Title: Lecture - QFT II, PHYS 603

Speakers: Francois David

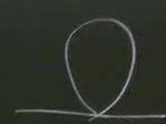
Collection/Series: Quantum Field Theory II (Core), PHYS 603, November 12 - December 11, 2024

Subject: Condensed Matter, Particle Physics, Quantum Fields and Strings

Date: December 10, 2024 - 9:00 AM

URL: https://pirsa.org/24120005

Gauge Theories SU(2)

ϕ^4  $\rightarrow \frac{1}{2-d}$ SU(2)

- UV divergences at $d=4$
- Organize P.T Gauge Part / Helicity structure

2pt 3pt, 4pt

AA AAA $AAAA$

$\bar{c}c$ $\bar{c}Ac$

$$\log\Lambda^2 \rightarrow \frac{1}{8\pi^2}\frac{1}{4-d}$$

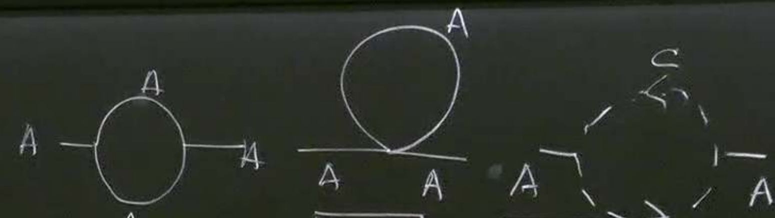
$$\Lambda^2 \rightarrow \frac{1}{2\pi}\frac{1}{2-d}$$

- dimensional regularization

Poincare, Gauge sym., BRST, ~~Scale invariance~~

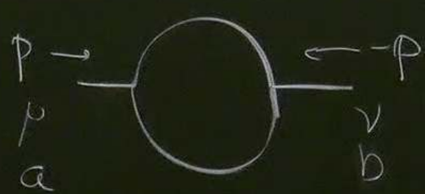
$\frac{1}{-d}$

SU(2) Λ^2 potentially present



cancell ? if not $\boxed{-1}$ $\boxed{+2}$ $\boxed{-1}$

mass generated for A. $m \sim g\Lambda$ $\Rightarrow$ Causality + Unitarity



$$\boxed{\frac{1}{2\pi}\frac{1}{2-d}} \Leftarrow (-1)\,\delta^{\mu\nu}\,\delta^{ab}\int\frac{d^dk}{(2\pi)^d}\frac{1}{k^2} \qquad -1+2-1=0 \;!$$

divergence $d \to 2$

$A^\mu A^\mu \frac{1}{2-d}$ not gauge invariant

Massive Vector boson A_μ

$$(\partial_\mu A_\nu)^2 + b(\partial_\mu A_\mu)^2 + M^2 A_\mu A_\nu$$

3 $\perp$ modes mass M

1 // mode mass $M_\ell = \sqrt{1+b}\, M$

but − sign

$(-+++)$ propagator

$$A_\mu K_{\mu\nu} A_\nu \qquad K_{\mu\nu}(p) = h_{\mu\nu}(p^2+M^2) + b\, p_\mu p_\nu$$

$$\sim\sim \qquad (K^{-1})_{\mu\nu} = \frac{-1}{p^2+M^2}\begin{pmatrix}0&&&\\&1&&\\&&1&\\&&&1\end{pmatrix} + \frac{-1}{(1+b)p^2+M^2}\begin{pmatrix}-1&&&\\&0&&\\&&0&\\&&&0\end{pmatrix}$$

