Title: Vortex lines and dg-shifted Yangians

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Abstract:

I'll discuss the representation theory of line operators in 3d holomorphic-topological theories, following recent work with Wenjun Niu and Victor Py. Examples of the line operators we have in mind include half-BPS lines in 3d N=2 supersymmetric theories (reinterpreted in a holomorphic twist). We compute the OPE of line operators, which endows the category with a meromorphic tensor product, and establish a perturbative nonrenormalization theorem for the OPE. Then, applying Koszul-duality methods of Costello and Costello-Paquette, we represent the category of lines as modules for a new sort of mathematical object, which we call a dg-shifted Yangian. This is an A-infinity algebra, with a chiral coproduct whose data includes a Maurer-Cartan element that behaves like an infinitesimal r-matrix. The structure is a cohomologically shifted version of the ordinary Yangians that represent lines in 4d holomorphic-topological theories.

Vortex lines & dg-shifted Yangians

Perimeter Institute, 5 Dec. 2024

Tudor Dimofte, University of Edinburgh
w/ Wenjun Niu (PI), Victor Py (Edinburgh)

Basic setup

locally $\mathbb{C}_z \times \mathbb{R}_t$

- Focus on holomorphic-topological (HT) QFT's in d = 3

  E.g. generic twist (minimal BPS sector) of 3d N=2 QFT

- Recall: local operators $\mathcal{A}$ form a (-1)-shifted Poisson vertex algebra [Oh-Yagi '19]

  Gradings: $r \in \mathbb{Z} \quad (\text{or } \mathbb{Q} \text{ or } \mathbb{R})$ cohomological (i.e. ghost or R-charge)

  $j \in \frac{1}{2}\mathbb{Z} \quad (\text{or } \mathbb{Q} \text{ or } \mathbb{R})$ spin (i.e. conformal grading)

  In 3d N=2, $\mathcal{A}$ = quarter-BPS local ops, counted in 3d index

$$I = \mathrm{Tr}_{\mathcal{A}}(-1)^r q^j$$

  N.b. $\mathcal{A}$ can have nontrivial higher operations… cf. [Costello-TD-Gaiotto '20] [Garner-Williams '23] [Gaiotto-Kulp-Wu '24] [Alfonsi-Kim-Young '24]

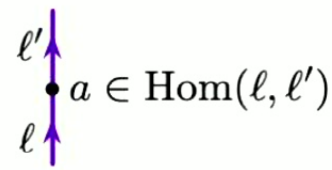
Basic setup

locally $\mathbb{C}_z \times \mathbb{R}_t$

- Focus on holomorphic-topological (HT) QFT's in d = 3
- Recall: local operators $\mathcal{A}$ form a (-1)-shifted Poisson vertex algebra $\begin{matrix} r \in \mathbb{Z} \\ j \in \frac{1}{2}\mathbb{Z} \end{matrix}$ (or $\mathbb{Q}$ or $\mathbb{R}$)

  N.b. $\mathcal{A}$ can have nontrivial higher operations…

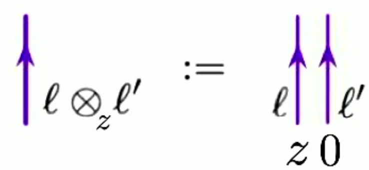
- Line operators, locally at $\{z\} \times \mathbb{R}_t$ (3d N=2: half-BPS lines)
  - form a category $\mathcal{C}$



(3d N=2: quarter-BPS junction)

$\ell, \ell' \in \mathrm{Ob}(\mathcal{C})$

  - with an OPE $\otimes_z : \mathcal{C} \boxtimes \mathcal{C} \to \mathcal{C}$



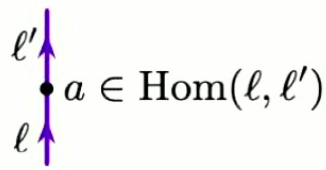
    a coherent *family* of functors over $z \in \mathbb{C}^*$

cf. "meromorphic tensor categories" [Soibelman '97]

