Title: Lecture - Relativity, PHYS 604

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Subject: Cosmology, Strong Gravity

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Abstract:

Production of GWs

$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ $\quad |h_{\mu\nu}| \ll 1$

$\rightarrow$ Donder gauge $\partial_\mu \bar{h}^{\mu\nu} = 0$

$T_{\mu\nu} = 0$; $\Box \xi^\nu = 0$

With S

E.M,

$\rightarrow A_\mu \rightarrow A$

$\rightarrow \Box \alpha = 0$

$A^0 = 0$,

$\rightarrow \alpha =$

With Sources

E.M, $\partial_\mu A^\mu = 0$

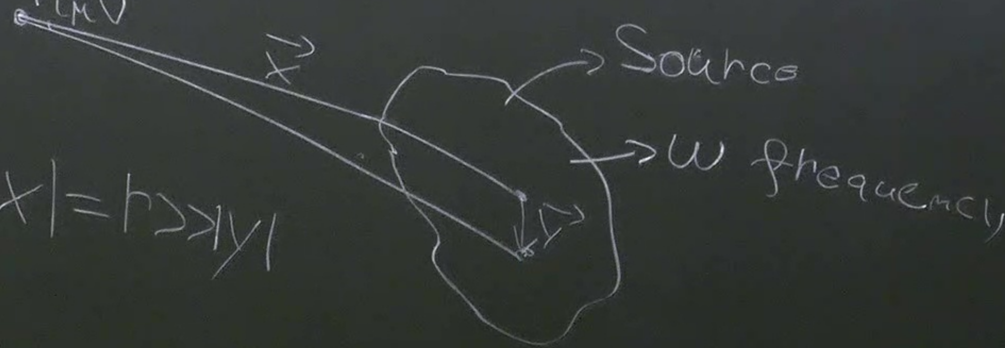
$\to A_\mu \to A_\mu + \partial_\mu \alpha$

$\to \Box \alpha = 0 \Rightarrow$ we still have $\partial_\mu A^\mu = 0$

$A^0 = 0$, $A_0 \to A_0 + \partial_0 \alpha = 0$

$\to \alpha = -\int dt\, A_0$, $\Box \alpha = -\int dt\, \Box A^0 = -\int dt\, \rho$

GW = Transverse
Traceless dof



$|x| = r >> |y|$

- Weak gravitational field → Background $= \eta_{\mu\nu}$
- expansion in v of the source

$$\frac{1}{2}\mu v^2 = \frac{G m \mu}{r}$$

$$\rightarrow v^2 = \frac{2Gm}{r}$$

$$\Box \bar{h}_{\mu\nu} = -16\pi G T_{\mu\nu}$$

$$\rightarrow h_{\mu\nu} = 4G \int d^3y \, \frac{T_{\mu\nu}(t - |x-y|, y)}{|x-y|}$$

$$r = |x|$$

$$|x-y| = (x^2 - 2x\cdot y + y^2)^{1/2}$$

$$\frac{1}{|x-y|} \simeq \frac{1}{r} + \frac{x\cdot y}{r^3} \simeq \frac{1}{r}$$

$$\bar{h}_{\mu\nu} = \frac{4G}{r} \int T_{\mu\nu}(t - r - \hat{n}\cdot y, y)\, d^3y$$

→ Source with size d.

→ $|y| \leq d \rightsquigarrow |y|\omega \leq d\omega = v \ll 1$

→ $y \cdot n\, \omega < v$

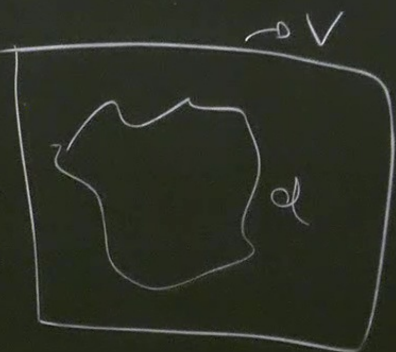
$$T_{\mu\nu}(t - r - \hat{n}\cdot y, y) \simeq T_{\mu\nu}(t - r, y) + \underbrace{n \cdot y}_{n^i y^i}\, \dot{T}_{\mu\nu}(t - r, y) + \frac{1}{2}\ddot{T}(t - r, y)\,(y \cdot n)^2$$

