

**Title:** Lecture - Relativity, PHYS 604

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**Collection/Series:** Relativity (Core), PHYS 604, November 12 - December 11, 2024

**Subject:** Cosmology, Strong Gravity

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**Abstract:**

Production of GWs

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad |h_{\mu\nu}| \ll 1$$

→ Dondet gauge  $\partial_{\mu} \bar{h}^{\mu\nu} = 0$

$$T_{\mu\nu} = 0, \quad \square \bar{g}^{\mu\nu} = 0$$

With

E.M.,

$$\rightarrow A_{\mu} \rightarrow A$$

$$\rightarrow \square \alpha = 0$$

$$A^0 = 0,$$

$$\rightarrow \alpha =$$

With Sources

$$\text{E.M.}, \partial_\mu A^\mu = 0$$

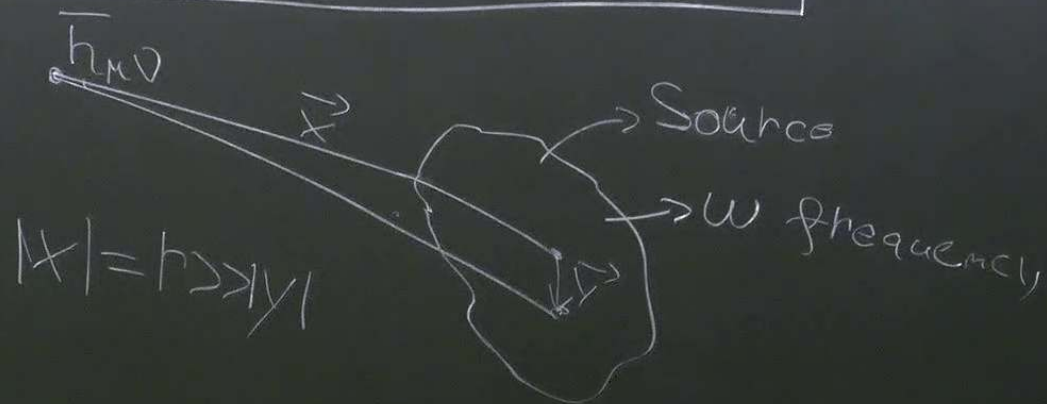
$$\rightarrow A_\mu \rightarrow A_\mu + \partial_\mu \alpha$$

$$\rightarrow \square \alpha = 0 \rightarrow \text{we still have } \partial_\mu A^\mu = 0$$

$$A^0 = 0, A_0 \rightarrow A_0 + \partial_0 \alpha = 0$$

$$\rightarrow \alpha = -\int dt A_0, \square \alpha = -\int dt \square A^0 = -\int dt \rho$$

GW = Transverse  
Traceless dof



• Weak gravitational field  $\rightarrow$  Background  $= \gamma_{\mu\nu}$

• expansion in  $v$  of the source

$$\frac{1}{2} M V^2 = \frac{G m \mu}{r}$$

$$\rightarrow V^2 = \frac{2Gm}{r}$$

$$\square \bar{h}_{\mu\nu} = -16\pi G T_{\mu\nu}$$

$$\rightarrow h_{\mu\nu} = 4G \int d^3y \frac{T_{\mu\nu}(t-|x-y|, y)}{|x-y|}$$

$$p = |x-y|$$
$$|x-y| = (x^2 - 2xy + y^2)^{1/2}$$
$$\frac{1}{|x-y|} \approx \frac{1}{r} + \frac{x \cdot y}{r^3} \approx \frac{1}{r}$$

$$\bar{h}_{\mu\nu} = \frac{4G}{r} \int T_{\mu\nu}(t-r-\hat{m} \cdot \mathbf{y}, \mathbf{y}) d^3\mathbf{y}$$

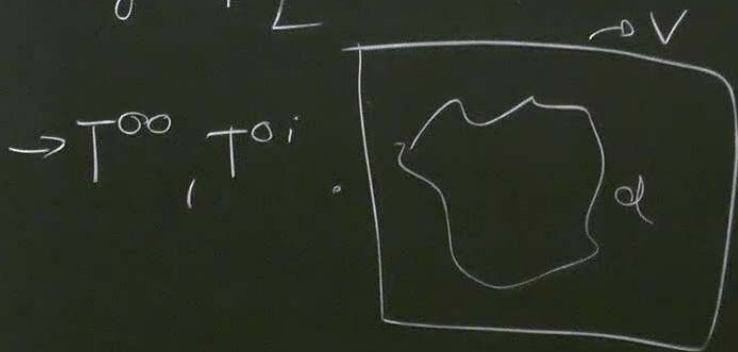
→ Source with size  $d$ .  
 →  $|y| \leq d \sim |y|/\omega \leq d\omega = v \ll 1$   
 →  $y \cdot m\omega < v$

$$T_{\mu\nu}(t-r-\hat{m} \cdot \mathbf{y}, \mathbf{y}) \approx T_{\mu\nu}(t-r, \mathbf{y}) + \underbrace{m \cdot y}_{m_i y_i} \dot{T}_{\mu\nu}(t-r, \mathbf{y}) + \frac{1}{2} \ddot{T}(t-r, \mathbf{y}) (y \cdot m)^2$$

