Title: Lecture - Relativity, PHYS 604

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Subject: Cosmology, Strong Gravity

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Abstract:

$$\begin{cases} \Box \bar{h}_{\mu\nu} = -16\pi G T_{\mu\nu} \\ \partial^\mu \bar{\bar{h}}_{\mu\nu} = \end{cases} \qquad h(x) \qquad \partial_t h = 0$$

$$T_{00} = \rho(x) \qquad T_{0i} = T_{ij} = 0$$

$$\Box \bar{h}_{00} = -16\pi G \rho(x) \Rightarrow \nabla^2 \bar{h}_{00} = -16\pi G \rho(x)$$

$$\Box \bar{h}_{0i} = \Box \bar{h}_{ij} = 0 \Rightarrow \bar{h}_{0i} = \bar{h}_{ij} = 0$$

$h = 0$

$)$

$\nabla^2 \bar{h}_{00} = -16\pi G \rho_{(m)}$

0

$$\nabla^2 \phi = 4\pi G \rho_{(m)} = -4 \nabla^2 \bar{h}_{00} \Rightarrow \bar{h}_{00} = -4\phi$$

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} h \eta_{\mu\nu} \Rightarrow \bar{h} = -h$$

$$h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{1}{2} \bar{h} \eta_{\mu\nu}$$

$$\bar{h} = 4\phi \Rightarrow h = -4\phi$$

$$h_{00} = \bar{h}_{00} - \frac{1}{2}(4\phi)\eta_{00} = -2\phi$$

$$h_{0i} = 0 - \frac{1}{2}(4\phi)\eta_{0i} = 0$$

$$h_{ij} = 0 - \frac{1}{2}(4\phi$$

$$1G\,\rho_{(m)} = -4\,\nabla^2 \bar{h}_{00} \Rightarrow \bar{h}_{00} = -4\phi$$

$$h\,\eta_{\mu\nu} \Rightarrow \bar{h} = -h$$

$$\frac{1}{2}\bar{h}\,\eta_{\mu\nu}$$

$$\Rightarrow h = -4\phi$$

$$(4\phi)\,\eta_{00} = -2\phi$$

$$(4\phi)\,\eta_{0r} = 0$$

$$h_{ij} = 0 - \tfrac{1}{2}(4\phi)\,\eta_{ij} = -2\phi\,\delta_{ij}$$

$$\boxed{ds^2 = -(1+2\phi)\,dt^2 + (1-2\phi)\,\delta_{ij}\,dx^i\,dx^j}$$

$$\phi = -\frac{GM}{c^2 r} \ll 1$$

Gravitomagnetism

Electromagnetism

$$T_{0\mu} = (\rho, \vec{j}) = j_\mu$$

$$A^\mu = (\phi, \vec{A}) = \frac{1}{4}\bar{h}^{0\mu}$$

$$A_0 = -\phi = \frac{1}{4}\bar{h}_{00}$$

$$\Box \bar{h}^{0\mu} = -16\pi G j^\mu$$

$h_{ij} = 0$

$h_{00} = \bar{h}_{00} - \frac{1}{2}(4\phi)\eta_{00} = -2\phi$

$h_{0i} = 0 - \frac{1}{2}(4\phi)\eta_{0i} = 0$

$$\boxed{\Box A^{\mu} = 4\pi G j^{\mu}}$$ Maxwell's equations

$$\partial_\mu \bar{h}^{\mu\nu} = 0 \Rightarrow \boxed{\partial_\mu A^\mu = 0} \longrightarrow$$ gravitomagnetism

$$h_{0i} = \bar{h}_{0i} - \frac{1}{2}(4\phi)\cancel{\eta_{0i}} = A_i$$

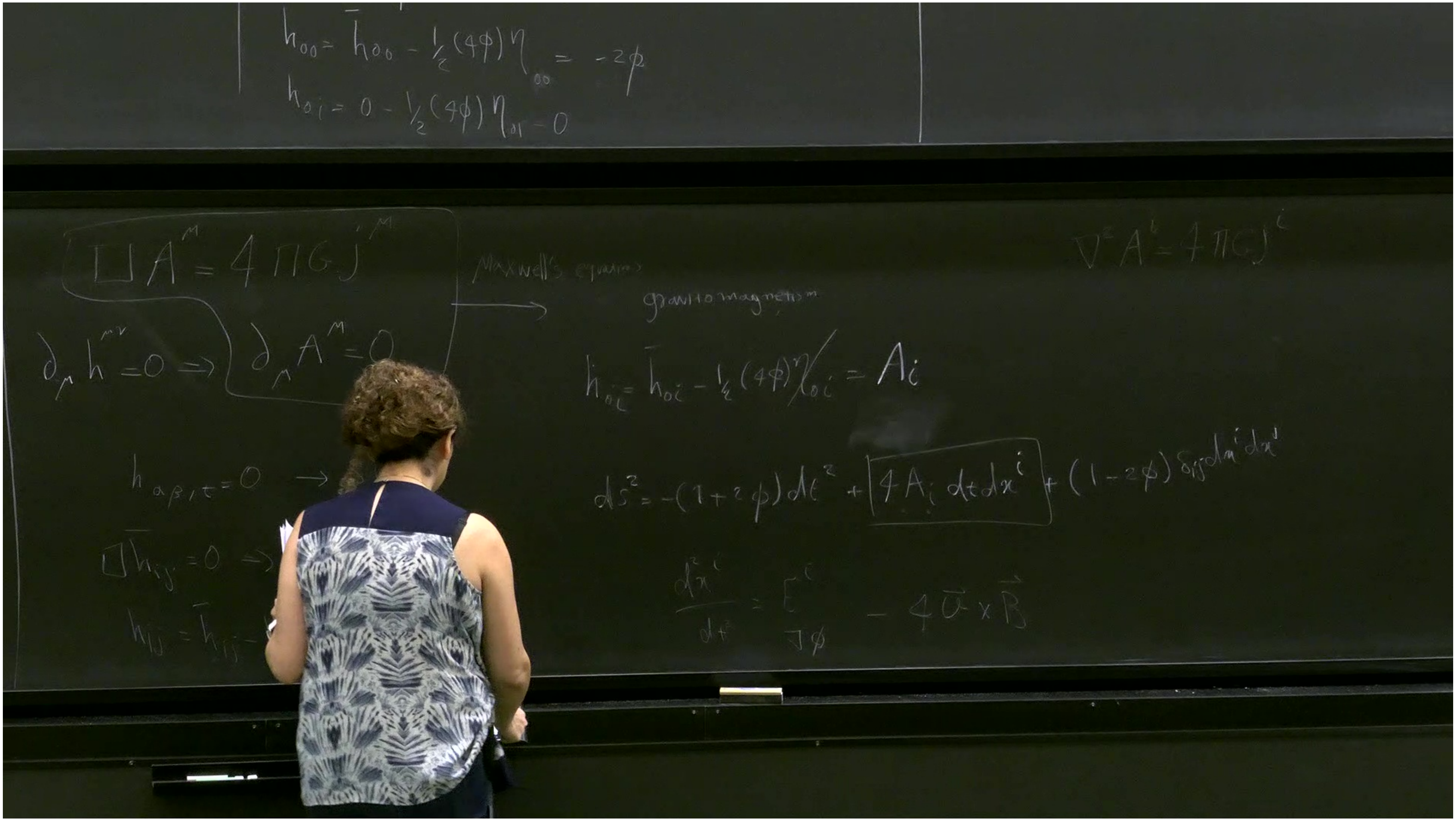
$h_{\alpha\beta,t} = 0 \rightarrow$ static

