Title: Lecture - QFT II, PHYS 603

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Collection/Series: Quantum Field Theory II (Core), PHYS 603, November 12 - December 11, 2024

Subject: Condensed Matter, Particle Physics, Quantum Fields and Strings

Date: December 04, 2024 - 9:00 AM

URL: https://pirsa.org/24120003

- R G. equation : Scale anomaly ϕ^4_4

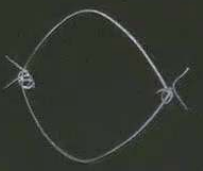
- Perturbative Renormalization $\longleftrightarrow$ Wilsonian Renormalization
  Stat. Phys

g_R $\longleftrightarrow$ renormalization scale choice

effective c.c. $g(E)$ running Beta Function

E ↑ energy

$$\mu \frac{\partial}{\partial \mu} g(\mu) = \beta(g(\mu)) = \frac{3}{(4\pi)^2} g^2 + \dots$$



massless in d dim classical

$$\int d^d x \left[\frac{1}{2}(\partial_\mu \phi)^2 + \frac{g}{4!}\phi^4\right] \qquad x^\mu \to x^\mu + \epsilon^\mu$$

Poincaré / Euclidean $\Rightarrow$ Current $T^\mu{}_\nu = \partial^\mu \phi \partial_\nu \phi - \delta^\mu{}_\nu \left(\frac{1}{2}(\partial\phi)^2 + \frac{g}{4!}\phi^4\right)$

Noether Th. stress-energy tensor

$\partial_\mu T^\mu{}_\nu = 0 \Leftarrow$ e.o.m $\qquad \lambda > 0$

dim of ϕ $\quad [\phi] = \frac{d-2}{2}$ $\quad S_0[\phi]$ $\quad \phi(x) \to \phi_\lambda(x) = \lambda^{\frac{d-2}{2}} \phi(\lambda x)$ $\quad x \to \lambda x$

$S_0[\phi_\lambda]$ $\quad g = 0$ $\quad$ scale invariance $\Rightarrow$ Conformal Invariance

$[g] = 4-d$ $= 0$ only if $d=4$

$S[\phi] = S[\phi_\lambda]$ in 4 dim

$\phi_\lambda(x) \to \lambda^{\frac{d-2}{2}} \phi(\lambda x)$ scale invariance (conformal) classically

$$J^\nu_{\text{scale}}(x) = T^\nu_{\ \nu}(x) x^\nu + \frac{d-2}{2} \phi \partial^\nu \phi$$

$$\partial_\mu J^\mu_{\text{scale}} = T^\mu_{\ \mu} + \frac{d-2}{2} \partial_\mu (\phi \partial^\nu \phi) = \frac{d-2}{2} \phi \Delta\phi - \frac{d}{2} g \phi^4$$

e.o.m $-\Delta\phi + \frac{g}{6}\phi^3 = 0$

$$\partial_\mu J^\mu_{\text{scale}} = (d-4)\frac{g}{4!}\phi^4 = \frac{d-4}{4}\phi\cdot\Delta\phi$$

$\neq 0$ if $d \neq 4$

$= 0$ if $d = 4$

Quantum Theory ?

not scale invariant

Renormalization

$\frac{1}{4}\phi\cdot\Delta\phi$

$T^{\mu}_{\ \nu}$, $\phi\partial^{\nu}\phi$ → quantum operators Composite operators involving ϕ, $\partial_\nu\phi$

$(d-4)\frac{g_R}{4!}\phi^4(x_0)$

$\uparrow$

$-g_R\frac{1}{4!}\phi^4(z)$ g_R

$\int\frac{d^dk}{(2\pi)^d}\frac{1}{(k^2)^2}$ → dimensional regularization

x_0 z →

$(-g)^{3/2}$

x_0 z

$\frac{1}{(4\pi)^2}\frac{2}{4-d}$ ← universal

divergent diagram

$\partial_\mu J^{\mu}_{\text{Scale}} = (d-4)\frac{g_R}{4!}\phi^4\left[1+(-g_R)^{3/2}\frac{1}{(4\pi)^2}\frac{2}{4-d}+\cdots\right]$

$$\mu \frac{\partial}{\partial \mu} g(\mu) = \beta(g(\mu)) = \frac{\quad}{(4\pi)^2} g + \dots$$

