Title: Lecture - Beautiful Papers

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Subject: Other

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$$H = \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \qquad \vec{n}^2 = 1$$

$$\Big\downarrow\ s \gg 1$$

$$\mathcal{L} = \frac{1}{2g^2} \partial \vec{n}^2 + \theta \times \frac{\epsilon^{\mu\nu}}{8\pi} \vec{n} \cdot (\partial_\mu \vec{n} \times \partial_\nu \vec{n})$$

$= O(3)$ σ-model with θ angle

$g = 2/s$, $\theta = 2\pi s$

$L \to \infty$

$$H = \sum_{i=1}^{L} \vec{S}_i \cdot \vec{S}_{i+1}$$

Haldane: Gapped for integer spin s
Gapless for half-integer s

$H =$ Solvable models (AKLT [11]) realize various scenarios.

$H = \sum \vec{S}_i \cdot \vec{S}_{i+1}$ gapless $s = 1/2$

$H = \sum \vec{S}_i \cdot \vec{S}_{i+1} + \underbrace{\frac{1}{2}}_{\beta} \vec{S}_i \cdot \vec{S}_{i+2}$ has gap and degenerate vacua!

Lieb-Schultz-Mattis Theorem:

Any Local H, half-integer spin-chain with translation and rot sym
has (A) no gap (ie $m=0$ ptcs) or
(B) degenerate ground-states (Spontaneously broken parity)

$$H = J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j = J \sum_i \Big(\tfrac{1}{2} S_i^+ S_j^- + \tfrac{1}{2} S_i^- S_j^+ + \underbrace{S_i^z S_j^z}_{\text{es}} \Big)$$

$$\left[S^a_{/s}, S^b_{/s} \right] = i \underbrace{\frac{1}{s}}_{\text{spins become classical as } s \to \infty} \varepsilon^{abc} S^c_{/s}$$

$$\vec{S}^2 = s(s+1)$$

$$\vec{S} \sim s \quad \text{if } s \gg 1$$

neel state

$$\longrightarrow \Big| \overbrace{\underbrace{\underset{+s}{\uparrow}\underset{-s}{\downarrow}\underset{+s}{\uparrow}\underset{-s}{\downarrow}}_{\text{close to g.s.}} \uparrow\downarrow\uparrow\downarrow \ldots}^{\text{neel state}} \Big\rangle = |NS\rangle$$

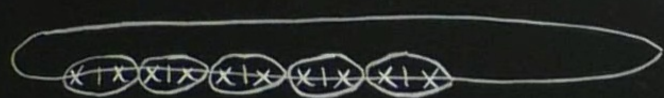
$\downarrow$ spin wave analysis

$$H = \sum \underbrace{\varepsilon_{\vec{k}}}_{\text{gapless, } \varepsilon \sim \vec{k}} c^\dagger_{\vec{k}} c_{\vec{k}}$$

" $\vec{S}_i = \pm s\, \vec{n}_i + \vec{l}_i$ "

(1) $\vec{n}\left(2i+\frac{1}{2}\right) \equiv \dfrac{\vec{S}_{2i+1} - \vec{S}_{2i}}{2s}$, $\vec{\ell}\left(2i+\frac{1}{2}\right) \equiv \dfrac{\vec{S}_{2i+1} + \vec{S}_{2i}}{2}$

