black Hole – Step 4 & 5

Step 4 – Boundary Term analysis:

- Black hole spacetime has asymptotic boundary at infinity
- Impose asymptotic flat boundary conditions on canonical variables
- Non-perturbative expression for the boundary term

Step 5 - The physical Hamiltonian (for one asymptotic end):

- Asymptotic evaluation of the stability condition possible non-perturbatively
- Combination of exact results for f_*, g_* and the boundary term gives the physical Hamiltonian:

$$H = \lim_{r\to\infty} \frac{2\pi}{\kappa r} p_v^2 = \lim_{r\to\infty} \frac{2\pi}{\kappa r}\left((p_v^{(0)})^2 + 2p_v^{(0)}p_v^{(2)} + O(3)\right),$$

  where $\kappa = 16\pi$ is the gravitational coupling constant
- Zeroth order: $H_0 = M$ (ADM mass)
- Second order:

$$H_2 = \frac{1}{\kappa}\sum_{l\geq 2,m,I}\int_{\mathbb{R}+} \mathrm{d}r\, N^3 \tilde{Y}^I_{lm}\partial_r \tilde{X}^I_{lm} + \frac{N}{2}\left((\tilde{Y}^I_{lm})^2 + (\partial_r \tilde{X}^I_{lm})^2 + V_I(\tilde{X}^I_{lm})^2\right),$$

  where $N^3 = \sqrt{\frac{2M}{r}}$, $N = 1$ and $\tilde{X}, \tilde{Y}$ related to X, Y via canonical transformation
- V_I: Regge-Wheeler-Zerilli potential

Discussion of the Classical Perturbation Theory

Black Hole perturbation theory well established to second order both in Lagrangian [Regge, Wheeler, Zerilli,...] and Hamiltonian formulation [Moncrief,Brizuela,Martín-García,...]

Standard Perturbation Theory:

- Perturbation theory on full phase space
- Definition of gauge-invariance order by order
- No consensus in the literature beyond second order
- Backreaction neglected

Our Approach:

- Non-perturbative definition of observables
- Perturb only with respect to fully gauge-invariant observables
- Well-defined observables to any order in perturbation theory
- Backreaction included

Next step: Quantisation of the observables with respect to Gullstrand Painlevé free-falling observer [work in progress]

- Quantise the perturbations using a Fock quantisation
- Mode functions: eigenvalue equation similar to Schrödinger equation for singular potential
- Possilbe regularisation at the singularity ($r = 0$):

→ New orthonormal basis for singular Schrödinger operators [JN & TT]

→ Methods of LQC type quantisation of Kantowski Sachs [Ashtekar, Bodendorfer, Gambini, Haggard, Olmedo, Pullin, Rovelli, Singh, Vidotto]

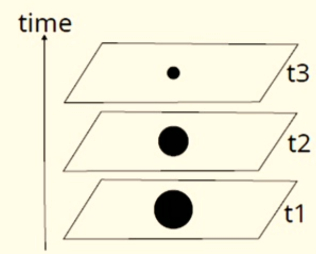
→ Methods of dust collapse models [Fazzini, Giesel, Husain, Kelly, Liu, Santacruz, Weigl, Wilson-Ewing]

Physics of Evaporating Black Hole

Perturbative access to **apparent horizon**; observer-dependent alternative to event horizon

- Consider a foliation by Cauchy surfaces Σ_t and closed, oriented 2-surfaces $S_t \subset \Sigma_t$
- Σ_t has normal n, and S_t has normal s tangential to Σ_t
- Future outward/inward oriented null vectors $l_\pm = n \pm s$ define outward/inward null geodesic congruences with expansion $\theta_\pm$
- Trapped surfaces in S_t: $\theta_+ = 0$; apparent horizon: union of all trapped surfaces
- Perturbative evaluation of the trapping condition, calculation of the shape of the apparent horizon
- Area of the apparent horizon interesting observable:

$$A = 4\pi r_s^2 + r_s \int_0^{r_s} m(r)\,\mathrm{d}r$$



- $m(r)$: local mass density depending on non-symmetric degrees of freedom
- For additional matter (scalar field, neutrinos, ...) more background observables (backreaction)
- Classical Theory: Area cannot decrease (area theorem)
- Quantum Theory: Decrease of area (Hawking effect, quantum violation of energy inequalities)

Physics of Evaporating Black Hole

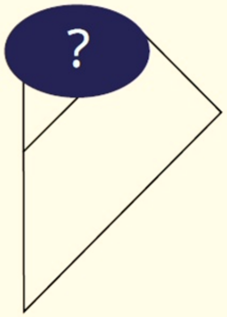
Perturbative access to **effective metric**:

