Title: Lecture - Beautiful Papers

Speakers: Pedro Vieira

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Subject: Other

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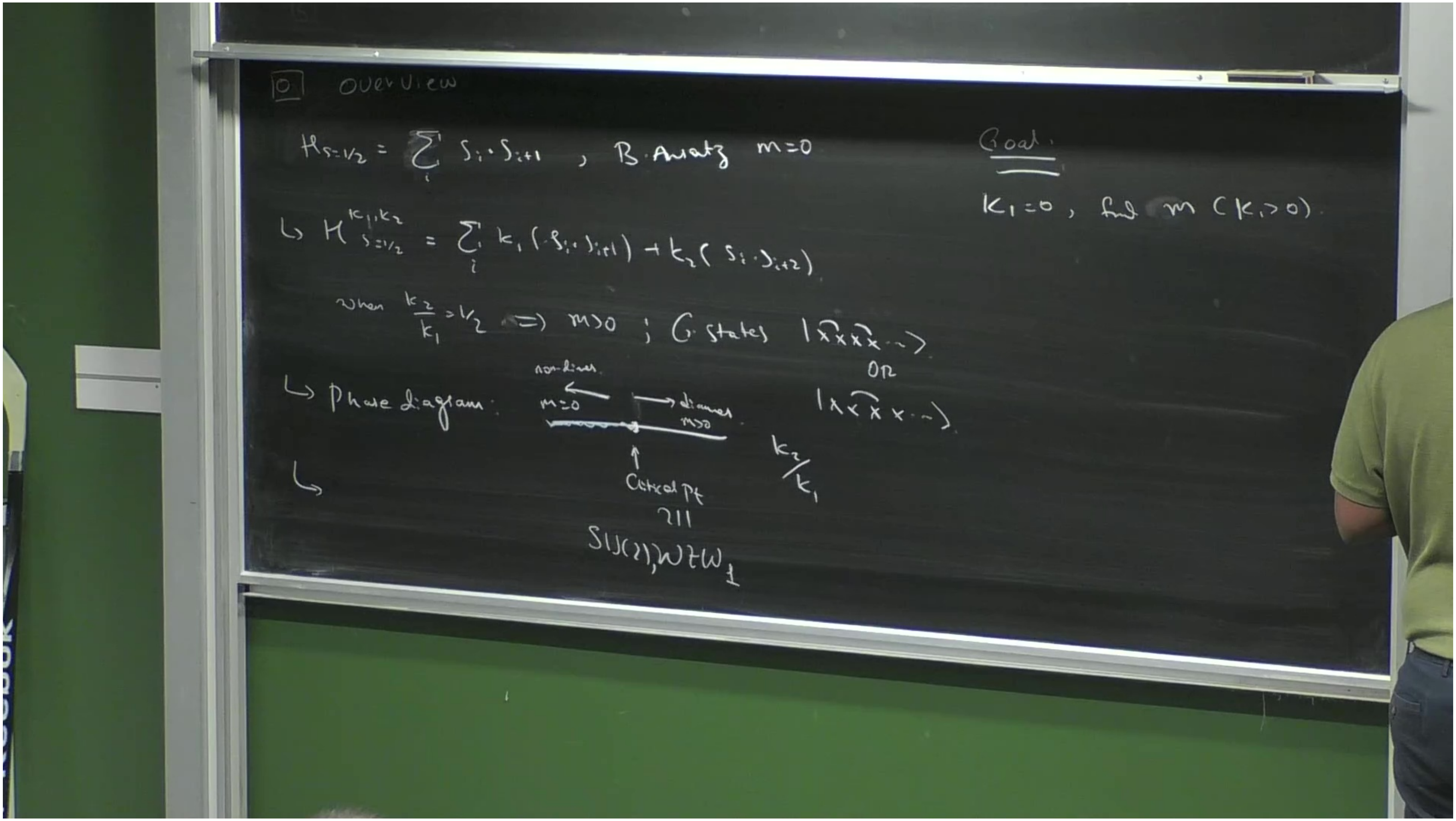
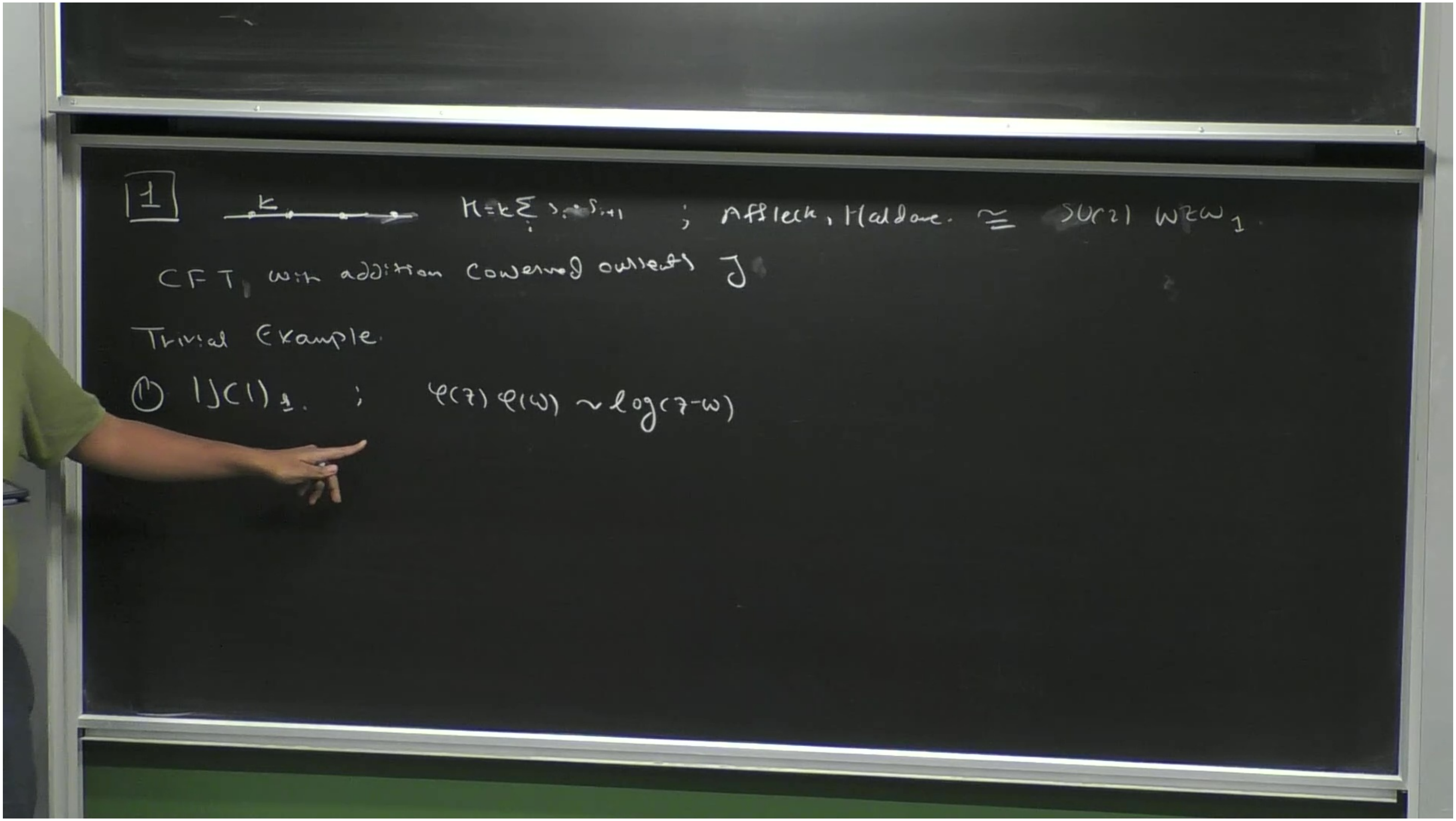