(2) Massive Vector boson A_μ

$$(\partial_\mu A_\nu)^2 + b(\partial_\mu A_\mu)^2 + M^2 A_\nu A_\nu$$

3 $\perp$ modes mass M

1 $\parallel$ mode mass $M_\ell = \sqrt{1+b}\, M$

but $-$ sign

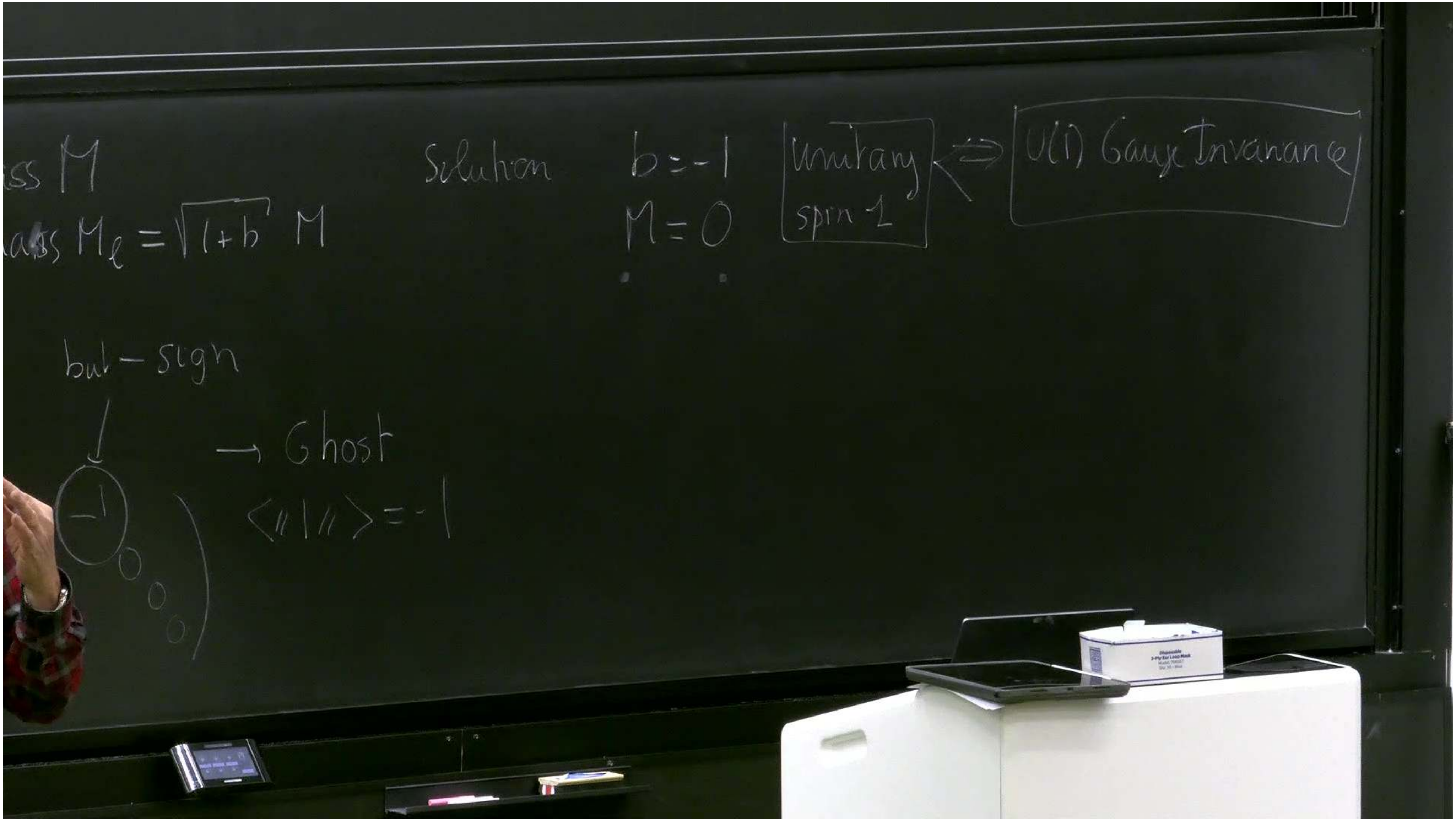
$(-+++)$ propagator

$$A_\mu K_{\mu\nu} A_\nu \qquad K_{\mu\nu}(p) = h_{\mu\nu}(p^2+M^2) + b\, p_\mu p_\nu$$

$\to$ Gh

$\langle \parallel | \parallel$

$$(K^{-1})_{\mu\nu} = \frac{-1}{p^2+M^2}\begin{pmatrix} 0 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} + \frac{-1}{(1+b)p^2+M^2}\begin{pmatrix} -1 & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{pmatrix}$$


ss M
ass $M_\ell = \sqrt{1+b}\ M$
Solution
$b = -1$
$M = 0$
Unitary spin-1
⟺
U(1) Gauge Invariance
but - sign
→ Ghost
$\langle u | u \rangle = -1$

Poincaré, Gauge Sym., BRST, Scale invariance

③ Another method to compute the β-function

1 loop Effective action

in "background Field" Gauge Fixing Method

$$\underset{\text{quantum}}{A_\mu} = \underset{\text{classical}}{A^{bg}_\mu} + \underset{\text{quantum}}{\tilde{A}_\mu} \qquad \text{enforce G.F on } \tilde{A}_\mu \quad D^{bg\,\mu}\tilde{A}_\mu = 0$$

covariant derivative in the bg Field A^{bg}_μ

$2-d$ not gauge invariant

$$\Gamma[A^{bg}] = S_{YM}[A^{bg}] + \frac{1}{2}\,\mathrm{tr}\left[\mathrm{Log}\left(S''[A^{bg}]\right)\right] - \mathrm{tr}\,\mathrm{Log}\left(-\partial^{\mu} D^{bg}_{\mu}\right)$$

quantum effective action

Hessian

gauge field

ghosts

ghost action Hessian

Diff operator acting on $\hat{A}$ but depending on A^{bg}

$\Downarrow$

Heat Kernel Expansion in short "time" for Diff Operators (Seeley-DeWitt)

important in Math $\rightarrow$ Index Theorems . . . .