$s >> 1$, $\vec{n}\cdot\vec{\ell} = 0$, $\boxed{\vec{n}^{\,2} = 1} + O(1/s)$



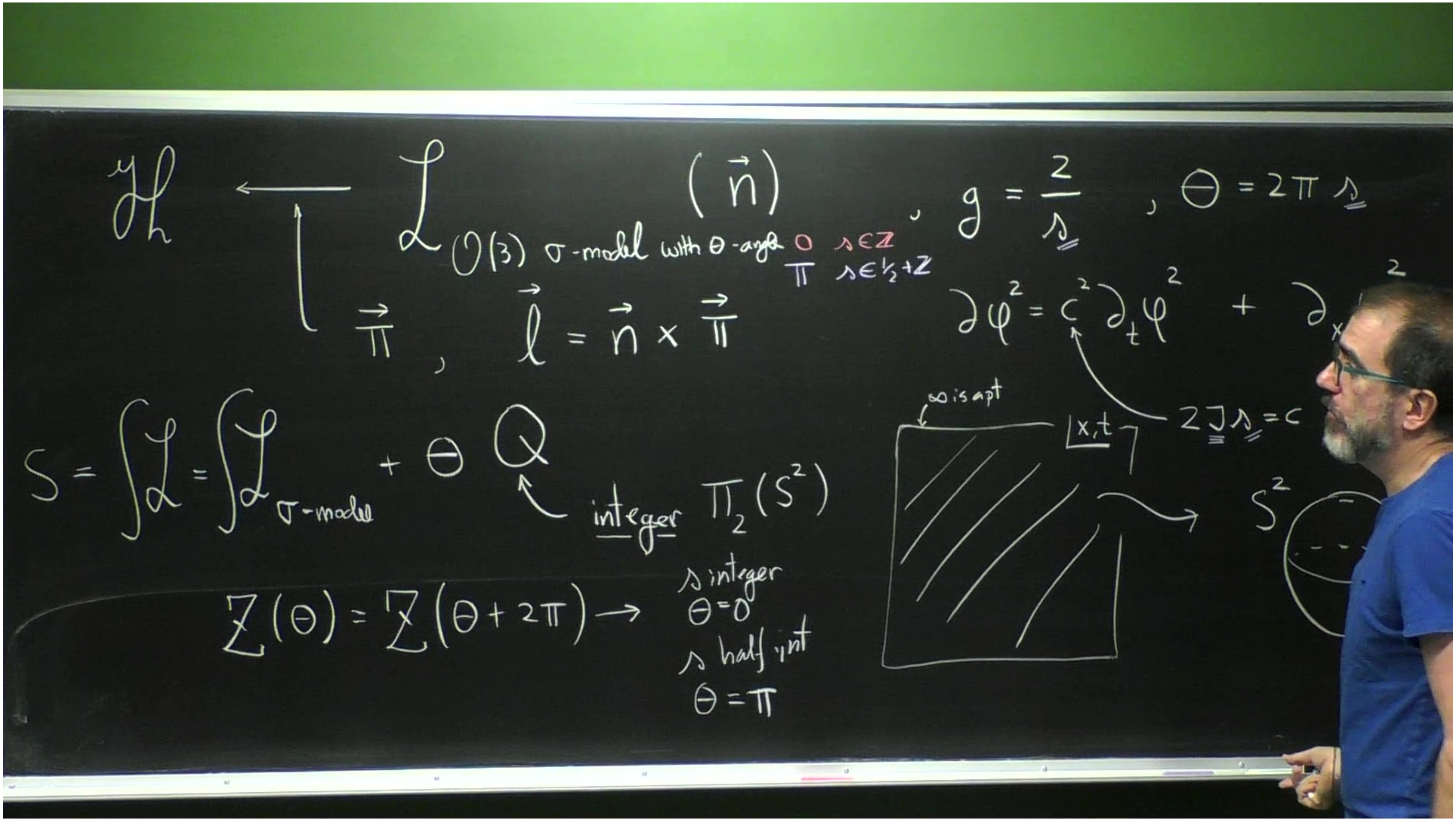
Large s limit of $H = J \sum_i \vec{S}_i\, \vec{S}_{i+1}$

$\longrightarrow H = \sum_i \cdots = \int dx\, \mathcal{H}(\vec{\ell}, \vec{n})$

cont limit

$i = 1 \ldots L$

$x = 0 \ldots 1$, $x = i/L$



$\mathcal{H} \longleftarrow \mathcal{L}(\vec{n})$, $g = \frac{2}{s}$, $\Theta = 2\pi s$
O(3) σ-model with Θ-angle 0 $s \in \mathbb{Z}$
π $s \in \frac{1}{2} + \mathbb{Z}$
$\vec{\pi}$, $\vec{l} = \vec{n} \times \vec{\pi}$
$\partial \varphi^2 = c^2 \partial_t \varphi^2 + \partial_x$
∞ is a pt
x, t
$2Js = c$
$S = \int \mathcal{L} = \int \mathcal{L}_{\sigma\text{-model}} + \Theta Q$
integer $\pi_2(S^2)$
$Z(\Theta) = Z(\Theta + 2\pi) \rightarrow$ s integer $\Theta = 0$
s half int $\Theta = \pi$
S^2

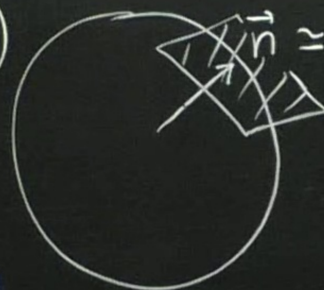
$O(3)$ σ-model with $\theta = 0$ is gapped "$\Longrightarrow$" $H_{s \in \mathbb{Z}}$ is gapped!

