Title: Lecture - Relativity, PHYS 604

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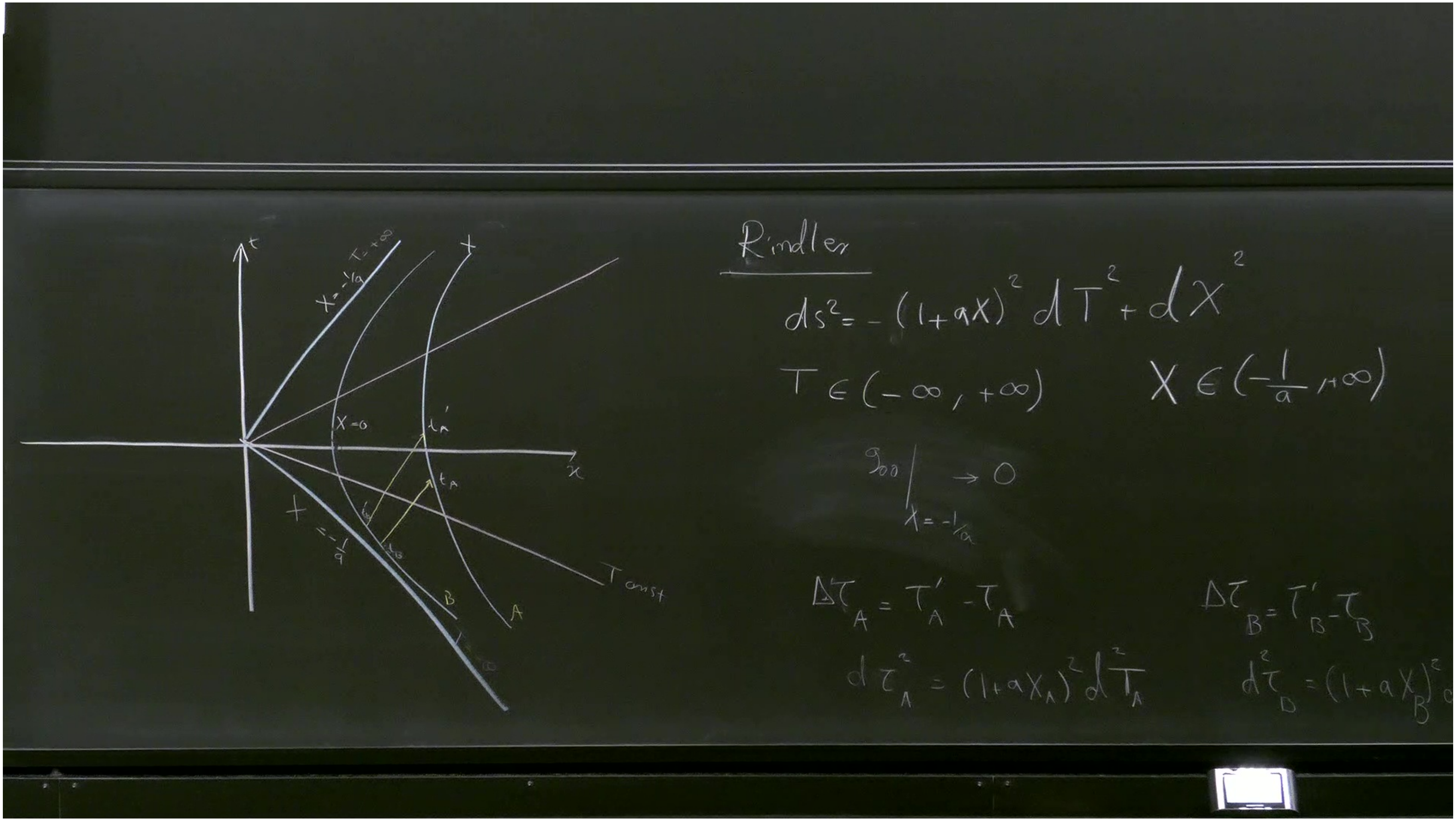
Collection/Series: Relativity (Core), PHYS 604, November 12 - December 11, 2024