"chiral categories" [Beilinson-Drinfeld '04, Gaitsgory '08, Raskin '13]

Lots of lines

$\mathbb{C}_z \times \mathbb{R}_t$: - form a category $\mathcal{C}$



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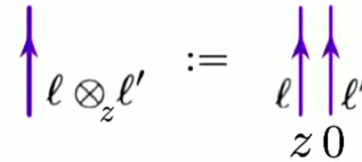


Compare… 3d fully topological theory

$\mathbb{R}^2 \times \mathbb{R}_t$: - lines form a category, w/ a *locally constant* family $\otimes_z \quad z \in \mathbb{C}^*$

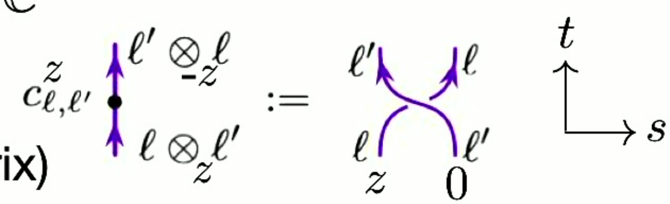
= a single tensor product and its braiding

e.g. reps of a quantum group (bialgebra + R-matrix)

4d HT QFT

$\mathbb{C}_z \times \mathbb{R}_s \times \mathbb{R}_t$: - lines form a category, w/ an OPE $\otimes_z \quad z \in \mathbb{C}^*$

and a spectral braiding

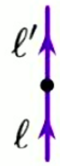


e.g. reps of a Yangian (vertex coalgebra + spectral R-matrix)

[Costello '13, Costello-Witten-Yamazaki '17,'18]

Lots of lines

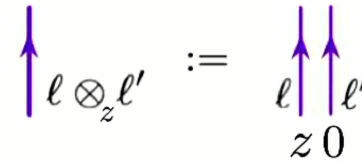
$\mathbb{C}_z \times \mathbb{R}_t$: - form a category $\mathcal{C}$



$a \in \mathrm{Hom}(\ell, \ell')$

$\ell, \ell' \in \mathrm{Ob}(\mathcal{C})$

- with an OPE $\otimes_z : \mathcal{C} \boxtimes \mathcal{C} \to \mathcal{C} \quad z \in \mathbb{C}^*$



Goals:

How to describe $\mathcal{C}$ concretely, e.g. in twists of 3d N=2 theories ?

How to compute the OPE? (What does it even mean?)

What algebraic object represents lines in a 3d HT QFT?

Answer: a dg-shifted Yangian (to be defined)

- A perturbative HT theory, quantized in BV-BRST, takes the schematic form

$$S = \int_{\mathbb{C}\times\mathbb{R}} \mathbf{p}_i d'\mathbf{q}^i + W(\mathbf{p},\mathbf{q},\partial) \qquad d' = \underset{\text{odd}}{\partial_t dt} + \underset{\text{odd}}{\partial_{\bar z} d\bar z} \qquad \partial = \underset{\text{even}}{\partial_z dz}$$

where **p**, **q** are multiforms that collect the free fields and their HT descendants

(for each *i*)

$$\mathbf{q} \in \Omega'^{\bullet}(\mathbb{C}\times\mathbb{R})dz^{j(\mathbf{q})}[r(\mathbf{q})]$$
$$= (q^{(0)} + q^{(1)}_{\bar z} d\bar z + q^{(1)}_t dt + q^{(2)}_{\bar z t} d\bar z dt) dz^j$$
$$\mathbf{p} \in \Pi\Omega'^{\bullet}(\mathbb{C}\times\mathbb{R})dz^{j(\mathbf{p})}[r(\mathbf{p})]$$

$$r(d') = 1 \qquad j(\partial) = 1$$
$$r(\text{action}) = 2 \qquad j(\text{action}) = 1$$
$$\Rightarrow \quad r(\mathbf{p}) + r(\mathbf{q}) = 1 \qquad r(W) = 2$$
$$j(\mathbf{p}) + j(\mathbf{q}) = 1 \qquad j(W) = 1$$