$$d \to 4 \qquad \partial_\mu J^\mu_{scale} = \frac{1}{4!} \frac{3}{(4\pi)^2} g_R^2 \, \phi^4 = \boxed{\beta_g(g_R) \cdot \frac{1}{4!} \phi^4 = \partial_\mu J^\mu_{scale}}$$

dim 4 operator, dim 4 operator

Scale anomaly $\neq 0$

Beta Fund = coeff of the scale anomaly

divergence of the current

$$d = 3 \quad [g] = 1 \qquad \partial_\mu J^\mu_{scale} = \left(-g_R + c\, g_R^2 + \dots \right) \frac{1}{4!} \phi^4$$

$\beta_{\phi^4_B}$

WF fixed point

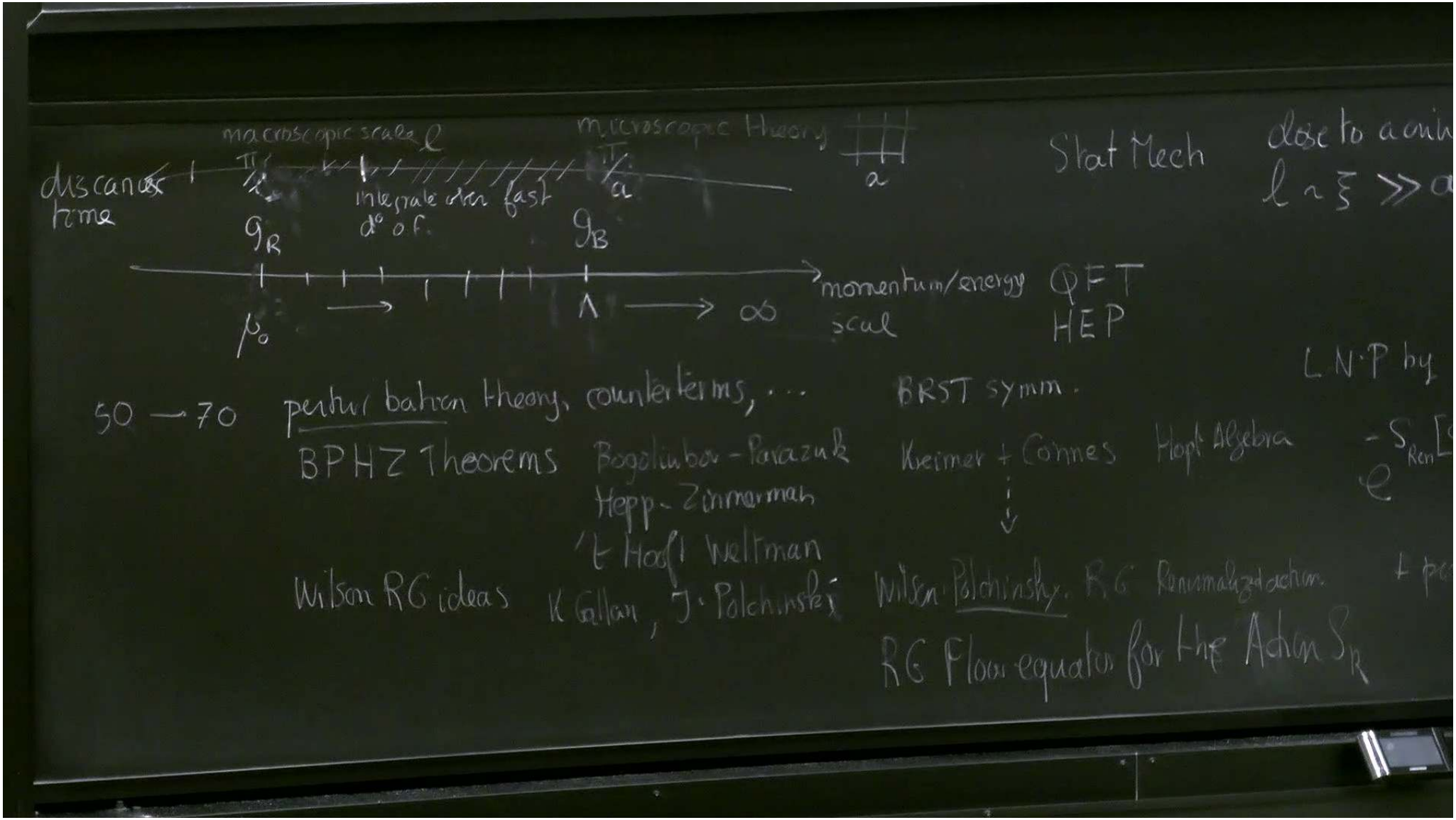
Chose a reference energy/momentum scale μ_0

