Title: Symplectic singularities, Phase diagrams, and Magnetic Quivers

Speakers: Amihay Hanany

Collection/Series: Colloquium

Subject: Other

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Abstract:

Over the past 10 years we faced an impressive progress in the understanding of theories with 8 supercharges. This was through the introduction of theoretical tools which help analyze hypermultiplet moduli spaces in a whole host of gauge theories. Coulomb branches of 3d N=4 theories are now computed very easily through the so called “monopole formula”. The resulting moduli spaces are symplectic singularities which are characterized into different families — closures of nilpotent orbits, intersections of Slodowy slices, orbifolds, slices in the affine Grassmanian, and more. All these names should henceforth enter the physics vocabulary in studies of theories with 8 supercharges.

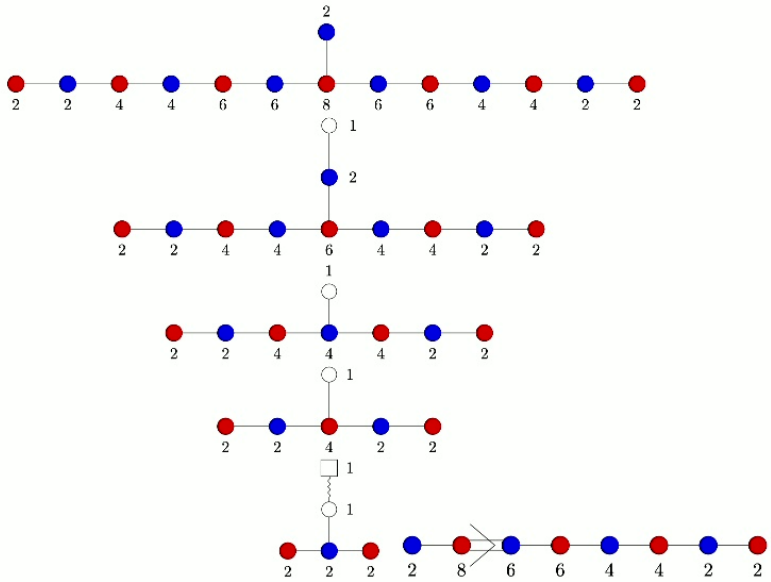
The phase diagrams of symplectic singularities give a further characterization. They are computed with the help of combinatorial tools such as quiver subtraction. This helps distinguish simple/complicated moduli spaces and extends the notion of the Higgs mechanism to theories that admit no Lagrangians.

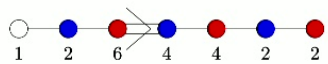
A major ingredient in the success of the recent progress is the use of brane systems for theories with 8 supercharges. Magnetic quivers are computed using these brane systems, and solve long standing problems in finding Higgs branches in regimes where Lagrangian techniques are not available. This sheds light on tensionless strings in 6d, massless instantons in 5d and Argyres Douglas theories in 4d.

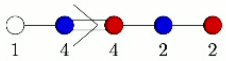
The talk aims to review the progress in understanding theories with 8 supercharges and to give a taste to the new tools and to the new terminology that rose as a result of this study.

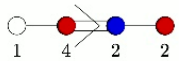
Branes Quivers and Supersymmetric Gauge Theories

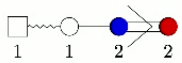
Status Update on theories with 8 supercharges 2024











Amihay Hanany

Over the past 10 years we faced an impressive progress in the understanding of theories with 8 supercharges

combining knowledge from the physics of branes in superstring theory, the quantum dynamics of supersymmetric gauge theories, and the structure of geometric singularities

This talk will try to share some of the excitement we have in developing the new techniques

Branes

objects in Type II superstrings

- Extended objects in space time
- Solutions of classical equations of motion
- Have fields on their world volume
- gauge fields, scalars, fermions
- Preserve some of the supersymmetry
- The low energy dynamics is characterized by a SQFT in the respective dimensions

This gives a very nice dual role:
Study gauge theories to learn about the dynamics of branes
Study branes to learn about the dynamics of gauge theories

Branes

The branes for today

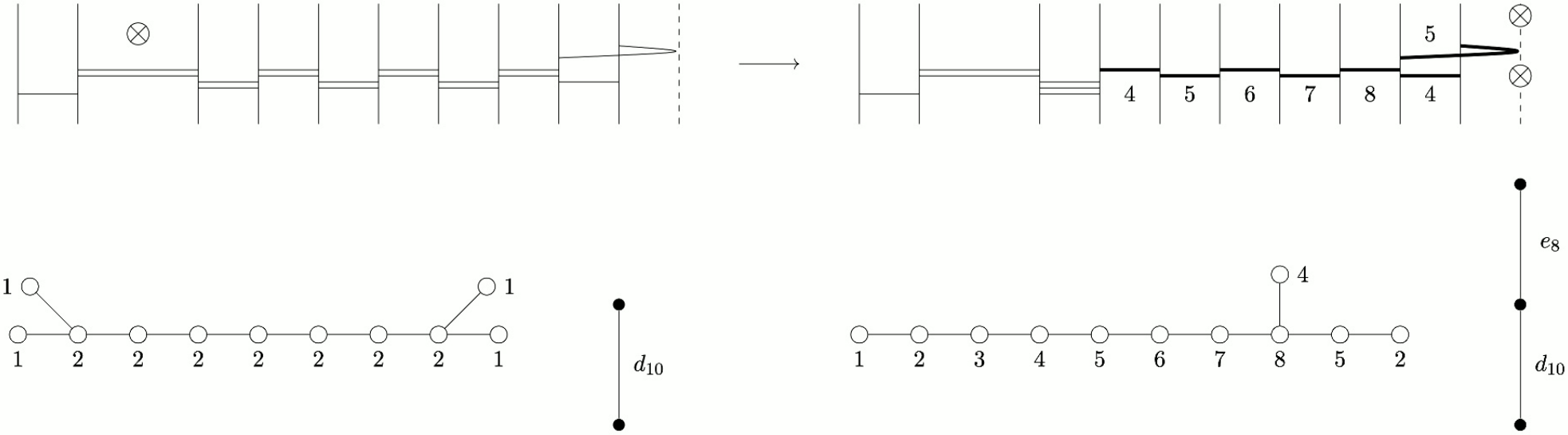
- Dp branes $-1 \leq p \leq 9$: QFT in $p+1$dimensions
- NS5 brane : QFT in 5+1 d
- Fundamental string F1 : 2d QFT
- Bound states of 5 branes forming Brane Webs : 5d QFT