→ $\mu, \nu = i, j$

$$S^{ij} = \int d^3x\, T^{ij}, \quad S^{ij,k} = \int T^{ij} x^k\, d^3x, \quad S^{ij,kl} = \int d^3x\, T^{ij} x^k x^l$$

$$h_{ij} = \frac{4G}{r}\left[S^{ij} + \dot{S}^{ij,k} n^k + \ddot{S}^{ij,kl} n^k n^l + \dots \right]$$

$$\to T^{00}, T^{0i}$$



$$M^{ijk\dots} = \int d^3x \, T^{00} x^i x^j x^k \dots$$

$$P^{ijk\dots} = \int d^3x \, T^{0i} x^j x^k \dots$$

$\eta_{jk\ell}\, n^k n^\ell + \ldots]$

$$\dot{M} = \int_V d^3x\ \dot{T}^{00}$$

$$\to \partial_\mu T^{\mu\nu} = 0 \to \partial_0 T^{00} = -\partial_i T^{0i}$$

$$\dot{M} = -\int_V d^3x\ \partial_i T^{0i} = \int_{\partial V} T^{0i} n_i\, dA = 0$$

$$\dot{M}^i = \int d^3x\ \dot{T}^{00} x^i = -\int d^3x\ \partial_j T^{0j} x^i = \int d^3x\, T^{0j} \underbrace{\partial_j x^i}_{\delta^i_j}$$

$$= \int d^3x\, T^{0i} = P^i$$

$$\underbrace{\partial_j x^i}_{\delta^i_j}$$

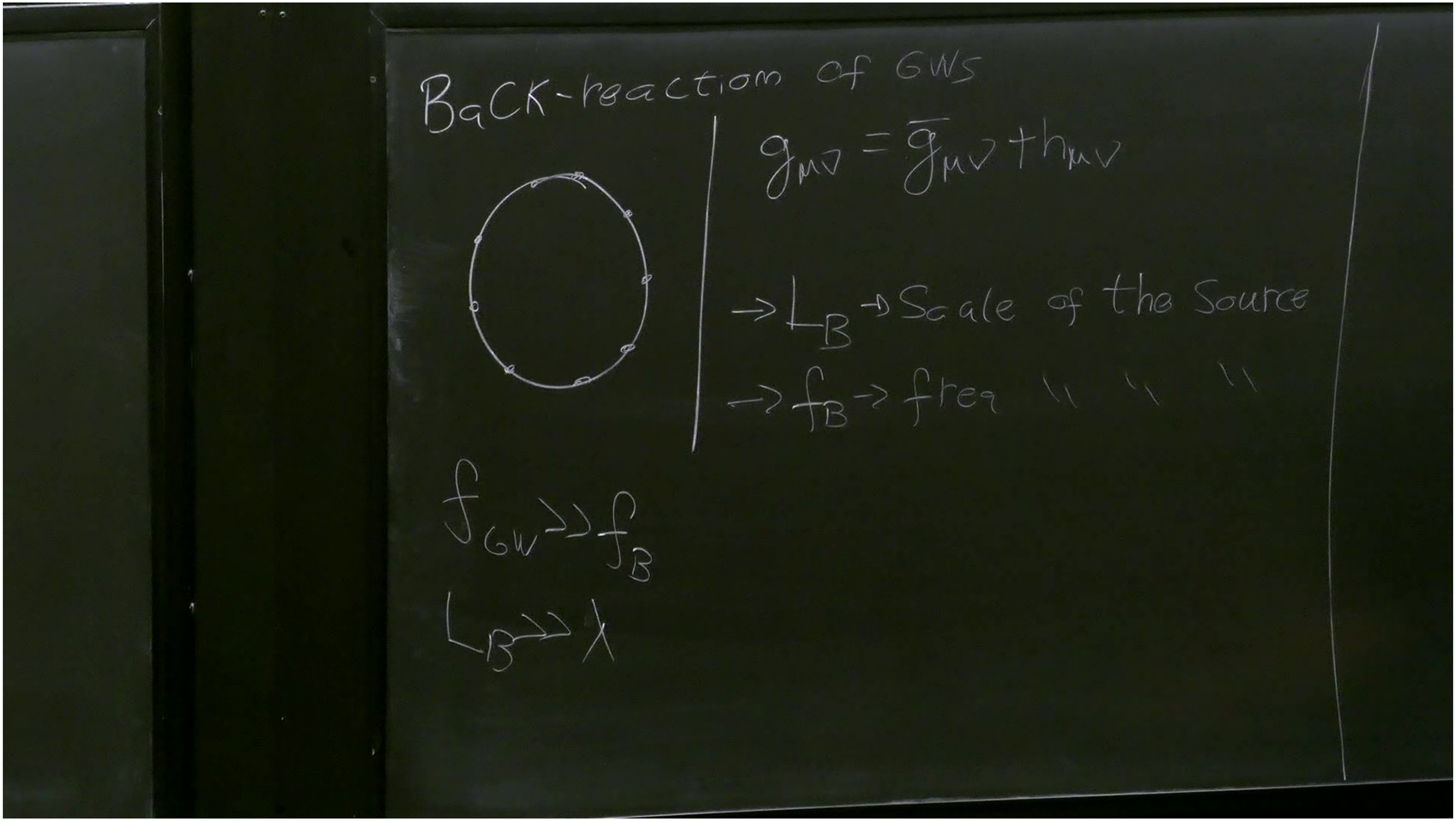
$$\dot{M}^i = P^i \,, \quad \dot{M}^{ij} = \dot{P}^{i,j} + \dot{P}^{j,i} \qquad \dot{M}^{ijk} = \ddot{P}^{(ij,k)}$$