0 Overview
H_{S=1/2} = Σ' S_i · S_{i+1} , B. Ansatz m=0
↳ H^{k1,k2}_{S=1/2} = Σ_i k1 (S_i · S_{i+1}) + k2 (S_i · S_{i+2})
when k2/k1 = 1/2 ⇒ m>0 ; G. states |xxxx...⟩
OR
|xxxx...⟩
↳ Phase diagram:
non-dimer
m=0
dimer
m>0
Critical pt
SU(2)_1 WZW
k2/k1
Goal:
K1=0, find m (K1>0)


1
Affleck, Haldane.
SU(2) WZW 1
CFT with addition conserved currents J
Trivial Example.
U(1)
φ(z) φ(w) ~ log(z-w)

CFT with addition conserved currents J

Trivial Example

(1) $U(1)_1$; $\varphi(z)\varphi(w) \sim \log(z-w)$, $i\partial\varphi(z)\, i\partial\varphi(w) \sim \frac{1}{(z-w)^2}$

$J(z) = i\partial\varphi(z)$

Stress-tensor

Alg: $J(z)J(w) \sim \frac{1}{(z-w)^2} + \frac{0}{(z-w)}$ → $f_{abc} = 0$, $U(1)$

level

$T(z) = (i\partial\varphi\, i\partial\varphi) = (JJ)$

↑ Sugawara Const

$SU(2)_1$ WZW

• Alg $\{J^a, \bar{J}^a\}$

$$J^a(z)\, J^b(w) \sim \frac{\frac{1}{2}\delta^{ab}}{(z-w)^2} + \frac{i\epsilon^{abc} J^c(w)}{(z-w)}$$