Branes in string theory make a big lab for studying supersymmetric gauge theories and other phenomena in theoretical physics

Type IIA brane system with an effective 6d theory

Example — SU(2) with 10 flavors

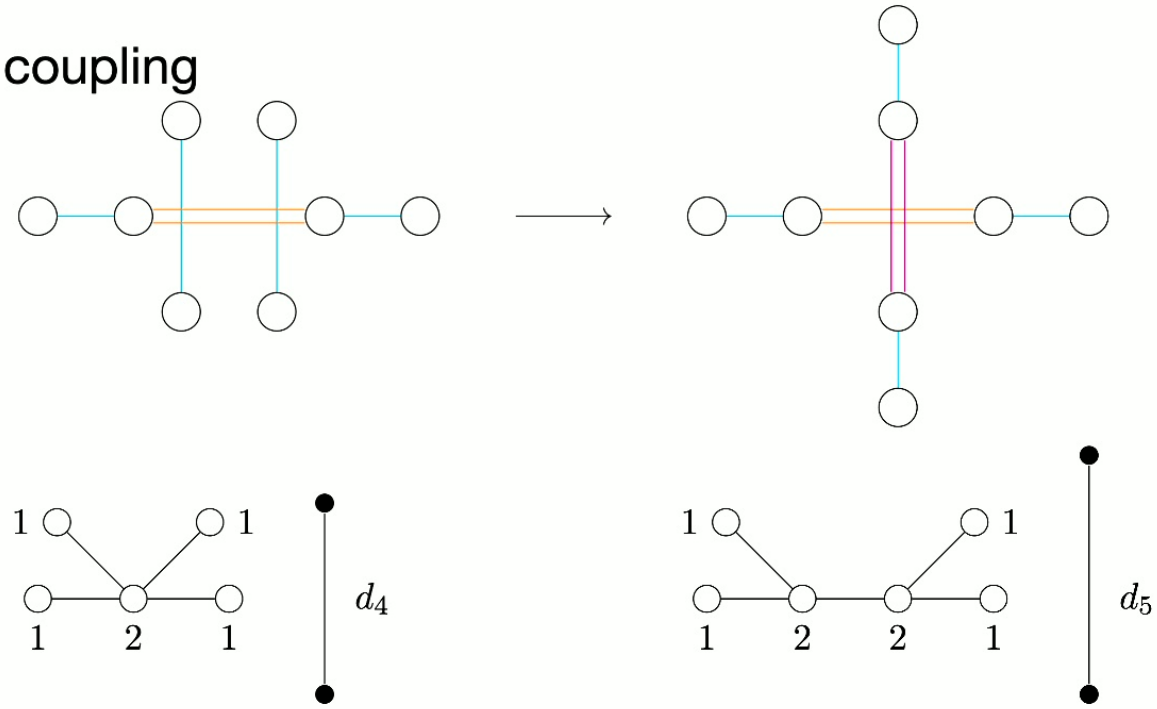
- 1NS brane (point), 10 D8 branes (vertical lines),
- 2 D6 branes (horizontal lines) per interval



Type IIB brane system with an effective 5d theory

Brane Web for SU(2) with 4 flavors

- 2 NS branes (vertical), 8 7 branes (points), 2 D5 branes (horizontal)
- finite / infinite coupling



