Title: Lecture - QFT II, PHYS 603

Speakers: Francois David

Collection/Series: Quantum Field Theory II (Core), PHYS 603, November 12 - December 11, 2024

Subject: Condensed Matter, Particle Physics, Quantum Fields and Strings

Date: December 09, 2024 - 9:00 AM

URL: https://pirsa.org/24120004

Renormalisation of Gauge Theories $SU(2)$

without Fermions $d = 4$

$A^a_\mu \quad a = 1, 2, 3$ Adj Rep.

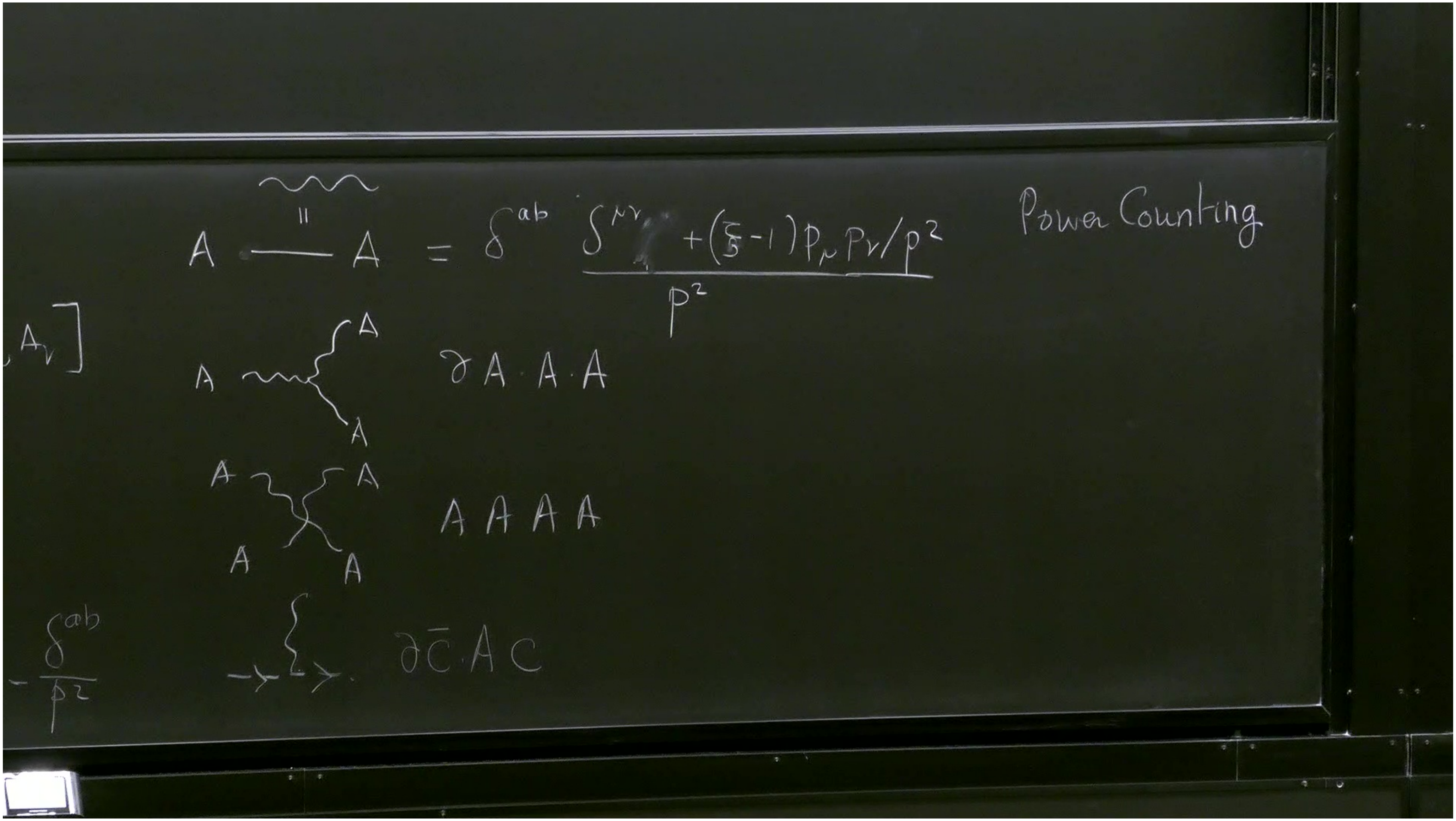
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i\,[A_\mu, A_\nu]$$

$$D_\mu \cdot = \partial_\mu \cdot + i\,[A_\mu ; \ ]$$

$$L[A] = \frac{1}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu} + \partial_\mu \bar{C}_a D_\mu C_a + \frac{1}{2g\xi} (\partial_\mu A_\mu)^2$$

<u>Euclidean</u>

Feynman Gauge $\quad \xi = 1$


Power Counting
$A - A = \delta^{ab} \frac{\delta^{\mu\nu} + (\xi-1)p_\mu p_\nu/p^2}{p^2}$
$\partial A \cdot A \cdot A$
$A\,A\,A\,A$
$\partial\bar{c}\cdot A\,c$
$\frac{\delta^{ab}}{p^2}$
$A_\nu]$

SU(2) gauge theory

Perturbation theory

&

Renormalization

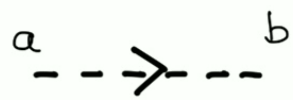
(1 loop)

Gauge propagator $\dfrac{\delta_{\mu\nu}\delta_{ab}}{p^2}$ 

SU(2) structure $a = 1, 2, 3$

δ_{ab} 

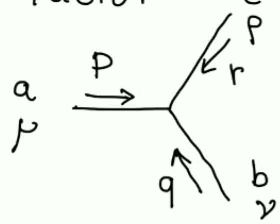
Helicity structure $\delta_{\mu\nu}$ 

Ghost propagator $\dfrac{\delta_{ab}}{p^2}$ 

SU(2) structure δ_{ab} 

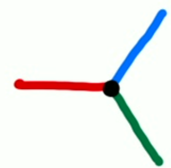
Gauge AAA 3-vertex

Factor

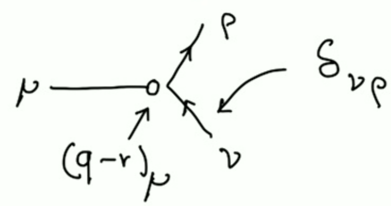


$$ig\,\epsilon_{abc}\left[\delta_{\mu\nu}(p-q)_\rho + \delta_{\nu\rho}[q-r]_\mu + \delta_{\rho\mu}(r-p)_\nu\right]$$