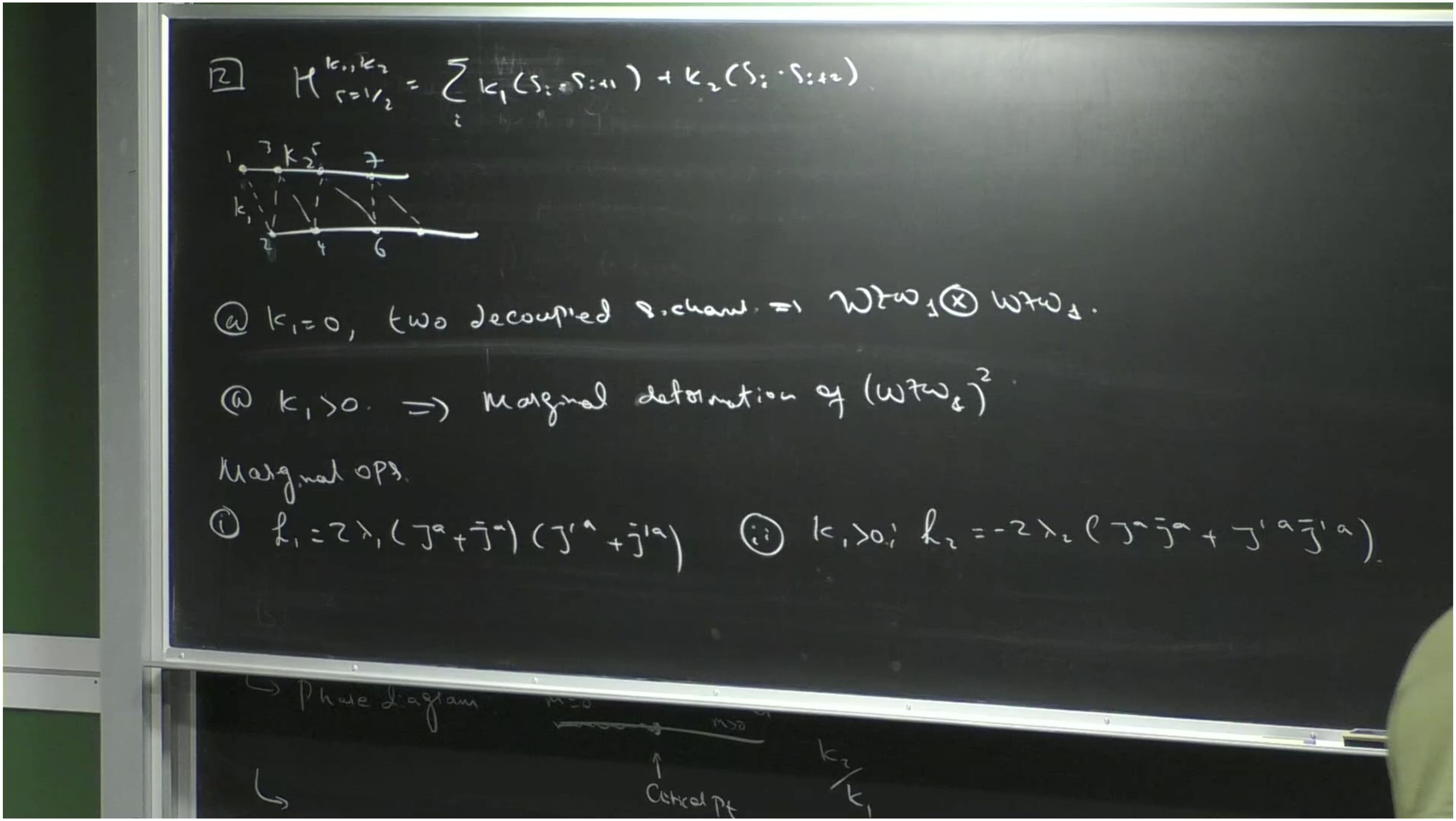
($\frac{1}{2}$ → level-1, ϵ^{abc} → levi-civita)

↳ same thing for $\bar{J}^a \bar{J}^b$

Cons. current

$\bar{\partial} J = 0, \quad \partial \bar{J} = 0$

• Sug $T(z) = (J^a J^a)$

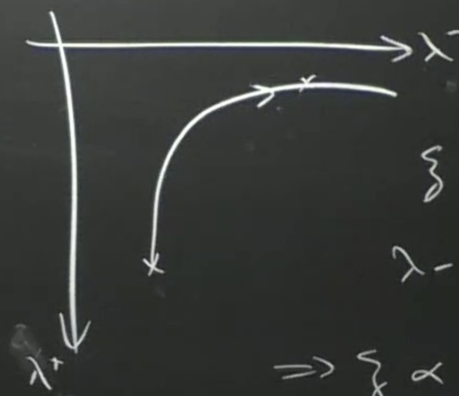

2 H^{k1,k2}_{S=1/2} = Σ_i k1(S_i · S_{i+1}) + k2(S_i · S_{i+2})
1 3 k2 5 7
k1
2 4 6
(a) k1=0, two decoupled s.chains ⇒ WZW_1 ⊗ WZW_1.
(b) k1>0. ⇒ marginal deformation of (WZW_1)^2
Marginal ops.
(i) L_1 = 2λ_1 (J^a + J̄^a)(J'^a + J̄'^a)
(ii) k1>0: L_2 = -2λ_2 (J^a J̄^a + J'^a J̄'^a)
Phase diagram
m=0
m>0
Critical pt
k2/k1