Electric Quivers

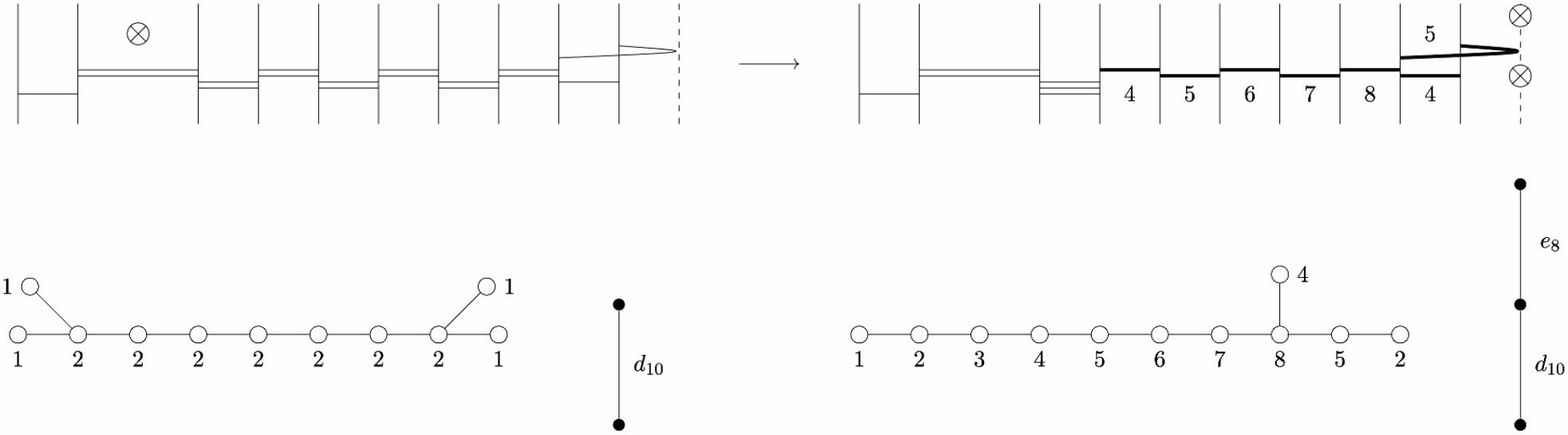
A quiver is a short way to write a Lagrangian

- Each brane segment carries a gauge field on its world volume
- As there are many brane segments, there are many gauge groups
- The Lagrangian contains all the information about the dynamics of the fields living on the branes
- gauge fields, matter fields, kinetic terms, interaction terms
- The quiver collects all of this data in an elegant graphical way

From a brane system to a quiver

intervals map to nodes, vertical lines map to edges

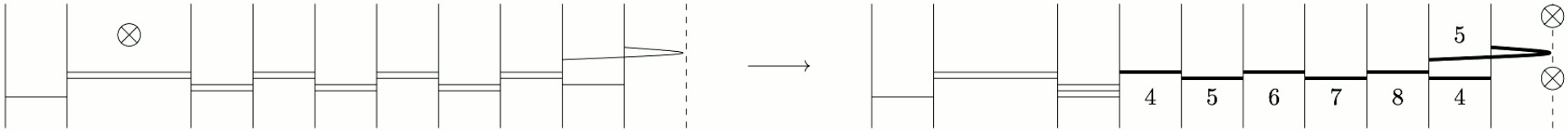
- Each interval with n D branes contributes a node of label n in the quiver
- Each vertical line contributes an edge in the quiver

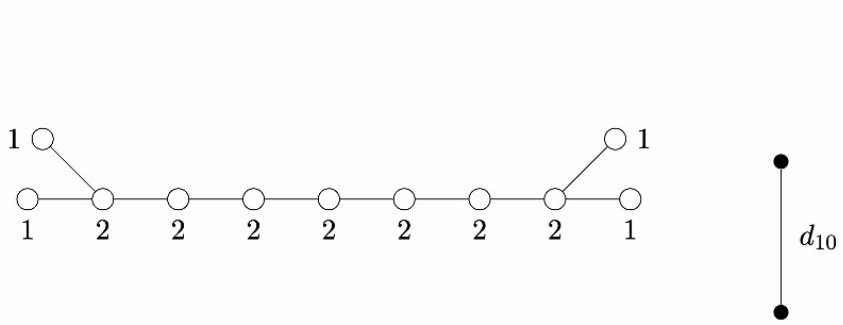


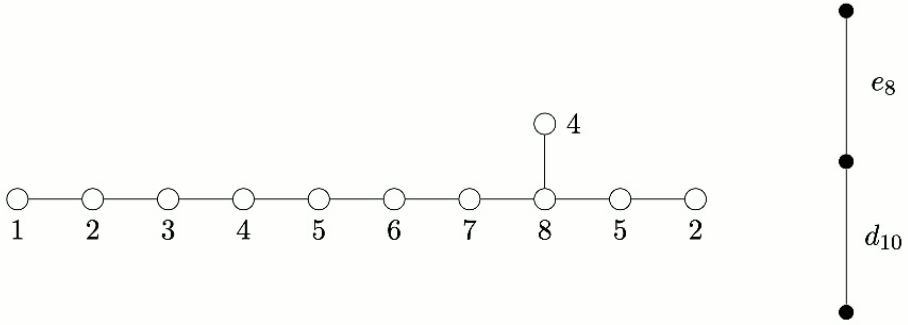
From a quiver to a Lagrangian

nodes and edges — vector multiplets and hypermultiplets

- Each node of label n represents (supersymmetric) U(n) gauge theory
- Each edge represents matter fields in the bi-fundamental representation







