

Title: Emergent Modified Gravity: Covariant framework for effective (Loop) Quantum Gravity

Speakers: Erick Duque

Collection/Series: Quantum Gravity

Subject: Quantum Gravity

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Abstract:

Emergent Modified Gravity (EMG) is a post-Einsteinian theory of canonical gravity. In this formulation, modified constraints are required to preserve an algebra of hypersurface deformation form and will in general imply modified structure functions. This procedure leads to the conclusion that spacetime is an emergent object with a nontrivial dependence on the gravitational phase space variables through the modified structure functions. Consistency conditions are imposed on the modified constraints and the emergent spacetime metric to ensure general covariance. The resulting modifications allowed by EMG go beyond those obtained from adding higher curvature terms and can result in nonpolynomial dependencies on extrinsic curvature components. In this talk, we discuss how a particular interpretation of such modifications as holonomy terms makes it possible to use EMG as a covariant framework for effective (loop) quantum gravity. We then focus on dynamical solutions of the spherically symmetric model which include nonsingular black holes, new effects to gravitational collapse, and MOND-like effects at intermediate scales.

Emergent Modified Gravity: Covariant framework for effective (Loop) Quantum Gravity

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Advisor: Martin Bojowald

The Pennsylvania State University

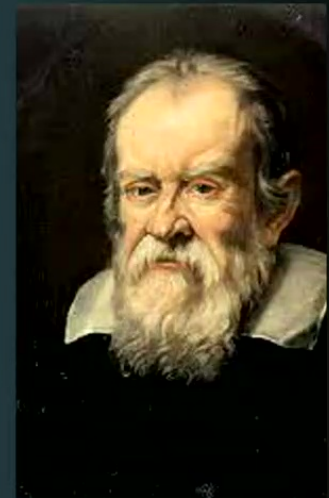
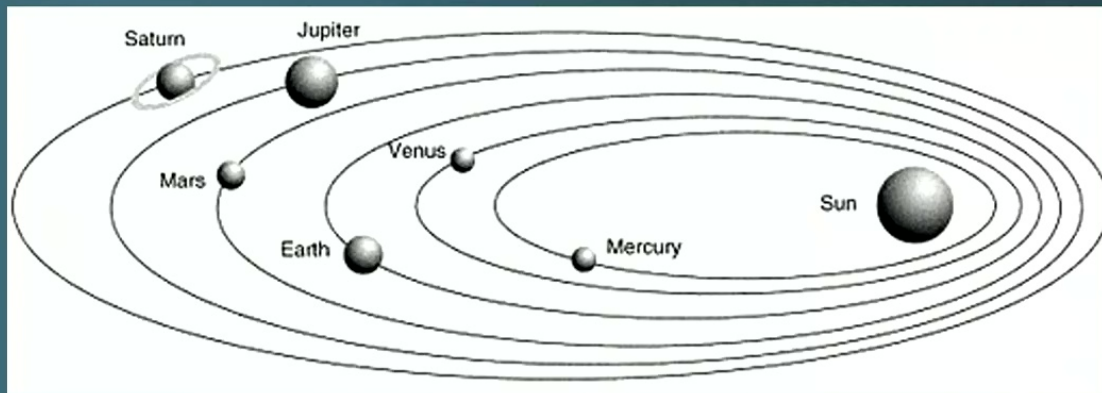
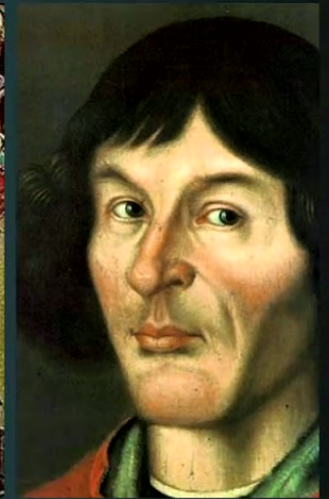
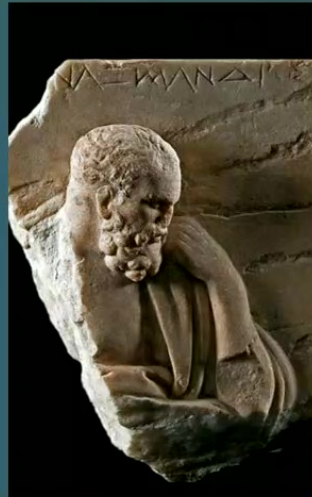
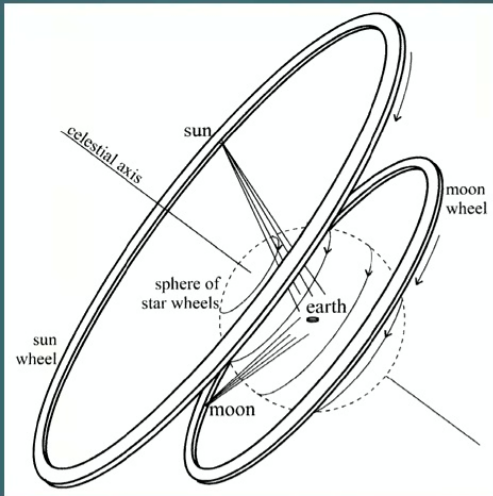
Perimeter Institute Quantum Gravity Seminar

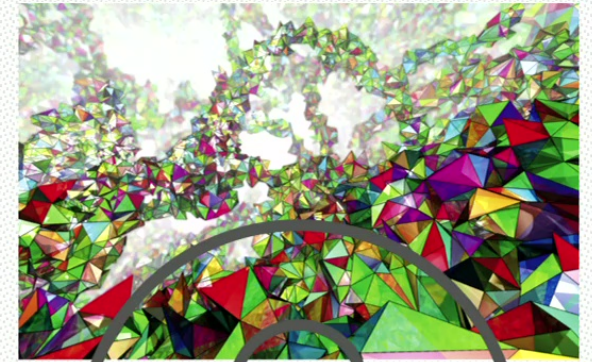
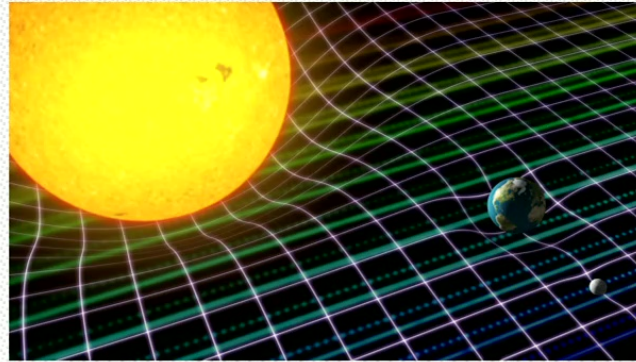
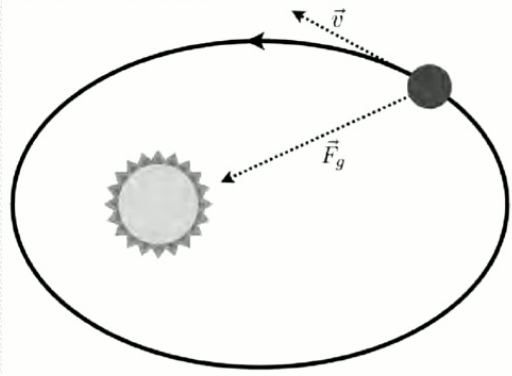
December 5th, 2024