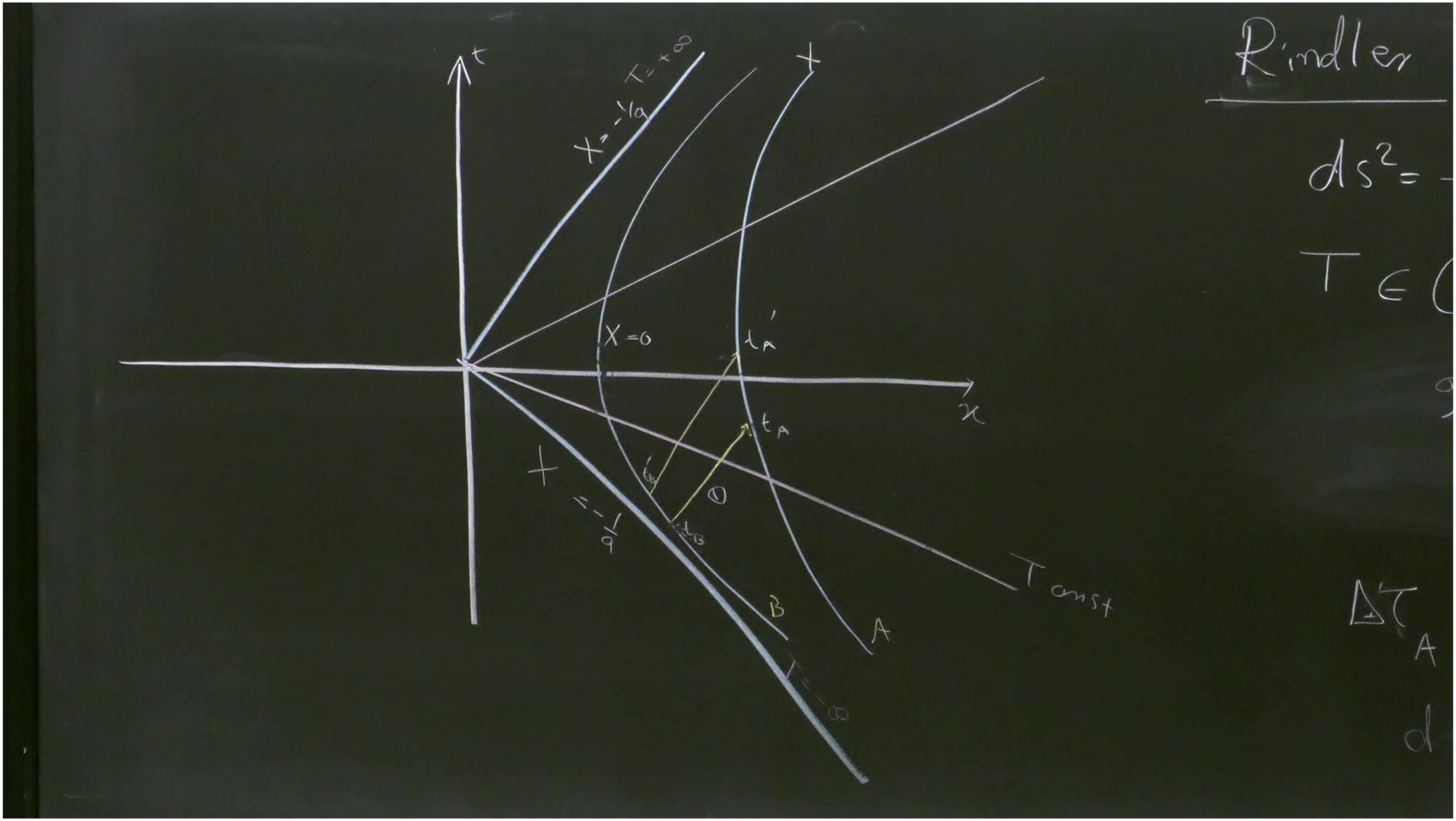
Subject: Cosmology, Strong Gravity

Date: December 10, 2024 - 10:45 AM

URL: https://pirsa.org/24120013

Abstract:


Rindler
$ds^2 = -(1+aX)^2 dT^2 + dX^2$
$T \in (-\infty, +\infty)$
$X \in (-\frac{1}{a}, +\infty)$
$g_{00}\big|_{X=-1/a} \to 0$
$\Delta\tau_A = \tau_A' - \tau_A$
$\Delta\tau_B = \tau_B' - \tau_B$
$d\tau_A^2 = (1+aX_A)^2 dT_A^2$
$d\tau_B^2 = (1+aX_B)^2$
$X=-1/a$
$T=+\infty$
$X=0$
T const
A
B
x
t