- Examples: 3d N=2 chirals X^i w/ superpotential W(X)

$$S = \int_{\mathbb{C}\times\mathbb{R}} \Psi_i d' X^i + W(X)$$

3d N=2 gauge theory + CS + matter

$$A:\ r = 1,\ j = 0,\ \text{odd}$$
$$B:\ r = 0,\ j = 1,\ \text{even}$$

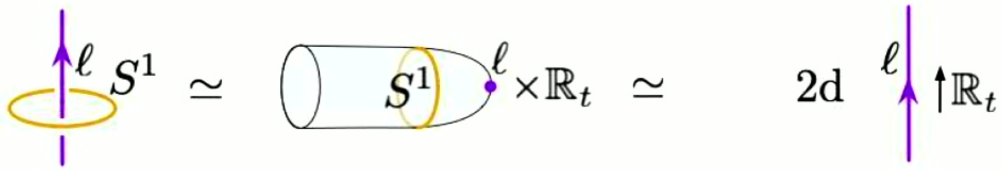
$$S = \int_{\mathbb{C}\times\mathbb{R}} BF'(A) + k\mathrm{Tr}(A\partial A) + \Psi d'_A X + W(X) \ = \int_{\mathbb{C}\times\mathbb{R}} Ad'B + \Psi d' X + B[A,A] + k\mathrm{Tr}(A\partial A) + \ldots$$

Lines via state-op

Reduce on the link of $\{0\} \times \mathbb{R}_t$ (where putative line ops would be inserted)
(a circle)

to get an effective 2d topological theory $\mathcal{T}_{2d}$.

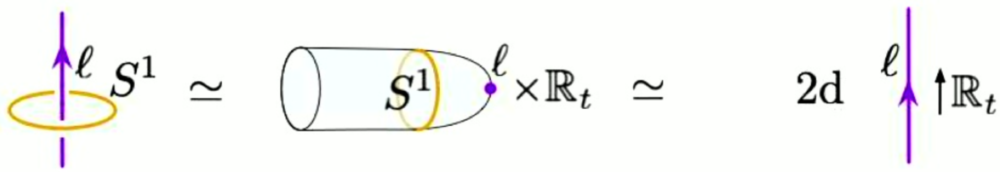
Expect: $\mathcal{C} \simeq$ bdy conditions ($\mathcal{T}_{2d}$) *as a category, naively losing the OPE*



- Roughly, $\mathcal{T}_{2d}$ is a 2d B-model whose target is the loop space of the 3d “target”

Lines via state-op

$\mathcal{C} \simeq$ bdy conditions ($\mathcal{T}_{2d}$)



- Roughly, $\mathcal{T}_{2d}$ is a 2d B-model whose target is the loop space of the 3d "target"
- Free chiral $\displaystyle\int_{\mathbb{C}\times\mathbb{R}} \Psi d'X = \int_{\mathbb{R}_+\times\mathbb{R}_t} \sum_{n+m=-1} \Psi_n dX_m$

$$\mathcal{C} \approx \mathrm{Coh}(L\mathbb{C})$$
$$\simeq \mathbb{C}[X_n]_{n\in\mathbb{Z}}\text{-(dg)mod}$$

$$X = \sum_{n\in\mathbb{Z}} X_n z^{-n-1} \qquad \Psi = \sum_{n\in\mathbb{Z}} \Psi_n z^{-n-1}$$

all but fin. many $X_{\geq 0}$ = 0,
all but fin. many $X_{<0}$ free.