contains Log Λ diverges or $\frac{1}{4-d}$ poles $\propto$ original action $S_{YM}[A^{bg}]$

$\Rightarrow$ renormalized action CT

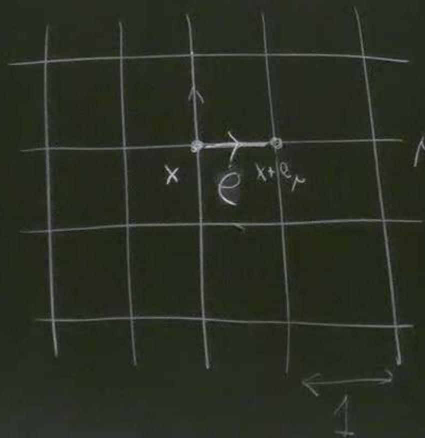
quantum

covariant derivative
in the bg Field A_μ^{bg}

Heat Kernel
for Diff Operat...
important in ...

Wilson Loops , Lattice Gauge Theories $\boxed{\text{K Wilson (74)}}$
& confinement

F Wegner (72...
3d Ising ←
dua...



lattice $\mathbb{Z}^d$ (square)

How to introduce gauge field

Matter Fields ϕ in fundamental representation

$\phi = (\phi^i)_{i=1,2}$ for $SU(2)$ $e_\mu = (0 \dots 1 \dots 0)$ ($\uparrow \mu$)

e_μ elementary vector in direction μ

$A_\mu dx^\mu$ ← connection

$\phi(x) \to \phi(y)$ covariant derivative

$D_\mu = \partial_\mu - A_\mu$

ordinary derivative

$\phi(x+e_\mu) - \phi(x) = \dots$

quantum

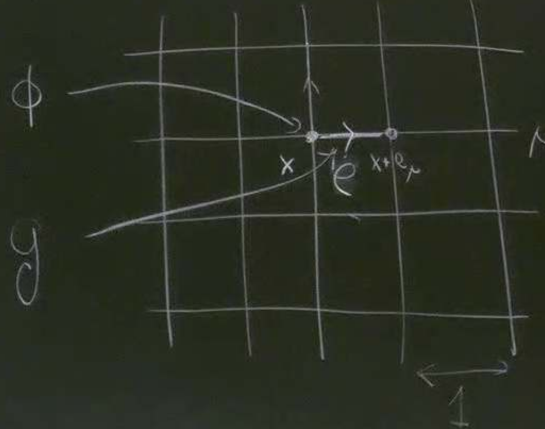
covariant derivative
in the bg. Field A_μ^{bg}

Heat Kernel
for Diff Operat[...]
important in [...]

Wilson Loops & confinement , Lattice Gauge Theories $\boxed{\text{K Wilson (74)}}$

F Wegner (72)

3d Ising ← dual



lattice $\mathbb{Z}^d$ (square)

How to introduce gauge field

Matter Fields ϕ in fundamental representation

$\phi = (\phi^i)_{i=1,2}$ for $SU(2)$ $\quad e_\mu = (0 \cdot 1 \cdot 0)$, $\uparrow \mu$

e_μ elementary vector in direction μ

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$\phi(x) \to \phi(y)$ covariant derivative

$D_\mu = \partial_\mu - A_\mu$

ordinary derivative

$\phi(x+e_\mu) - \phi(x) =$

Heat Kernel Expansion in short "time" $\sim \frac{1}{4-d}$ poles $S_{YM}[A^{hg}]$

for Diff Operators (Seeley-DeWitt) $\Rightarrow$ renormalized action CT

important in Math $\rightarrow$ Index Theorems . . . .