(at least for s large enough)

$$\mathcal{L} = \frac{1}{2g} \partial \vec{n}^{\,2} + \lambda(\vec{n}^{\,2} - 1)$$

$$= \frac{1}{2g} \sum_{i=1}^{2} (\partial n_i)^2$$

massless!



$$\vec{n} \simeq (0, 0, 1) + \text{fluct}$$

$$\vec{n} = \Big(n_1,\ n_2,\ \underbrace{\sqrt{1 - n_1^2 - n_2^2}}_{\simeq 1 - \underbrace{\frac{1}{2}\sum n_i^2}_{\langle \delta S_z \rangle} + \dots}\Big)$$

$$\langle \vec{S}(0) \cdot \vec{S}(\vec{x}) \rangle = \pm s^2 \left(1 + \frac{1}{\pi s} \log(x\mu) + \dots \right)$$

$$\text{log div} = \int \underbrace{d^2\vec{k}}_{k\,dk} \frac{1}{\vec{k}^2} =$$

$\uparrow\downarrow\uparrow\downarrow\uparrow \ \ldots\ldots \ \uparrow\uparrow\uparrow\uparrow$

$\longleftrightarrow\ \ell = e^{\pi s} \longleftrightarrow m = e^{-\pi s}$

$$\mathcal{L} = \frac{1}{2g} \partial \vec{n}^2 + \lambda \left(\vec{n}^2 - 1 \right) \qquad \underbrace{e^{-2\pi/g}}_{m^2} \qquad \vec{n} = 1 \dots 3 \to N \gg 1$$

$\lambda = \lambda_{SP}$

$O(N)$ Sigma model

$$\mathcal{L}_\lambda = -\lambda + N \log \det \left(-\frac{1}{2g} \partial^2 + \lambda \right) \to \lambda_{SP} = e^{-2\pi/g} + O\left(\frac{1}{N}\right)$$

$N - 1 = 2$ $\underbrace{\text{gapless goldstone bosons}}_{\text{Wrong}}$ $\longrightarrow$ $N = 3$ very light $m > 0$ particles and sym is not broken!

$$H = \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \qquad \vec{n}^2 = 1$$

$\downarrow s \gg 1$

$$\mathcal{L} = \frac{1}{2g^2} \partial \vec{n}^2 + \theta \times \frac{\varepsilon^{\mu\nu}}{8\pi} \vec{n} \cdot (\partial_\mu \vec{n} \times \partial_\nu \vec{n})$$

= O(3) σ-model with θ angle

$g = 2/s \ , \ \theta = 2\pi s$

We know from Bethe ansatz

$s = 1/2$ is gapless!

$L \to \infty$

$$H = \sum_{i=1}^{L} \vec{S}_i \cdot \vec{S}_{i+1}$$

Haldane: Gapped for integer spin s

Gapless for half-integer s

H = Solvable models

(AKLT [11]) realize

various scenarios.

$H = \sum \vec{S}_i \cdot \vec{S}_{i+1}$ gapless $s = 1/2$

$H = \sum \vec{S}_i \cdot \vec{S}_{i+1} + \underbrace{\tfrac{1}{2}}_{\beta} \vec{S}_i \cdot \vec{S}_{i+2}$ has gap and degenerate vacua!

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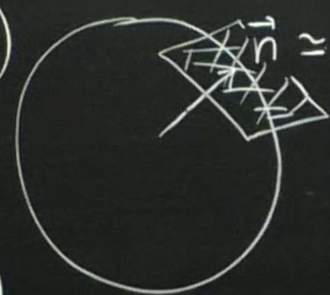
$O(3)$ σ-model with $\theta = 0$ is gapped "$\Longrightarrow$" $H_{s\in\mathbb{Z}}$ is gapped!