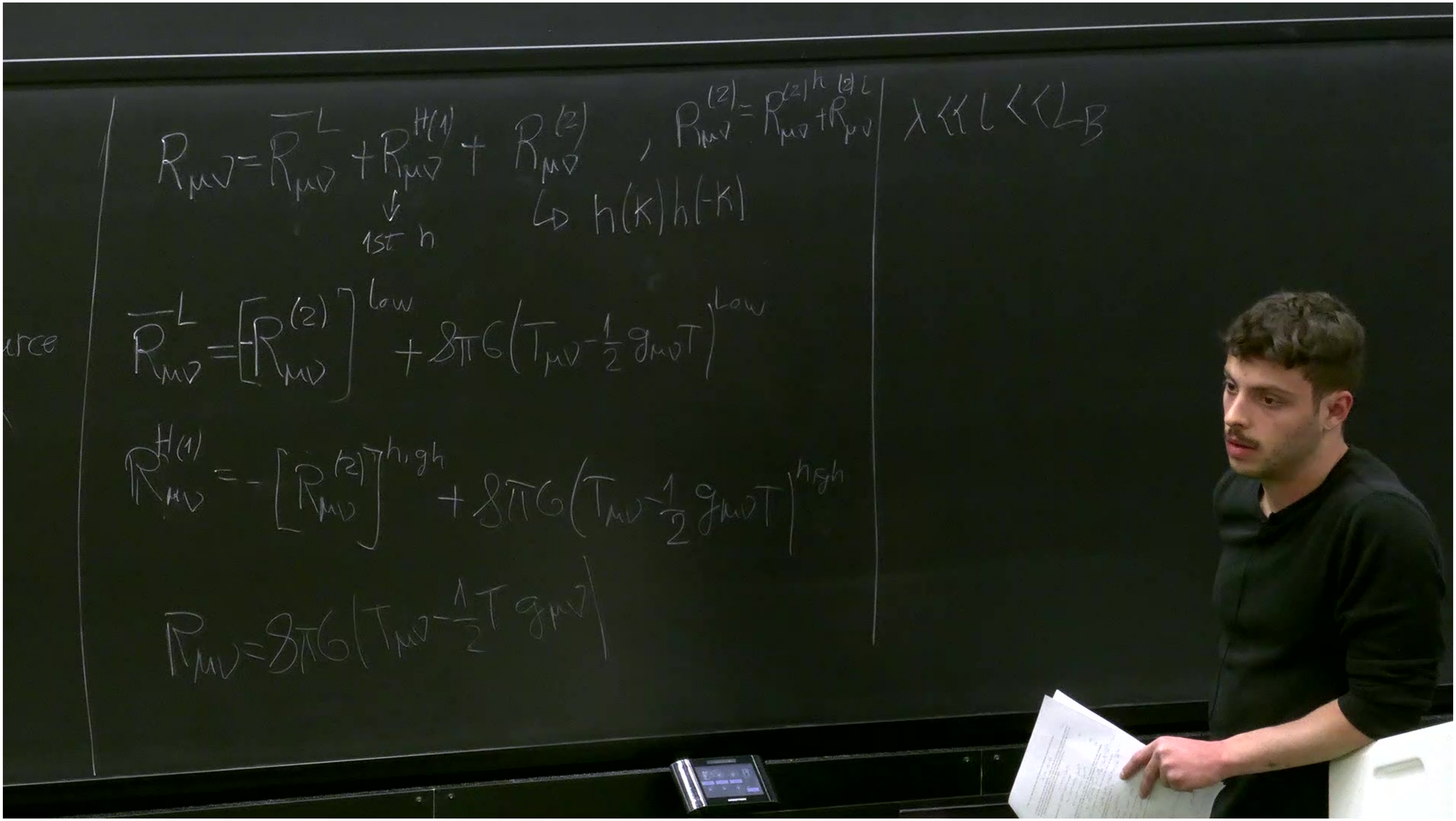
$$\dot{P}^i = 0 \,, \quad \ddot{P}^{i,j} = S^{ij} \qquad \ddot{P}^{i,jk} = S^{ij,k} + S^{ik,j}$$