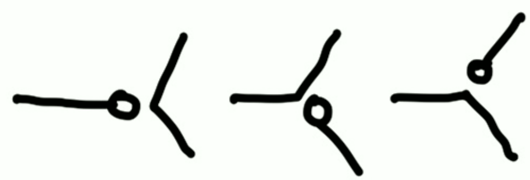
SU(2) structure (1 vertex) ϵ_{abc}



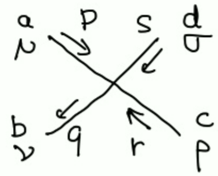
Helicity structure $\delta_{\nu\rho}\,(q-r)_\mu$



3 vertices

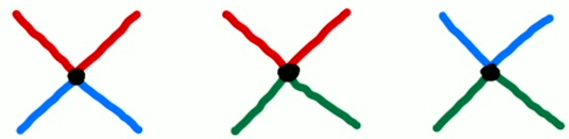


Gauge AAAA 4-vertex

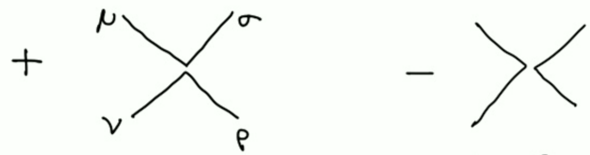


$$-g^2\left[(\delta_{ac}\delta_{bd}-\delta_{ad}\delta_{bc})\,\delta_{\mu\rho}\delta_{\nu\sigma} + \text{permutations}\right]$$

SU(2) structure (3 vertices)

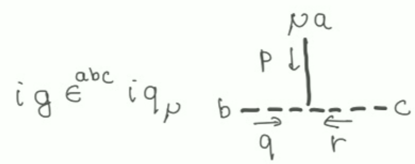


Helicity structure

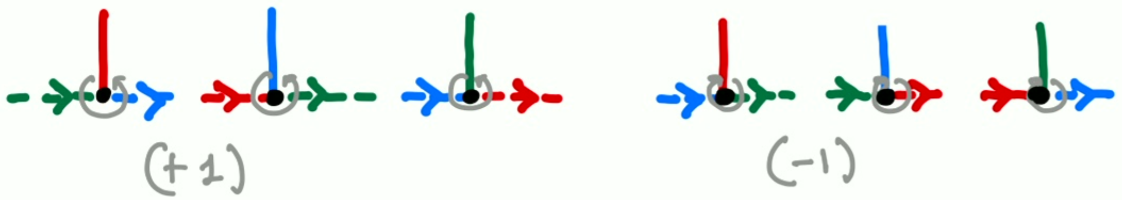


$\delta_{\mu\sigma}\,\delta_{\nu\rho}$ $\quad - \delta_{\mu\nu}\,\delta_{\rho\sigma}$