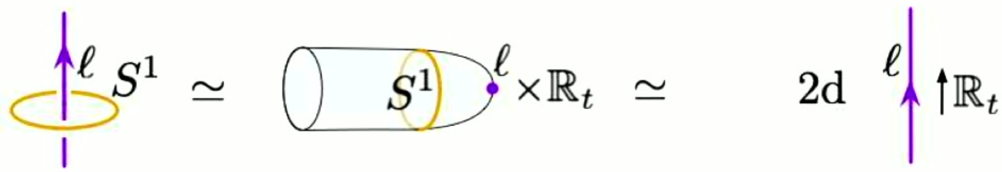
trivial line $\mathbb{1} \simeq \mathcal{O}_{L_+\mathbb{C}} = \mathbb{C}[X_n]_{n<0}$ (regular locus)

elementary vortices $V_N \simeq \mathcal{O}_{z^N\mathbb{C}[\![z]\!]} = \mathbb{C}[X_n]_{n+N<0}$

$$\begin{cases} X \sim z^N \\ \Psi \sim z^{-N} \end{cases} \text{near } z = 0$$

Lines via state-op

$\mathcal{C} \simeq$ bdy conditions ($\mathcal{T}_{2d}$)



- Roughly, $\mathcal{T}_{2d}$ is a 2d B-model whose target is the loop space of the 3d "target"
- d chirals w/ W $\quad \int_{\mathbb{C}\times\mathbb{R}} \Psi_i d' X^i + W(X) \;=\; \int_{\mathbb{R}_+\times\mathbb{R}_t} \sum_{n+m=-1} \Psi_{in} dX^i_m \; + \mathrm{Res}_0 W(X(z))$

$$X^i = \sum_{n\in\mathbb{Z}} X^i_n z^{-n-1} \qquad \Psi_i = \sum_{n\in\mathbb{Z}} \Psi_{in} z^{-n-1}$$

$$\mathcal{C} \approx \mathrm{MF}(L\mathbb{C}^d, \mathrm{Res}_0 W)$$

trivial line $\quad \mathbb{1} \simeq \mathcal{O}_{L_+\mathbb{C}^d} = \mathbb{C}[X^i_n]^{1\le i\le d}_{n<0}$ $\quad$ (regular locus)

elementary vortices $\quad V_{\vec{N}} \simeq \mathbb{C}\big[X^i_n\big]^{1\le i\le d}_{n+N_i<0}$

only valid if the singularity $\left\{ \begin{array}{l} X^i \sim z^{N_i} \\ \Psi_i \sim z^{-N_i} \end{array} \right.$ near $z=0$ keeps W residue-free

Lines via QM

cf. [Costello '13, Costello-Paquette '20]

To access the OPE, we use a second description, valid (at least) for *perturbative* QFT's.

- Assume any line ℓ can be engineered by introducing 1d topological QM

  state space V_ℓ (dg vec space, may be infinite-dimensional)

  coupled to bulk via couplings in a 1d action, encoded in a MC element

$$\mu_\ell \in \mathrm{End}(V_\ell \otimes \mathbb{1}) \simeq \mathrm{End}(V_\ell) \otimes \mathcal{A} \qquad \mathcal{A} = \mathrm{End}_{\mathcal{C}}(\mathbb{1}) = \text{bulk local ops}$$

$$\ell \quad \longleftrightarrow \quad (V_\ell, \mu_\ell)$$

- Assumption amounts to $\mathcal{C}$ being generated by $\mathbb{1}$ = structure sheaf on reg. locus
- Now expect:

$$\ell \otimes_z \ell' \quad \longleftrightarrow \quad (V_\ell \otimes V_{\ell'}, \mu_\ell(z) + \mu_{\ell'}(0) + r_{\ell,\ell'}(z, z^{-1}))$$

Lines via QM

Example: free chiral

$$V_1(z) = \exp \int_{\{z\}\times\mathbb{R}_t} \bar{\alpha} d_t \alpha + \alpha X \qquad V_{-1}(z) = \exp \int_{\{z\}\times\mathbb{R}_t} \bar{a} d_t a + a\Psi$$