$\mathcal{L}_{int} = \lambda_+ K_+ + \lambda_- K_-$.

RG: $\dfrac{d\lambda_\pm}{d\log L} = (\lambda_\pm)^2$.

Initial conds.

$K_1 \ll K_2 \Rightarrow \lambda_+ < 0, \ \lambda_- > 0$.

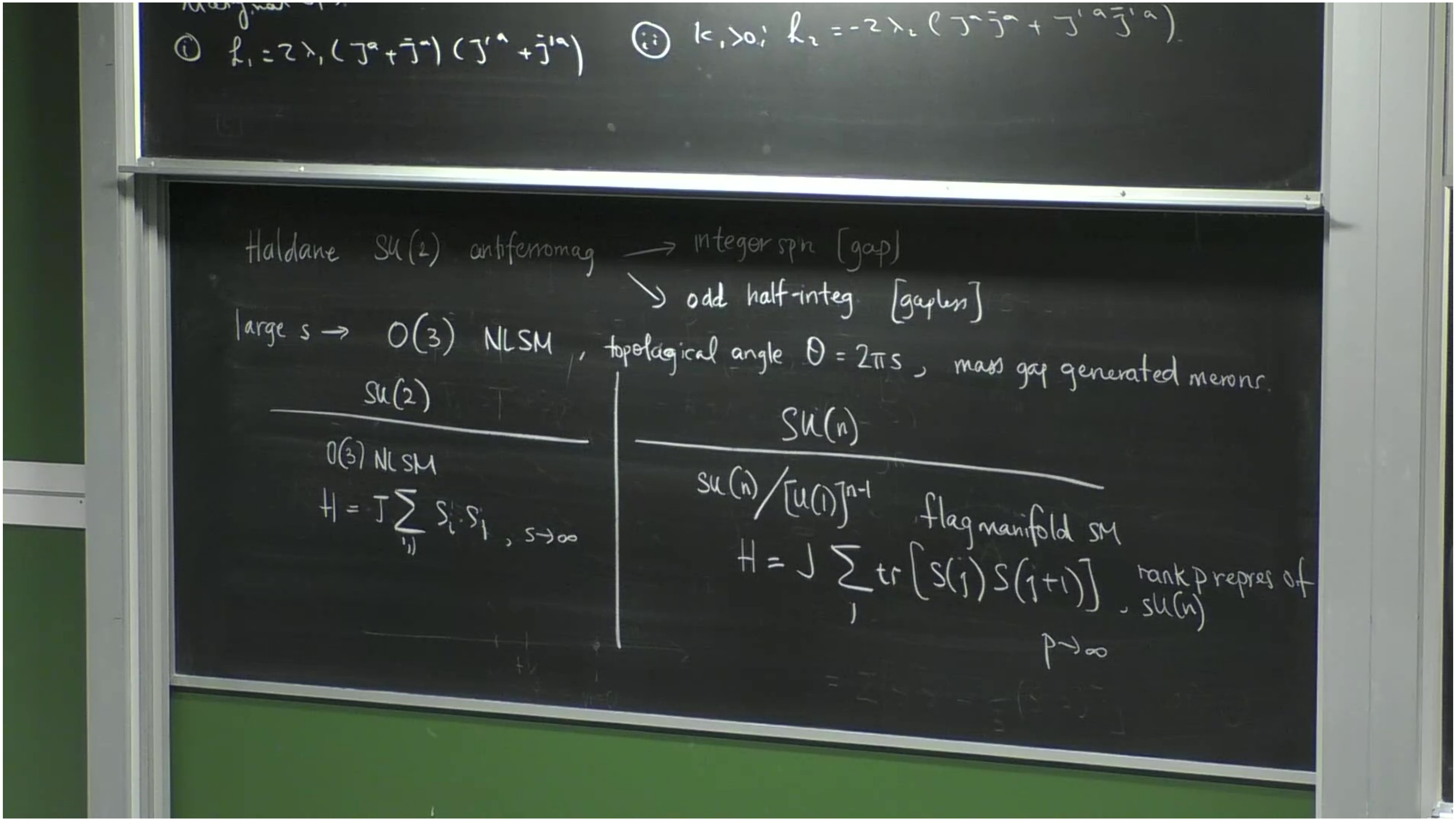


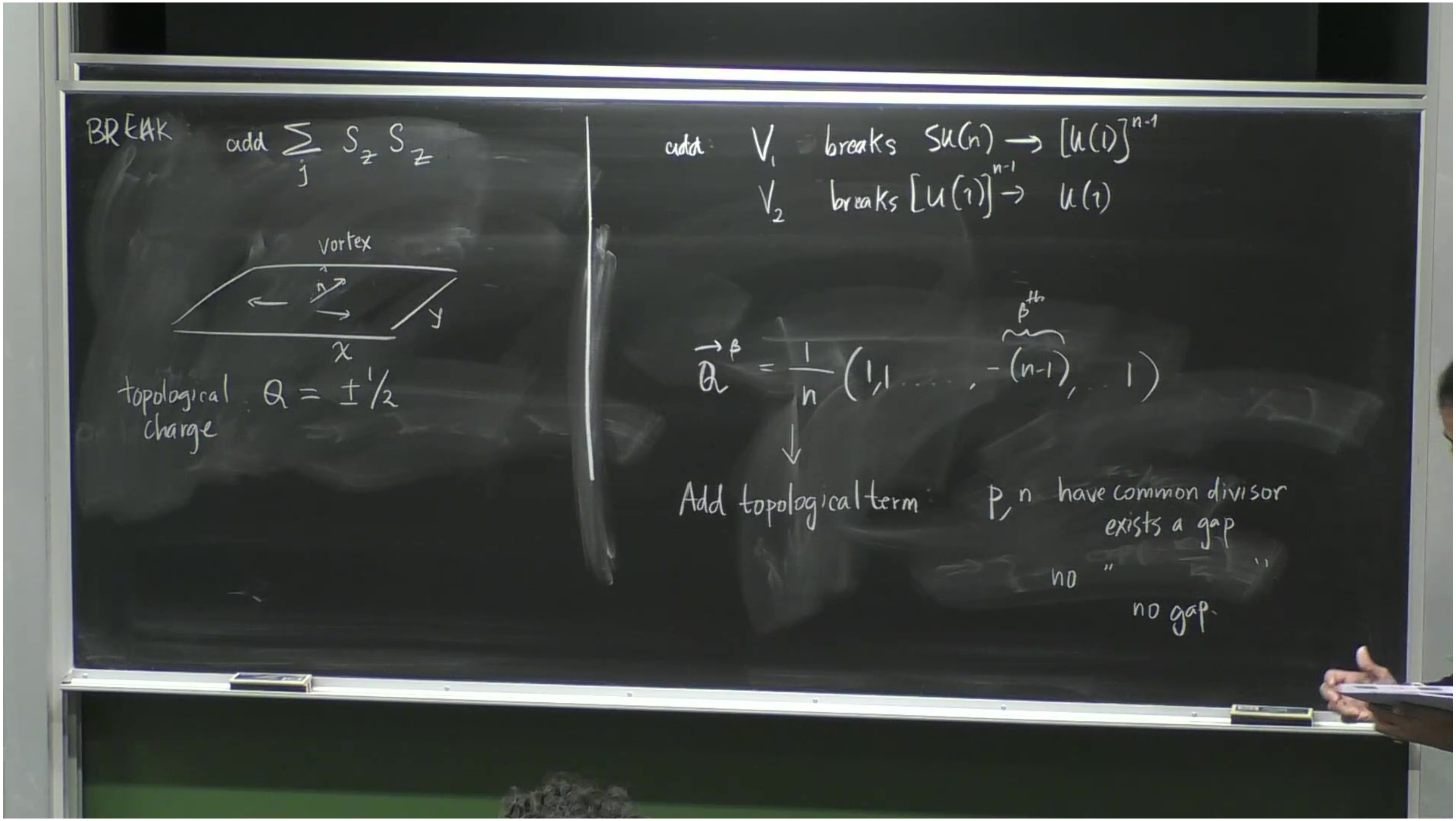
$\xi \propto \exp(-1/\lambda_-)$

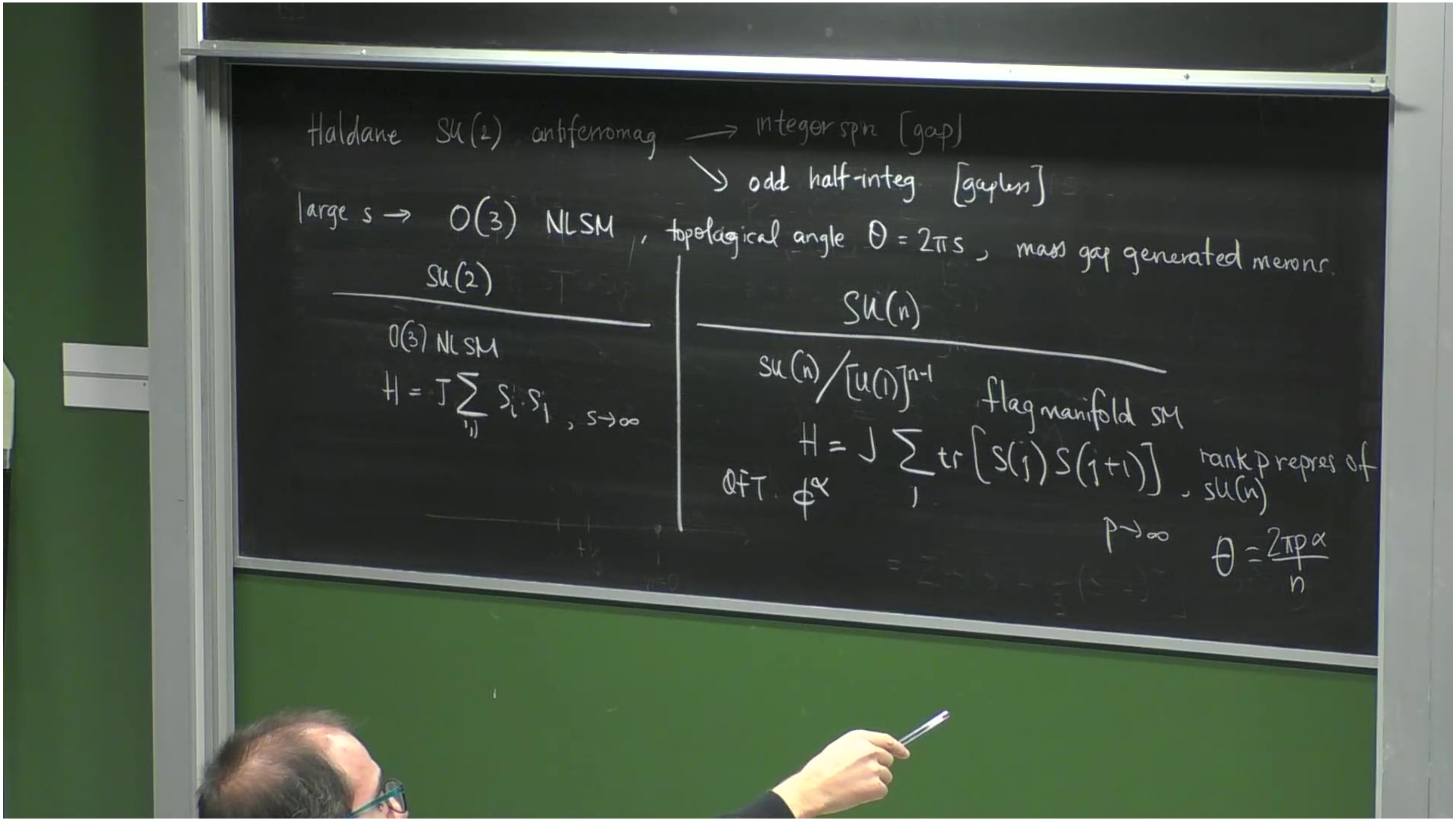
$\lambda_- = K_2/K_1$

$\Rightarrow \xi \propto \exp\left(-\dfrac{K_2}{K_1}\right)$