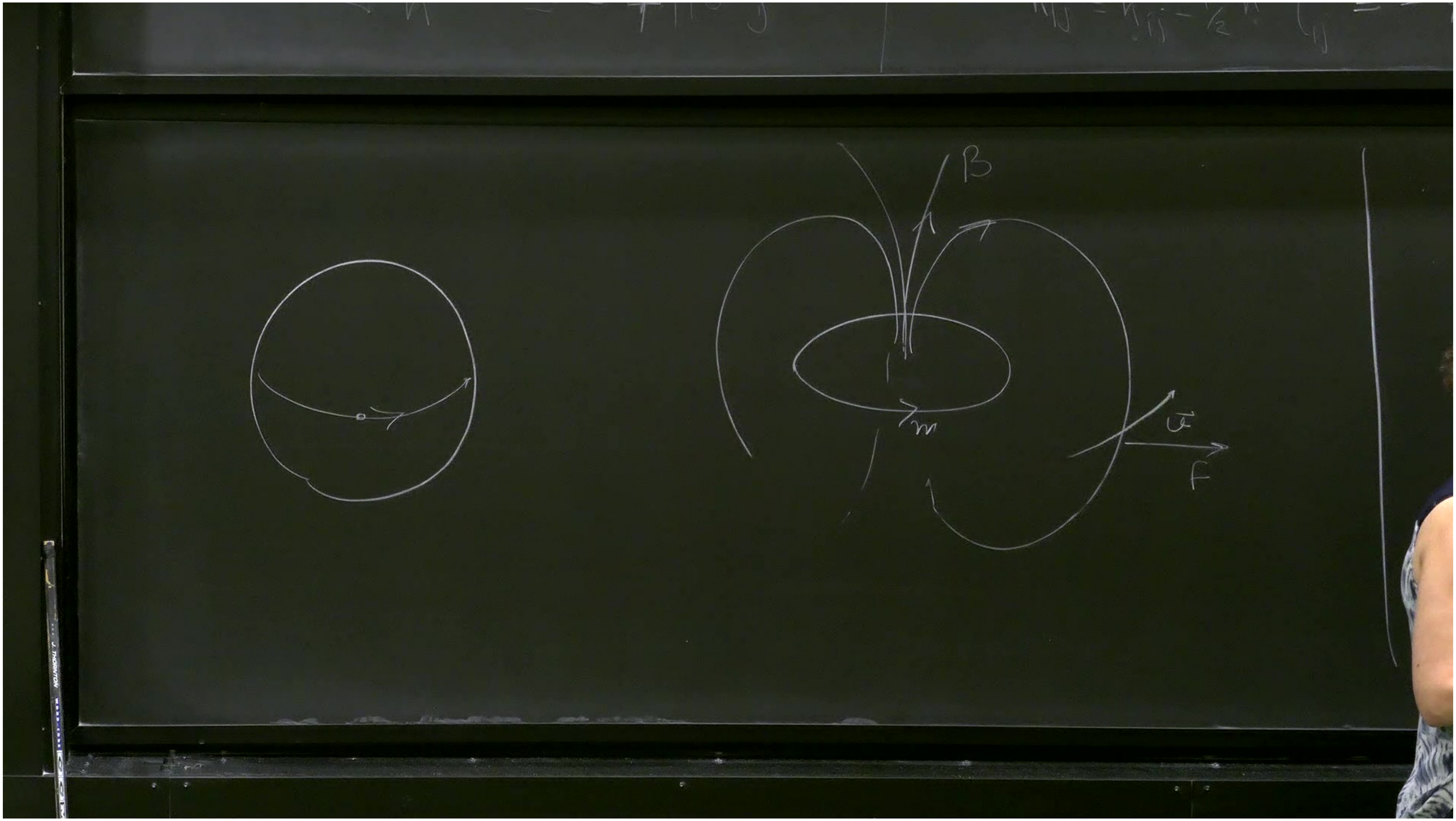
$$ds^2 = -(1+2\phi)dt^2 + \boxed{4A_i\, dt\, dx^i} + (1-2\phi)\delta_{ij}dx^i dx^j$$