$\Lambda_{QCD}^{\overline{MS}} \simeq$ [illegible] Mev Energy scale at which $\alpha_{QCD} \simeq 1$
Renormalization scheme $\overline{MS}$

$\Lambda \gg \mu_0$ action $S_R[\phi] = \int (\partial\phi)^2 + \frac{g}{4!}\phi^4$ $g \sim$ bare coupling g_B

g_R renormalize coupling at scale μ_0 $g_R = g_R(g_B, \Lambda) \Leftrightarrow g_B(g_R, \Lambda)$

UV finite thory adjust $g_B = g_B(g_R, \Lambda)$ so that g_R stays fixed


macroscopic scale ℓ
microscopic theory
a
discrete time
integrate over fast d° of f.
g_R
g_B
momentum/energy scal
μ_0
Λ
∞
Stat Mech
close to a crit
$\ell \sim \xi \gg a$
QFT
HEP
50 — 70 perturbation theory, counterterms, ...
BPHZ Theorems Bogoliubov - Parasiuk
Hepp - Zimmermann
't Hooft Weltman
Wilson RG ideas K Gallan, J. Polchinski
BRST symm.
Kreimer + Connes Hopf Algebra
Wilson-Polchinsky. RG Renormalized action.
RG Flow equation for the Action S_R
L.N.P by
$-S_{Ren}[$
e
+ pe

$S_0[\phi_\lambda]$ $g=0$ scale invariance $\Rightarrow$ Conformal Invariance

Chose a reference energy/momentum scale μ_0 Note g

$\Lambda_{QCD}^{\overline{MS}} \sim$ Mev Energy scale at which $\alpha_{QCD} \simeq 1$

Renormalization scheme $\overline{MS}$

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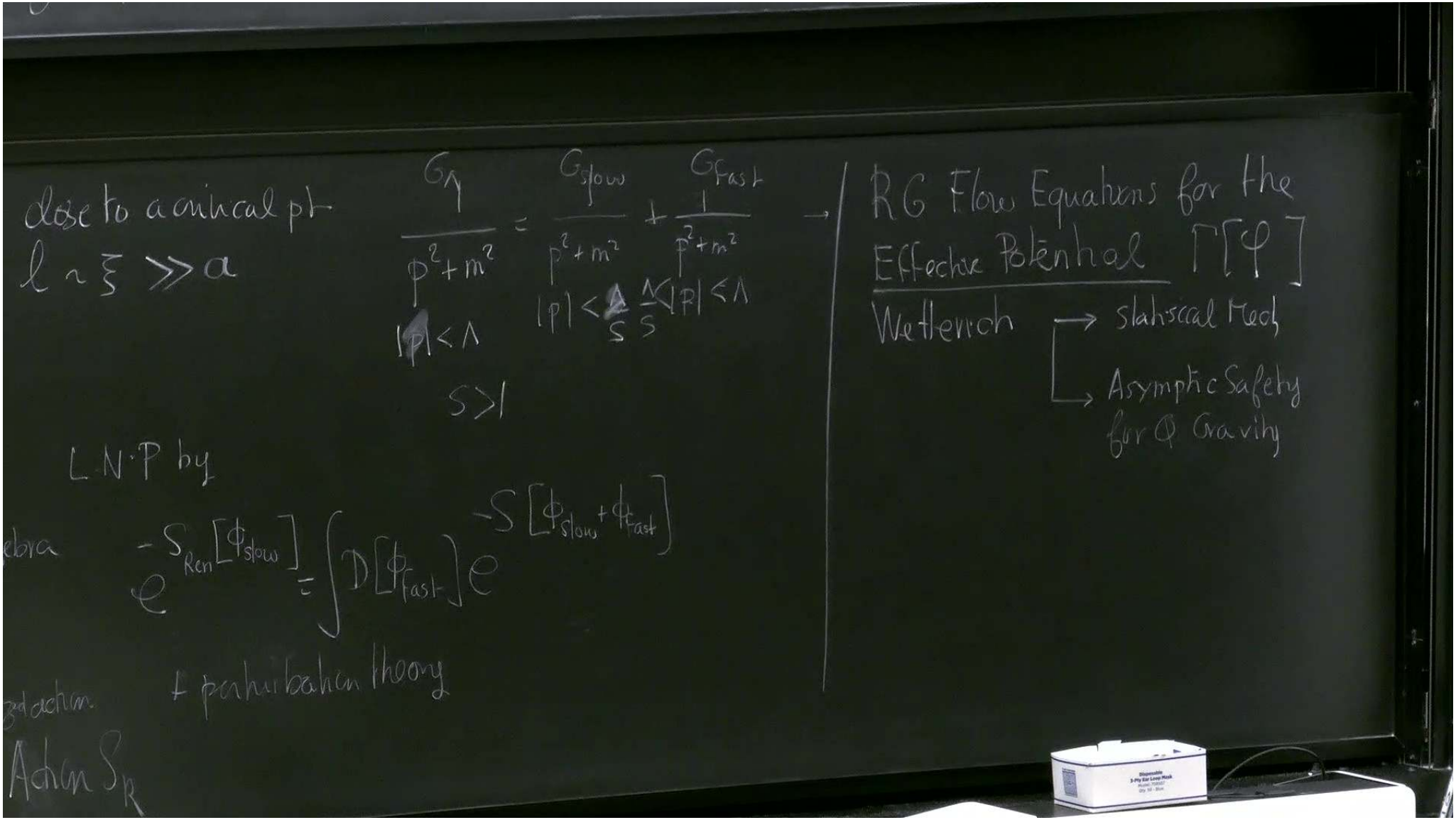
$\uparrow$ introduce Counterterms

