Title: Lecture - QFT II, PHYS 603

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Collection/Series: Quantum Field Theory II (Core), PHYS 603, November 12 - December 11, 2024

Subject: Condensed Matter, Particle Physics, Quantum Fields and Strings

Date: December 04, 2024 - 9:00 AM

URL: https://pirsa.org/24120003

- R G. equation; <u>Scale anomaly</u> ϕ^4_4
- Perturbative Renormalization $\longleftrightarrow$ Wilsonian Renormalization

  Stat. Phys

g_R <---> renormalization scale choice

effective c.c. $g(E)$ running <u>Beta</u> Function

energy

$$\mu \frac{\partial}{\partial \mu} g(\mu) = \beta(g(\mu)) = \frac{3}{(4\pi)^2} g^2 + \dots$$

massless
in d. dim
classical

$$\int d^d x \left[\frac{1}{2}(\partial_\mu \phi)^2 + \frac{g}{4!}\phi^4\right]$$

$$x^\mu \to x^\mu + \epsilon^\mu$$

Poincare / Euclidean $\Rightarrow$ Current $\quad T^\mu{}_\nu = \partial^\mu\phi\,\partial_\nu\phi - \delta^\mu_\nu\left(\frac{1}{2}(\partial\phi)^2 + \frac{g}{4!}\phi^4\right)$

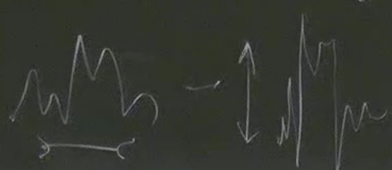
Noether Th. $\quad$ stress-energy tensor

$$\partial_\mu T^\mu{}_\nu = 0 \Leftarrow \text{e.o.m}$$

$\lambda > 0$

dim of ϕ $\quad [\phi] = \frac{d-2}{2}$

$$S_0[\phi] \quad \phi(x) \to \phi_\lambda(x) = \lambda^{\frac{d-2}{2}}\phi(\lambda x) \quad x \to \lambda x$$



$$S_0[\phi_\lambda]$$

$g=0$ $\quad$ scale invariance $\Rightarrow$ Conformal Invariance

$\boxed{[g] = 4-d}$ $= 0$ only if $d = 4$

$S[\phi] = S[\phi_\lambda]$ in 4 dim

$\phi_\lambda(x) \to \lambda^{\frac{d-2}{2}} \phi(\lambda x)$ scale invariance (conformal) classically

$$\boxed{J^\mu_{scale}(x) = T^\mu_{\ \nu}(x)\, x^\nu + \frac{d-2}{2}\phi\, \partial^\nu \phi}$$

$$\partial_\mu J^\mu_{scale} = T^\mu_{\ \mu} + \frac{d-2}{2}\partial_\mu(\phi\, \partial^\nu \phi) = \frac{d-2}{2}\phi\, \Delta\phi - \frac{d}{2} g \phi^4$$

e.o.m $-\Delta\phi + \frac{g}{6}\phi^3 = 0$

$$\boxed{\partial_\mu J^\mu_{scale} = (d-4)\frac{g}{4!}\phi^4 = \frac{d-4}{4}\phi \cdot \Delta\phi}$$

$\neq 0$ if $d \neq 4$

$= 0$ if $d = 4$

Quantum Theory?

not scale invariant

Renormalization

$-4\ \phi\cdot\Delta\phi$

$T^{\mu}{}_{\nu}$, $\phi\partial^{\nu}\phi$ → quantum operators

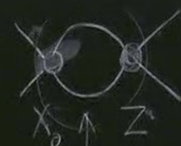
Composite operators involving ϕ, $\partial_\mu\phi$

$$(d-4)\frac{g_R}{4!}\phi^4(x_0)$$

$$-g_R\frac{1}{4!}\phi^4(z)$$

g_R

x_0 z →



$(-g)\,3/2$

divergent diagram

$$\int\frac{d^dk}{(2\pi)^d}\frac{1}{(k^2)^2} \to$$ dimensional regularization

$$\frac{1}{(4\pi)^2}\ \frac{2}{4-d}$$ ← universal

$$\partial_\mu J^{\mu}_{scale} = (d-4)\frac{g_R}{4!}\phi^4\left[1+(-g_R)\frac{3}{2}\frac{1}{(4\pi)^2}\frac{2}{4-d}+\dots\right]$$

$$\mu \frac{\partial}{\partial \mu} g(\mu) = \beta(g(\mu)) = \frac{\cdot}{(4\pi)^2} g + \dots$$

$$d \to 4 \qquad \partial_\mu J^\mu_{scale} = \frac{1}{4!}\frac{3}{(4\pi)^2} g_R^2 \, \phi^4 = \boxed{\beta_g(g_R)\cdot \frac{1}{4!}\phi^4 = \partial_\mu J^\mu_{scale}}$$