(at least for s large enough)

$$\mathcal{L} = \frac{1}{2g}\,\partial\vec{n}^{\,2} + \lambda(\vec{n}^{\,2}-1)$$



$$\vec{n} \simeq \overbrace{(0,0,1)}^{\text{sym broken}} + \text{fluct}$$

$$\vec{n} = \Big(n_1,\ n_2,\ \underbrace{\sqrt{1-n_1^2-n_2^2}}_{\simeq 1 - \frac{1}{2}\underbrace{\Sigma n_i^2}_{\langle \delta S_z\rangle} + \cdots}\Big)$$

$$= \frac{1}{2g}\sum_{i=1}^{2}(\partial n_i)^2$$

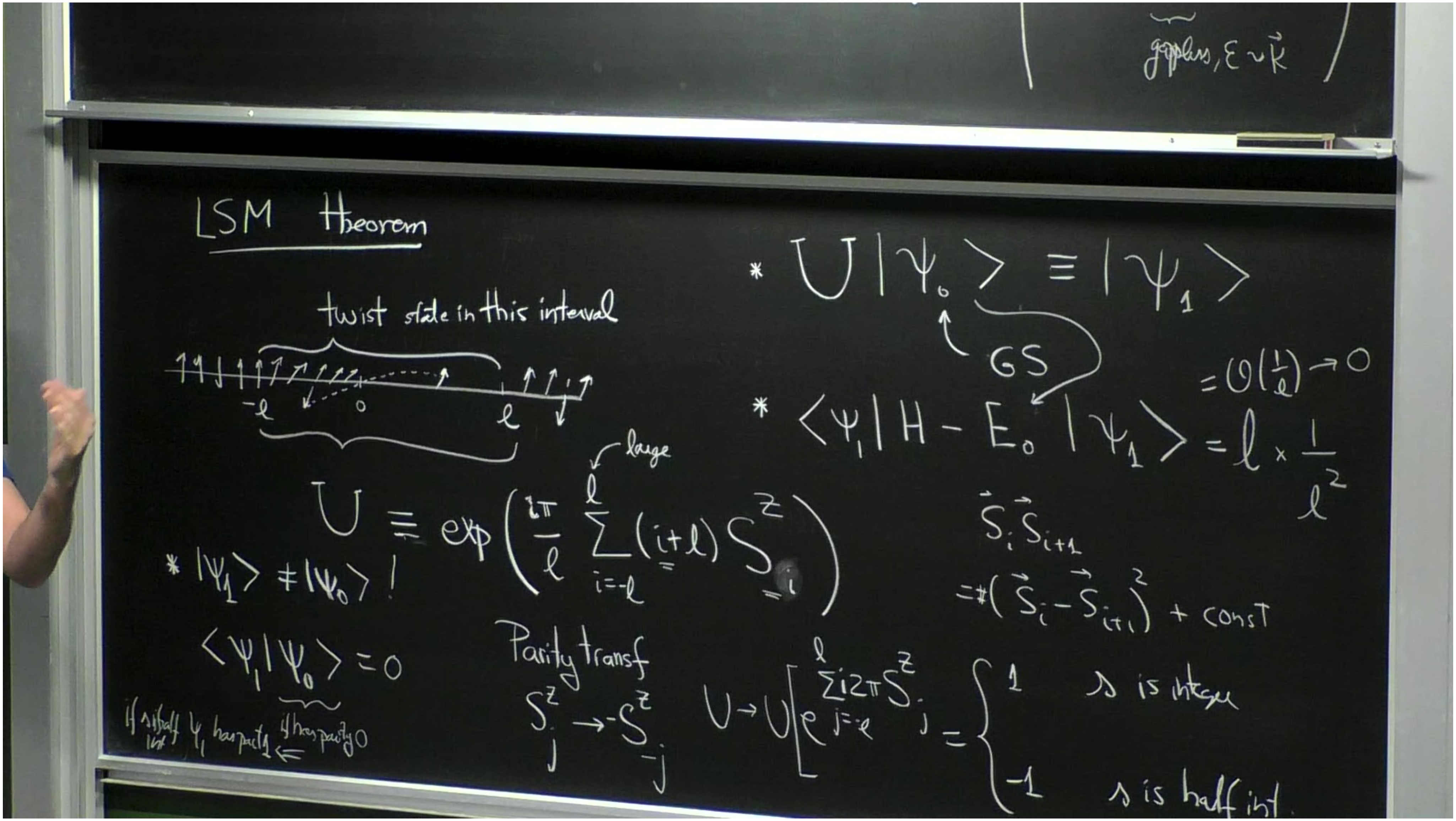
massless!