F Wegner (72)

3d. Ising $\xleftrightarrow[\text{dual}]{}$ Z_2 gauge theories

connection

covariant derivative

$D_\mu = \partial_\mu - A_\mu$

ordinary derivative

$\phi(x+e_\mu) - \phi(x) = \partial_\mu \phi$

ϕ lives on sites

$\partial_\mu \phi$ lives on links

$\xrightarrow{e}$ oriented link (edge)

$\xleftarrow[e^*]{}$ opposite oriented edge

$$\boxed{\begin{array}{ll} e \Leftrightarrow g(e) & g \in SU(2) \\ e^* & g(e^*) = g^{-1}(e) \end{array}}$$

G

$$D_\mu \phi(e_\mu) = \phi(x+e_\mu) - g(e_\mu)\phi(x)$$

$x \xrightarrow{e} x+e_\mu$

$$g(e_\mu) = 1 + i\epsilon A_\mu e_\mu + \cdots = e^{i\epsilon A_\mu dx^\mu)}$$

$\boxed{\text{small } \epsilon}$

ordinary gauge connection

What is a gauge transformation (local)

$\phi(x) \to h(x)\phi(x) \quad h \in SU(2)$

$\boxed{\phi \to h \circ \phi}$

matter fields

$\phi(x) \to [1 + i\epsilon\alpha(x)]\phi(x)$

what is it for $g(e)$

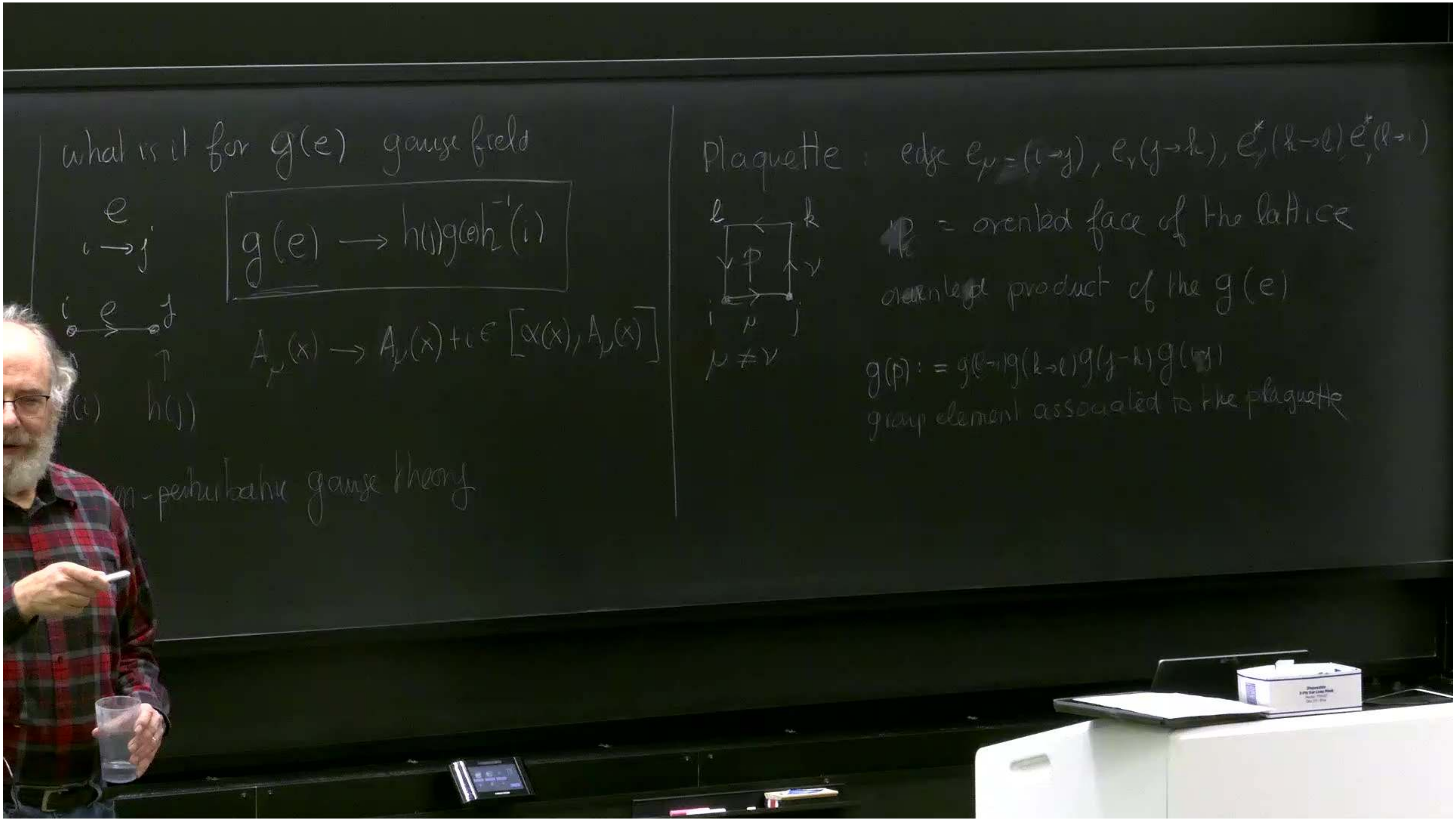
e

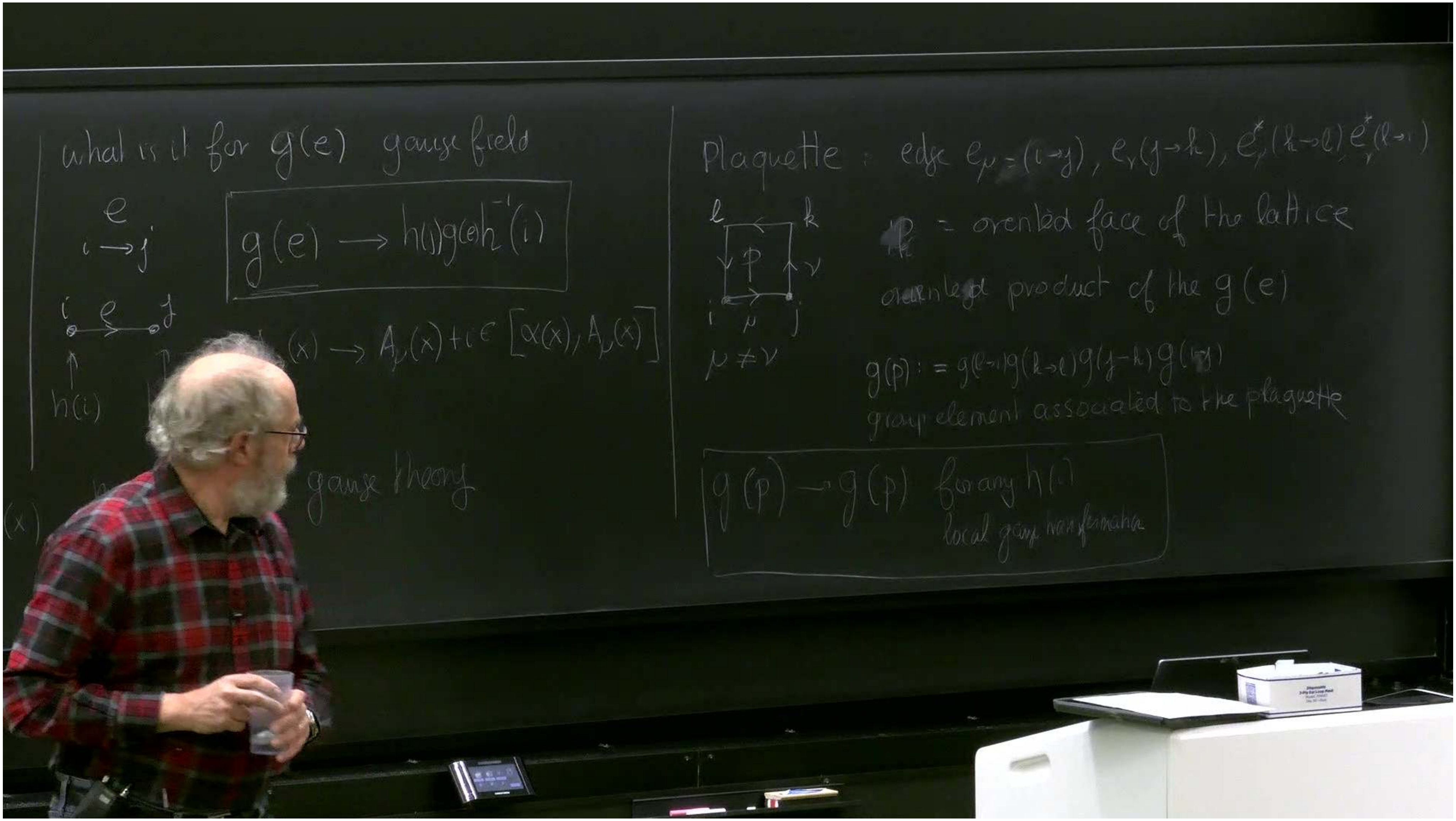
$i \to j$

$i \xrightarrow{e} j$

$h(i) \quad h(j)$

$g(e) \to$


what is it for g(e) gauge field
g(e) → h(i)g(e)h⁻¹(i)
Aμ(x) → Aμ(x) + iε [α(x), Aμ(x)]
non-perturbative gauge theory
Plaquette : edge eμ = (i→j), eν(j→k), eμ*(k→l), eν*(l→i)
p = oriented face of the lattice
oriented product of the g(e)
μ ≠ ν
g(p) := g(l→i) g(k→l) g(j→k) g(i→j)
group element associated to the plaquette