$\bar{\alpha}, \alpha \in \Pi\Omega^\bullet(\mathbb{R}_t)$ (w/ r,j shifts) $\qquad \bar{a}, a \in \Omega^\bullet(\mathbb{R}_t)$

$$V_{V_1} \simeq \mathbb{C}[\alpha^{(0)}] \simeq \mathbb{C}^2 \qquad V_{V_{-1}} \simeq \mathbb{C}[a^{(0)}] \simeq H^\bullet_{\bar{\partial}}(\mathbb{C})$$

$$V_1 \otimes_z V_{-1} := V_1(z)V_{-1}(0) = \exp \int_{\{0\}\times\mathbb{R}_t} \bar{\alpha} d_t \alpha + \bar{a} d_t a + \alpha X(z) + a\Psi(0) + \frac{1}{z}\alpha a$$

due to a contraction $X(z,t)\Psi(z',t') \sim \dfrac{\delta(t-t')(dt-dt')}{z-z'}$

Lines via QM

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due to a contraction $X(z,t)\Psi(z',t') \sim \dfrac{\delta(t-t')(dt-dt')}{z-z'}$

Claim: every perturbative OPE, in a perturbative HT theory, looks just like this!

Non-renormalization

Assume:

- general perturbative form of the bulk action + unbroken ghost number symmetry
- have represented line ops by coupling *linearly* to bulk fields & ∂ derivatives

e.g. $$\int_{\mathbb{R}} aX\Psi \rightsquigarrow \int_{\mathbb{R}} \alpha b + ab\Psi - \alpha X$$ (at the cost of introducing more QM fields & QM interactions)

Theorem: the only Feynman diagrams adding new couplings to $\ell \otimes_z \ell'$ have *zero* bulk vertices — they merely contract the (linear) bulk operators in the ℓ couplings pairwise with the bulk operators in the ℓ' couplings.

Proof: counting ghost number + distributing dt's from the propagators.

Koszul duality

Let's combine the approaches!

Following [Costello '13, Costello-Paquette '20] expect

$\mathcal{C} \xrightarrow{\sim} \mathcal{A}^!\text{-mod}$ where $\mathcal{A}^!$ is Koszul-dual to $\mathcal{A}$ (as A-infinity algebras)

- This representation comes from choosing an asymptotic b.c. at $z \to \infty$ s.t. the space of states on $\mathbb{C}$ with no ops inserted is one-dimensional.

Koszul duality

How to find $\mathcal{A}^!$?

- Intuitive, but tedious to implement:
  - learn everything about $\mathcal{A}$ (as an A-infinity algebra)
  - apply math or equivalently obtain $\mathcal{A}^!$ as universal couplings of putative QM to $\mathcal{A}$ (bar construction)
- More direct:
  - do circle reduction to 2d loop-space theory $\quad \mathcal{C} \approx \mathrm{MF}(L\mathcal{X})$
  - there's a canonical object $\mathcal{E}_\infty$ supported on the complement of the regular locus

$$\mathbb{1} = \mathcal{O}_{L_+\mathcal{X}}$$

$$\mathcal{E}_\infty = \mathcal{O}_{L_-\mathcal{X}}$$

By construction: $\mathrm{Hom}_{\mathcal{C}}(\mathcal{E}_\infty, \mathbb{1}) = \mathbb{C} \longleftrightarrow$ states on a large disc with $\mathbb{1}$ inserted $= \mathbb{C}$

Then $\mathcal{A}^! \simeq \mathrm{End}_{\mathcal{C}}(\mathcal{E}_\infty)$

Koszul duality

Example: free chiral $X = \sum_{n\in\mathbb{Z}} X_n z^{-n-1}$ $\Psi = \sum_{n\in\mathbb{Z}} \psi_n z^{-n-1}$

$\mathbb{1}$ supported on $n < 0$ modes

$\mathcal{E}_\infty$ supported on $n \geq 0$ modes $\mathcal{A}^! = \mathbb{C}[X_n, \psi_n]_{n\geq 0}$