- Perturbative solution of the bulk stability condition
- Metric known in terms of symmetric and non-symmetric degrees of freedom
- In quantum theory: Computation of expectation values in a given state

→ Effective classical metric

Calculation of **quantum Penrose diagram**:

- Calculate Penrose diagram of effective metric in the usual way
- Possible modifications of global picture and causal structure of spacetime
- Determine the fate of evaporating black holes
- Calculate expectation values of area of apparent horizon



Final Stage of Hawking Evaporation

Several conjectures for final stage of Hawking evaporation:

Complete Evaporation [Boluna, Profumo, Blé, Hennings (2024)]

- Black hole completely disappears
- What happens with the singularity?
- Resolution of the black hole information paradox?

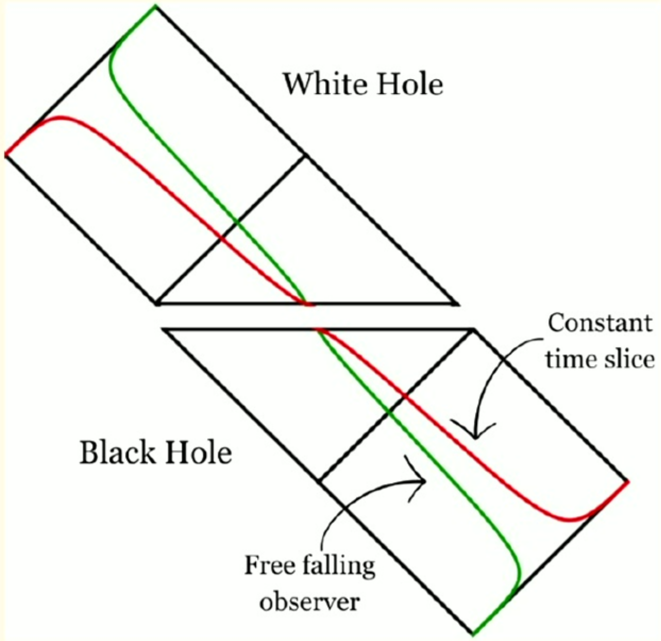
Stable Remnants [Rovelli, Vidotto (2014); Carr, Kuhnel (2021)]

- Black Hole reaches a minimal size called black hole remnant
- Remnant is stabilized by quantum effects
- Possible candidate for dark matter

Final Stage of Hawking Evaporation

Transition of black hole to white hole [Haggard, Rovelli (2015); Rovelli, Soltani (2023), Thiemann (2024)]

- GP coordinates cover out-/ingoing Eddington-Finkelstein coordinates
- Glue both solutions at the singularity (quantum transition region)
- Constant time slices: Cauchy surfaces
- Interpretation as Quantum tunneling of black to white hole



Black hole white hole transition [Thiemann (2024)]

Experimental Constraints on Primordial Black Holes

Important for observations:

- Neutrinos and gamma rays important part of Hawking radiation [Page (1976)]
- Study black holes that formed shortly after Big Bang (**primordial black holes** (PBH))
- PBH in final phase of evaporation process, possible explosion with a burst of radiation

Current observations:

- Upper bounds on PBH burst events from gamma ray observations: $\sim 2000\,\mathrm{pc}^{-3}\mathrm{year}^{-1}$
  [H.E.S.S., HAWC & Fermi-LAT collaborations]
- More data needed to further constrain PBH

Our plan: Derivation of **templates** for evaporation bursts from first principle computations

Conclusion

Summary:

- Potential observation of quantum gravity with evaporating black holes
- Development of a novel, first principle approach to black hole evaporation including
  - backreaction
  - exterior and interior
  - quantised gravity
  - perturbation-independent definition of observables
  - expansion to arbitrary order
- Study of quantum physics using apparent horizons and effective metric [still under development]
- So far: Agreement with classical perturbation theory without backreaction [Regge, Wheeler, Zerilli, ...]

Future Work:

- Generalisation to higher order perturbations: Interacting gravitational waves [work in progress]
- Study of boundary observables at infinity (BMS group) to constrain quantum theory
- Extension to Standard Model matter (neutrinos, scalar fields, ...)
- Generalisation to axial symmetry (Kerr black hole)
- Quantisation of background and perturbations
- Study the expectation values of the area of the apparent horizon in quantum theory
- Calculation of effective metric and quantum Penrose diagram
- Derivation of templates for the evaporation burst of evaporating black holes

Thank You!