→  $\mu, \nu = i, j$

$$S^{ij} = \int d^3x T^{ij}, \quad S^{ijk} = \int T^{ij} x^k d^3x, \quad S^{ijkl} = \int d^3x T^{ij} x^k x^l$$

$$h_{ij} = \frac{4G}{r} \left[ S_{ij} + \dot{S}_{ijlK} m^K + \ddot{S}_{ijlKl} m^K m^l + \dots \right]$$



$$M^{ijk\dots} = \int d^3x T^{00} x^i x^j x^k \dots$$

$$P^{ijk\dots} = \int d^3x T^{0i} x^j x^k \dots$$

$i, j, k, l, m, n, p, \dots$

$$\dot{M} = \int_V d^3x \dot{T}^{00}$$

$$\rightarrow \partial_m T^{m\nu} = 0 \rightarrow \partial_0 T^{00} = -\partial_i T^{0i}$$

$$\dot{M} = - \int_V d^3x \partial_i T^{0i} = \int_{\partial V} T^{0i} n_i dA = 0$$

$$\begin{aligned} \dot{M}^i &= \int d^3x \dot{T}^{00} x^i = \int d^3x \partial_j T^{0j} x^i = \int d^3x T^{0j} \underbrace{\partial_j x^i}_{\delta_j^i} \\ &= \int d^3x T^{0i} = P^i \end{aligned}$$

$$\dot{M}^i = P^i, \quad \dot{M}^{ij} = \dot{P}^{ij} + P^{\delta ij} \quad \dot{M}^{ijk} = \dot{P}^{ijk}$$

$$\dot{P}^i = 0, \quad \dot{P}^{ij} = S^{ij} \quad \dot{P}^{ijk} = S^{ijk} + S^{ikj}$$

$$-\Delta \bar{h}_{ij} = \frac{4G}{r} \left[ \underbrace{\frac{1}{2} \ddot{M}^{ij}}_{1\text{-st}} + \underbrace{\frac{1}{3} (\ddot{P}^{i\delta k} + \ddot{P}^{ikj} - 2\ddot{P}^{kji})}_{2\text{-nd ordst}} + \frac{1}{6} \ddot{M}^{ijk} \right]$$

$$\bar{Q}_{ij} = M_{ij} - \frac{1}{3} S_{ij} M_{kk}$$

$$\bar{h}_{ij}^{GW} = \frac{2G}{r} \bar{Q}_{ij} \Big|_{t=t_R=t-r}$$

of  $\frac{\partial y^i}{\partial x^j}$

BACK-reaction of GWS



$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

$\rightarrow L_B \rightarrow$  Scale of the Source

$\rightarrow f_B \rightarrow$  freq " " "

$$f_{GW} \gg f_B$$

$$L_B \gg \lambda$$

$$R_{\mu\nu} = \overline{R}_{\mu\nu}^L + \underbrace{R_{\mu\nu}^{H(1)}}_{\downarrow \text{1st } h} + R_{\mu\nu}^{(2)}, \quad R_{\mu\nu}^{(2)} = R_{\mu\nu}^{(2)h} + R_{\mu\nu}^{(2)l} \quad \lambda \ll L \ll L_B$$

$\hookrightarrow h(k)h(-k)$

$$\overline{R}_{\mu\nu}^L = \left[ R_{\mu\nu}^{(2)} \right]^{\text{low}} + 8\pi G \left( T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right)^{\text{low}}$$

$$R_{\mu\nu}^{H(1)} = - \left[ R_{\mu\nu}^{(2)} \right]^{\text{high}} + 8\pi G \left( T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right)^{\text{high}}$$

$$R_{\mu\nu} = 8\pi G \left( T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu} \right)$$

orce



$$\lambda \ll L \ll L_B$$

→ Average over  $L$

$$\rightarrow \overline{R_{\mu\nu}} = -\langle R_{\mu\nu}^{(2)} \rangle + \langle T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \rangle$$

$$\downarrow \epsilon_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T$$

$$t_{\mu\nu} = \frac{1}{32\pi G} \langle 2\partial_\mu h_{\rho\sigma}^{\text{TT}} \partial_\nu h_{\rho\sigma}^{\text{TT}} \rangle$$

$$P = \frac{\Delta E}{\Delta t} = \int d^3x \dot{t}_{00} = \frac{G}{5} \langle \ddot{Q}_{ij} \ddot{Q}_{ij} \rangle$$

Landau-Lifshitz

$$\partial_\mu (T^{\mu\nu} + t^{\mu\nu}) = 0$$