$$\langle \vec{S}(0)\cdot\vec{S}(\vec{x})\rangle = \pm s^2\Big(1 + \frac{1}{\pi s}\log(x\mu) + \cdots\Big)$$

$$\Big|_{\text{log div}} = \int \underbrace{d^2\vec{k}}_{KdK}\ \frac{1}{\vec{k}^{\,2}} = $$

↑↓↑↓↑ ...... ↗↗↑↗↗

$$\longleftrightarrow\ l = e^{\pi s} \longleftrightarrow m = e^{-\pi s}$$


gapless, $\varepsilon \sim \vec{k}$
LSM theorem
twist state in this interval
$-\ell$ 0 ℓ
large
$U \equiv \exp\left(\frac{i\pi}{\ell}\sum_{i=-\ell}^{\ell}(i+\ell)S_i^z\right)$
* $|\psi_1\rangle \neq |\psi_0\rangle$!
$\langle\psi_1|\psi_0\rangle = 0$
if s is half int ψ_1 has parity 1 $\Leftarrow$ if has parity 0
Parity transf
$S_j^z \to -S_{-j}^z$
$U \to U\left[e^{\sum_{j=-\ell}^{\ell} i2\pi S_j^z}\right] = \begin{cases} 1 & s \text{ is integer} \\ -1 & s \text{ is half int.}\end{cases}$
* $U|\psi_0\rangle \equiv |\psi_1\rangle$
GS
* $\langle\psi_1|H - E_0|\psi_1\rangle = \ell \times \frac{1}{\ell^2} = O\left(\frac{1}{\ell}\right) \to 0$
$\vec{S}_i \vec{S}_{i+1}$
$= \#(\vec{S}_i - \vec{S}_{i+1})^2 + \text{const}$

if s is half integer $\longrightarrow$ 2 vacua which brake parity

OR

1 vacum that breaks no sym with massless odd excitations under parity

if s is integer we can have vacuum non degen and $m>0$ (see before)

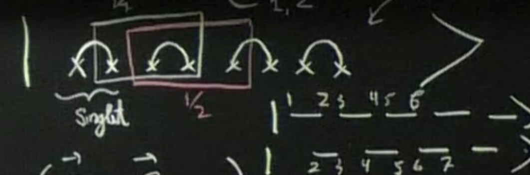
or $m=0$ (as well)

$$Z(\theta) = Z(\theta + 2\pi) \longrightarrow \begin{array}{l} s \text{ integer} \\ \theta = 0 \\ s \text{ half int} \\ \theta = \pi \end{array}$$

Solvable Cases

$$\cdots\ \varepsilon^{\alpha_2\beta_2}\,\varepsilon^{\alpha_1\beta_1}\,|\alpha_1\,\beta_1\,\alpha_2\,\beta_2\,\cdots\rangle$$

dimmer



$s = 1/2$ $\quad H = \sum S_i \cdot S_{i+1} + \lambda\, S_i \cdot S_{i+2} + \cdots$

$s = 1$ $\quad H = \sum S_i \cdot S_{i+1} - \underbrace{\beta\,(S_i \cdot S_{i+1})^2}_{s=1} + \lambda\,(\vec{S}_i \cdot \vec{S}_{i+2}) + \cdots$

$H_{1/2}^{\lambda=0}$ ← Bethe ansatz — single GS: $m=0$

$H_1^{\beta=1,\,\lambda=0}$

$$H_{1/2}^{\lambda=1/2} = \#\sum_i \left(\vec{S}_i + \vec{S}_{i+1} + \vec{S}_{i+2}\right)^2 + \#$$