erator

g_R

WF fixed point


close to a critical pt
$\xi \sim \xi \gg a$
$\frac{G_\Lambda}{p^2+m^2} = \frac{G_{slow}}{p^2+m^2} + \frac{G_{fast}}{p^2+m^2}$
$|p|<\Lambda$
$|p|<\frac{\Lambda}{s}$
$\frac{\Lambda}{s}<|p|<\Lambda$
$s>1$
RG Flow Equations for the Effective Potential $\Gamma[\varphi]$
Wetterich
Statistical Mech
Asymptic Safety for QG Gravity
L.N.P by
$e^{-S_{Ren}[\phi_{slow}]} = \int D[\phi_{fast}]\, e^{-S[\phi_{slow}+\phi_{fast}]}$
+ perturbation theory
Action S_R

Wilson RG ideas — 't Hooft Veltman — K Gallan, J. Polchinski — Wilson Polchinsky. RG Renormalization + perturbation theory

RG Flow equation for the Action S_R .

$$S_0[\phi]\int_x \frac{1}{2}(\partial_\mu\phi)^2 + V(\phi) \longrightarrow \Gamma[\varphi] = S_0[\varphi] + \frac{1}{2}\,\mathrm{Tr}\left[\mathrm{Log}\left[-\Delta + V''(\varphi)\right]\right] \quad 1\ \text{loop}$$

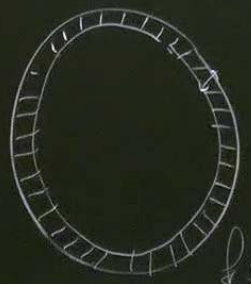
general potential

$\varphi(x) = \varphi$ cst fied

$$\int d^d x\, V(\varphi) = \int d^d x \int d^d k\ \mathrm{Log}\left[k^2 + V''(\varphi)\right]$$