Type IIA brane system with an effective 6d theory

Example — SU(2) with 10 flavors

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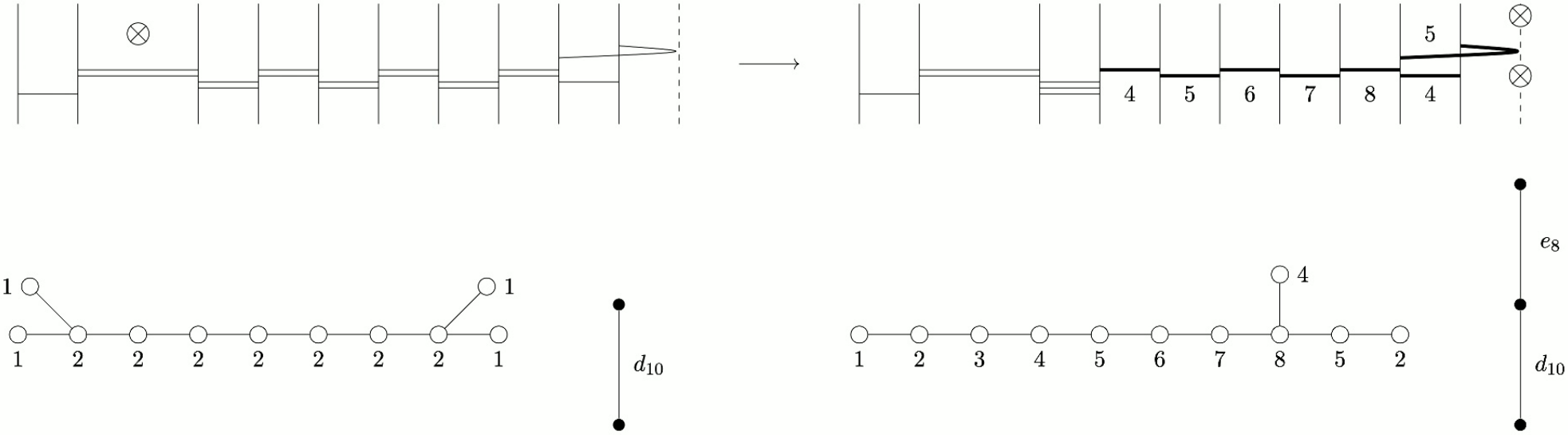

Type IIB brane system with an effective 5d theory

Brane Web for SU(2) with 4 flavors

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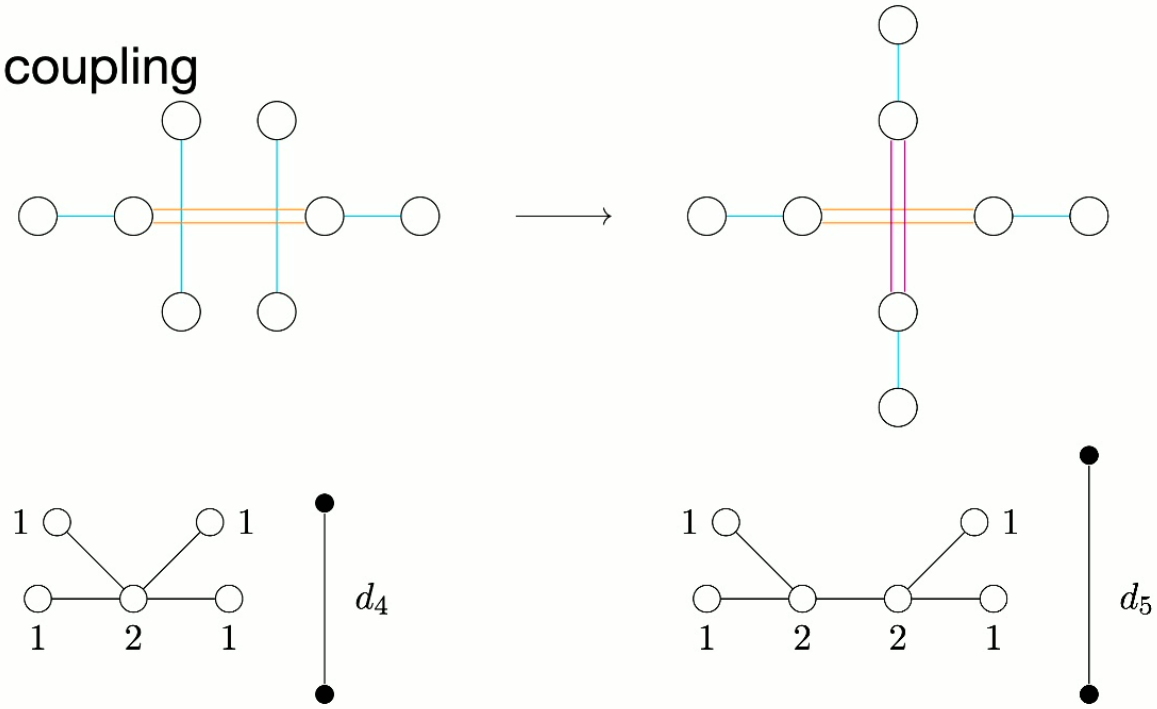
- finite / infinite coupling