Scale anomaly $\neq 0$

dim 4 operator, dim 4 operator

Beta Func = coeff of the scale anomaly
divergence of the current

$\beta_{\phi^4_3}$

WF fixed point

$$d = 3 \quad [g] = 1 \qquad \partial_\mu J^\mu_{Scale} = \left(-g_R + c\, g_R^2 + \dots\right)\frac{1}{4!}\phi^4$$

or

WF fixed point

$\to g_R$

Chose a reference energy/momentum scale μ_0

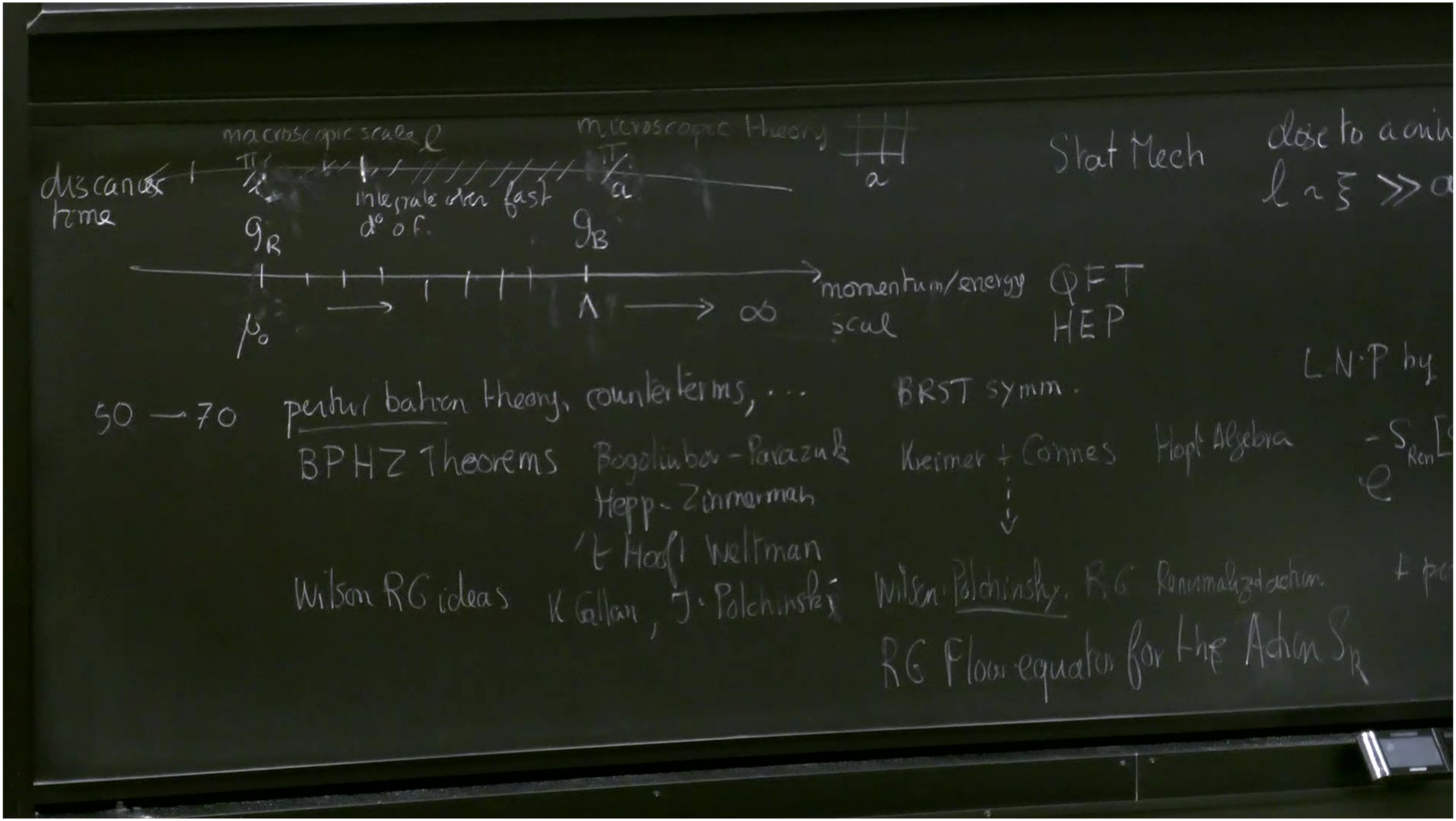
$\Lambda_{QCD}^{\overline{MS}} \sim \ldots$ MeV   Energy scale at which $\alpha_{QCD} \simeq 1$