Outline

1. Motivation for Quantum & Modified Gravity
2. Emergent Modified Gravity
3. Applications: Effective (L)QG, nonsingular black holes, gravitational collapse, MOND
4. Future





Universal gravitation



General relativity

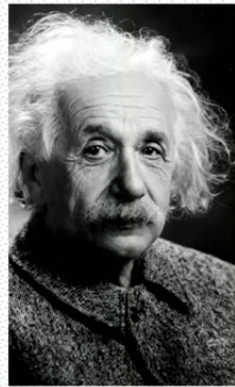


Quantum gravity



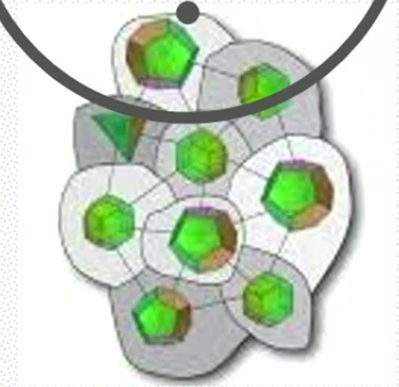
$$F_g = mg$$

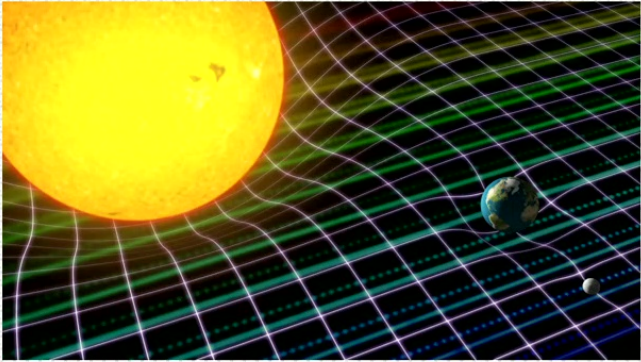
$$F = ma$$



$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$u^\nu \nabla_\nu u^\mu = a^\mu$$

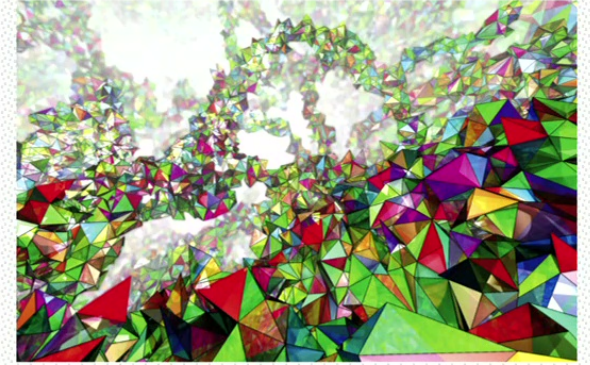




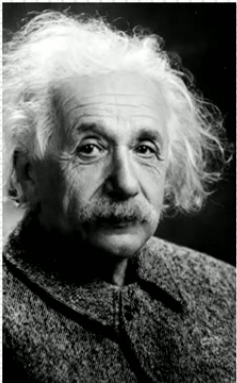
General relativity



Missing piece?

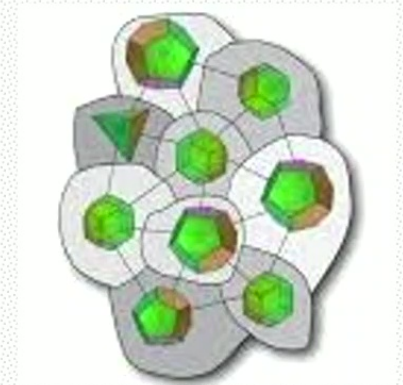
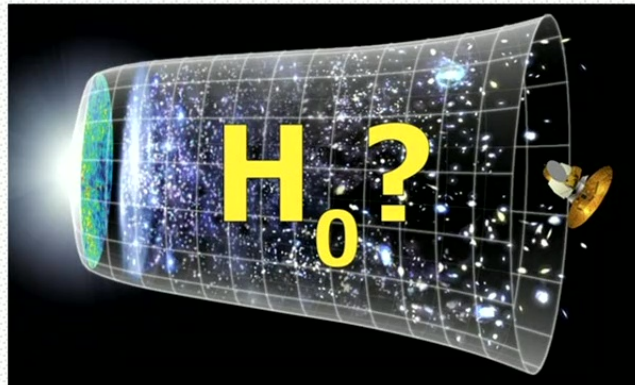