what is it for g(e) gauge field
e
i → j
g(e) → h(j)g(e)h^{-1}(i)
i e j
h(i)
(x) → A_μ(x) + iε [α(x), A_μ(x)]
gauge theory
Plaquette : edge e_μ = (i→j), e_ν(j→k), e*_μ(k→l), e*_ν(l→i)
l k
p
ν
i μ j
μ ≠ ν
= oriented face of the lattice
oriented product of the g(e)
g(p) := g(l→i)g(k→l)g(j→k)g(i→j)
group element associated to the plaquette
g(p) → g(p) for any h(i)
local group transformation

matter fields

$g(\square) \leftrightarrow$ curvature of the gauge connection

$g(\to) = 1 + i\epsilon A(\to)$ $\quad A(\underset{\mu}{\to}) = A_\mu$

then

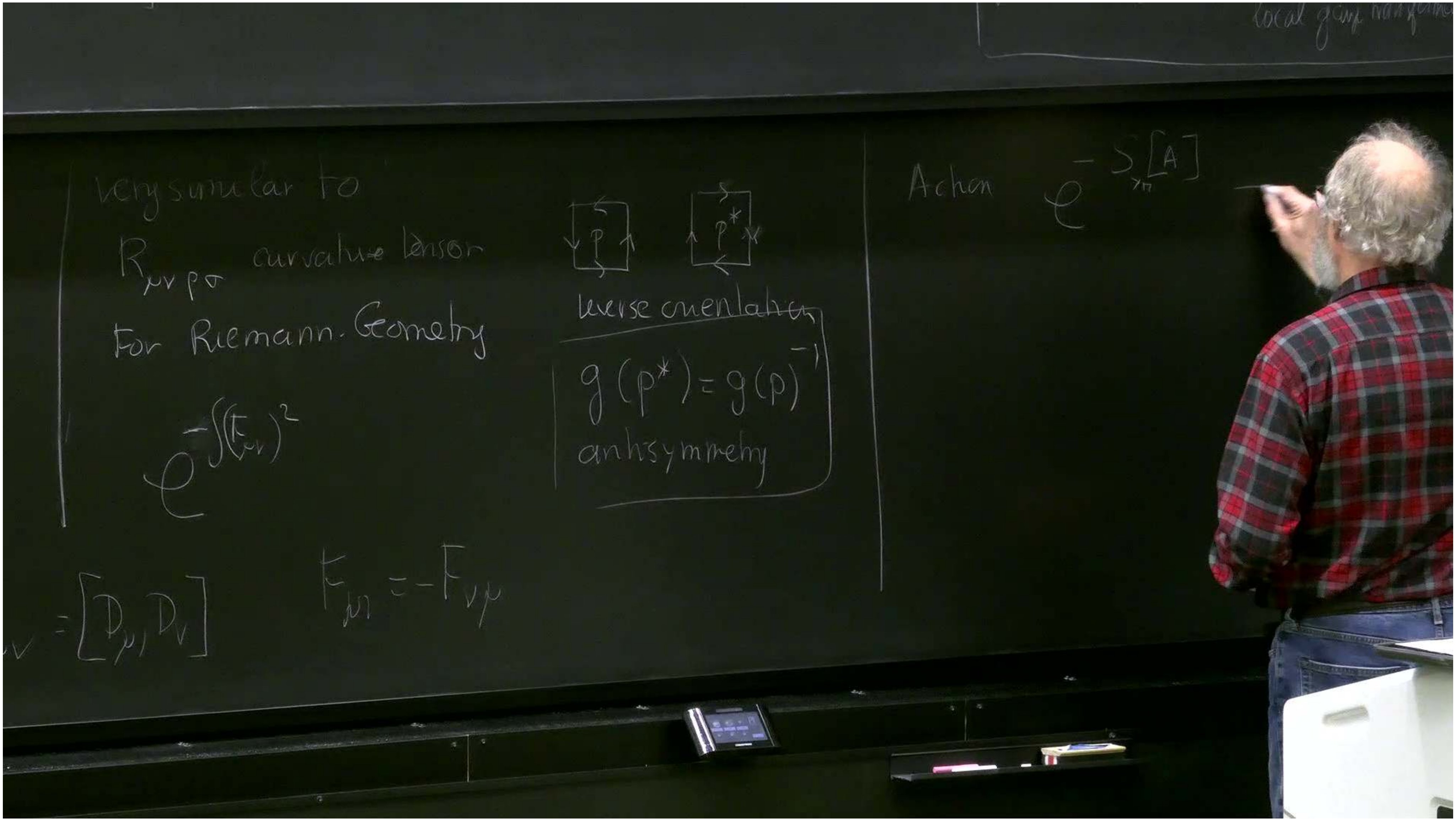
$g(\square_\nu) = 1 + i\epsilon F(\square)$