$$\rightarrow \bar{h}_{ij} = \frac{4G}{r}\left[\underbrace{\frac{1}{2}\ddot{M}^{ij}}_{\text{1-st}} + \underbrace{\frac{1}{3}\left(\ddot{P}^{i,jk} + \ddot{P}^{j,ik} - 2\ddot{P}^{k,ji}\right) + \frac{1}{6}\dddot{M}^{ijk}}_{\text{2-nd order}}\right]$$

$$\boxed{Q_{ij} = M_{ij} - \frac{1}{3}\delta_{ij} M^k_k}$$

$$\boxed{\bar{h}^{GW}_{ij} = \frac{2G}{r}\ddot{Q}_{ij}\Big|_{t=t_R=t-r}}$$


Back-reaction of GWs
$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$
$\rightarrow L_B \rightarrow$ Scale of the Source
$\rightarrow f_B \rightarrow$ freq " " "
$f_{GW} \gg f_B$
$L_B \gg \lambda$



$R_{\mu\nu} = \bar{R}^{L}_{\mu\nu} + R^{H(1)}_{\mu\nu} + R^{(2)}_{\mu\nu}$, $R^{(2)}_{\mu\nu} = R^{(2)h}_{\mu\nu} + R^{(2)L}_{\mu\nu}$
1st h
$\hookrightarrow h(k)h(-k)$
$\lambda \ll l \ll L_B$
urce
$\bar{R}^{L}_{\mu\nu} = -\left[R^{(2)}_{\mu\nu}\right]^{low} + 8\pi G\left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T\right)^{low}$
$R^{H(1)}_{\mu\nu} = -\left[R^{(2)}_{\mu\nu}\right]^{high} + 8\pi G\left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T\right)^{high}$
$R_{\mu\nu} = 8\pi G\left(T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu}\right)$

$\lambda <<< \ell <<< L_B$

$\rightarrow$ Average over ℓ

$$\rightarrow \bar{R}_{\mu\nu} = -\langle R^{(2)}_{\mu\nu} \rangle + \langle T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \rangle$$

$$\hookrightarrow t_{\mu\nu} - \frac{1}{2} g_{\mu\nu} t$$

$$t_{\mu\nu} = \frac{1}{32\pi G} \langle \partial_\mu h^{TT}_{\rho\sigma} \partial_\nu h^{TT}_{\rho\sigma} \rangle$$

$$P = \frac{\Delta E}{\Delta t} = \int d^3x \, \dot{t}_{00} = \frac{G}{5} \langle \dddot{Q}_{ij} \dddot{Q}_{ij} \rangle$$

Landau-Lishftz

$$\partial_\mu (T^{\mu\nu} + t^{\mu\nu}) = 0$$