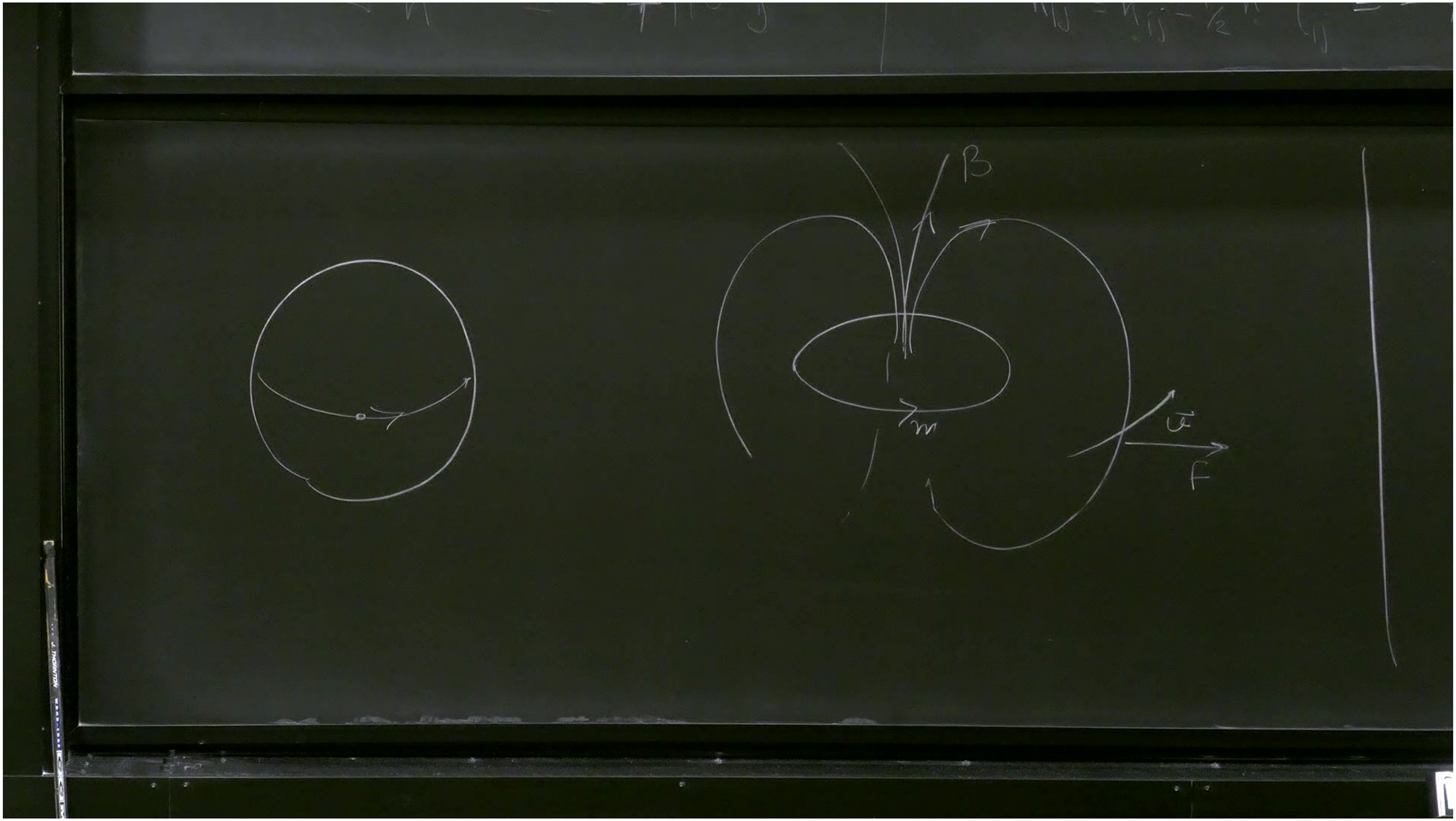
$\Box \bar{h}_{ij} = 0 \Rightarrow \bar{h}_{ij} = 0$

$$\frac{d^2x^i}{dt^2} = \Gamma^i \qquad \nabla\phi$$

$h_{ij} = \bar{h}_{ij} - \frac{1}{2}\bar{h}\,\eta_{ij} = -2\phi\,\delta_{ij}$


h_00 = h̄_00 − ½(4φ)η_00 = −2φ
h_0i = 0 − ½(4φ)η_0i = 0
□A^μ = 4πG J^μ
Maxwell's equations
gravitomagnetism
∂_μ h̄^μν = 0 ⇒ ∂_μ A^μ = 0
h_0i = h̄_0i − ½(4φ)η_0i = A_i
∇²A^i = 4πG J^i
h_αβ,t = 0 →
ds² = −(1+2φ)dt² + 4A_i dt dx^i + (1−2φ)δ_ij dx^i dx^j
□h̄_ij = 0 ⇒
h_ij = h̄_ij −
d²x^i/dt² = E^i − 4 v × B
∇φ


B
m
v
F


B
m
v
F

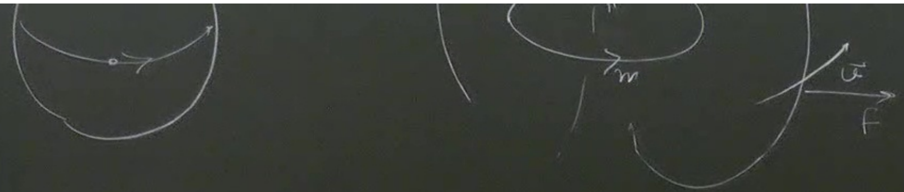
Schwarzschild Solution

- Exact solution to $\boxed{\text{Vacuum}}$ Einstein's Eq's
- Spherically symmetric
- asymptotically flat. $(\Lambda = 0)$

$$ds^2 = -f(r)\,dt^2 + \frac{1}{f(r)}\,dr^2 + r^2 d\Omega^2$$

$$f(r) = 1 - \frac{2M}{r}$$

$$d\Omega^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$$



- Spherically Symmetric
- asymptotically flat. $(\Lambda = 0)$

$$ds^2 = -f(r)\, dt^2 + \frac{1}{f(r)}\, dr^2 + r^2\, d\Omega^2$$

$f(r) = 1$ ...

$d\Omega^2 =$ ...

Birkhoff's theorem:

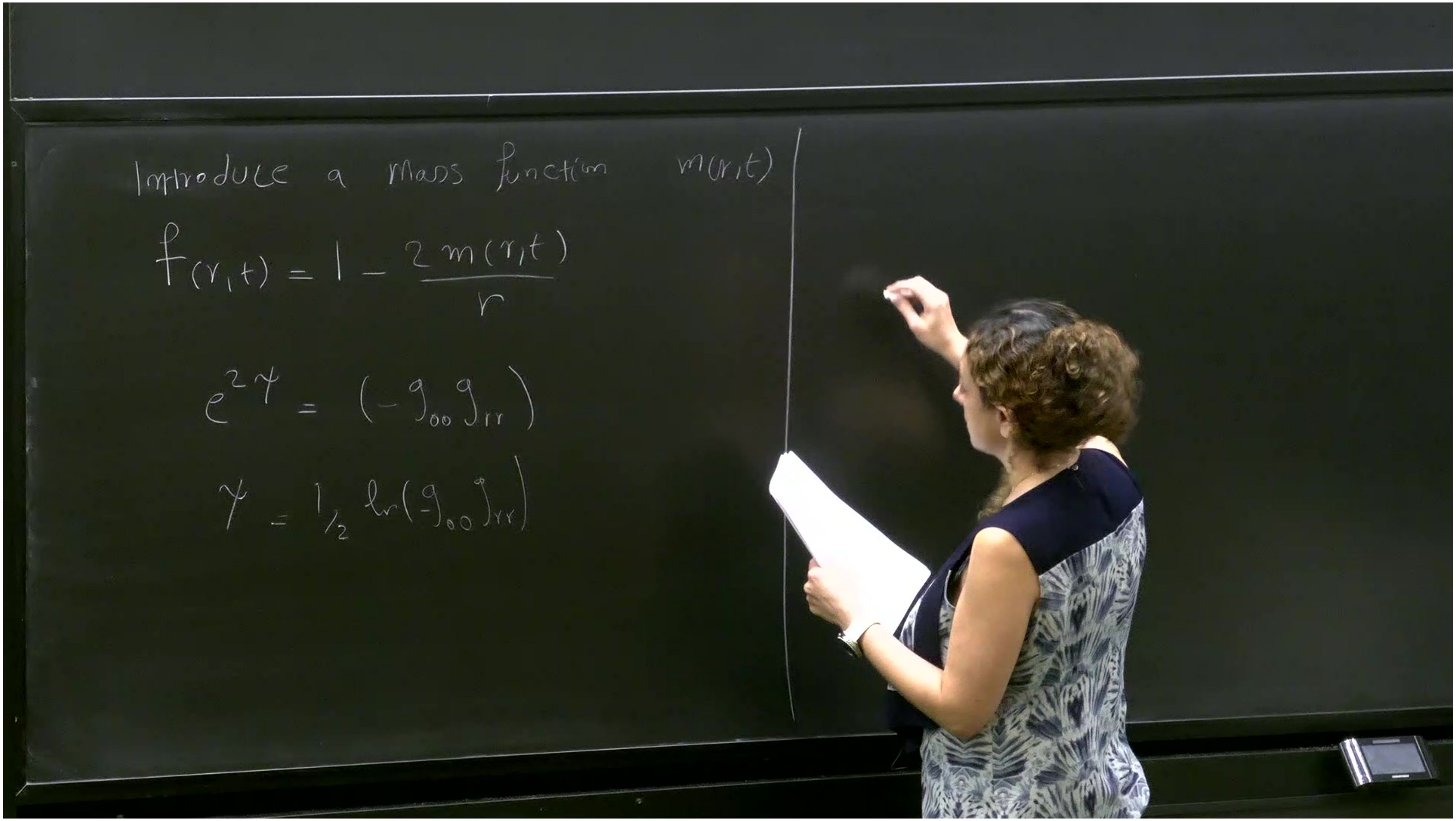
Any $\boxed{\text{spherically symmetric}}$ solution to the $\boxed{\text{Vacuum}}$ Einst Equations' must be $\boxed{\text{Static}}$

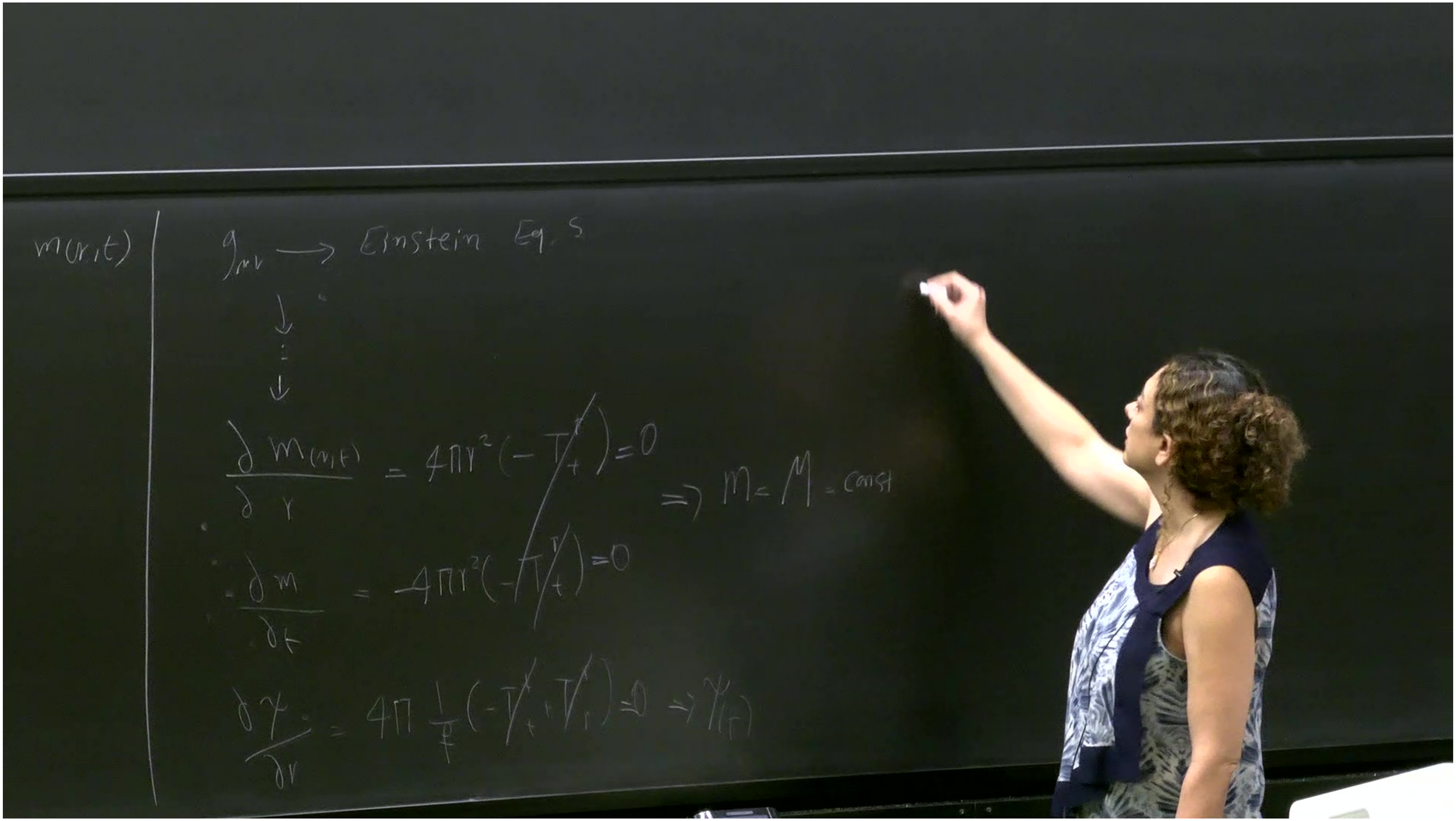
and if $\boxed{\text{asymptotically flat}}$ $(\Lambda = 0)$ then Uniquely described by $\boxed{\text{Shwarzschild}}$ solution.