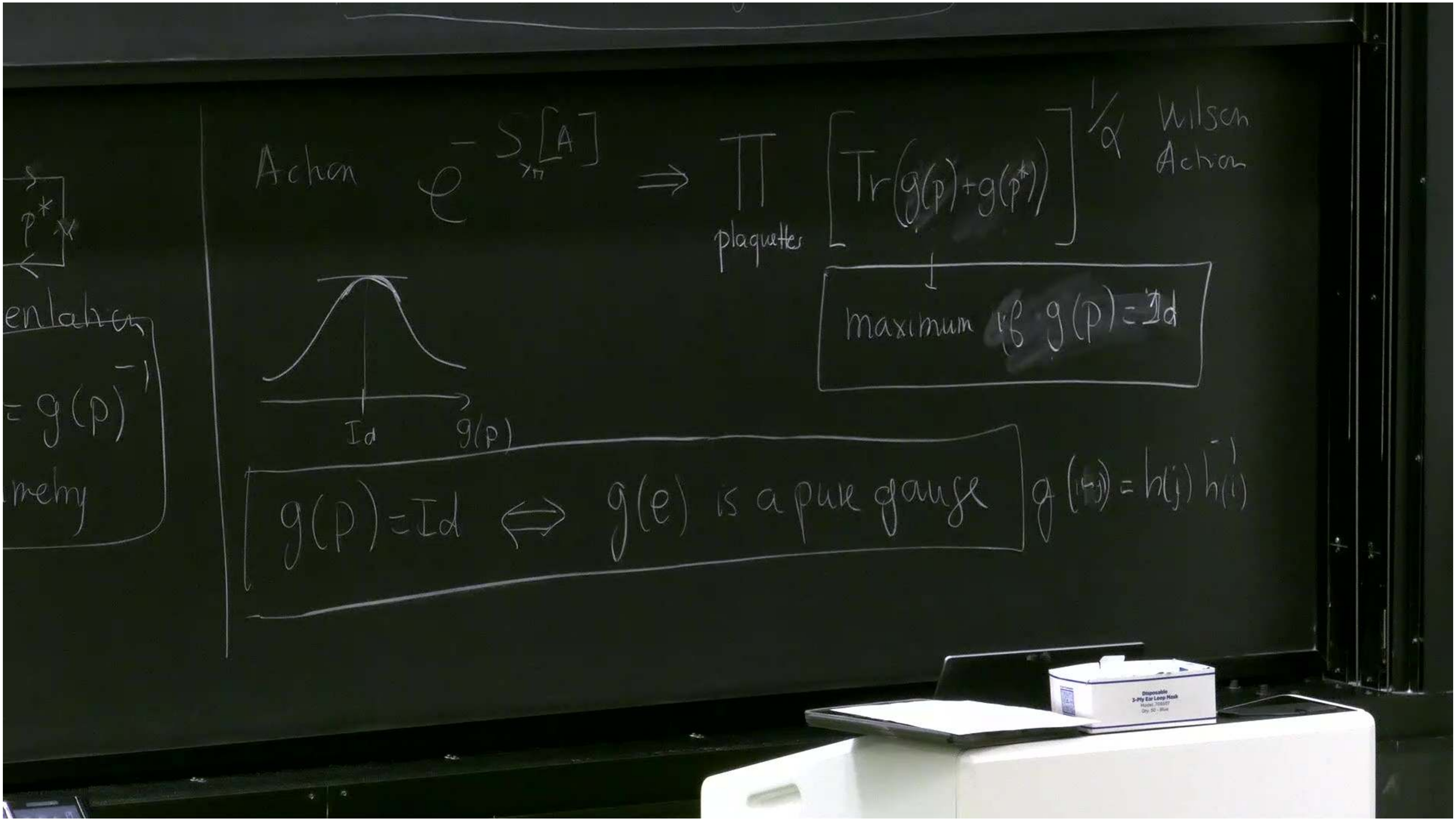
$F(\underset{\mu}{\square}_\nu) = F_{\mu\nu} = [D_\mu, D_\nu]$

very similar to

$R_{\mu\nu\rho\sigma}$ curvature tensor

For Riemann. Geometry

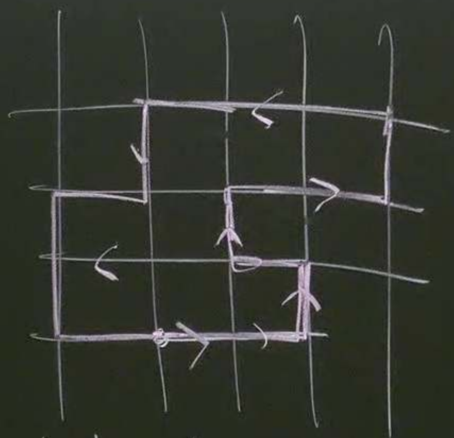

very similar to
$R_{\mu\nu\rho\sigma}$ curvature tensor
for Riemann Geometry
$e^{-\int (F_{\mu\nu})^2}$
inverse orientation
$g(p^*) = g(p)^{-1}$
antisymmetry
$= [D_\mu, D_\nu]$
$F_{\mu\nu} = -F_{\nu\mu}$
Action $e^{-S_{YM}[A]}$