Quantum gravity

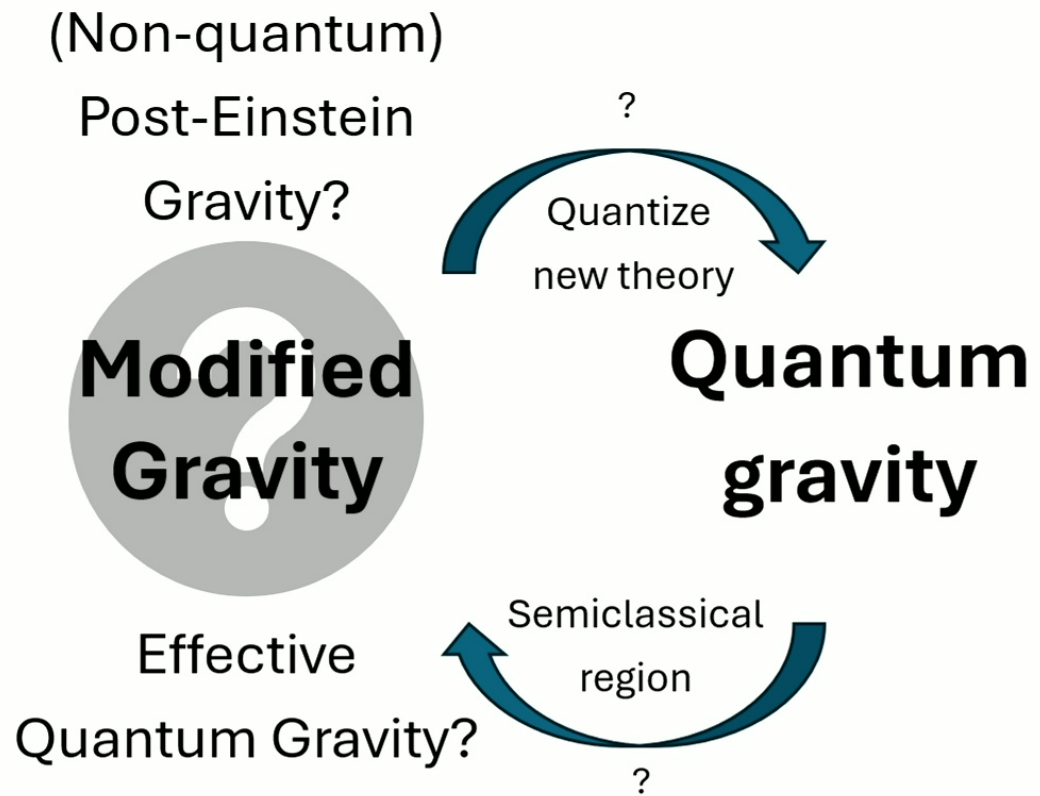


$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

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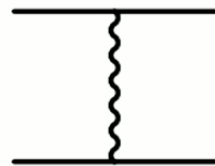


**General
relativity**



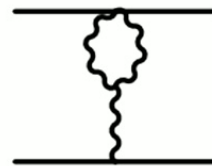
$$S = \int d^4x \sqrt{-g} \left[-\frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \dots \right]$$

**Perturbative
quantum
gravity**

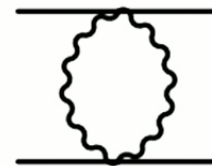


+

Graviton 1-loop
corrections
(renormalization)



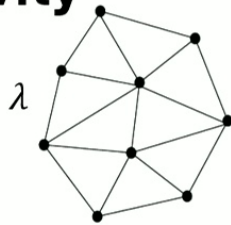
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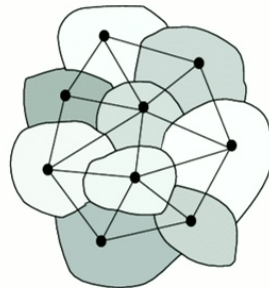
Higher curvature
(counter)terms
in effective action



**Loop
quantum
gravity**



Spin networks
(holonomies)



Holonomy terms
in Hamiltonian

$$A \rightarrow h_\lambda[A] = \exp \left(i \int_\lambda A \right)$$

Traditional modified gravity

Higher curvature terms

$$S[g] = \int dx^4 \sqrt{-\det g} (R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \dots)$$

Typical issues:

1. Additional degrees of freedom
2. Instabilities
3. Perturbative quantization beyond 1-loop?

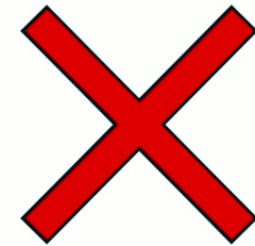
Effective LQG modified gravity

Holonomy modifications on Hamiltonian constraint

$$H[A] \rightarrow H \left[\exp \left(i \int_{\lambda} A \right) \right]$$

Typical issues:

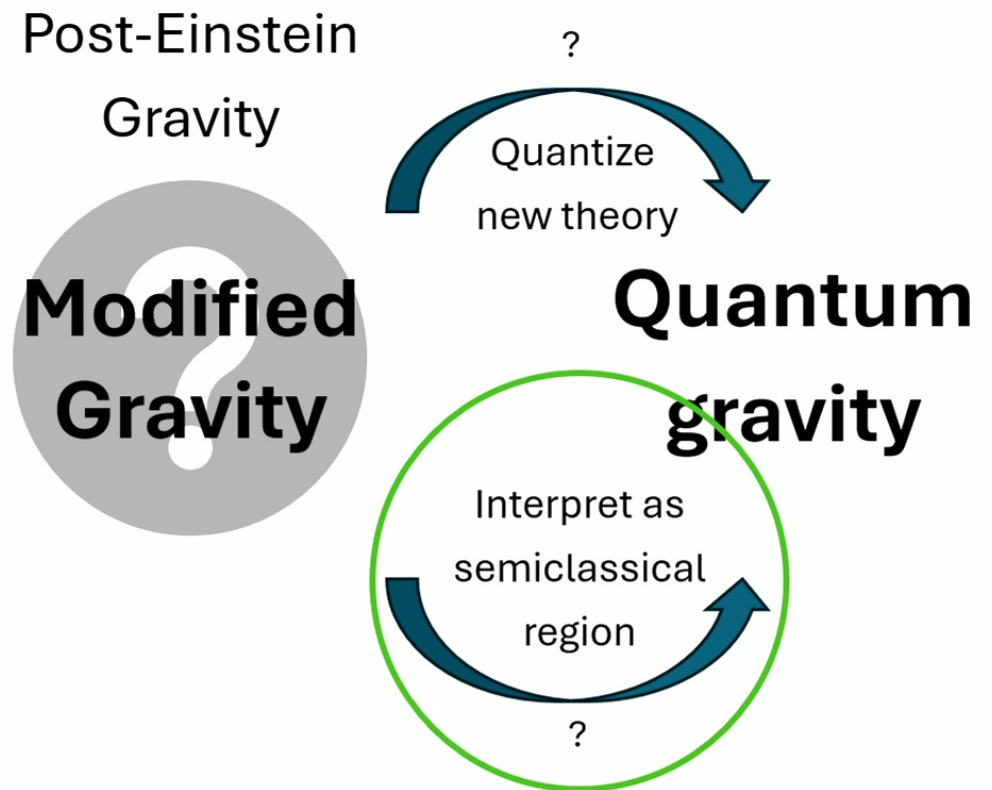
1. Several inequivalent black hole models
2. Anomalous constraint algebra
3. Break general covariance **[arXiv:2410.18807]**



Covariant modified gravity beyond higher curvature terms?