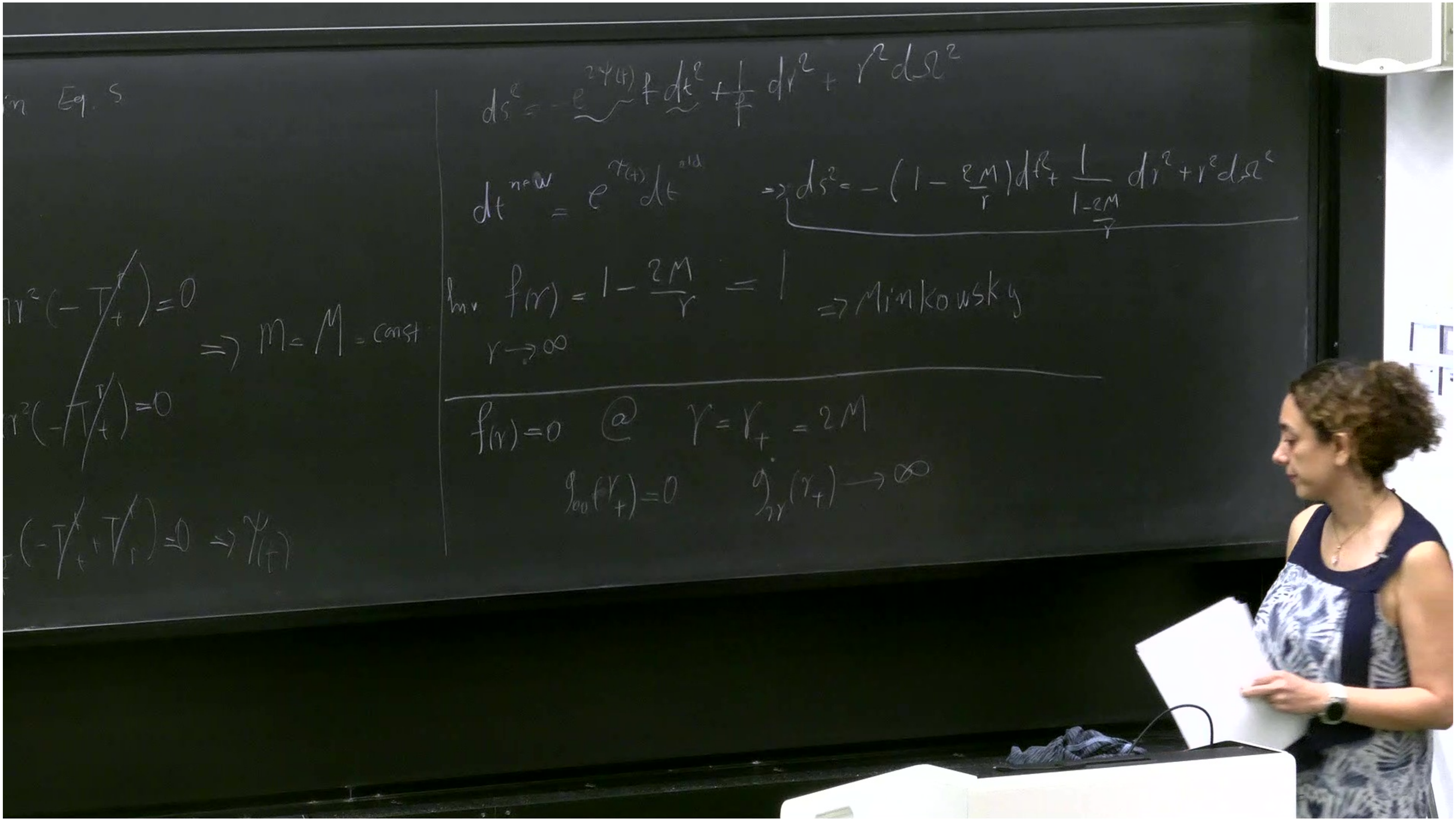
Proof:

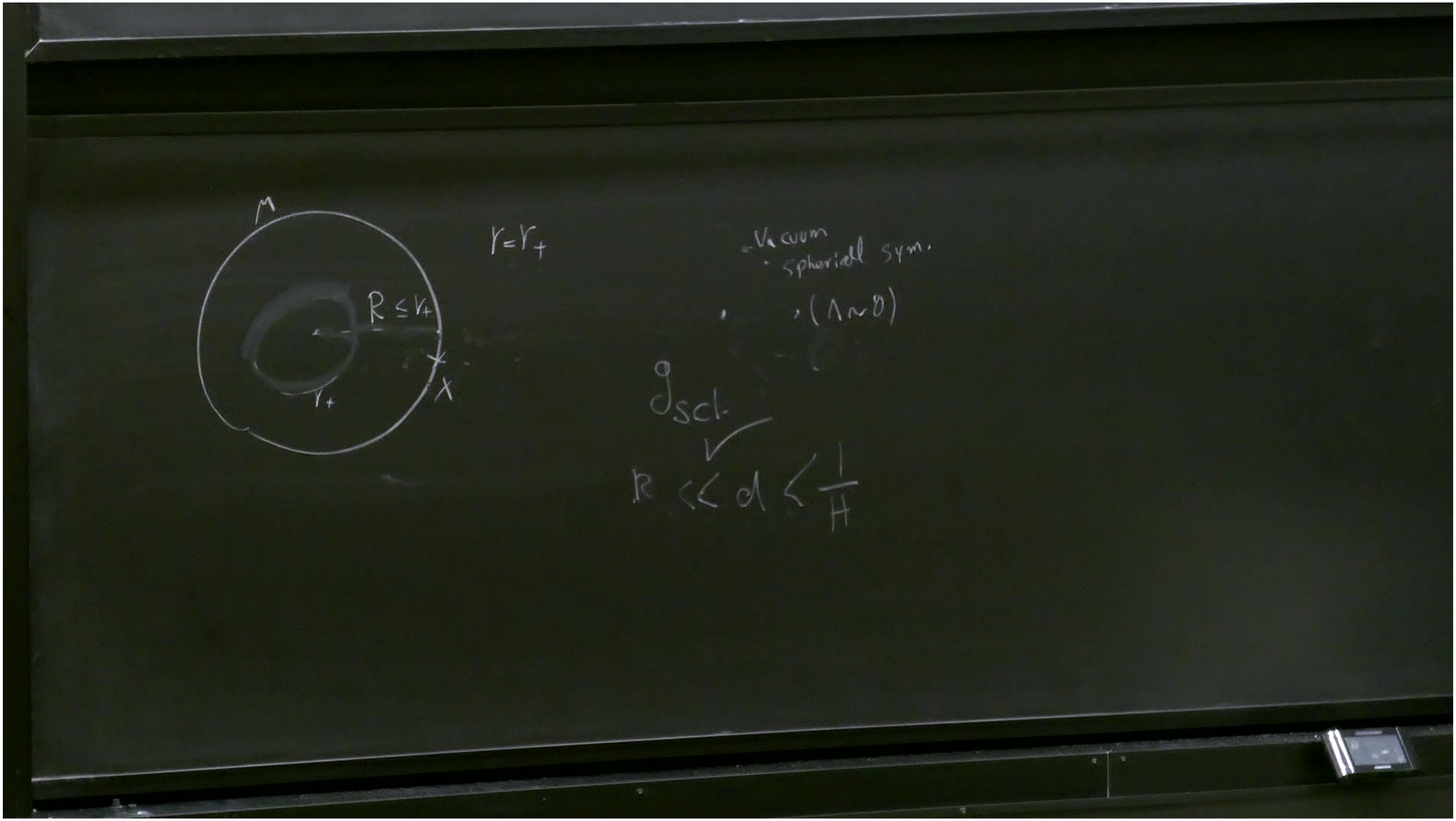
$$ds^2 = -e^{2\chi} f\, dt^2 + \frac{1}{f}\, dr^2 + \underbrace{r^2}_{e^{\Lambda(r)}}\, d\Omega^2 \rightarrow \text{(See MTW)}$$