Action $e^{-S_{YM}[A]} \Rightarrow \prod_{\text{plaquettes}} \left[\mathrm{Tr}(g(p)+g(p^*))\right]^{1/\alpha}$
Wilson Action
maximum if $g(p) = \mathrm{Id}$
Id
$g(p)$
$g(p) = \mathrm{Id} \Leftrightarrow g(e)$ is a pure gauge
$g(i \to j) = h(j)\, h(i)^{-1}$

$$\phi = (\phi^i)_{i=1,2} \text{ for } SU(2) \qquad e_\mu = (0 \cdots \underset{\uparrow \mu}{1} \cdots 0) \qquad \phi(x+e_\mu) - \phi(x) = \partial_\mu \phi$$

$\overset{\longleftrightarrow}{1}$

e_μ elementary vector on direction μ



Wilson Loop

any oriented circuit $\mathcal{C}$ on the Lattice $\Rightarrow$ all gauge invariant observables of the theory

$$g(\mathcal{C}) = \prod_{e \in \mathcal{C}} g(e)$$

ordered product

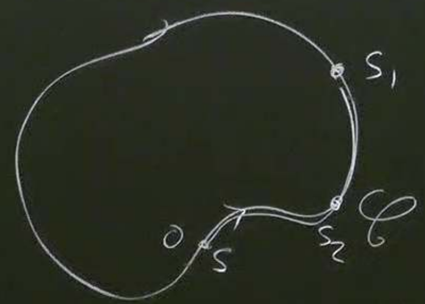
Gauge invariant quantity

Non local

Continuum

$\phi(x+e_\mu)-\phi(x) = \partial_\mu \phi \qquad x \xrightarrow{e} x+e_\mu$

Continuum



oriented smooth loop

$$g(\mathcal{C}) \longrightarrow P\left[\exp\left(i\int_{\mathcal{C}} dx^\mu A_\mu(x)\right)\right]$$

Wilson loop operator

path ordered integral

magnetic flux

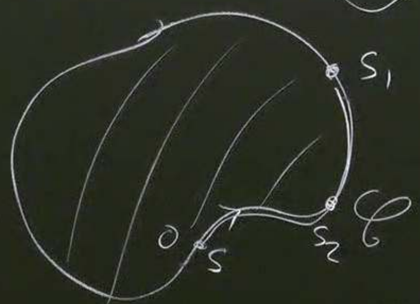
$$1 + i\int_0^1 ds\, \frac{dx^\mu}{ds} A_\mu(x) - \int_0^1 ds_1 \int_0^{s_1} ds_2\, \frac{dx^\mu(s_1)}{ds_1} A_\mu(x(s_1))\, \frac{dx^\nu}{ds_2}(s_2)\, A_\nu(x(s_2))$$

$s \to X(s)$

parametrization $s \in S^1$, $s \in [0,1]$

$(e_\mu) - \phi(x) = \partial_\mu \phi$ $\qquad x \xrightarrow{e} x + e_\mu$

Continuum  $\longrightarrow \square = F_{\mu\nu}^2$



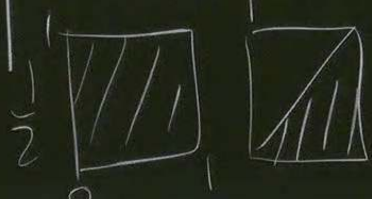
$$g(\mathcal{C}) \longrightarrow P\left[\exp\left(i\int_{\mathcal{C}} dx^\mu A_\mu(x)\right)\right]$$ Wilson loop operator

path ordered integral

magnetic flux

oriented smooth loop

$$1 + i\int_0^1 ds\, \frac{dx^\mu}{ds} A_\mu(x) - \int_0^1 ds_1 \int_0^{s_1} ds_2\, \frac{dx^\mu}{ds_1}(s_1) A_\mu(x(s_1))\, \frac{dx^\nu}{ds_2}(s_2) A_\nu(x(s_2))$$



$s \to X(s)$

parametrization $s \in S^1$, $s \in [0,1]$