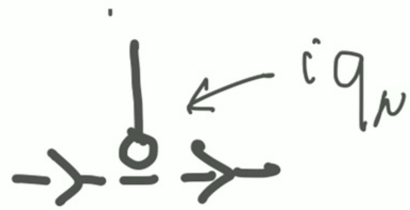
Ghost-Gauge CAC 3-vertex



SU(2) structure (6 vertices) ϵ_{abc}

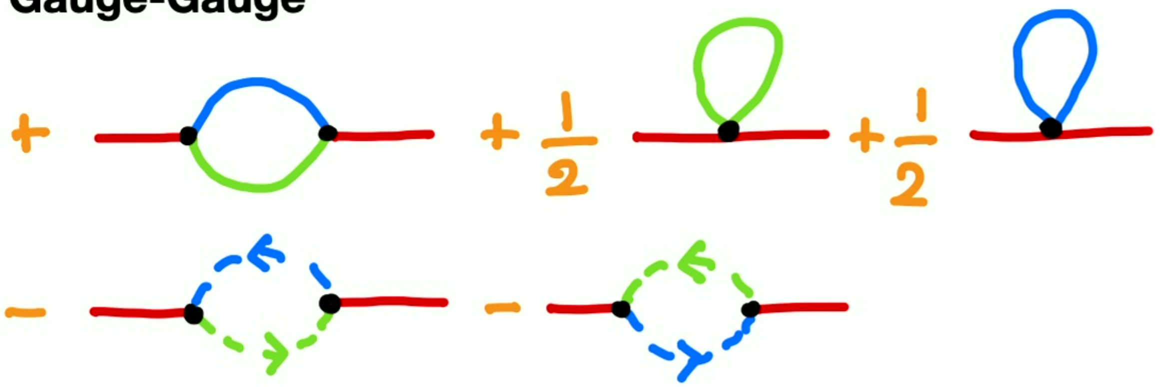


Helicity structure

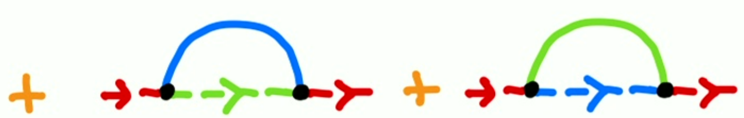


The 1-loop 2-point irreducible functions

Gauge-Gauge

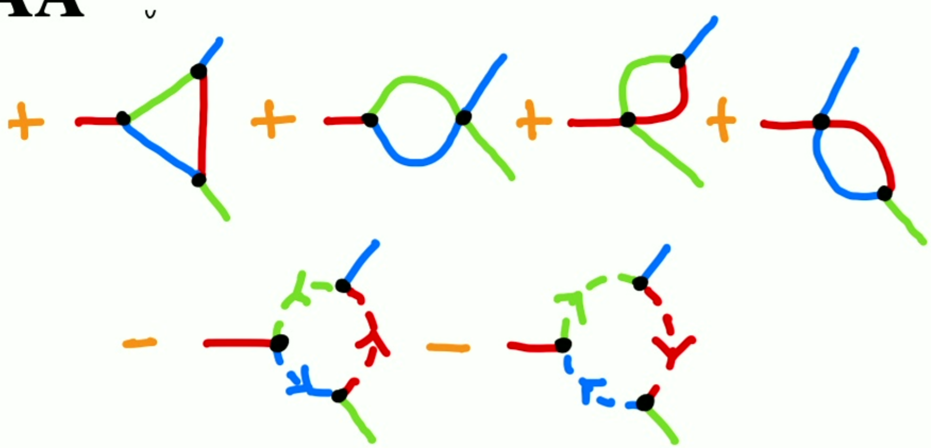


Ghost-antighost

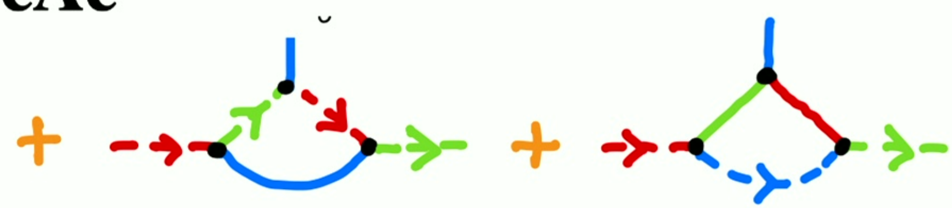


The 1-loop 3-point irreducible functions

AAA

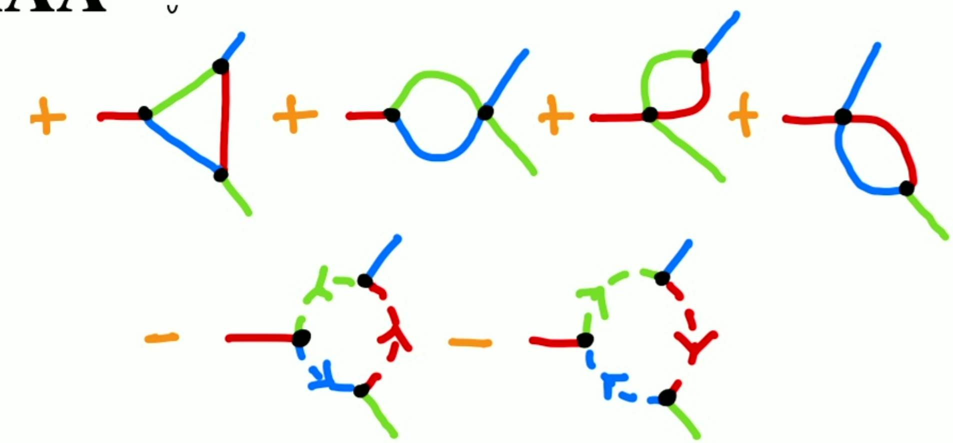


c̄Ac



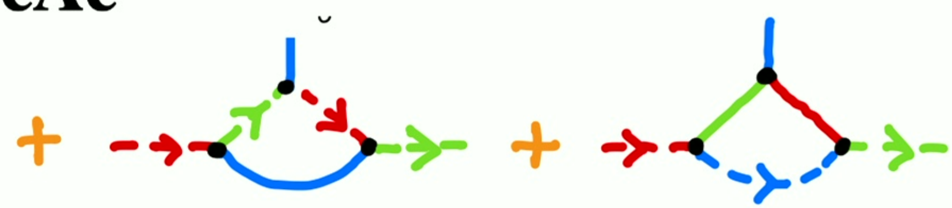
The 1-loop 3-point irreducible functions

AAA



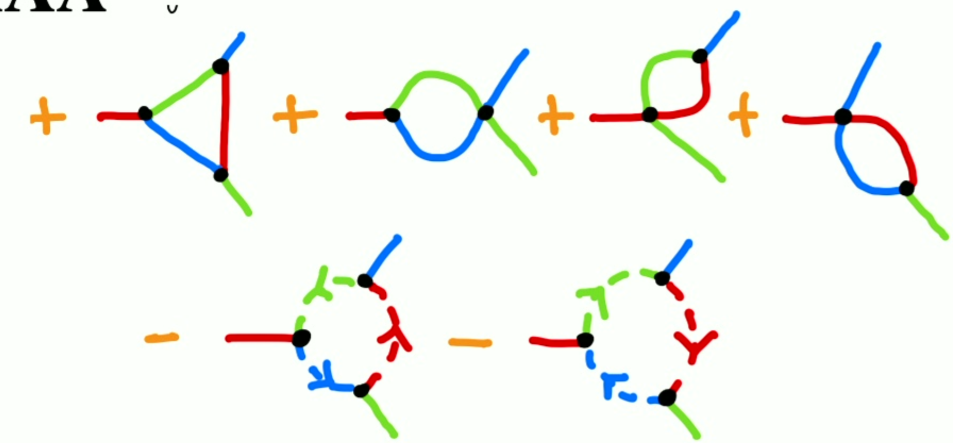
$$p \, \log \Lambda^2$$

c̄Ac



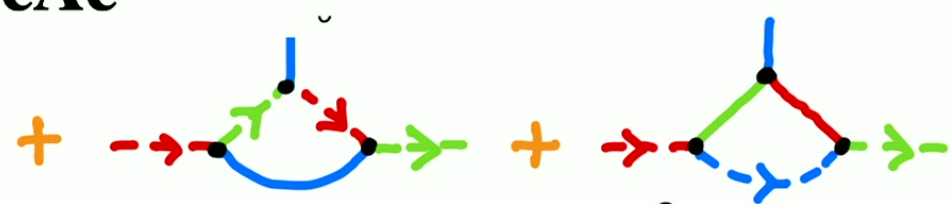
The 1-loop 3-point irreducible functions