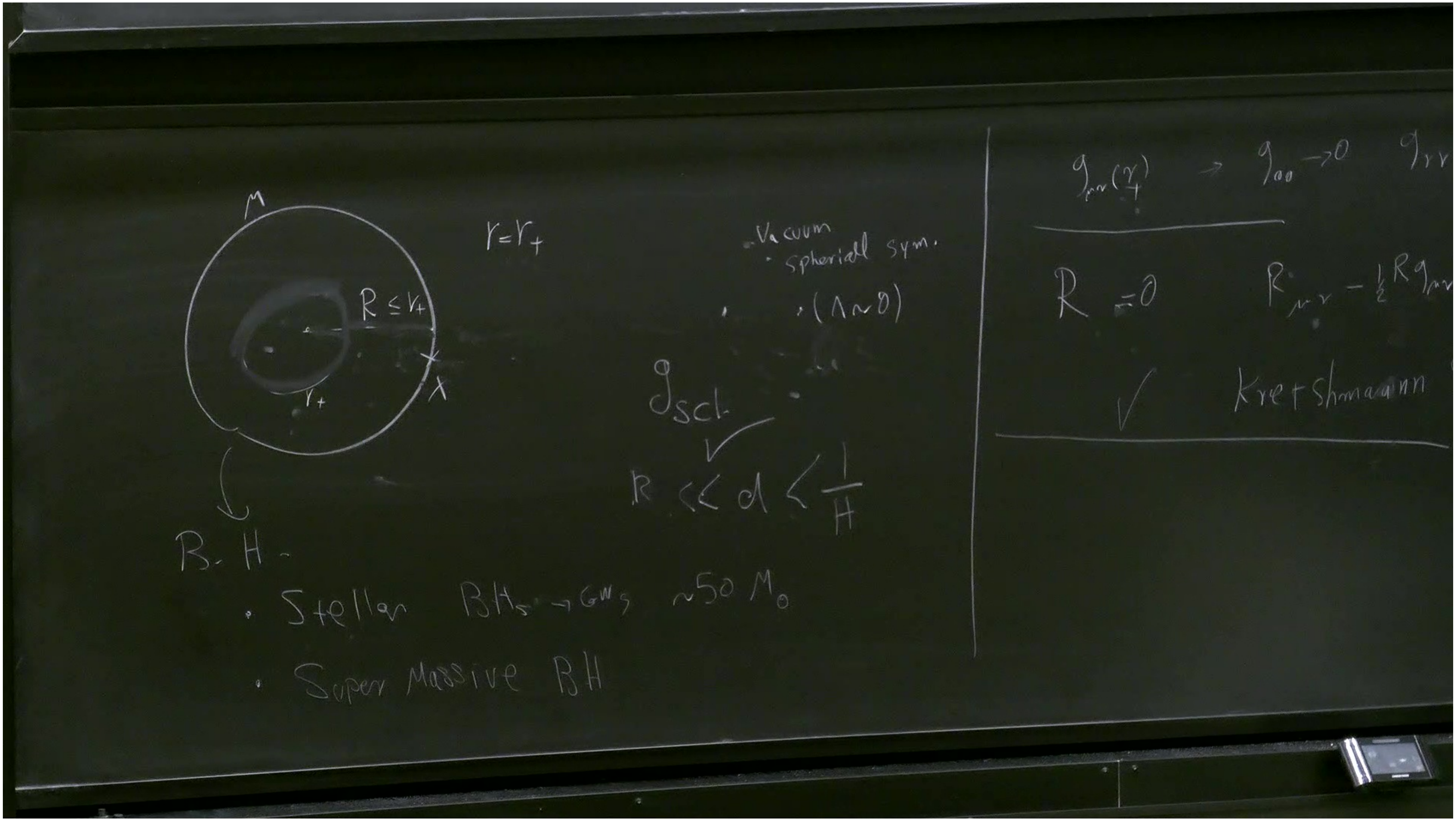
and $g_{0r} = 0$

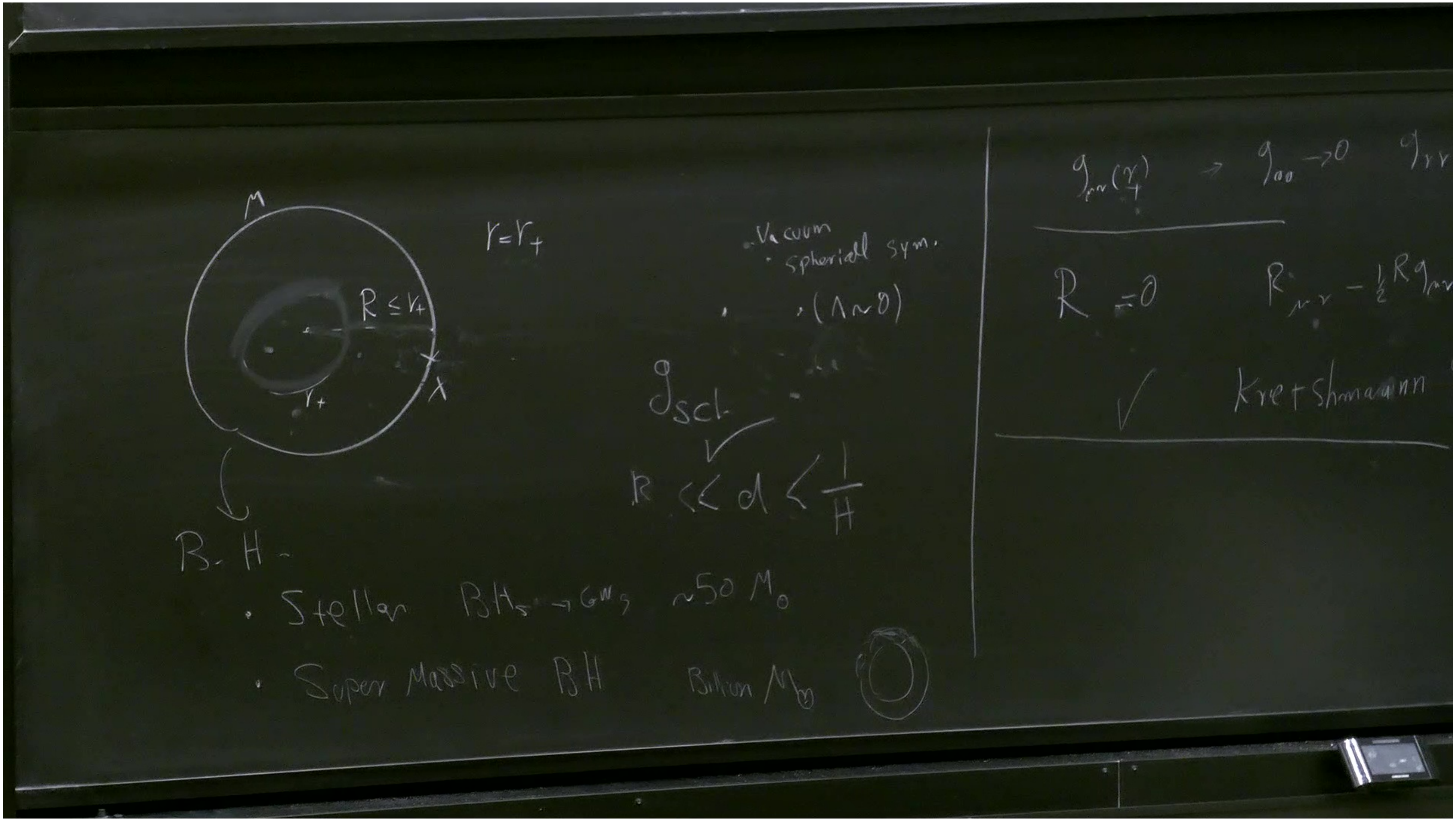

Introduce a mass function $m(r,t)$
$f(r,t) = 1 - \frac{2m(r,t)}{r}$
$e^{2\gamma} = (-g_{00}\,g_{rr})$
$\gamma = \frac{1}{2}\ln(-g_{00}\,g_{rr})$


m(r,t)
g_{μν} → Einstein Eq.s
∂m(r,t)/∂r = 4πr²(−T^r_t) = 0 ⇒ m = M = const
∂m/∂t = −4πr²(−T^r_t) = 0
∂γ/∂r = 4π (1/r)(−T^t_t + T^r_r) = 0 ⇒ γ(t)


Eq. 5
$ds^2 = -e^{2\gamma(t)} f\, dt^2 + \frac{1}{f} dr^2 + r^2 d\Omega^2$
$dt^{new} = e^{\gamma(t)} dt^{old}$
$\Rightarrow ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{1}{1-\frac{2M}{r}} dr^2 + r^2 d\Omega^2$
$\lim_{r\to\infty} f(r) = 1 - \frac{2M}{r} = 1$
$\Rightarrow$ Minkowsky
$\Rightarrow m = M = const$
$f(r) = 0 \;@\; r = r_+ = 2M$
$g_{00}(r_+) = 0 \qquad g_{rr}(r_+) \to \infty$


M
$r = r_+$
$R \le r_+$
r_+
x
· Vacuum
· spherical sym.
· $(\Lambda \sim 0)$
g_{scl}
$R \ll d < \frac{1}{H}$


M
r=r+
R ≤ r+
r+
λ
Vacuum
spherial sym.
(Λ~0)
gscl
R << d < 1/H
B. H.
Stellar BHs → GWs ~50 M⊙
Super Massive BH
gμν(r+) → g00 → 0 grr
R = 0 Rμν − ½ R gμν
Kretshmaann