level


(i) ℓ1 = 2λ1 (J^a + J̄^a)(J'^a + J̄'^a)
(ii) k1 > 0; ℓ2 = -2λ2 (J^a J̄^a + J'^a J̄'^a)
Haldane SU(2) antiferromag → integer spin [gap]
↘ odd half-integ [gapless]
large s → O(3) NLSM, topological angle θ = 2πs, mass gap generated merons
SU(2)
O(3) NLSM
H = J Σ_{i,j} S_i · S_j , s→∞
SU(n)
SU(n)/[U(1)]^{n-1} flag manifold SM
H = J Σ_j tr[S(j) S(j+1)] rank p repres of SU(n)
p→∞


BREAK
add $\sum_j S_z S_z$
vortex
x
y
topological charge $Q = \pm 1/2$
add: V_1 breaks $SU(n) \to [U(1)]^{n-1}$
V_2 breaks $[U(1)]^{n-1} \to U(1)$
β^{th}
$\vec{Q}^{\beta} = \frac{1}{n}(1, 1, \ldots, -(n-1), \ldots 1)$
Add topological term:
p, n have common divisor
exists a gap
no " "
no gap.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QFT φ^α
p→∞
θ = 2πpα/n

Large spin limit of $AdS_5 \times S^5$ string theory and low energy expansion of ferromagnetic spin chains