AAA



$$p \log \Lambda^2$$

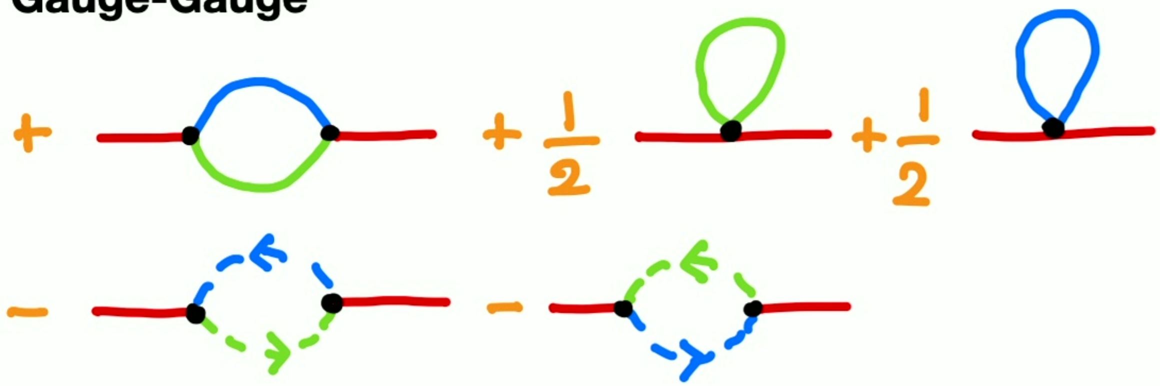
$\bar{\mathbf{c}}$Ac



$$p \log \Lambda^2$$

The 1-loop 2-point irreducible functions

Gauge-Gauge



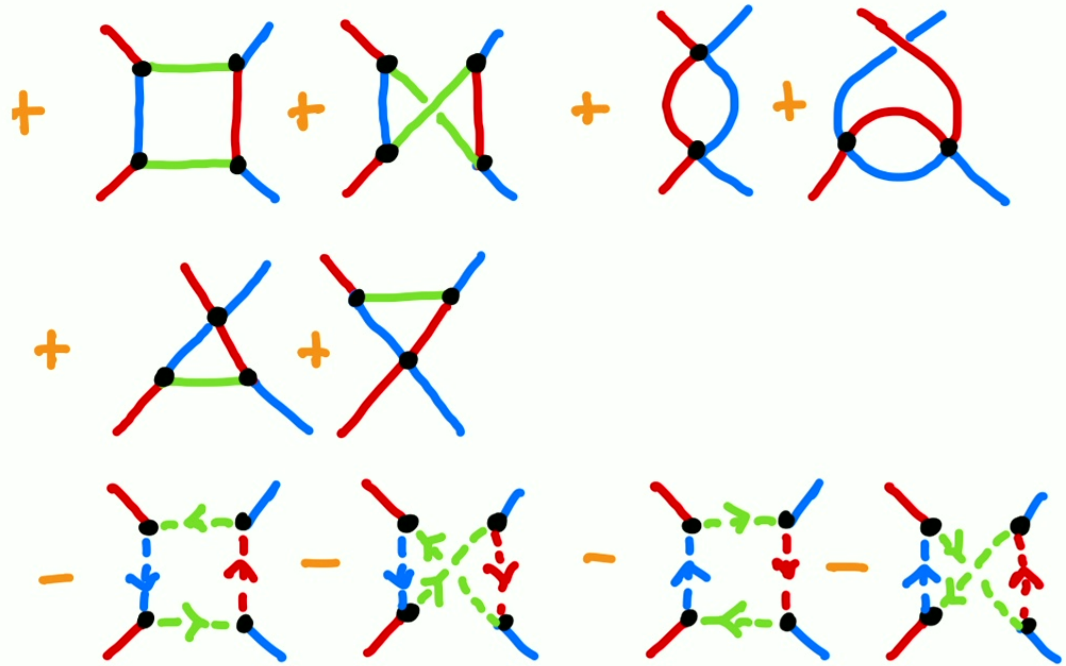
$$\Lambda^2 + p^2 \log \Lambda^2$$

Ghost-antighost

$$p^2 \log \Lambda^2$$

The 1-loop 4-point irreducible functions

AAAA



Analysis of UV divergences

The Λ^2 quadratic divergents must disappear (the vector field must stay massless

The helicity structure of the 3 and 4 vertices must be of the same form than for the vertices (gauge invariance)

The coefficients of the divergences must be related, so that the counterterms in the renormalized action respect gauge invariance

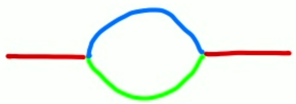
Convenient regularization ('t Hooft-Weltman): **dimensional regularization**

It respects gauge invariance and Poincaré/Euclidean symmetries

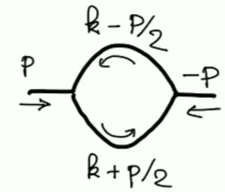
$$\int \frac{d^dk}{(2\pi)^d} \frac{1}{(k^2)} \to \frac{1}{2\pi} \frac{1}{2-d} \qquad \int \frac{d^dk}{(2\pi)^d} \frac{1}{(k^2)^2} \to \frac{1}{8\pi^2} \frac{1}{4-d}$$

The helicity terms and Lorentz indices must be considered

AA 2 point function



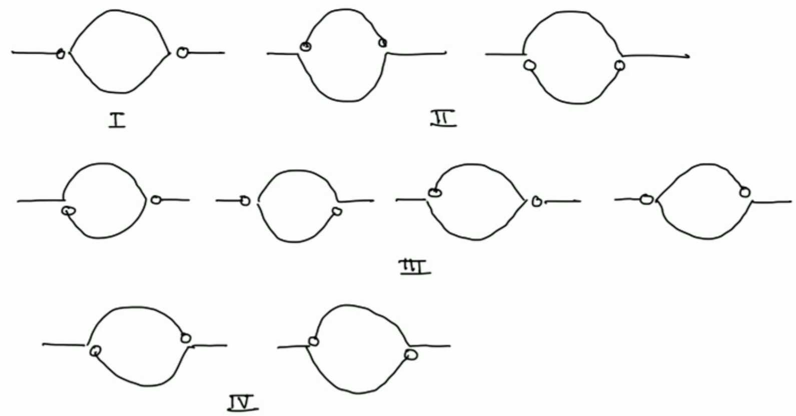
symmetric internal momenta

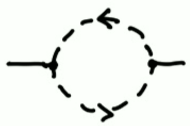


9 diagrams into
6 classes

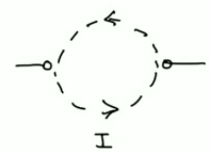
pole at d=4 with coeff

$$\delta_{\mu\nu}p^2\left(\frac{-19}{6}\right)+p_\mu p_\nu\left(\frac{22}{6}\right)$$





1 diagram



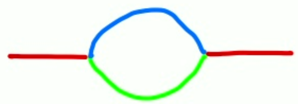
pole at d=4 with coeff

$$\delta_{\mu\nu}p^2\left(\frac{-1}{6}\right)+p_\mu p_\nu\left(\frac{-1}{3}\right)$$

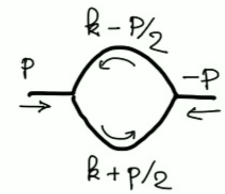
Add up in a
gauge invariant
term :-)

$$\frac{-11}{3}\left(\delta_{\mu\nu}p^2-p_\mu p_\nu\right)$$

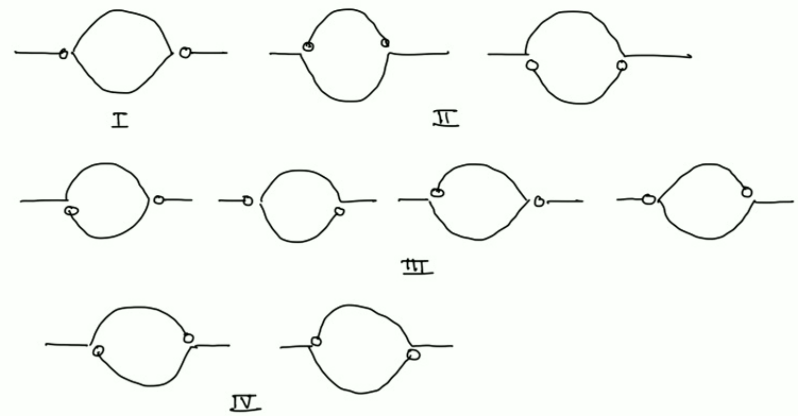
AA 2 point function



symmetric internal momenta

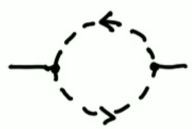


9 diagrams into
6 classes

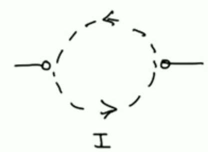


pole at d=4 with coeff

$$\delta_{\mu\nu}p^2\left(\frac{-7}{2}\right)+p_\mu p_\nu\,(4)$$



1 diagram



pole at d=4 with coeff

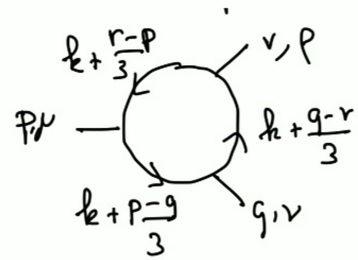
$$\delta_{\mu\nu}p^2\left(\frac{-1}{6}\right)+p_\mu p_\nu\left(\frac{-1}{3}\right)$$