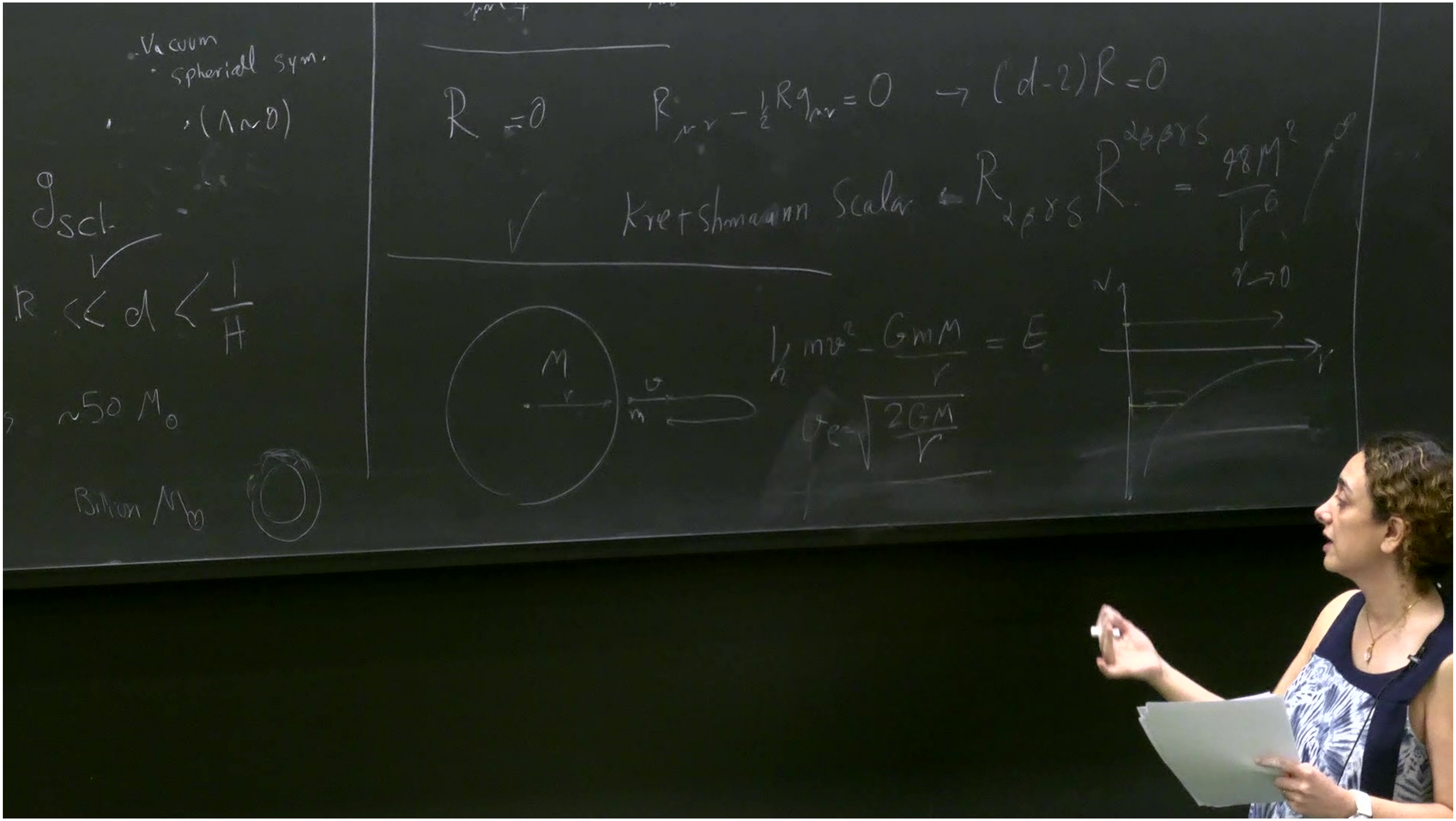

M
r=r+
Vacuum
spherical sym.
R ≤ r+
(Λ~0)
r+
λ
gscl
R << d < 1/H
B. H.
Stellar BHs → GWs ~50 M⊙
Super Massive BH Billion M⊙
gμν(r+) → g00 → 0 grr
R = 0 Rμν − ½ R gμν
Kretshmann

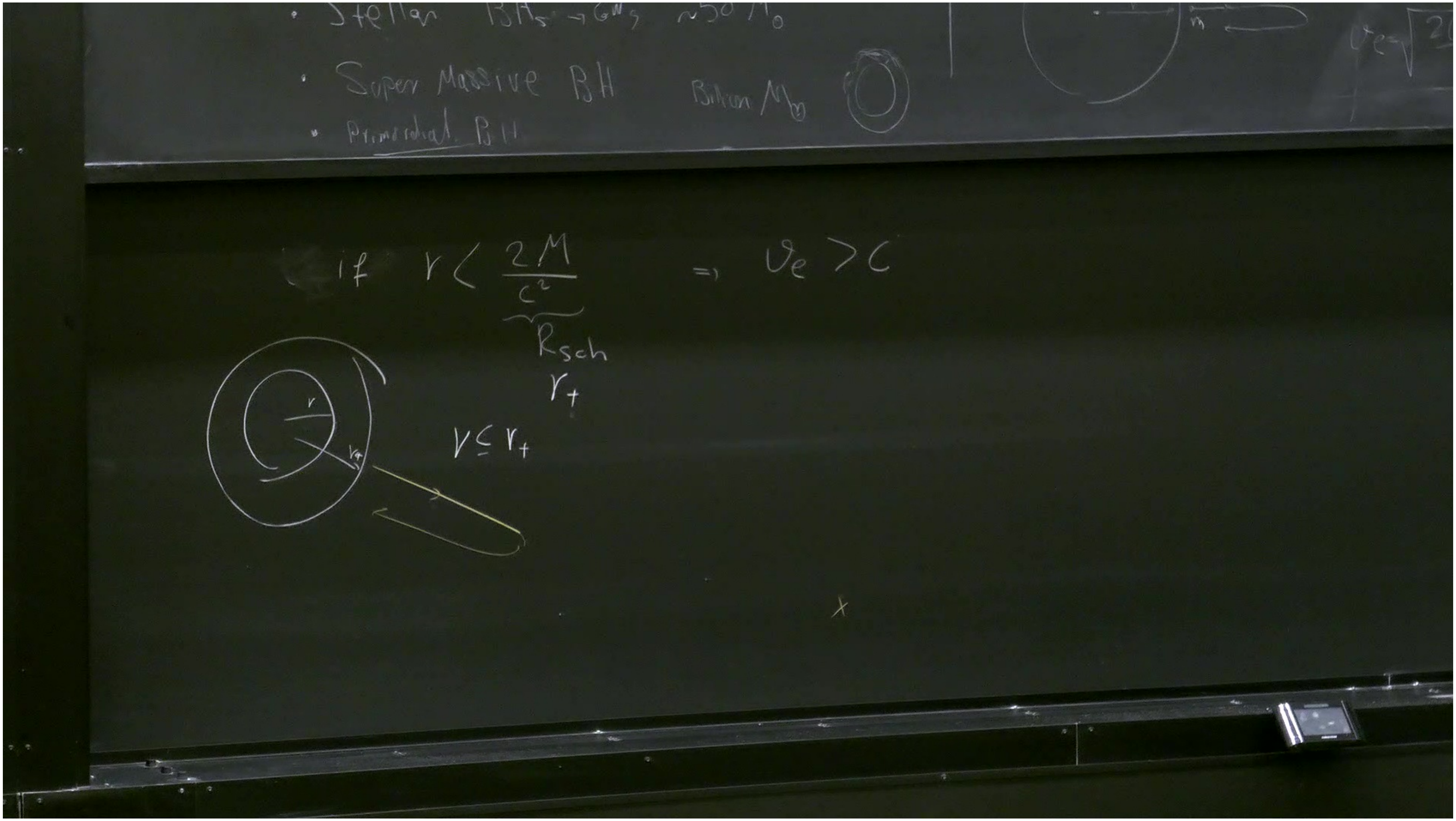
$g_{\mu\nu}(r) \to g_{00} \to 0 \quad g_{rr} \to \infty$

$R = 0 \qquad R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 0 \to (d-2)R = 0$

✓ Kretshmaann Scalar $= R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} = \frac{48M^2}{r^6} \to \infty$

$r \to 0$


Vacuum
spherical sym.
(Λ~0)
g_scl.
R << d < 1/H
~50 M_0
Billion M_0
R = 0
R_μν − ½ R g_μν = 0 → (d−2) R = 0
Kretschmann Scalar = R_αβγδ R^αβγδ = 48M²/r⁶
r→0
M
½ mv² − GmM/r = E
v_e = √(2GM/r)


Stellar BHs → GWs ~50 $M_\odot$
Super Massive BH
Billion $M_\odot$
Primordial BH
if $r < \underbrace{\frac{2M}{c^2}}_{R_{sch}}$ ⇒ $v_e > c$
r_+
$r \leq r_+$