M. Kruczenski, A.V. Ryzhov and A.A.Tseytlin

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Introduction

- We are interested in studying the relationship between highly spinning strings in $AdS^5 \times S^5$ and operators with large charge in $\mathcal{N} = 4$ SYM

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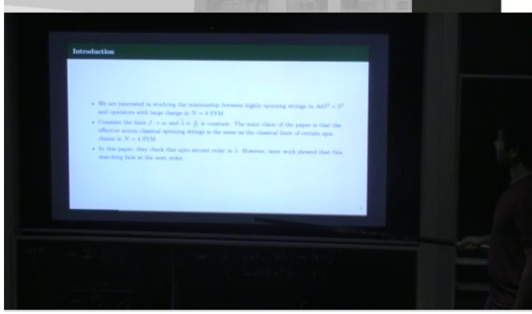
Introduction

- We are interested in studying the relationship between highly spinning strings in $AdS^5 \times S^5$ and operators with large charge in $\mathcal{N} = 4$ SYM
- Consider the limit $J \to \infty$ and $\tilde{\lambda} \equiv \frac{\lambda}{J^2}$ is constant. The main claim of the paper is that the effective action classical spinning strings is the same as the classical limit of certain spin chains in $\mathcal{N} = 4$ SYM
- In this paper, they check this upto second order in $\tilde{\lambda}$. However, later work showed that this matching fails at the next order.

1



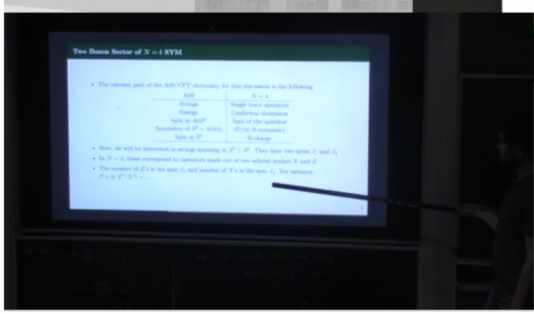
Two Boson Sector of $\mathcal{N} = 4$ SYM

- The relevant part of the AdS/CFT dictionary for this discussion is the following

AdS	$\mathcal{N} = 4$
Strings	Single trace operators
Energy	Conformal dimension
Spin in AdS^5	Spin of the operator
Symmetry of $S^5 = SO(6)$	$SU(4)$ R-symmetry
Spin in S^5	R-charge

- Here, we will be interested in strings spinning in $S^3 \subset S^5$. They have two spins J_1 and J_2
- In $\mathcal{N} = 4$, these correspond to operators made out of two adjoint scalars X and Z.
- The number of Z's is the spin J_1 and number of X's is the spin J_2. For instance, $\mathcal{O} = \text{tr } Z^{J_1} X^{J_2} + \ldots$

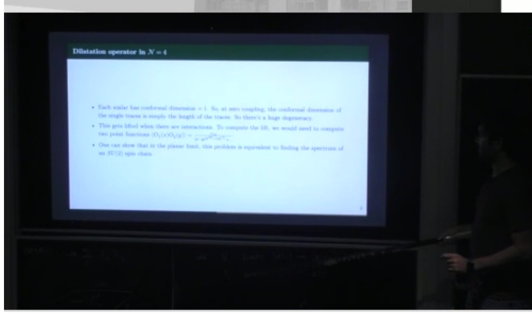
2



Dilatation operator in $\mathcal{N} = 4$

- Each scalar has conformal dimension $= 1$. So, at zero coupling, the conformal dimension of the single traces is simply the length of the traces. So there's a huge degeneracy.
- This gets lifted when there are interactions. To compute the lift, we would need to compute two point functions $\langle O_1(x) O_2(y) \rangle = \frac{c_{12}}{|x-y|^{\Delta^{(0)} + \lambda \Delta^{(1)}} + \ldots}$
- One can show that in the planar limit, this problem is equivalent to finding the spectrum of an $SU(2)$ spin chain.

3



$SU(2)$ spin chain

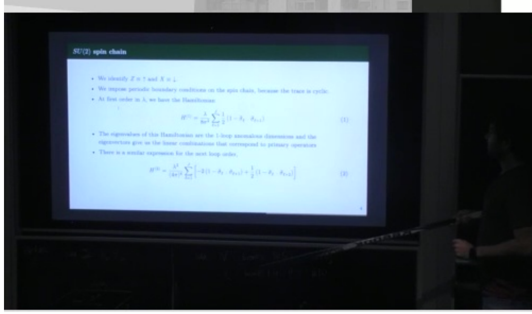
- We identify $Z \equiv \uparrow$ and $X \equiv \downarrow$.
- We impose periodic boundary conditions on the spin chain, because the trace is cyclic.
- At first order in λ, we have the Hamiltonian

$$H^{(1)} = \frac{\lambda}{8\pi^2} \sum_{\ell=1}^{J} \frac{1}{2} \left(1 - \vec{\sigma}_\ell \, . \, \vec{\sigma}_{\ell+1}\right) \tag{1}$$