Add up in a
gauge invariant
term :-)

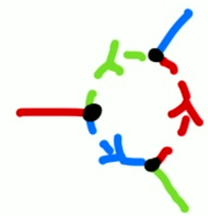
$$\frac{-11}{3}\left(\delta_{\mu\nu}p^2-p_\mu p_\nu\right)$$

AAA 3 point function

symmetric internal momenta



Ghost loop: one diagram

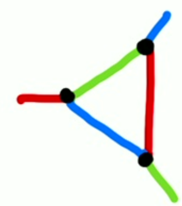


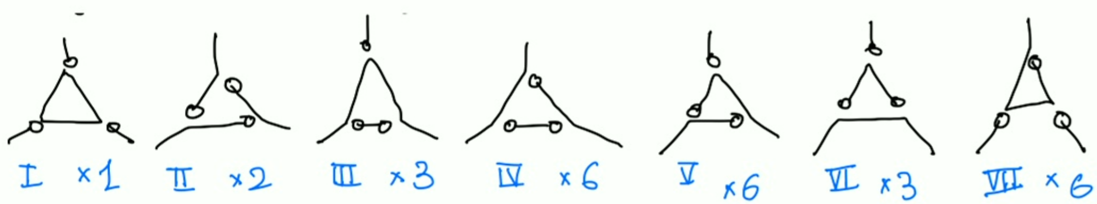
pole at d=4

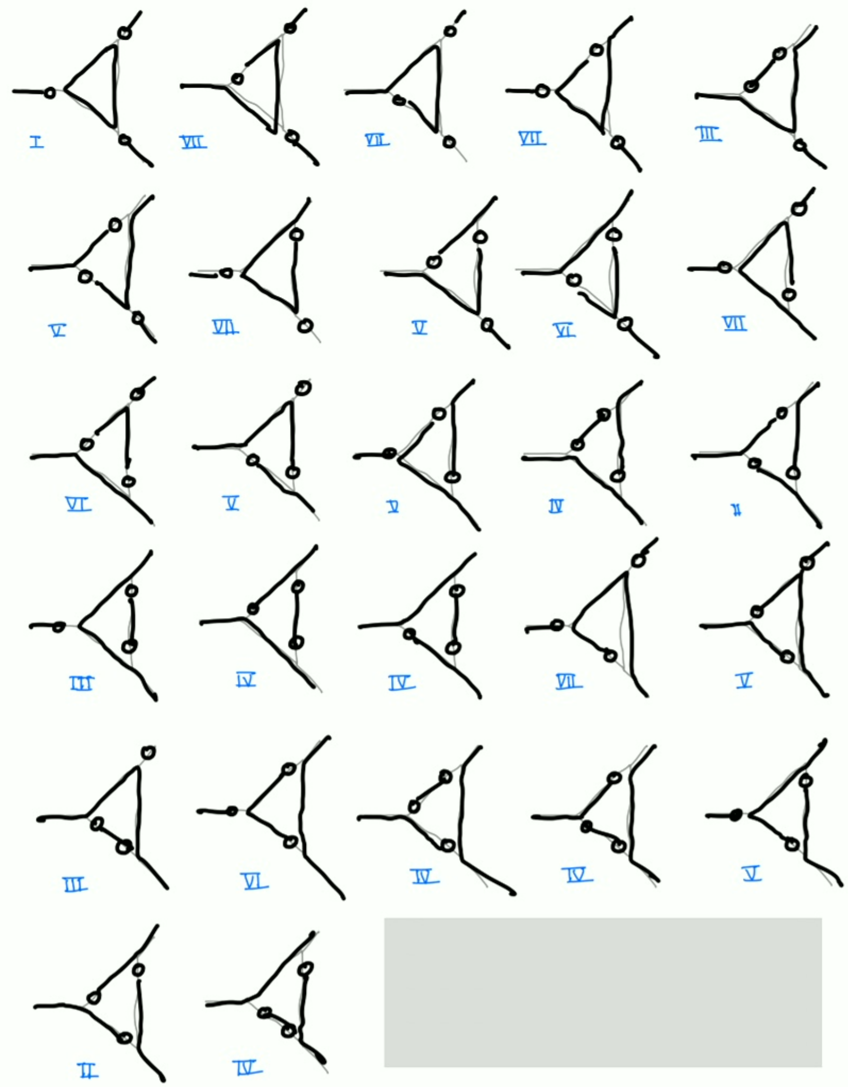
$$\frac{1}{12}\left(\delta_{\mu\nu}(q-p)_\rho + \delta_{\nu\rho}(r-q)_\mu + \delta_{\rho\mu}(p-r)_\nu \right)$$

Same helicity structure as the AAA vertex ! :-)

For the gluon triangle : 27 diagrams into 7 classes





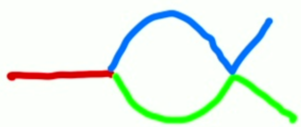


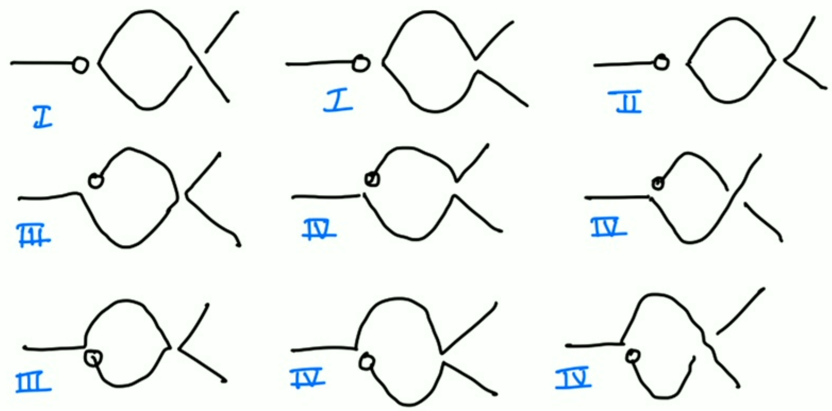
Non trivial result: each class gives a term with the good helicity structure :-) but with various coefficients.