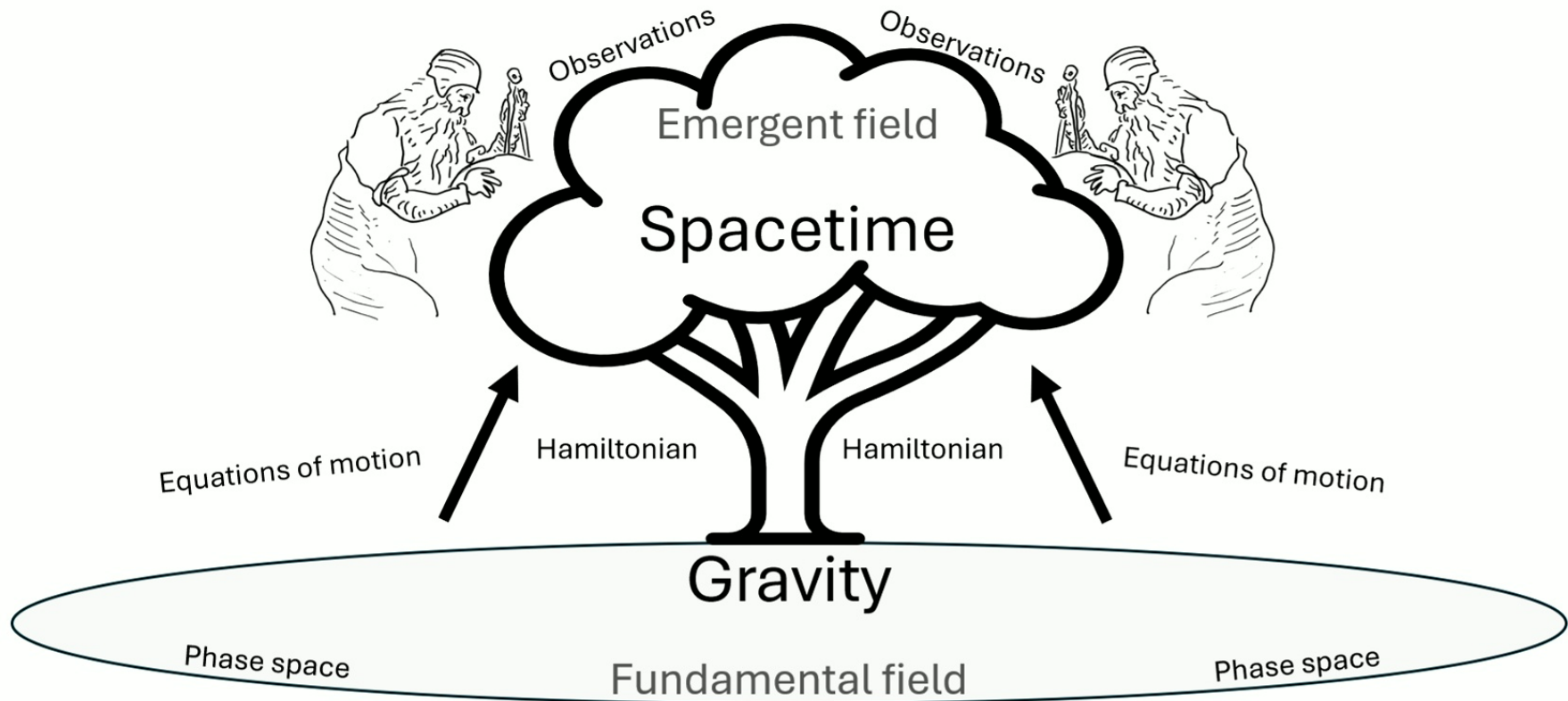
Properties:

1. No additional degrees of freedom
2. No instabilities
3. Anomaly-free constraint algebra
4. Generally Covariant



Emergent modified gravity

Distinction between spacetime and gravity



Canonical gravity

ADM decomposition

$$t^\mu = Nn^\mu + N^a s_a^\mu$$

$$ds^2 = -N^2 dt^2 + q_{ab}(dx^a + N^a dt)(dx^b + N^b dt)$$

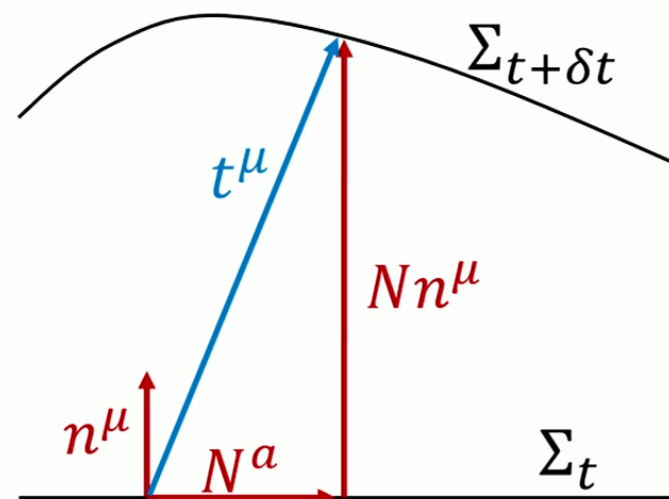
[Arnowitt, Deser, Misner 1962]

$$S_{\text{EH}}[g] = \int d^4x \sqrt{-\det g} R = \int dt d^3x (p^{ab} \dot{q}_{ab} - N^\mu H_\mu)$$

Configuration variable: q_{ab}

Lagrange multipliers: $N^\mu = (N, N^a)$ $H[N] = \int d^3x NH$

Constraints: $H_\mu = (H, H_a) \mid H_\mu = 0 \rightarrow \dot{A} = \{A, H_\mu[N^\mu]\} \quad \delta_\epsilon A = \{A, H_\mu[\epsilon^\mu]\}$