Rindler
ds²=
T ∈
X = −1/a
T = +∞
X = 0
t
x
T const
B
A
ΔT
A

$$-dx^2$$

$$X \in \left(-\frac{1}{a}, +\infty\right)$$

$$ds^2\Big|_{①} = 0 = -(1+aX)^2 dT^2 + dX^2 = 0$$

$$T_A - T_B = \int_B^A dT = \int_B^A \frac{dX}{1+aX} = \int_B^A dT = T_A' - T_B'$$

①　②

$$T_A' - T_A = T_B' - T_B \Rightarrow dT_A = dT_B$$

$$\Delta\tau_B = \tau_B' - \tau_B$$

$$d\tau_B^2 = (1+aX_B)^2 dT_B^2$$

$$\frac{\Delta\tau_A}{\Delta\tau_B} = \frac{(1+aX_A)\,\cancel{dT_A}}{(1+aX_B)\,\cancel{dT_B}}$$

$$\frac{d\tau_A}{d\tau_B} = \frac{\sqrt{-g_{00}(A)}}{\sqrt{-g_{00}(B)}}$$

T_A

$$d\tau_A^2 = (1+aX_A)^2 dT_A^2 \qquad d\tau_B^2 = ($$

$$\frac{\nu_A}{\nu_B} = \frac{1+aX_B}{1+aX_A}$$

$$z = \frac{\lambda_A - \lambda_B}{\lambda_A} = \frac{\nu_B}{\nu_A} - 1 > 0$$

If

$$ds^2 = 0 \qquad \frac{\nu_A}{\nu_B} < 1 \qquad \frac{\nu_B}{\nu_A} > 1$$

$$dT = \pm \frac{1}{1+ax} dx$$

$$T = \pm \frac{1}{a} \log(1+ax) + \text{const.}$$

$$U \equiv T - \frac{1}{a} \log(1+ax)$$

$$V \equiv T + \frac{1}{a} \log(1+ax)$$

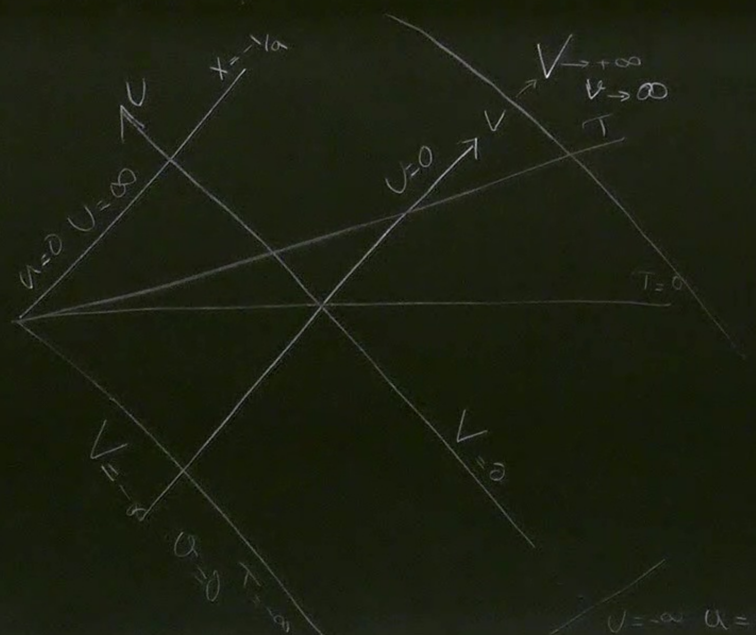
$$\Delta\tau_B = \tau'_B - \tau_B$$

$$d\tau_B^2 = (1+aX_B)^2\, dT_B^2$$

$$\frac{\Delta\tau_A}{\Delta\tau_B} = \frac{(1+aX_A)\,\cancel{dT_A}}{(1+aX_B)\,\cancel{dT_B}}$$