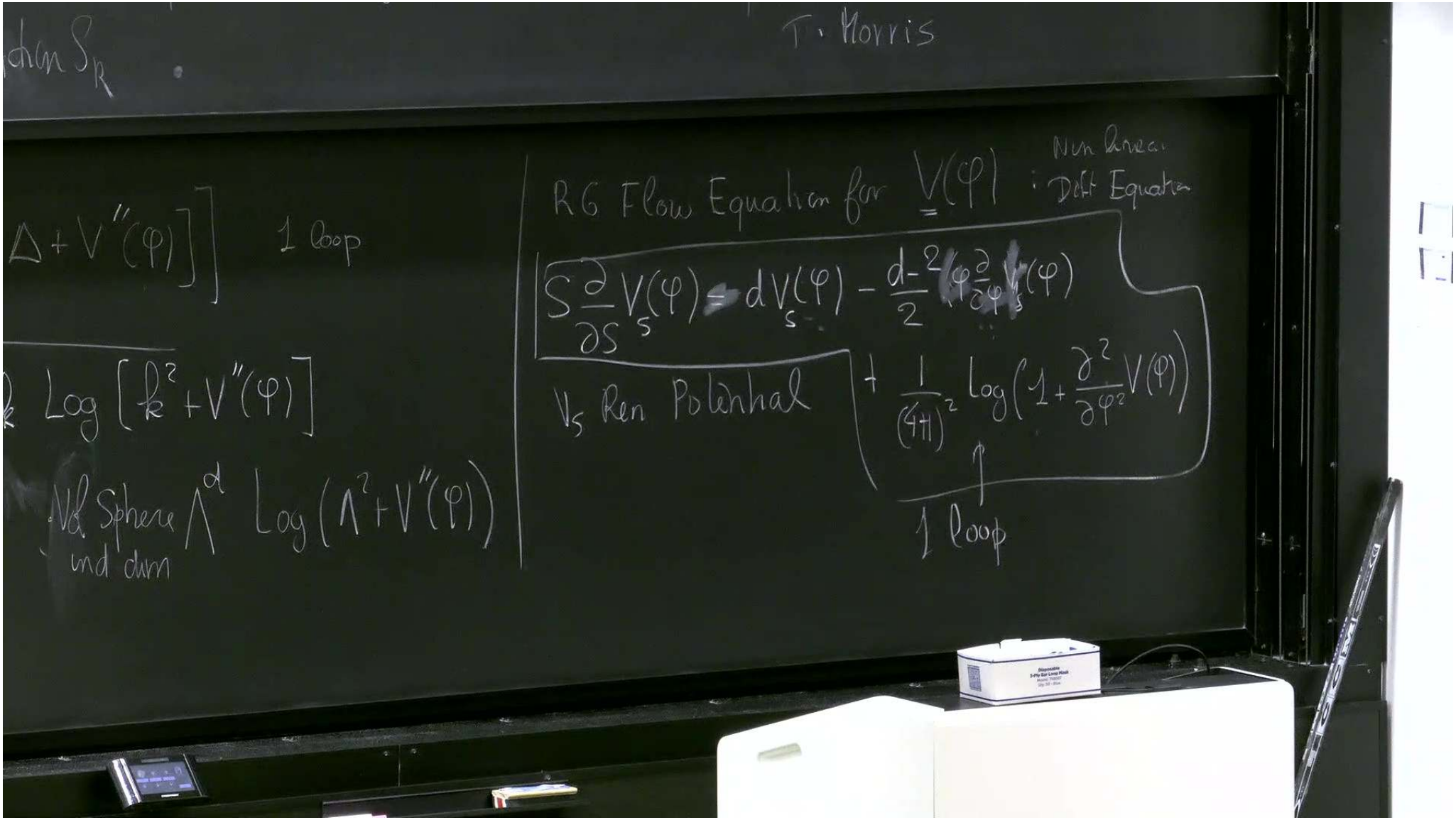
$$\in \text{Vol Sphere in } d \text{ dim}\ \Lambda^d\ \mathrm{Log}\left(\Lambda^2 + V''(\varphi)\right)$$