The hypersurface deformation algebra

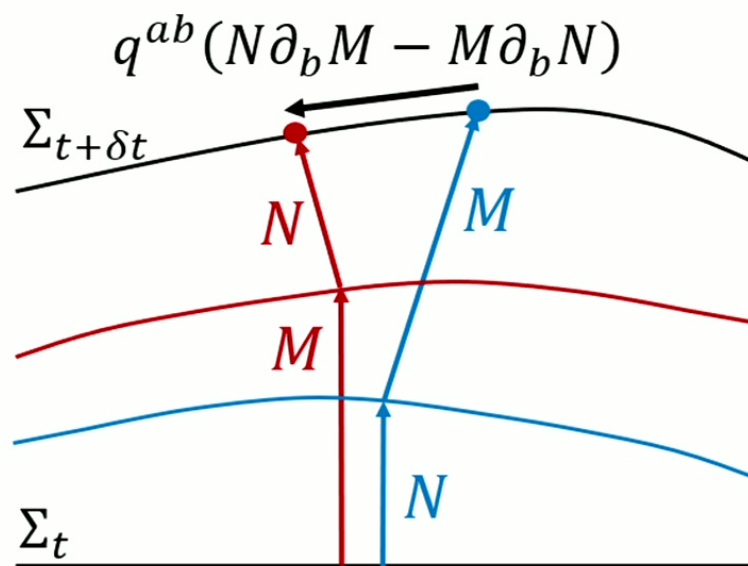
Anomaly freedom

$$\{\vec{H}[\vec{N}], \vec{H}[\vec{M}]\} = -H_a [M^b \partial_b N^a - N^b \partial_b M^a]$$

$$\{H[N], \vec{H}[\vec{M}]\} = -H[M^a \partial_a N]$$

$$\{H[N], H[M]\} = -H_a [q^{ab} (M \partial_b N - N \partial_b M)]$$

[Hojmann, et.al. 1994]



Covariance

$$\delta_\epsilon g_{\mu\nu} = \mathcal{L}_\xi g_{\mu\nu}$$

[arXiv:2310.06798]

- Uniqueness result: Algebra implies GR [Hojmann, et.al. 1994].
- Assumption: The spatial metric q_{ab} is a configuration variable.

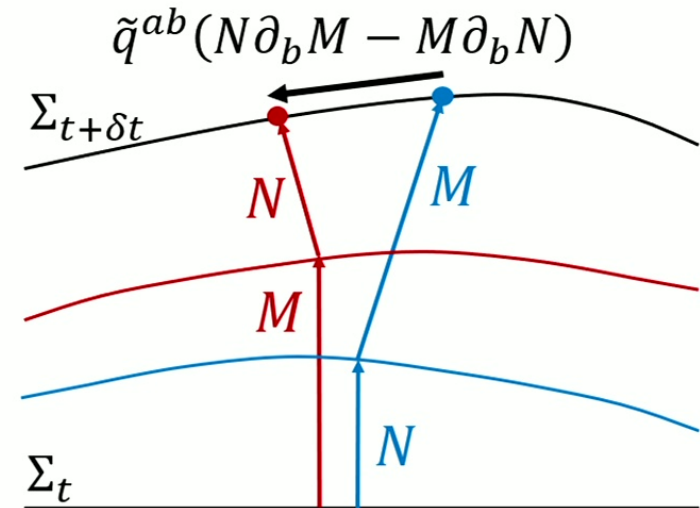
Emergent modified gravity

Anomaly freedom:

$$\{\vec{H}[\vec{N}], \vec{H}[\vec{M}]\} = -H_a [M^b \partial_b N^a - N^b \partial_b M^a]$$

$$\{\tilde{H}[N], \vec{H}[\vec{M}]\} = -\tilde{H}[M^a \partial_a N]$$

$$\{\tilde{H}[N], \tilde{H}[M]\} = -H_a [\tilde{q}^{ab} (M \partial_b N - N \partial_b M)]$$



Emergent spacetime: $ds^2 = -N^2 dt^2 + \tilde{q}_{ab} (dx^a + N^a dt)(dx^b + N^b dt)$

Covariance condition: $\delta_\epsilon \tilde{g}_{\mu\nu} = \mathcal{L}_\xi \tilde{g}_{\mu\nu}$

Spatial metric: \tilde{q}_{ab}
 Gravitational field: q_{ab}, p^{ab}

- The spatial metric $\tilde{q}_{ab} \neq q_{ab}$ is **not** a configuration variable.
- \tilde{q}_{ab} is a function of the phase space derivable from the constraint algebra.

[arXiv:2310.06798]

Spherical symmetry

Smaller phase space

$$\{K_x(x), E^x(y)\} = \{K_\varphi(x), E^\varphi(y)\} = \delta(x - y)$$

1. **Constraint ansatz:** \tilde{H} up to second-order spatial derivatives

2. **Anomaly-freedom:**

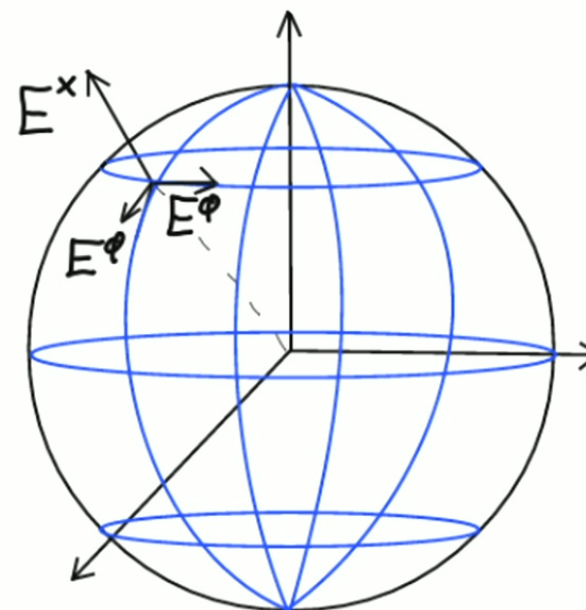
$$\{H_x[N^x], H_x[M^x]\} = -H_x[M^x(N^x)' - N^x(M^x)']$$

$$\{\tilde{H}[N], H_x[M^x]\} = -\tilde{H}[M^x N']$$

$$\{\tilde{H}[N], \tilde{H}[M]\} = -H_x[\tilde{q}^{xx}(MN' - NM')]$$

Emergent spacetime: $ds^2 = -N^2 dt^2 + \tilde{q}_{xx}(dx + N^x dt)^2 + E^x d\Omega^2$

3. **Covariance condition:** $\delta_\epsilon \tilde{g}_{\mu\nu} = \mathcal{L}_\xi \tilde{g}_{\mu\nu}$



[arXiv:2310.06798]

Spherical symmetry

$$\tilde{q}^{xx} = \left(\left(f + \left(\frac{\lambda(E^x)'}{E\varphi} \right)^2 \right) \cos^2(\lambda K_\varphi) - 2q \frac{\sin(2\lambda K_\varphi)}{2\lambda} \right) \chi^2 \frac{E^x}{(E\varphi)^2}$$

$$q_{(classical)}^{xx} = \frac{E^x}{(E\varphi)^2}$$

Classical limit: $\lambda, q \rightarrow 0$, $f, \chi \rightarrow 1$
 $\alpha_0, \alpha_2 \rightarrow 1$