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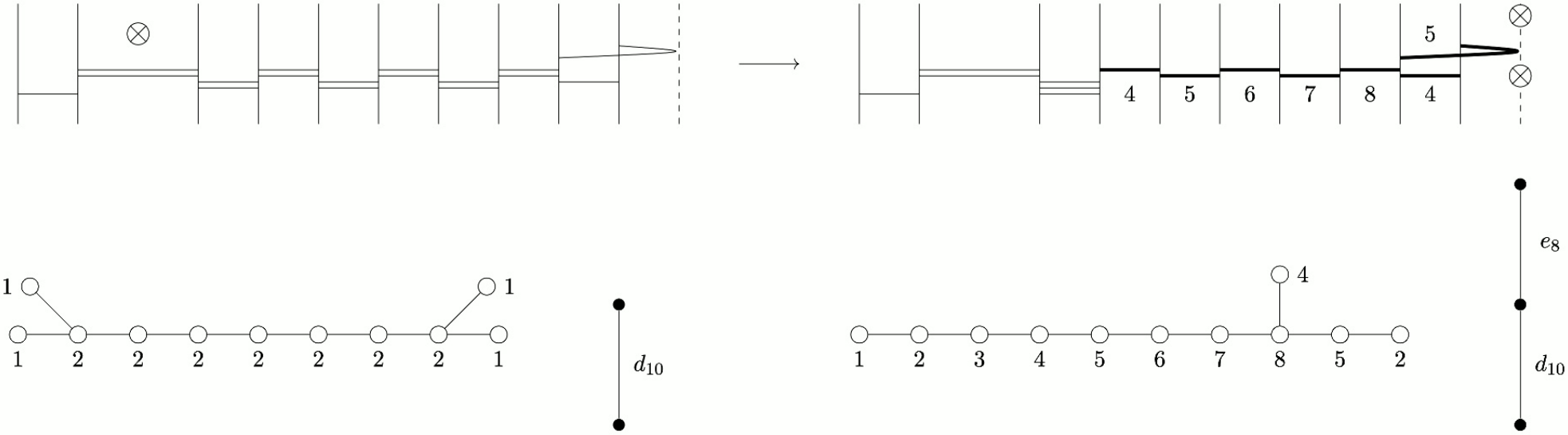


Quivers divide in two types:
Electric Quiver
Magnetic Quiver

For electric quivers Dp branes are stretched between NS5 branes

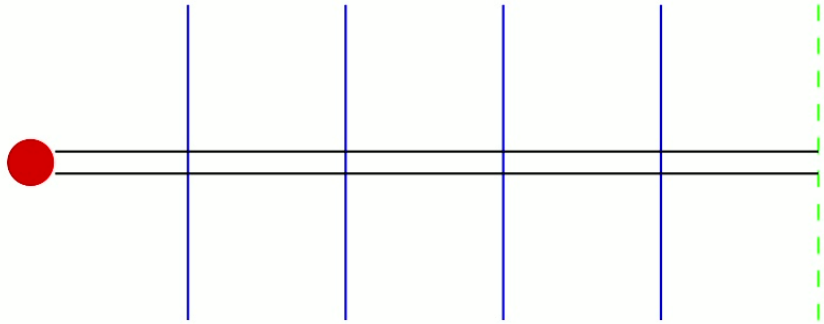
the theory is in p space time dimensions

For magnetic quivers Dp branes are stretched between D($p+2$) branes

A brane system for an electric quiver

D3 (Horizontal) D5 (vertical) NS5 (point) — 3d $\mathcal{N} = 4$ gauge theory

- The D3 branes are stretched between NS5 branes
- SU(2) with 4 flavors



A brane system for a magnetic quiver

Example — SU(2) with 10 flavors in 6d; D6 stretched between D8

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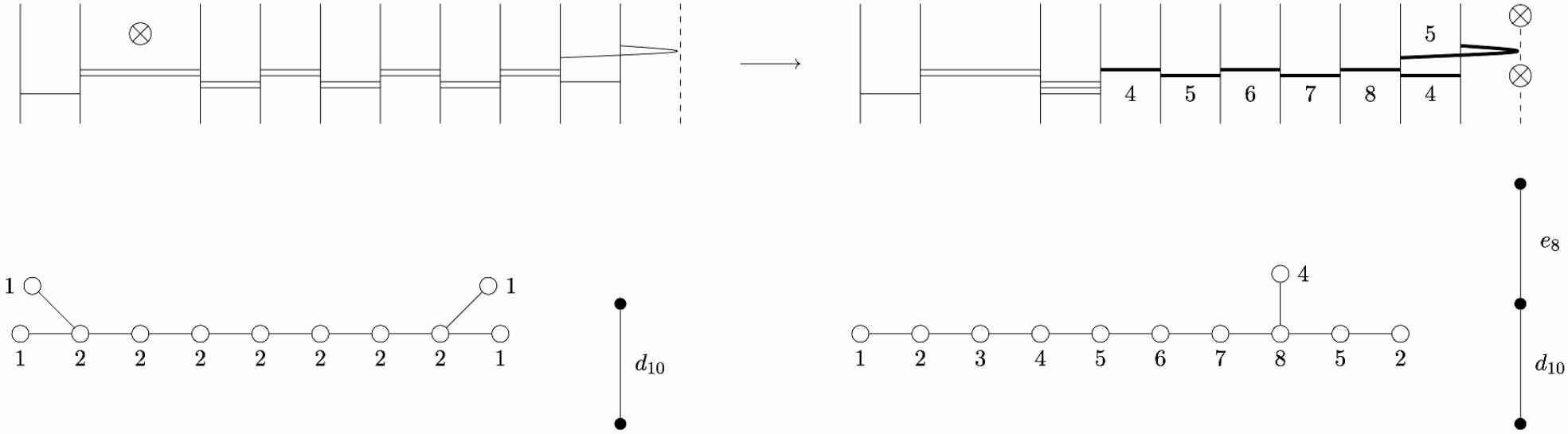
- 2 D6 branes (horizontal lines) per interval