$$\frac{d\tau_A}{d\tau_B} = \frac{\sqrt{-g_{00}(A)}}{\sqrt{-g_{00}(B)}}$$

if $X_B \to -1/a \Rightarrow z \to +\infty \Rightarrow$ infinite redshift



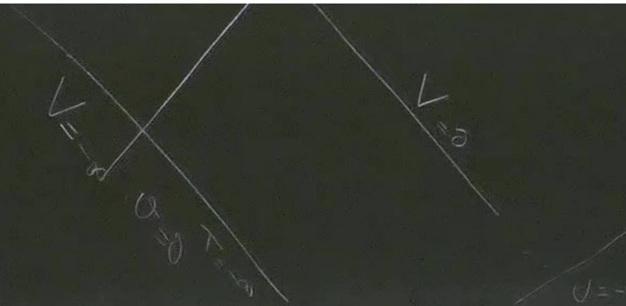
$$U \in (-\infty, +\infty)$$

$$V \in (-\infty, +\infty)$$

$$u = \lambda_{in} = -\frac{1}{a} e^{-aU}$$

$$v = \lambda_{out} = \frac{1}{a} e^{aV}$$

$$u \in (-\infty, 0) \qquad v \in (0, \infty)$$



$v = \lambda_{out} = \frac{1}{a} e^{aV}$

$u \in (-\infty, 0)$ $\quad v \in (0, \infty)$

$\to ds^2 = -e^{a(V-U)}\, dU\, dV \quad \Rightarrow \quad ds^2 = -du\, dv$

$\to u \in (-\infty, +\infty) \quad v \in (-\infty, \infty)$

$x \equiv \frac{v-u}{2} \qquad t = \frac{u+v}{2}$

$\Rightarrow ds^2 = -dt^2 + dx^2$ Minkowsk space-time

Schwarzschild Blackhole

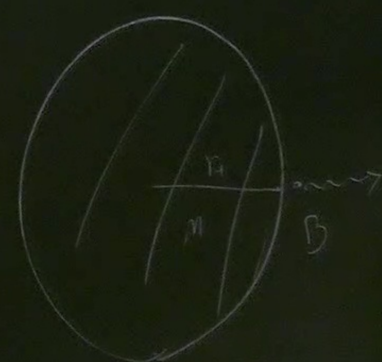
$$ds^2 = -f(r)\,dt^2 + f(r)^{-1}\,dr^2 + r^2\,d\Omega^2$$