Emergent spacetime: $ds^2 = -N^2 dt^2 + \tilde{q}_{xx} (dx + N^x dt)^2 + E^x d\Omega^2$

[arXiv:2310.06798]

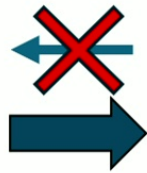
Effective loop quantum gravity

- The λ modifications may be interpreted as effective corrections from loop quantum gravity, corresponding to the angular holonomy length of links of discretized spheres:

$$h_\varphi = \exp(i \int d\varphi K_\varphi) = \exp(i\lambda K_\varphi)$$

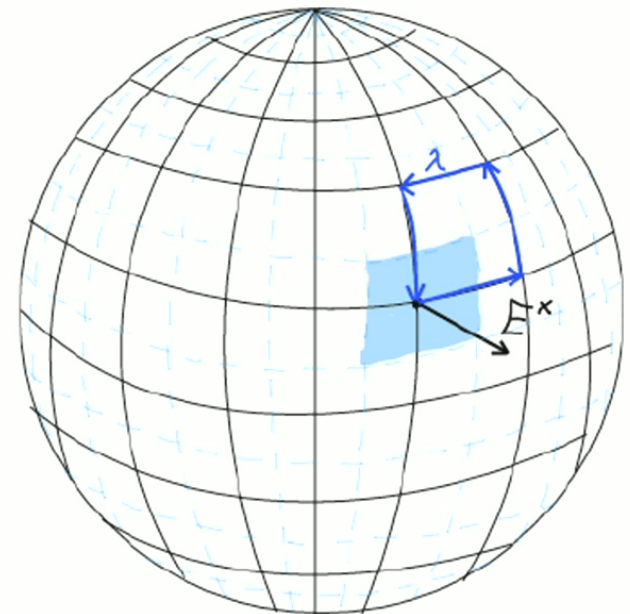
- No radial holonomies $h_x = \exp(i \int dx K_x) \dots$

**Emergent
Modified
Gravity**

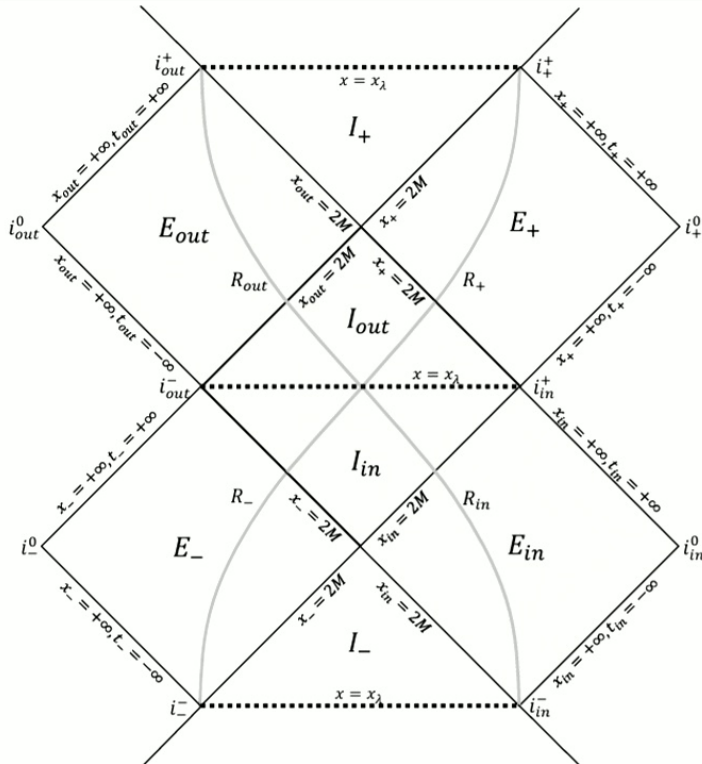


*Interpret as
semiclassical region*

**(Loop)
Quantum
Gravity**



Nonsingular black hole solution in vacuum



[Alonso-Bardaji, Brizuela 2021, 2022]
[with Belfaqih, Bojowald, Brahma arXiv:2407.12087]

$$f = \alpha_i = 1, q = 0, \chi = 1$$

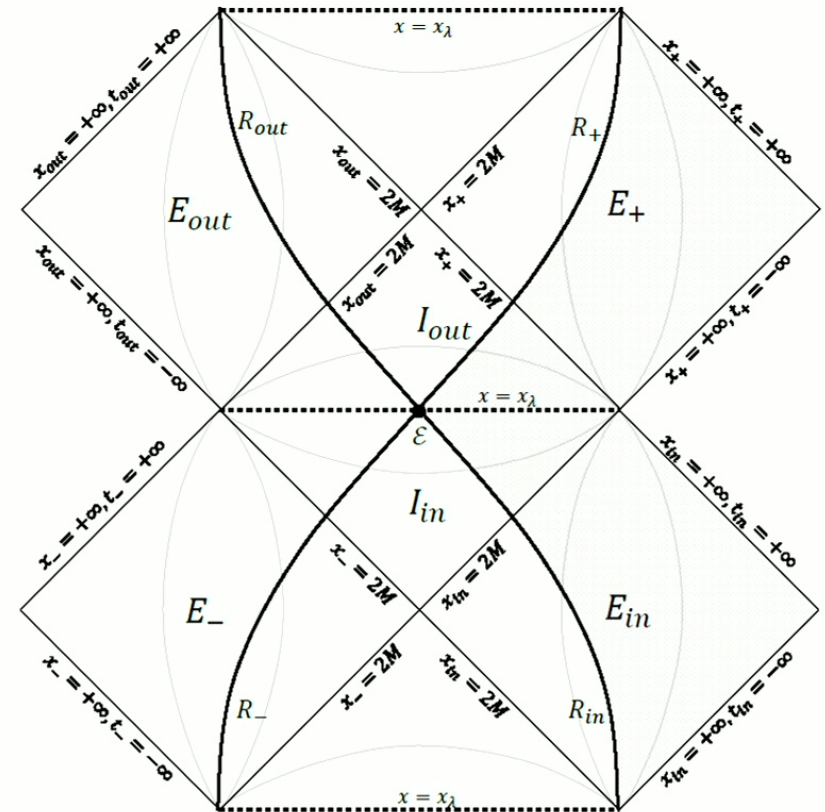
$$ds^2 = -\left(1 - \frac{2M}{x}\right) dt^2 + \left(1 - \frac{2M}{x}\right)^{-1} \left(1 + \lambda^2 \left(1 - \frac{2M}{x}\right)\right)^{-1} dx^2 + x^2 d\Omega^2$$