Standard techniques in QFT

Theoretical tools

- Given a Lagrangian, specified by an electric quiver
- Can apply the standard techniques in field theory to compute various physical quantities
- perturbative field theory — expand around free fields
- requires the gauge coupling, or any other perturbative parameter to be very small
- For large values of the gauge coupling, perturbative techniques fail and there is a need for other methods

brane systems and strong coupling

magnetic quivers are tools for understanding non perturbative physics

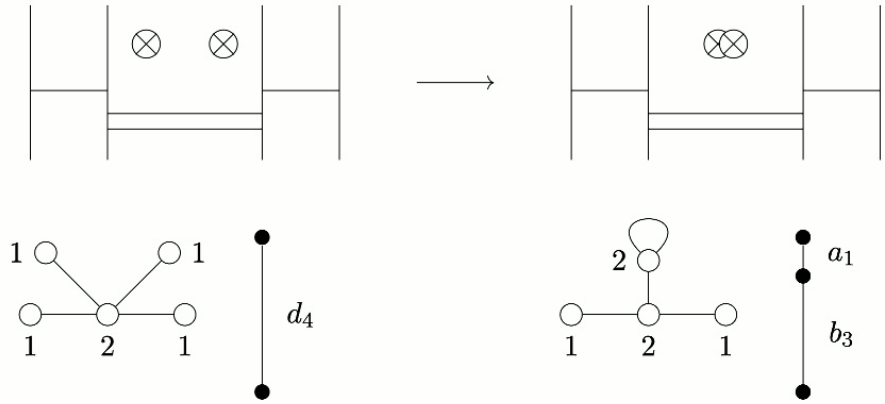
- For a brane system, the gauge coupling is given by L, the distance between NS5 branes
- $\frac{1}{g^2} \propto L$
- For small L the gauge coupling is large and perturbative techniques fail
- In particular, coincident NS5 branes reflect infinite coupling phenomena
- important, for example, in 5d and in 6d
- The magnetic quiver critically changes as one moves from finite to infinite coupling

There are at least two Magnetic Quivers. One reproduces the information given by the electric quiver. Another gives the information for strong coupling

Magnetic quiver in 6 dimensions

Physics of tensionless strings

- BPS strings become tensionless at special points in the moduli space
- like the origin of the Tensor Branch
- What appears to be infinitely many massless states gets replaced by an effective description using a new Higgs branch, given by a magnetic quiver



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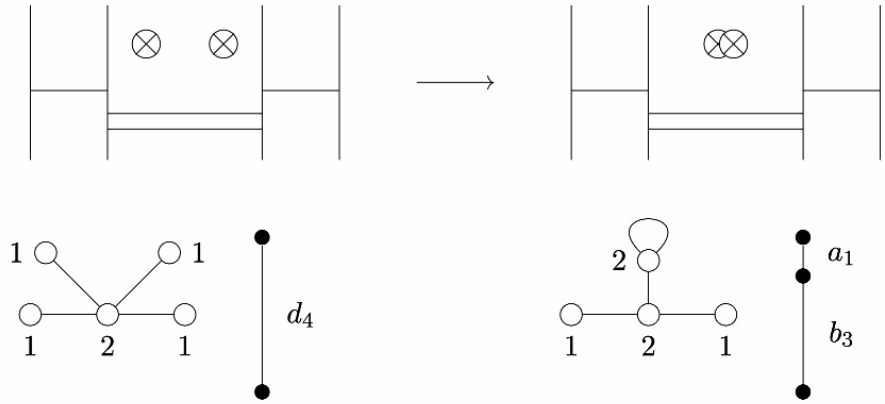
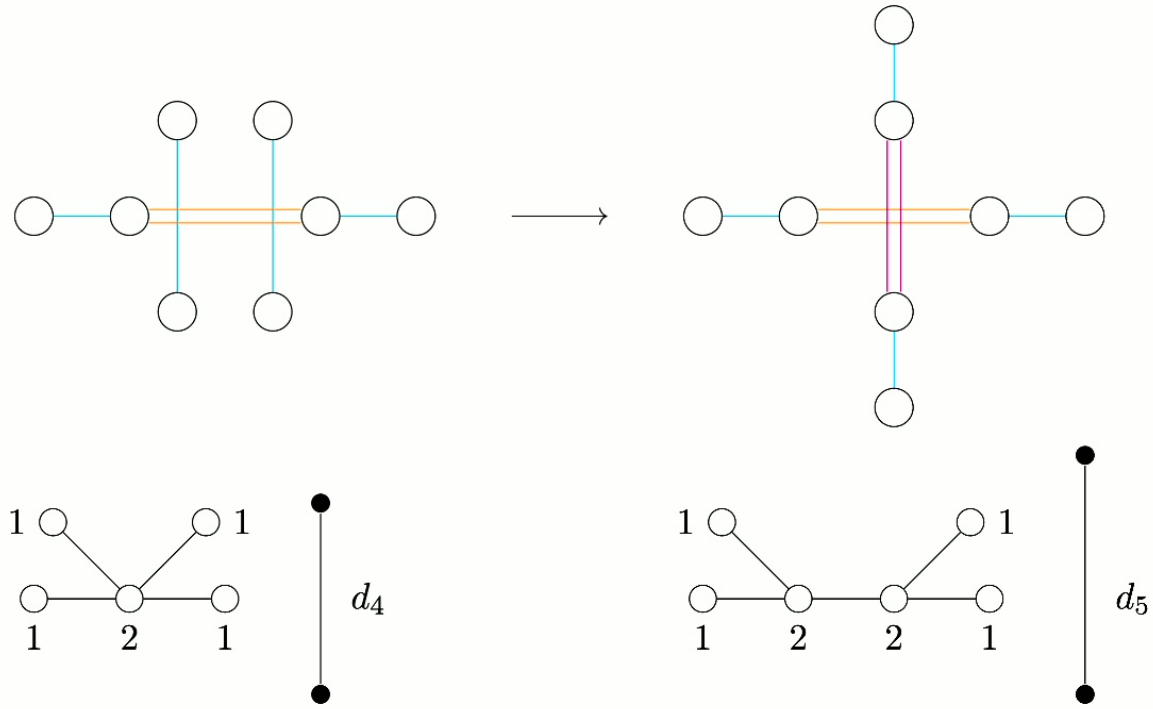