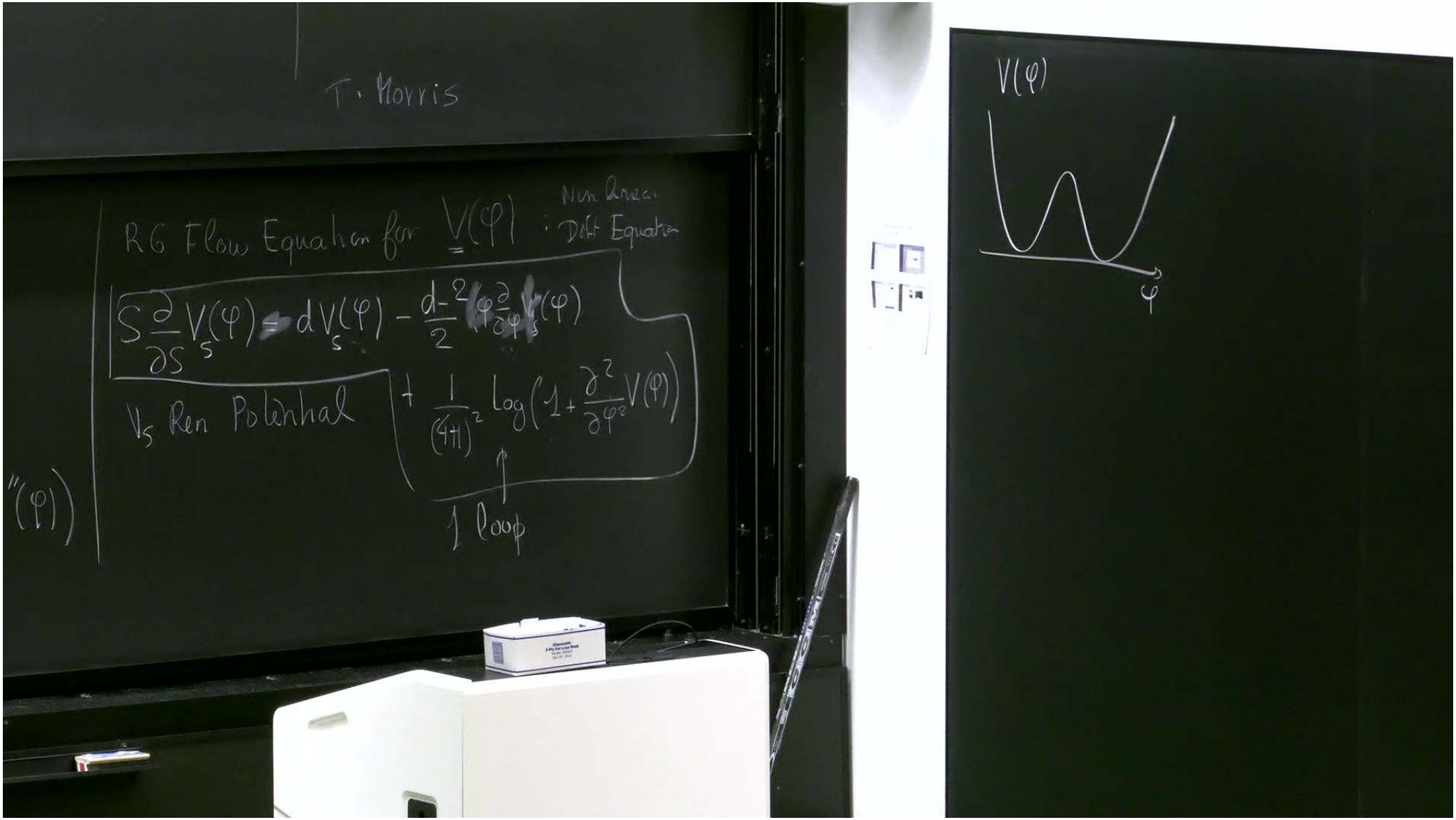
$|k| < \Lambda$



RG transf

$\Lambda(1-\epsilon) < |k| < \Lambda$


T. Morris
RG Flow Equation for $V(\varphi)$: Non linear Diff Equation
$\frac{\partial}{\partial s} V_s(\varphi) = d V_s(\varphi) - \frac{d-2}{2} \varphi \frac{\partial}{\partial \varphi} V_s(\varphi)$
V_s Ren Potential
$+ \frac{1}{(4\pi)^2} \log\left(1 + \frac{\partial^2}{\partial \varphi^2} V(\varphi)\right)$
1 loop
$\Delta + V''(\varphi)$
1 loop
$\log\left[k^2 + V''(\varphi)\right]$
Vol Sphere ind dim $\Lambda^d \log\left(\Lambda^2 + V''(\varphi)\right)$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1 Loop
$V(\varphi)$
φ