- Minimum radius x_λ , solving $1 + \lambda(x)^2 \left(1 - \frac{2M}{x}\right) = 0$
- No physical singularity at x_λ
- Kruskal-type coordinates map spacetime of 'two black holes' joined at x_λ : This is a wormhole
- Infalling particles at rest at infinity take a time $\frac{8}{3} (M + x_\lambda) \sqrt{1 - x_\lambda/2M}$ to cross the interior (for constant $\lambda = \bar{\lambda}$)

Gravitational collapse

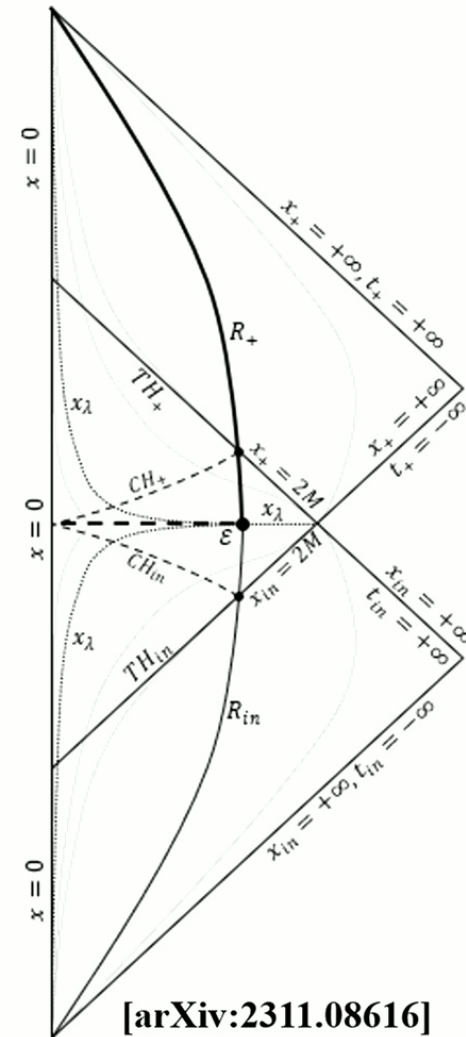
- Dust coupling possible by treating it as a collection of particles in the emergent spacetime $\delta_\epsilon u_\mu = \mathcal{L}_\xi u_\mu$
- Star's surface R follows a geodesic
- Exact solution for the interior of the star (for constant $\lambda = \bar{\lambda}$)

[arXiv:2311.08616]



Formation of a wormhole

- Two asymptotic regions joined by the interior
- The collapsing matter in the first universe forms a black hole, crosses the wormhole's interior, and emerges as an evaporating white hole in the next universe
- The star's surface takes a proper time $\frac{8}{3}(M + x_{\bar{\lambda}})\sqrt{1 - x_{\bar{\lambda}}/2M}$ to cross the interior of the wormhole
- $(\tau_{\odot} \approx \frac{8GM_{\odot}}{3c^3} \approx 0.01s)$



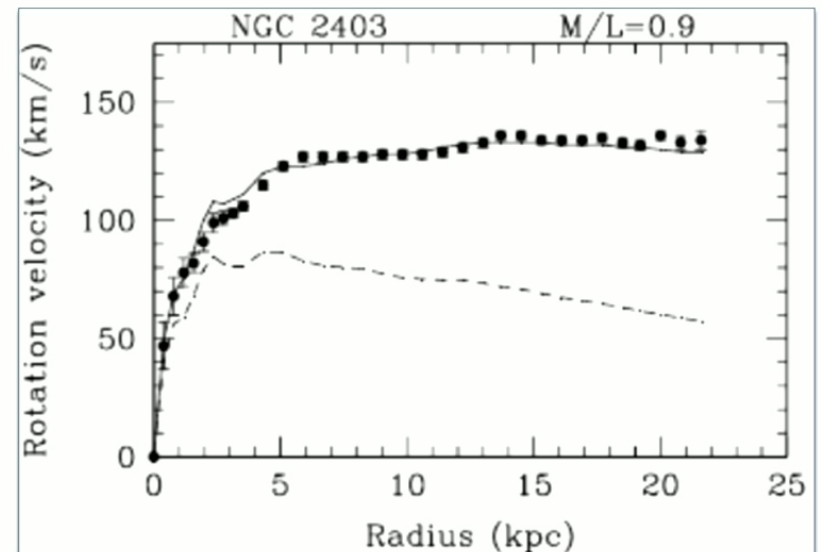
MOND (Modified Newtonian Dynamics)

Tully-Fisher relation: $v^4 = a_0 M$

Centripetal force: $F_c = -\frac{mv^2}{x}$

Circular motion $F_c = F_g$ implies flat rotation curves if

$$F_g = -\frac{m\sqrt{a_0 M}}{x} \rightarrow U_g = m\sqrt{a_0 M} \ln(x)$$



[Begeman, Broeils, Sanders 1991]

[Extended to 750kpc, McGaugh et al 2024]

MOND from EMG

Relativistic realization of MOND via logarithmic modification motivated by quantum gravity procedure [with Martin Bojowald arXiv:2310.19894]

- **Problem for QG:** Constraint algebra not a Lie algebra due to structure function
- **Idea:** Define operator $H^{(A)}$ replacing \tilde{H} such that new algebra is free of structure functions [Gambini, Pullin 2013]

Consider a linear combination of the already modified constraint:

$$H^{(A)} = B\tilde{H} + AH_x$$

- **Anomaly freedom:** Impose closure of algebra

$$\{H_x[N^x], H_x[M^x]\} = -H_x[M^x(N^x)' - N^x(M^x)']$$

$$\{H^{(A)}[N], D[N^x]\} = -H^{(A)}[N^x N']$$

$$\{H^{(A)}[N], H^{(A)}[M]\} = 0$$

MOND from EMG

Result

$$B = \frac{\tan(\lambda K_\varphi)}{\lambda}, \quad A = -\frac{\sqrt{E^x}}{2(E^\varphi)^2} \frac{\partial E^x}{\partial x}$$