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  - there's a canonical object $\mathcal{E}_\infty$ supported on the complement of the regular locus $\mathbb{1} = \mathcal{O}_{L_+\mathcal{X}}$ $\mathcal{E}_\infty = \mathcal{O}_{L_-\mathcal{X}}$

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$\mathbb{1}$ supported on $n < 0$ modes

$\mathcal{E}_\infty$ supported on $n \geq 0$ modes $\mathcal{A}^! = \mathbb{C}[X_n, \psi_n]_{n\geq 0} = \mathbb{C}[X(s)_-, \Psi(s)_-]$

define polar parts $X(s)_- = \sum_{n\geq 0} X_n s^{-n-1}$ $\Psi(s)_- = \sum_{n\geq 0} \psi_n s^{-n-1}$

lines as modules: $\mathbb{1} = \mathbb{C}$ $V_1 = \mathbb{C}[\psi_0]$ $V_2 = \mathbb{C}[\psi_0, \psi_1]$

$V_{-1} = \mathbb{C}[X_0]$

Warning: with W's (or in gauge theory) there are A-infinity operations, up to $\mu_{\deg(W)-1}$

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$\mathbb{1}$ supported on $n < 0$ modes

$\mathcal{E}_\infty$ supported on $n \geq 0$ modes

$$\mathcal{A}^! = \mathbb{C}[X_n, \psi_n]_{n\geq 0} = \mathbb{C}[X(s)_-, \Psi(s)_-]$$

Warning: with W's (or in gauge theory) there are A-infinity operations, up to $\mu_{\deg(W)-1}$

Example: chirals (X^i, Ψ_i) with $W(X)$ $\mathcal{A}^! \simeq \mathbb{C}[X^i_n, \psi_{i,n}]_{n\geq 0} \simeq \mathbb{C}[X^i(s)_-, \Psi_i(s)_-]$

deformed by

$$\mu_1 \Psi_i(s)_- = \mathrm{Res}_{z=0} \frac{\partial_i W(z)_-}{z-s} = \partial_i W(s)_-$$

$$\mu_2\big(\Psi_i(s)_-, \Psi_j(t)_-\big) = \mathrm{Res}_{z=0} \frac{\partial_i\partial_j W(z)_-}{(z-s)(z-t)} = -\frac{\partial_i\partial_j W(s)_- - \partial_i\partial_j W(t)_-}{s-t}$$

$$\mu_3\big(\Psi_i(s)_-, \Psi_j(t)_-, \Psi_k(u)_-\big) = \mathrm{Res}_{z=0} \frac{\partial_i\partial_j\partial_k W(z)_-}{(z-s)(z-t)(z-u)}$$

etc.!

dg-shifted Yangians

Example: free chiral

$$X = \sum_{n\in\mathbb{Z}} X_n z^{-n-1} \qquad \Psi = \sum_{n\in\mathbb{Z}} \psi_n z^{-n-1}$$

$$\mathcal{A}^! = \mathbb{C}[X_n, \psi_n]_{n\geq 0} \quad = \mathbb{C}[X(s)_-, \Psi(s)_-]$$

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Translate:

$$V_1 \otimes_z V_{-1} = \mathbb{C}[\psi_0, X_0]$$

w/ action

$$X_0 = X_0,\ X_1 = 0,\ X_2 = 0, \ldots$$

$$\psi_0 = \psi_0,\ \psi_1 = z\psi_0,\ \psi_2 = z^2\psi_0, \ldots$$

& new differential $r = \dfrac{\psi_0 X_0}{z}$

Encode: $\Delta_z : \mathcal{A}^! \to \left(\mathcal{A}^! \otimes \mathcal{A}^![\![z^{-1}, z]\!],\ r(z)\right)$

$$\Delta_z X(s) = X(s+z) \otimes X(s)$$

$$\Delta_z \Psi(s) = \Psi(s+z) \otimes \Psi(s)$$

(non-interacting)