The final result for the residue at d =4 is

$$\frac{11}{4}\left(\delta_{\mu\nu}(q-p)_\rho+\delta_{\nu\rho}(r-q)_\mu+\delta_{\rho\mu}(p-r)_\nu\right)$$

For the gluon bubble : 9 diagrams into 4 classes





Only class IV gives a non zero term, with the good helicity structure

$$-3\left(\delta_{\mu\nu}(q-p)_\rho+\delta_{\nu\rho}(r-q)_\mu+\delta_{\rho\mu}(p-r)_\nu\right)$$

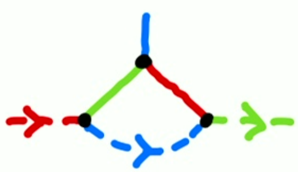
It remains to treat the $\bar{c}c$ 2-point function

2 diagram in 1 class

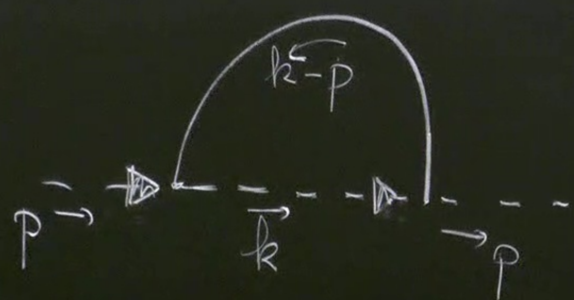
Coeff. pole at d=4 $\quad -g^2\ p^2$

And the $\bar{c}Ac$ 3-point function

1 diagram



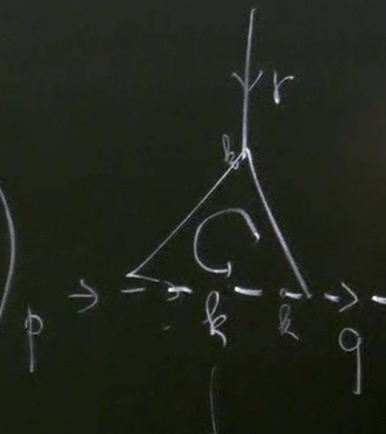
3 diagrams in two classes



$$(k-p)^2 = \underline{k^2 - 2kp} + p^2$$

$$\frac{1}{(k-p)^2} = \frac{1}{k^2}\left(1 + 2\frac{k\cdot p}{k^2} + \dots\right)$$

$$k \nearrow \infty$$

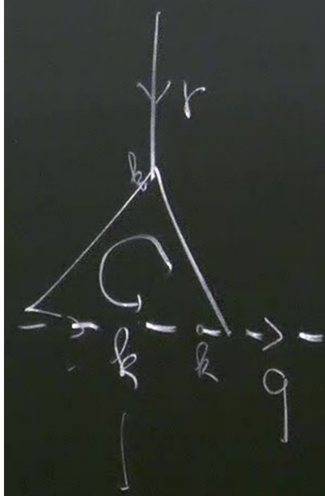


Rot Lorentz inv

$$p_\nu \int d^d k \, \frac{1}{(k-p)^2} \, \frac{k_\nu}{k^2}$$

$$= p_\nu \int d^d k \left(\frac{k_\nu}{(k^2)^2} + 2\,\frac{k_\nu k_\nu \cdot p_\nu}{(k^2)^3} \right) \to 2 p_\mu p_\nu \int \frac{k_\mu k_\nu}{(k^2)^2} d^d k \; ; \; k_\mu k_\nu \to \delta_{\mu\nu} \frac{k^2}{d}$$

$$\frac{k_\nu}{(k^2)^2} \text{ term } = 0$$



$$\frac{k^2}{(k^2)^3} \qquad \boxed{\Lambda^2} \qquad \cancel{\Lambda} \qquad \boxed{\log \Lambda}$$

parity $k \to -k$

$\Lambda^2 \; A \cdot A$ term

Lorentz inv

$$\boxed{k_\mu k_\nu \to \delta_{\mu\nu} \frac{k^2}{d}} \Rightarrow \frac{2p^2}{d} \underbrace{\boxed{\int d^dk \, \frac{1}{(k^2)^2}}}_{\text{div. } d=4} = \boxed{\frac{1}{8\pi^2} \cdot \frac{1}{4-d}} \times \frac{p^2}{2} \leftarrow \text{div. at } d=4$$

Feynman Gauge $\xi = 1$ p^2

UV divergences $\rightarrow$ poles $\frac{1}{4-d}$ $\alpha = g^2$

coefficients respect gauge invariance
in the different terms

Instead of 5 independent divergences $\longrightarrow$ only 3 Renor

Fields Renorm.

$A \longrightarrow Z_A A$ $Z_A = 1 + \circ\,\alpha \frac{1}{4-D}$ $\alpha \longrightarrow Z_g\, \alpha$

$c, \bar{c} \rightarrow Z_c\, c, \bar{c}$ $Z_c = 1 + *\,\alpha \frac{1}{4-D}$ $Z_g = 1 + \square \frac{1}{4-D}$

Feynman Gauge $\xi = 1$

UV divergences $\rightarrow$ poles $\frac{1}{4-d}$ $\qquad \alpha = g^2$ = loop expansi

coefficients respect gauge invariance
in the different terms

Instead of 5 independent divergences $\longrightarrow$ only 3

Renormalization
of the coupling constant α

Fields
Renorm.

$$A \longrightarrow Z_A A \qquad Z_A = 1 + \bullet\, \alpha \frac{1}{4-D}$$

$$c, \bar{c} \rightarrow Z_c\, c, \bar{c} \qquad Z_c = 1 + *\, \alpha \frac{1}{4-D}$$

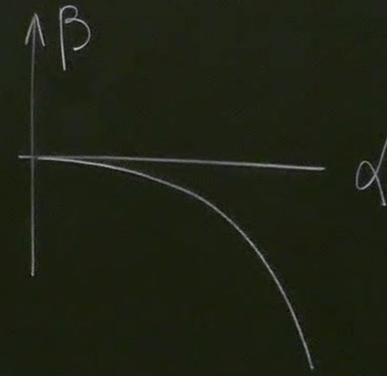
$$\alpha \longrightarrow Z_g\, \alpha$$

$$Z_g = 1 + \square \frac{1}{4-D}$$

ansion

β-function for α gauge coupling $\underline{SU(2)}$

$$\beta(\alpha) = \underline{\alpha}^2 \frac{1}{(4\pi)^2}\left(-\frac{11}{3}\right) \leftarrow \text{negative sign}$$