Kinematical divergence at $\lambda K_\varphi = \pi/2$ (consider constant λ for simplicity)

$$H^{(A)} \supset -\frac{\sqrt{E^x} \tan(\bar{\lambda} K_\varphi)}{2 \bar{\lambda}} E^\varphi \left(-\Lambda + \frac{1}{E^x} + \frac{\sin^2(\bar{\lambda} K_\varphi)}{\bar{\lambda}^2} \left(\frac{f}{E^x} + 2 \frac{\partial f}{\partial E^x} \right) \right)$$

Alleviated by

$$f(E^x) = 1 + \frac{\bar{\lambda}^2}{2} \left(\Lambda E^x - \ln \left(\frac{E^x}{c_0} \right) \right) \quad \text{Logarithmic terms expected from renormalization}$$

MOND from EMG

Structure function includes this new function

$$\tilde{q}^{xx} = \chi^2 \left(f(E^x) + \frac{\bar{\lambda}^2}{4e_2^2} \left(\frac{\partial E^x}{\partial x} \right)^2 \right) \cos^2(\bar{\lambda}K_\varphi) \frac{E^x}{(E\varphi)^2}$$

Dynamical solution includes logarithmic terms

$$ds^2 = - \left(1 - \frac{2M}{x} - \frac{\Lambda x^2}{3} \right) dt^2 + \left(1 + \bar{\lambda}^2 \left(1 - \frac{2M}{x} + \frac{\Lambda x^2}{6} - \ln \left(\frac{x}{\sqrt{c_0}} \right) \right) \right)^{-1} \left(1 - \frac{2M}{x} - \frac{\Lambda x^2}{3} \right)^{-1} \frac{dx^2}{\chi^2} + x^2 d\Omega^2$$

At intermediate scales $M/x, \Lambda x^2 \ll 1$, logarithmic term dominates: MOND effect recovered. [with Martin Bojowald arXiv:2310.19894]

Further applications

- **Scalar matter**
[with Martin Bojowald arXiv:2311.10693]
- **Scalar quasinormal modes**
[with Bojowald, Shankaranarayanan
arXiv:2410.17501]
- **(Emergent) Electromagnetism**
[arXiv:2407.14954]
- **Gowdy systems**
[with Martin Bojowald arXiv:2407.1353]

Future work

- **Tensor QNMs:** Test GWs observations
- **MOND/DM:** Astrophysical observations
- **Cosmology:** Promising applications to Hubble tension, MONDian effects, CMB, matter distribution, etc...
- **Nonlocality:** Beyond symmetry reduced models likely require nonlocal formulation
- **Quantum gravity:** If EMG is post-Einstein gravity (not effective QG), then quantization of EMG (not GR) leads to QG

Thank you for your attention

1. M. Bojowald and E. I. Duque, *Emergent modified gravity: Covariance regained*, Phys. Rev. D, **108**, 084066 (2023), [arXiv:2310.06798].
2. M. Bojowald and E. I. Duque, *Emergent modified gravity*, Class. Quant. Grav., **41**, 095008 (2024), [arXiv:2404.06375]
3. M. Bojowald and E. I. Duque, *MONDified gravity*, Phys. Lett. B, **108**, 084066 (2023), [arXiv:2310.19894].
4. E. I. Duque, *Emergent modified gravity: The perfect fluid and gravitational collapse*, Phys. Rev. D, **109**, 044014 (2024), [arXiv:2311.08616].
5. M. Bojowald and E. I. Duque, *Emergent modified gravity coupled to scalar matter*, Phys. Rev. D, **109**, 084006, (2024), [arXiv:2311.10693].
6. M. Bojowald, E. I. Duque, and D. Hartmann, *A new type of large-scale signature change in emergent modified gravity*, Phys. Rev. D, **109**, 084001, (2024), [arXiv:2312.09217].
7. I. H. Belfaqih, M. Bojowald, S. Brahma, and E. I. Duque, *Black holes in effective loop quantum gravity: Covariant holonomy modifications*, **(Submitted)**, [arXiv:2407.12087].
8. M. Bojowald and E. I. Duque, *Emergent modified gravity: Polarized Gowdy model on a torus*, Phys. Rev. D, **(To appear)**, [arXiv:2407.13583].
9. E. I. Duque, *Emergent electromagnetism*, **(To appear)**, [arXiv:2407.14954].
10. M. Bojowald, E. I. Duque, and S. Shankaranarayanan, *Scalar quasinormal modes in emergent modified gravity*, **(Submitted)** [arXiv:2410.17501].
11. I. H. Belfaqih, M. Bojowald, S. Brahma, and E. I. Duque, *Lessons for loop quantum gravity from emergent modified gravity*, **(Submitted)** [arXiv:2410.18807].

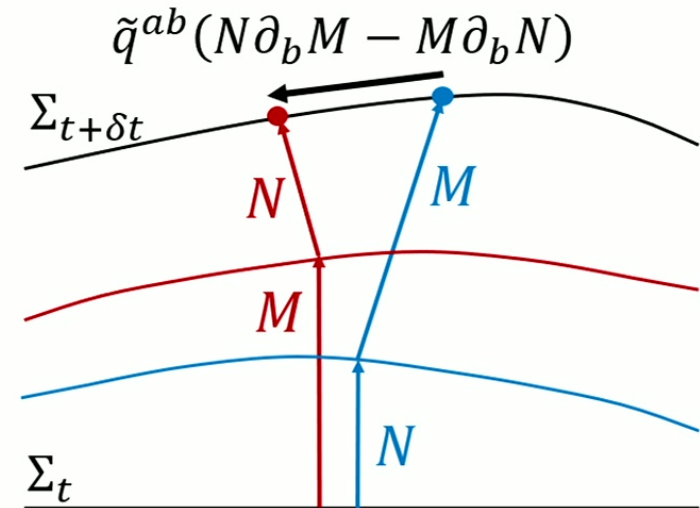
Emergent modified gravity

Anomaly freedom:

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$$\{\tilde{H}[N], \vec{H}[\vec{M}]\} = -\tilde{H}[M^a \partial_a N]$$

$$\{\tilde{H}[N], \tilde{H}[M]\} = -H_a [\tilde{q}^{ab} (M \partial_b N - N \partial_b M)]$$



Emergent spacetime: $ds^2 = -N^2 dt^2 + \tilde{q}_{ab} (dx^a + N^a dt)(dx^b + N^b dt)$

Covariance condition: $\delta_\epsilon \tilde{g}_{\mu\nu} = \mathcal{L}_\xi \tilde{g}_{\mu\nu}$

Spatial metric: \tilde{q}_{ab}
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