5d — massless instantons

SU(2) with 4 flavors

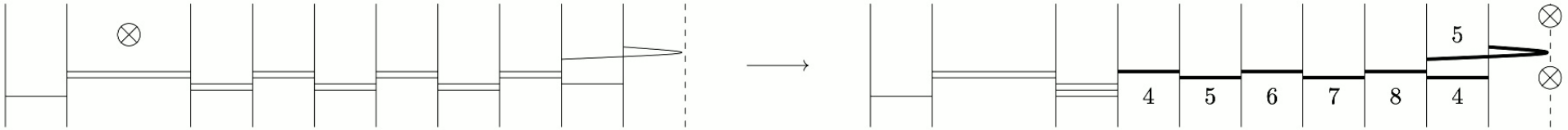
- finite / infinite coupling

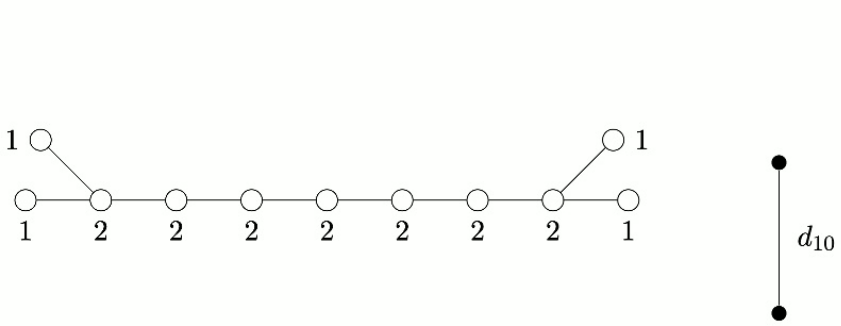


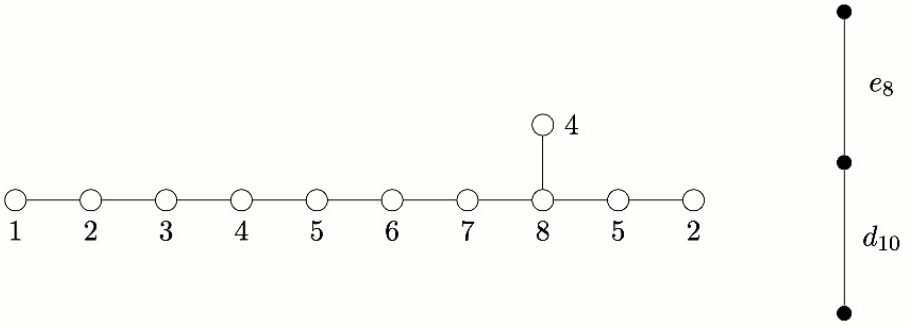
6d — small instanton transition

SU(2) with 10 flavors

- The Classical Higgs branch — minimal nilpotent orbit of SO(20)
- The moduli space of 1 SO(20) instanton on $\mathbb{C}^2$



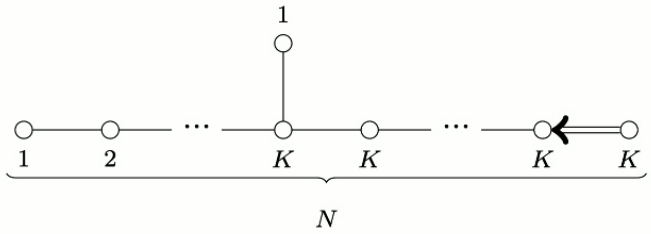




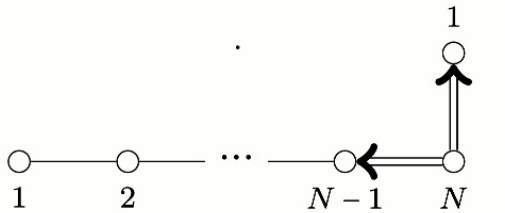
Magnetic quivers — SO(K) with N flavors in 5d

3 cases at finite coupling

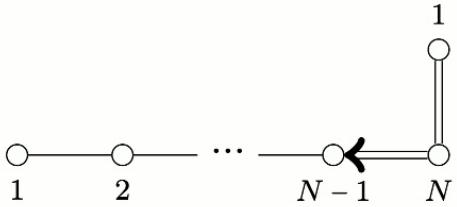
- N>K



- N=K



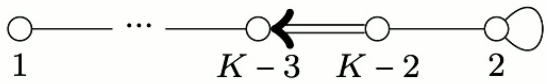
- N<K



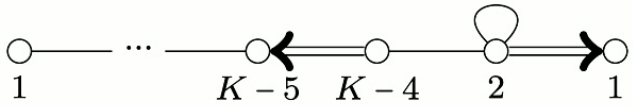
Magnetic quivers — SO(K) with N flavors in 5d