α

⚠ Gauge fixing

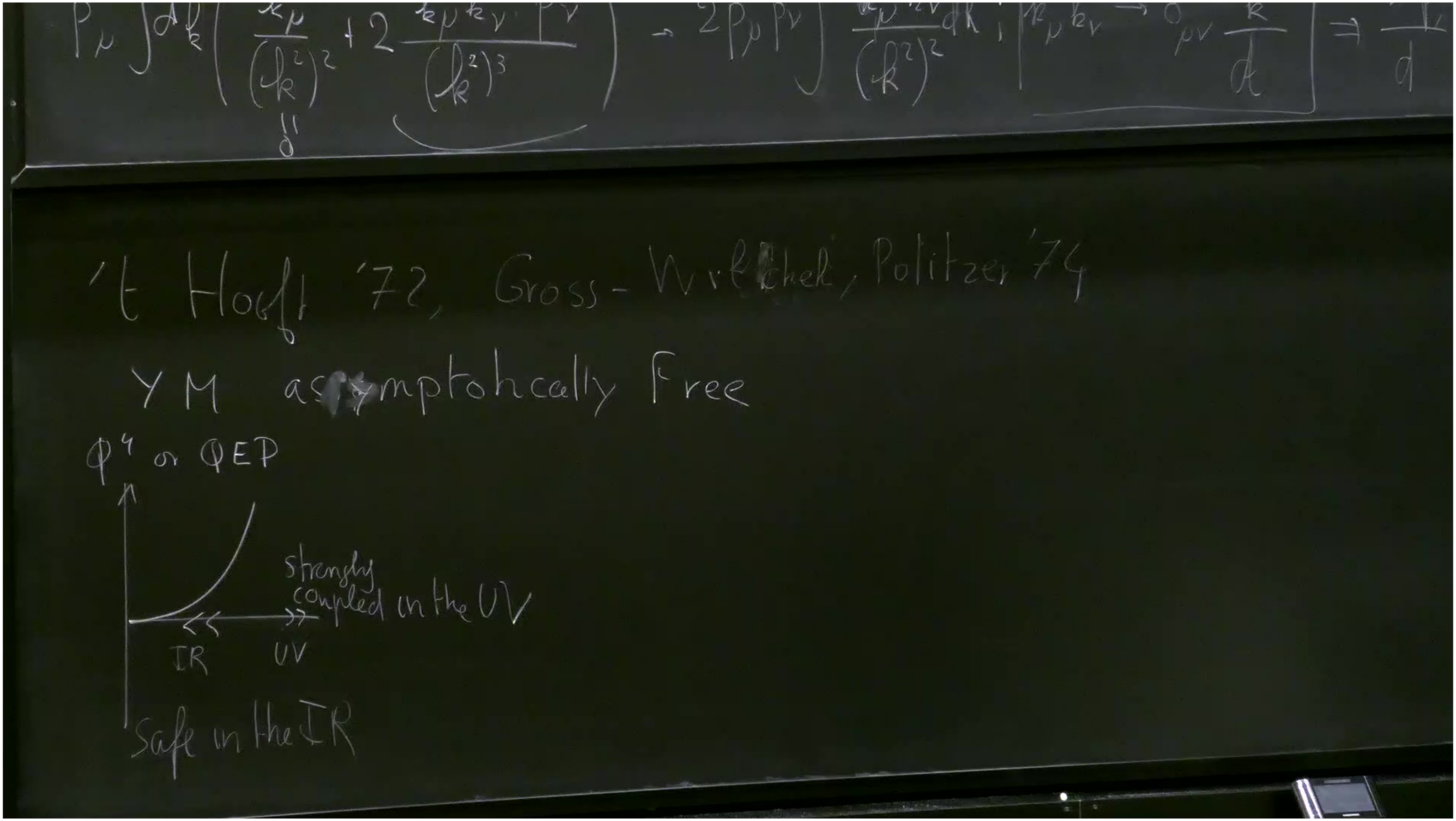
ξ here

Renormalization of ξ ?

Ward Identities

or

BRST Symmetry


't Hooft '72, Gross-Wilczek, Politzer '74
YM asymptotically free
φ⁴ or QED
strongly coupled in the UV
IR
UV
safe in the IR

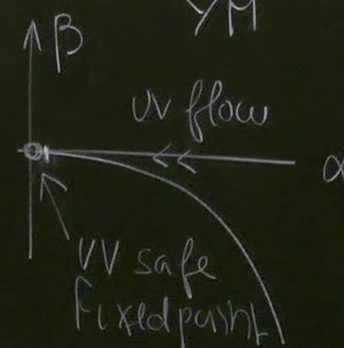
$$\cdots\!\to - \frac{\delta^{ab}}{p^2}\ \bar{C}C, \quad \partial\bar{C}\cdot A\, C$$

= loop expansion

β - function for α gauge coupling $\underline{SU(2)}$

$$\beta(\alpha) = \alpha^2 \frac{1}{(4\pi)^2}\left(-\frac{11}{3}\right) \leftarrow \text{negative sign}$$