- The eigenvalues of this Hamiltonian are the 1-loop anomalous dimensions and the eigenvectors give us the linear combinations that correspond to primary operators
- There is a similar expression for the next loop order,

$$H^{(2)} = \frac{\lambda^2}{(4\pi)^4} \sum_{\ell=1}^{J} \left[-2\left(1 - \vec{\sigma}_\ell \, . \, \vec{\sigma}_{\ell+1}\right) + \frac{1}{2}\left(1 - \vec{\sigma}_\ell \, . \, \vec{\sigma}_{\ell+2}\right)\right] \tag{2}$$

4



Spin chain action

- To obtain the effective action, we define coherent states

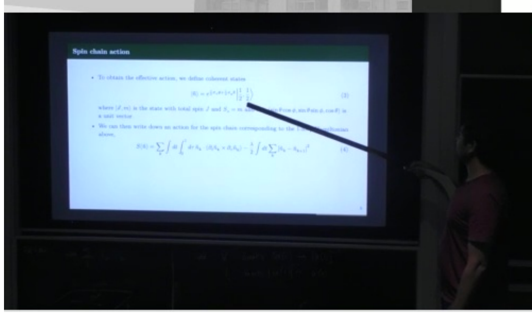
$$|\vec{n}\rangle = e^{\frac{i}{2}\sigma_z\phi + \frac{i}{2}\sigma_y\theta}\left|\frac{1}{2}, \frac{1}{2}\right\rangle \tag{3}$$

where $|J, m\rangle$ is the state with total spin J and $S_z = m$ and $\vec{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ is a unit vector.

- We can then write down an action for the spin chain corresponding to the 1-loop Hamiltonian above,

$$S(\vec{n}) = \sum_k \int dt \int_0^1 d\tau\, \vec{n}_k \cdot (\partial_t \vec{n}_k \times \partial_\tau \vec{n}_k) - \frac{\lambda}{2}\int dt \sum_k [\vec{n}_k - \vec{n}_{k+1}]^2 \tag{4}$$

5



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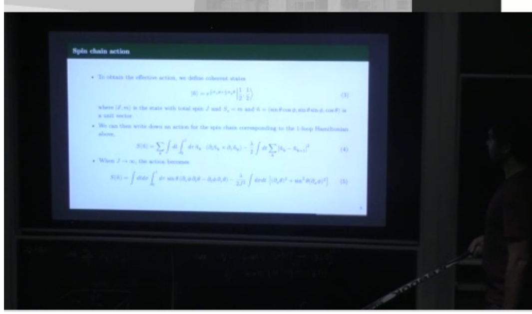
- When $J \to \infty$, the action becomes

$$S(\vec{n}) = \int dt d\sigma \int_0^1 d\tau\, \sin\theta\,(\partial_\tau\phi\,\partial_t\theta - \partial_t\phi\,\partial_\tau\theta) - \frac{\lambda}{2J^2}\int d\sigma dt\, \left[(\partial_\sigma\theta)^2 + \sin^2\theta(\partial_\sigma\phi)^2\right] \tag{5}$$

5



String Action

- The string is moving on $R \times S^3$. The metric in some coordinates is given by

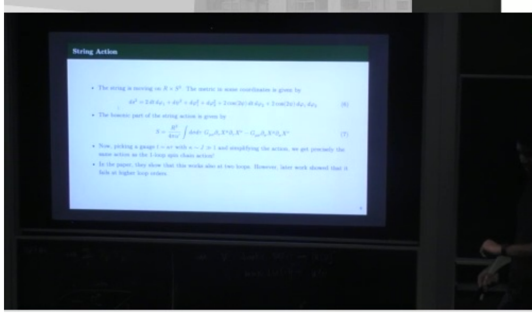
$$ds^2 = 2\,dt\,d\varphi_1 + d\psi^2 + d\varphi_1^2 + d\varphi_2^2 + 2\cos(2\psi)\,dt\,d\varphi_2 + 2\cos(2\psi)\,d\varphi_1\,d\varphi_2 \tag{6}$$

- The bosonic part of the string action is given by

$$S = \frac{R^2}{4\pi\alpha'}\int d\sigma d\tau\; G_{\mu\nu}\partial_\tau X^\mu \partial_\tau X^\nu - G_{\mu\nu}\partial_\sigma X^\mu \partial_\sigma X^\nu \tag{7}$$

- Now, picking a gauge $t = \kappa\tau$ with $\kappa \sim J \gg 1$ and simplifying the action, we get precisely the same action as the 1-loop spin chain action!
- In the paper, they show that this works also at two loops. However, later work showed that it fails at higher loop orders.

6



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