new MC element

$$r(z) = \sum_{m,n\geq 0} (-1)^m \binom{m+n}{m} \frac{X_m \otimes \psi_n - \psi_m \otimes X_n}{z^{m+n+1}}$$

$$\in \mathcal{A}^! \otimes \mathcal{A}^![\![z^{-1}]\!]$$

dg-shifted Yangians

Definition: a dgsY is a bigraded A-infinity algebra $\mathcal{A}^!$ with

- a Maurer-Cartan element $r(z) \in \mathcal{A}^! \otimes \mathcal{A}^! [\![z^{-1}, z]\!]$ $\quad (j = 0, r = 1)$
- an A-infinity morphism $\Delta_z : \mathcal{A}^! \to \big(\mathcal{A}^! \otimes \mathcal{A}^! [\![z^{-1}, z]\!],\, r(z)\big)$
- a translation operator (cf. Yangians, or VOA's)
- satisfying mostly standard conditions, in particular co-associativity of Δ_z :

$$(\Delta_z \otimes 1)\Delta_w = (1 \otimes \Delta_w)\Delta_{z+w}$$

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$$(\Delta_z \otimes 1)\Delta_w = (1 \otimes \Delta_w)\Delta_{z+w}$$

$$(1 \otimes \Delta_w) r(z+w) = r_{13}(z+w) + r_{12}(z)$$

$$(\Delta_z \otimes 1) r(w) = r_{13}(z+w) + r_{23}(w)$$

(log)YB!

Encode: $\Delta_z : \mathcal{A}^! \to \big(\mathcal{A}^! \otimes \mathcal{A}^! [\![z^{-1}, z]\!],\, r(z) \big)$

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(non-interacting)

new MC element

$$r(z) = \sum_{m,n \geq 0} (-1)^m \binom{m+n}{m} \frac{X_m \otimes \psi_n - \psi_m \otimes X_n}{z^{m+n+1}}$$

$$\in \mathcal{A}^! \otimes \mathcal{A}^! [\![z^{-1}]\!]$$

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- a Maurer-Cartan element $r(z) \in \mathcal{A}^! \otimes \mathcal{A}^! [\![z^{-1}, z]$ $(j = 0, r = 1)$
- an A-infinity morphism $\Delta_z : \mathcal{A}^! \to \big(\mathcal{A}^! \otimes \mathcal{A}^! [\![z^{-1}, z],\, r(z)\big)$
- a translation operator (cf. Yangians, or VOA's)
- satisfying mostly standard conditions, in particular co-associativity of Δ_z :

$$(\Delta_z \otimes 1)\Delta_w = (1 \otimes \Delta_w)\Delta_{z+w}$$

$$(1 + \Delta_w) r(z+w) = r_{13}(z+w) + r_{12}(z)$$

$$(\Delta_z \otimes 1) r(w) = r_{13}(z+w) + r_{23}(w)$$

(log)YB!

Conjecture: any (perturbative?) 3d HT QFT defines a dgsY $\mathcal{A}^!$

More so, non-renorm => there's a presentation of $\mathcal{A}^!$ such that

$$\Delta_z x(s) = x(s+z) \otimes x(s)$$

for all generators

&

$$r(z) = \sum_{m,n \geq 0} (-1)^m \binom{m+n}{m} \frac{q_m \otimes p_n - p_m \otimes q_n}{z^{m+n+1}}$$

Future wish list

- Nonperturbative corrections to OPE: any control?
- NP version of $\mathcal{A}^!$ itself, in gauge theory (accounting for monopoles in $\mathcal{A}$)
- Connecting with circle reductions and quantum K-theory

For 3d theory that flows to sigma-model to $\mathcal{X}$

$$HH^q_\bullet(\mathcal{C}) \overset{?}{\simeq} QK^\bullet(\mathcal{X})$$

$$\otimes_z \rightsquigarrow *$$

Thanks!!