YM



...gation
...pling constant α

...g α

$\frac{1}{4-D}$

⚠ Gauge fixing

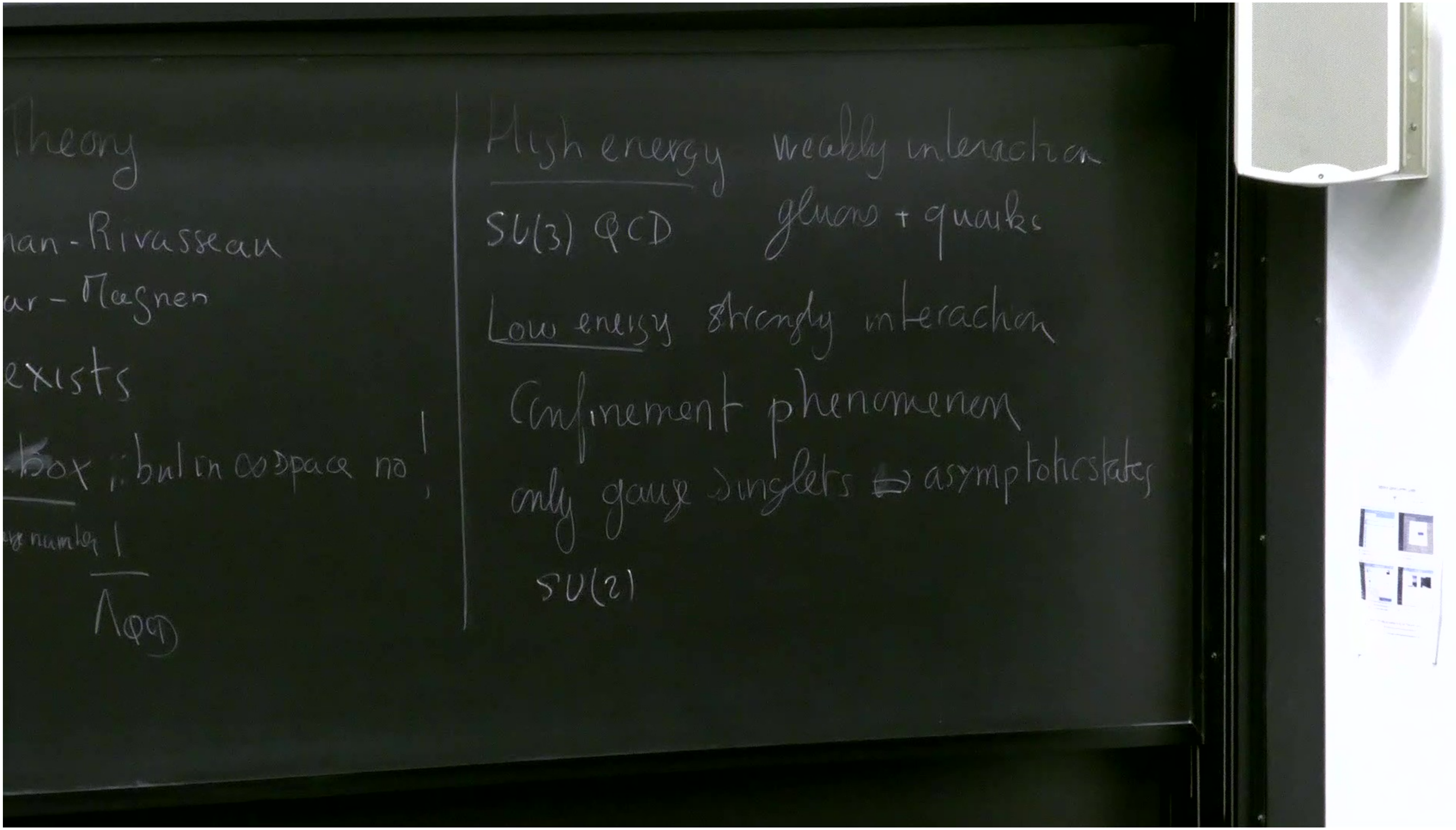
ξ here

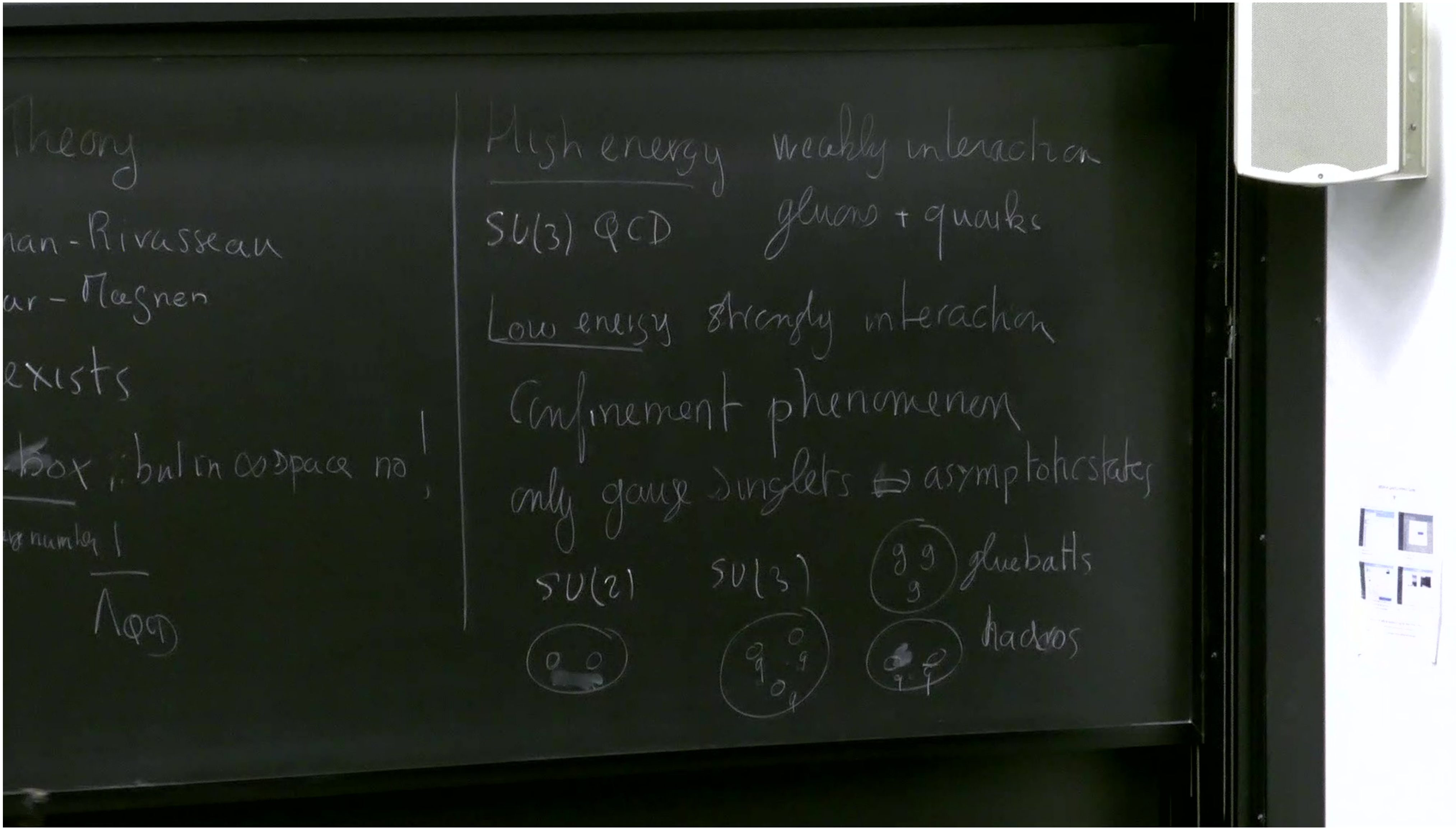
Renormalization of ξ ?

Ward Identities

or

BRST Symmetry


Theory
nan-Rivasseau
ar-Magnen
exists
box ; but in ∞ space no !
number 1
Λ_QCD
High energy weakly interaction
SU(3) QCD gluons + quarks
Low energy strongly interaction
Confinement phenomenon
only gauge singlets ↔ asymptotic states
SU(2)


Theory
nan - Rivasseau
ar - Magnen
exists
box ; but in ∞ space no !
number
Λ_QCD
High energy weakly interaction
SU(3) QCD gluons + quarks
Low energy strongly interaction
Confinement phenomenon
only gauge singlets ⇔ asymptotic states
SU(2)
SU(3)
glueballs
hadrons

Feynman Gauge $\quad \xi = 1 \quad \dashrightarrow -\frac{g}{p^2}$

UV divergences $\to$ poles $\frac{1}{4-d}$ $\qquad \alpha = g^2$ = loop exp

coefficients respect gauge invariance
in the different terms

Instead of 5 independent divergences $\longrightarrow$ only 3

Renormalization
of the coupling constant α

Fields
Renorm.

$$A \longrightarrow Z_A A \qquad Z_A = 1 + \circ\, \alpha \frac{1}{4-D} \qquad \alpha \longrightarrow Z_g \alpha$$

$$c, \bar{c} \to Z_c\, c, \bar{c} \qquad Z_c = 1 + * \alpha \frac{1}{4-D} \qquad Z_g = 1 + \square \frac{1}{4-D}$$