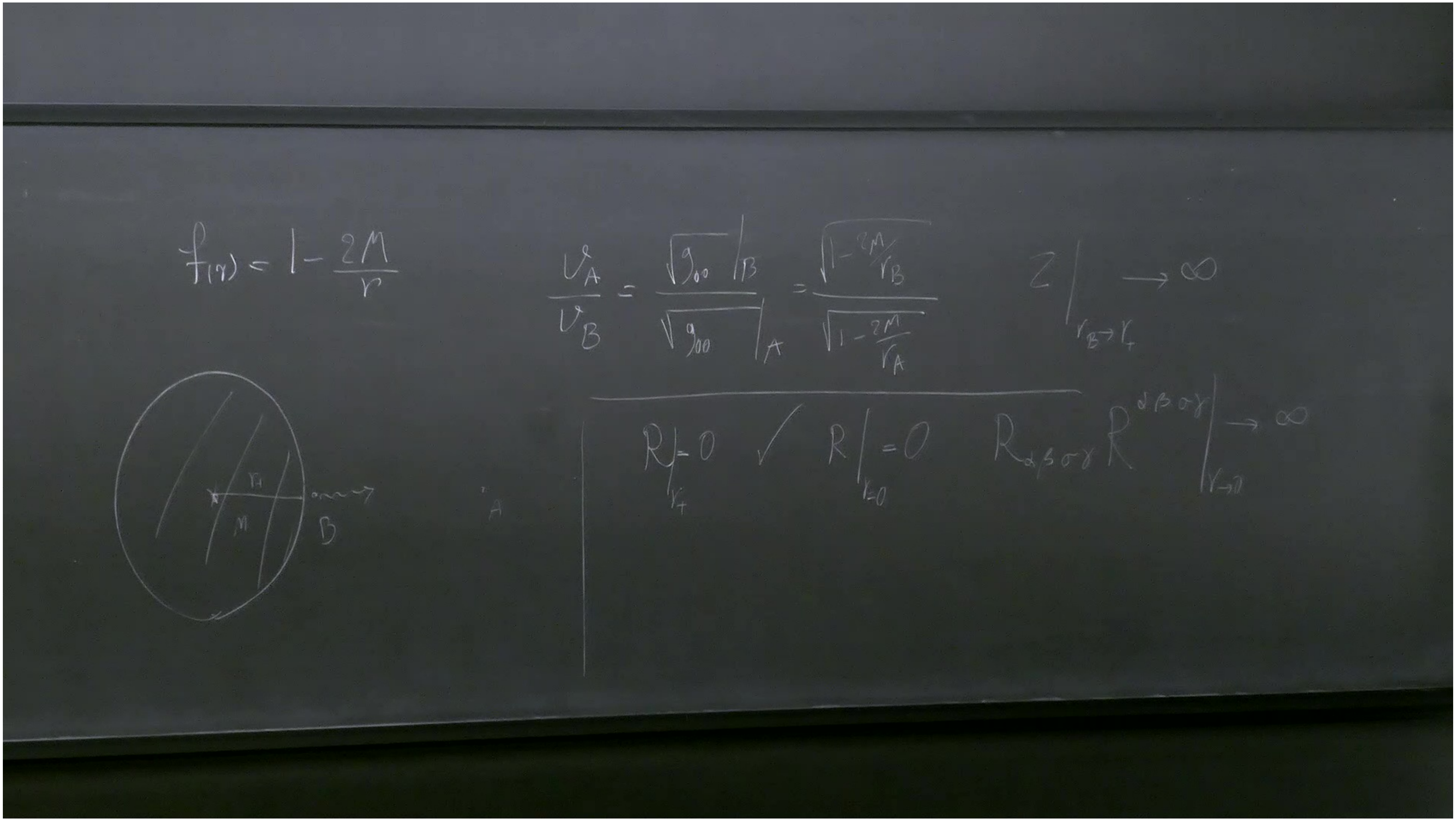
$$f(r) = 1 - \frac{2M}{r}$$

$r \to r_+ = 2M$ $\qquad g_{00}(r_+) \to 0$ $\qquad g_{rr}(r_+) \to \infty$

$r \in (2M, +\infty)$ $\qquad t \in (-\infty, +\infty)$

Earth $\to r_+ = 9\,mm$ $\qquad$ Sun $r_+ = 3\,km$




$f(r) = 1 - \frac{2M}{r}$
$\frac{v_A}{v_B} = \frac{\sqrt{g_{00}}\big|_B}{\sqrt{g_{00}}\big|_A} = \frac{\sqrt{1-2M/r_B}}{\sqrt{1-\frac{2M}{r_A}}}$
$Z\big|_{r_B \to r_+} \to \infty$
r_A
M
B
A
$R\big|_{r_+} = 0$ ✓ $R\big|_{r=0} = 0$ $R_{\alpha\beta\sigma\gamma}R^{\alpha\beta\sigma\gamma}\big|_{r\to 0} \to \infty$

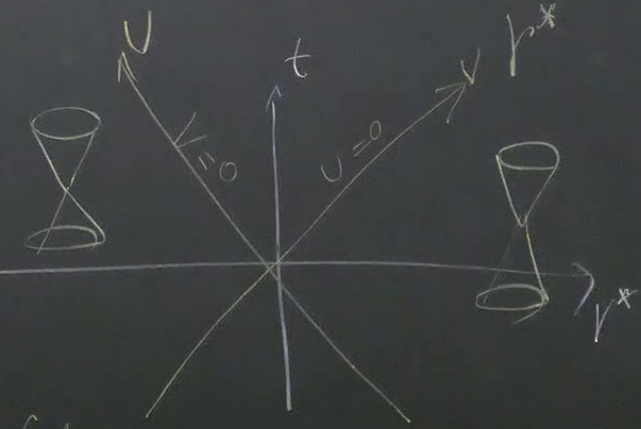
$\Rightarrow dt = \pm \frac{dr}{f(r)}$

$\frac{2M}{r}$

(radial)

$t = \pm \int \frac{dr}{f(r)} + \text{cst} = \pm \underbrace{\left(r + 2M \ln \left| \frac{r}{2M} - 1 \right| \right)} + \text{const}$

t, r^*:



$r^* = -\infty$

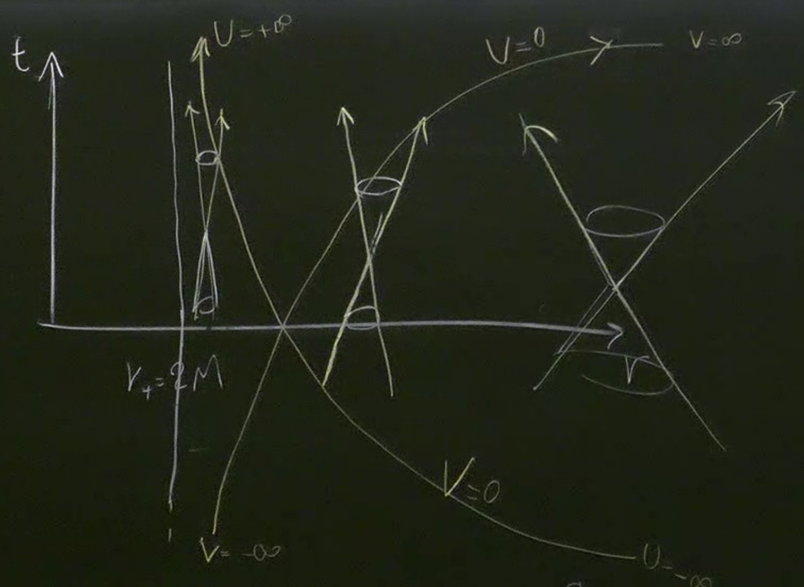
$r = 2M$

$t - r^* = U \rightarrow$ outgoing

$t + r^* = V \rightarrow$ ingoing

$\Rightarrow ds^2 = -f(r)\, dU\, dV$

$r_+ (r^*) \Rightarrow f(r) \rightarrow 0$



$r = 2M+\delta \quad \rightarrow g_{sch} \simeq g_{rindler}$

$$ds^2=0$$

$$d\Omega^2=0$$

$$dt^2 = \frac{1}{f^2}dr^2 \Rightarrow dt = \pm\frac{dr}{f(r)}$$

$$\frac{dt}{dr} = \pm\frac{1}{1-\frac{2M}{r}}$$

Kruskal extension, 1960

- following null rays (radial)
- affine parametrization

$r = 2M + \delta \;\rightarrow\; g_{sch} \simeq g_{rindler}$

• affine parametrization

$r^* = \frac{1}{2}(V - U)$

$e^{r^*/2M} = e^{r/2M}\left|\frac{r}{2M} - 1\right|$

$e^{r^*/2M} = e^{\frac{V-U}{4M}}$

$$\begin{cases} u = \underset{\rightarrow r<2M}{\mp}\, e^{-U/4M} & \Rightarrow du = \pm\, e^{-U/4M} \times \frac{1}{4M}\, dU \\ v = e^{V/4M} & dv = \dots \quad dV \end{cases}$$

$r = 2M + \delta \to g_{sch} \simeq g_{rindler}$

• affine parametrization

$r^* = \frac{1}{2}(V - U)$

$e^{r_*/2M} = e^{r/2M}\left|\frac{r}{2M} - 1\right|$

$e^{r_*/2M} = e^{\frac{V-U}{4M}}$

$$\begin{cases} u = \mp e^{-U/4M} & \Rightarrow du = \pm e^{-U/4M} \times \frac{1}{4M}\, dU \\ \quad \to r < 2M & \\ v = e^{V/4M} & dv = \dots\, dV \end{cases}$$

$r > 2M \to u \in (-\infty, 0)$, $v \in (0, +\infty)$

extension

$u \in (-\infty, +\infty)$, $v \in (-\infty, +\infty)$

$\times \frac{1}{4M}\, dU$

dV

$$\Rightarrow\ ds^2 = \frac{-32M^3}{r}\, e^{-r/2M}\, \underbrace{du\,dv}_{(dT^2 - dX^2)} + r^2 d\Omega^2 \ \checkmark$$

$$T = \frac{1}{2}(v+u)$$

$$X = \frac{1}{2}(v-u)$$

$$U \equiv T - \frac{1}{a} \log(1+aX)$$

$$V \equiv T + \frac{1}{a} \log(1+aX)$$

$$T = \frac{1}{2}(U+V) \qquad 1+aX = e^{\frac{a(V-U)}{2}} \quad \rightarrow \quad ds^2 =$$