3 cases at infinite coupling

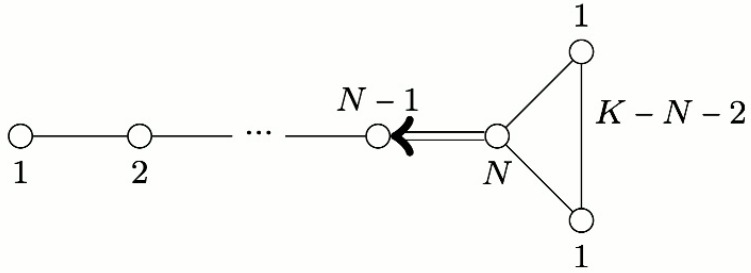
- N=K-3



- N=K-4



- N<K-4



Electric quivers and moduli spaces of vacua

Symplectic singularities

- Given a gauge theory specified by an electric quiver
- Each edge represents matter fields — many scalar fields
- Each scalar field can admit a vacuum expectation value
- We have a continuous set of vacua — called moduli space of vacua
- They form a geometric singularity
- called symplectic singularity

Magnetic quivers and geometric singularities

Symplectic singularities

- The magnetic quiver contains all data needed to specify the symplectic singularity
- strong connection between mathematics and physics
- Representation theory and supersymmetric gauge theories

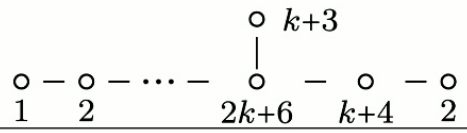
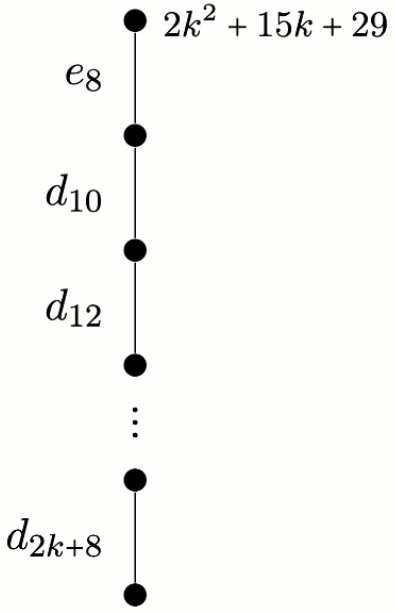
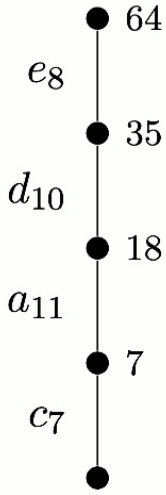
Symplectic singularities and phase diagrams

phase structure of supersymmetric theories

- A moduli space of vacua is divided into disjoint union of sets of vacua
- Each set is characterized by a collection of massless states
- As we move within a fixed set, massive states vary but massless states do not change
- A phase diagram consists of points and edges connecting them
- Each edge represents a symplectic singularity that specifies the transition from one collection of massless states to another

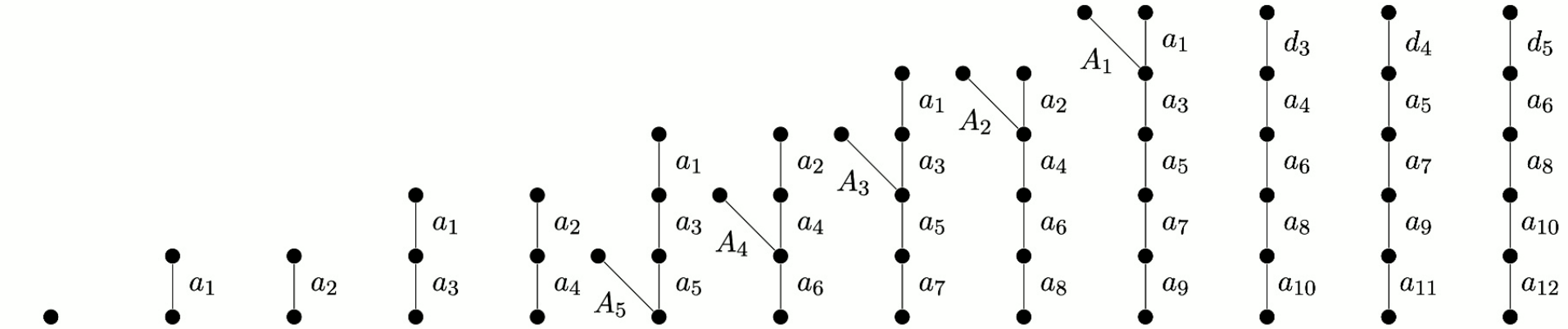
small instanton transition

finite - infinite coupling

$6d$ SCFT	$\mathrm{Sp}(k)$ with $N = 4k + 16$ flavours	G_2 with 7 flavours
Magnetic quiver		Not known
Hasse diagram		

SU(6) with fundamental matter

union of 2 cones



Summary

Changing the way we think

- Magnetic Quivers — encodes all data needed to understand strongly coupled moduli spaces
- Phase (Hasse) diagrams — changes the way we analyze symplectic singularities
- Brane systems — very instrumental in getting this progress
- Monopole formula — opened the window to all recent achievements

Thank you !