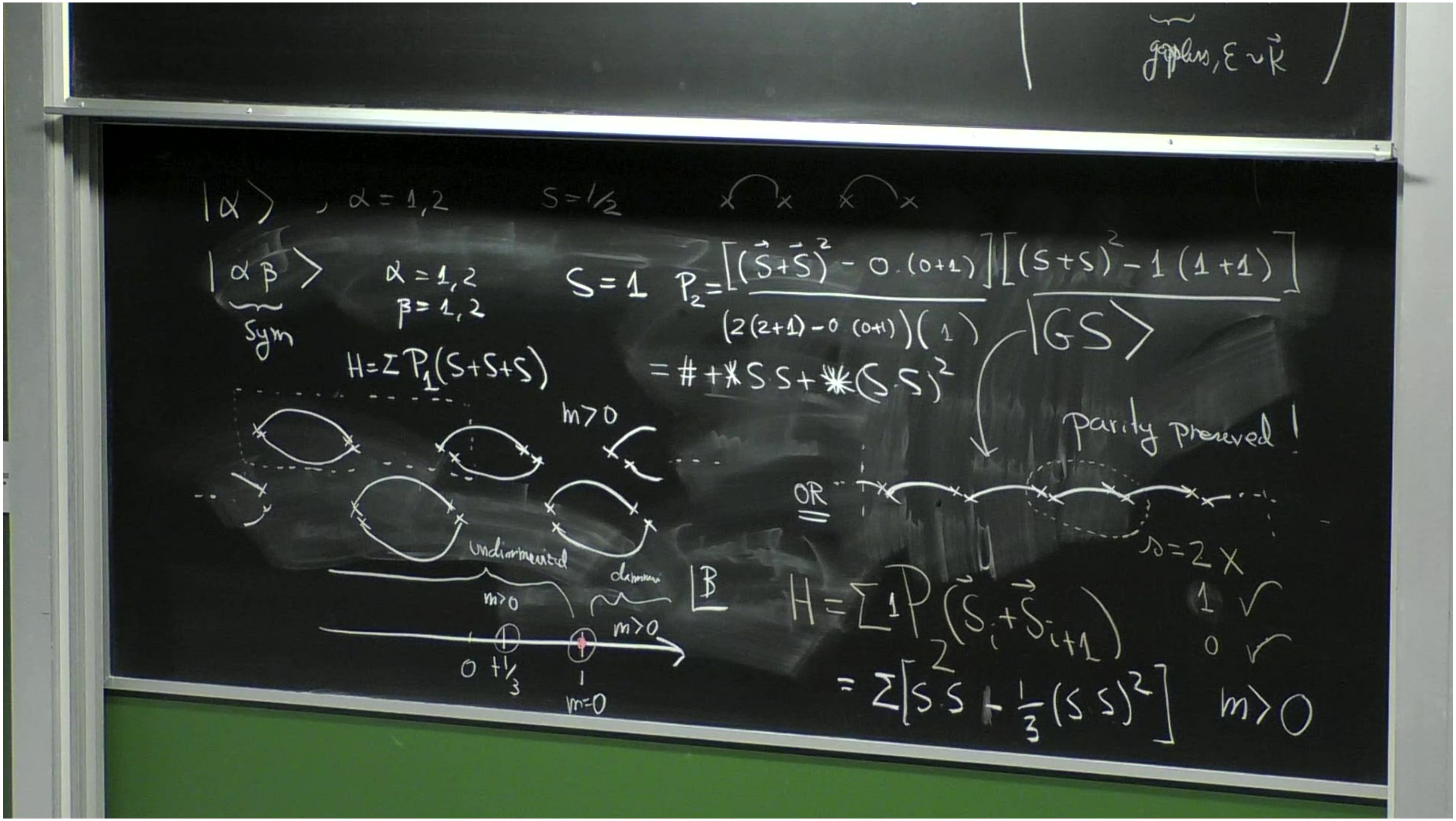
total spin $= \begin{cases} 3/2 \\ 1/2 \end{cases}$

GS always

2GS's, $m>0$

non-deg | dimmer → λ

$m=0$ | $m>0$


gapless, E ~ k
|α⟩ , α = 1,2   S = 1/2
|α β⟩   α = 1,2   β = 1,2
Sym
S = 1   P2 = [(S+S)^2 − 0.(0+1)] [(S+S)^2 − 1 (1+1)] / (2(2+1) − 0 (0+1))(1)
= # + * S S + * (S S)^2
|GS⟩
H = Σ P1(S+S+S)
m>0
parity preserved !
OR
s = 2 X
1 ✓
0 ✓
undimerized
dimerized
m>0
m>0
B
H = Σ 1 P2 (Si + Si+1)
= Σ[S S + 1/3 (S S)^2]   m>0
0 +1/3
1
m=0