Renormalization scheme $\overline{MS}$

$\Lambda \gg \mu_0$ action $S_R[\phi] = \int (\partial\phi)^2 + \frac{g}{4!}\phi^4$ $g \sim$ bare coupling g_B

g_R renormalized coupling at scale μ_0 $g_R = g_R(g_B, \Lambda) \Leftrightarrow g_B(g_R, \Lambda)$

UV finite theory   adjust $g_B = g_R(g_R; \Lambda)$ so that g_R stays fixed


macroscopic scale ℓ
microscopic theory
distances
time
integrate over fast d° o f.
g_R
g_B
a
Stat Mech
close to a crit
$\ell \sim \xi \gg a$
momentum/energy scale
μ_0
$\Lambda \longrightarrow \infty$
QFT
HEP
L.N.P by
50 → 70 perturbation theory, counterterms, ...
BRST symm.
BPHZ Theorems
Bogoliubov - Parasiuk
Hepp - Zimmerman
't Hooft Veltman
Kreimer + Connes
Hopf Algebra
$e^{-S_{Ren}}$
Wilson RG ideas
K Gallan, J. Polchinski
Wilson Polchinsky. RG Renormalized action
RG Flow equation for the Action S_R

$S_0[\phi\lambda]$ $g=0$ scale invariance $\Rightarrow$ Conformal Invariance

Chose a reference energy/momentum scale μ_0 ~~Note~~ g

$\Lambda_{QCD}^{\overline{MS}} \sim$ Mev Energy scale at which $\alpha_{QCD} \simeq 1$

Renormalization scheme $\overline{MS}$

...erator

$\Lambda \gg \mu_0$ action $S_R[\phi] = \int (\partial\phi)^2 + \frac{g}{4!}\phi^4$ $g \sim$ bare coupling g_B

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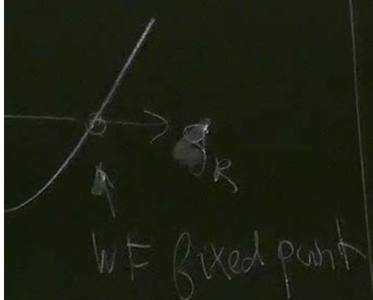
introduce Counterterms



close to a critical pt

$l \sim \xi \gg a$

$$\underbrace{\frac{1}{p^2+m^2}}_{|p|<\Lambda}^{G_\Lambda} = \underbrace{\frac{1}{p^2+m^2}}_{|p|<\frac{\Lambda}{s}}^{G_{slow}} + \underbrace{\frac{1}{p^2+m^2}}_{\frac{\Lambda}{s}<|p|<\Lambda}^{G_{fast}} \rightarrow$$

$s > 1$

L.N.P by

ebra

$$e^{-S_{Ren}[\phi_{slow}]} = \int D[\phi_{fast}]\, e^{-S[\phi_{slow}+\phi_{fast}]}$$

+ perturbation theory

ed action

Action S_R

RG Flow Equations for the Effective Potential $\Gamma[\varphi]$

Wetterich $\rightarrow$ statistical Mech

$\rightarrow$ Asymptotic Safety for Q Gravity 't Hooft Veltman

Wilson RG ideas   K Callan, J. Polchinski   Wilson-Polchinsky. RG Renormalization   + perturbation theory

RG Flow equation for the Action S_R

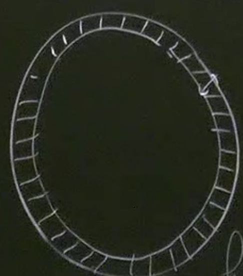
$$S_0[\phi] = \int_x \frac{1}{2}(\partial_\mu \phi)^2 + V(\phi) \longrightarrow \Gamma[\varphi] = S_0[\varphi] + \frac{1}{2}\,\mathrm{Tr}\Big[\mathrm{Log}\big[-\Delta + V''(\varphi)\big]\Big] \quad \text{1 loop}$$

general potential

$\varphi(x) = \varphi$ | cst fixed

$$\int d^dx\, V(\varphi) = \quad \int d^dx \int d^dk\, \mathrm{Log}\left[k^2 + V''(\varphi)\right]$$

$|k| < \Lambda$

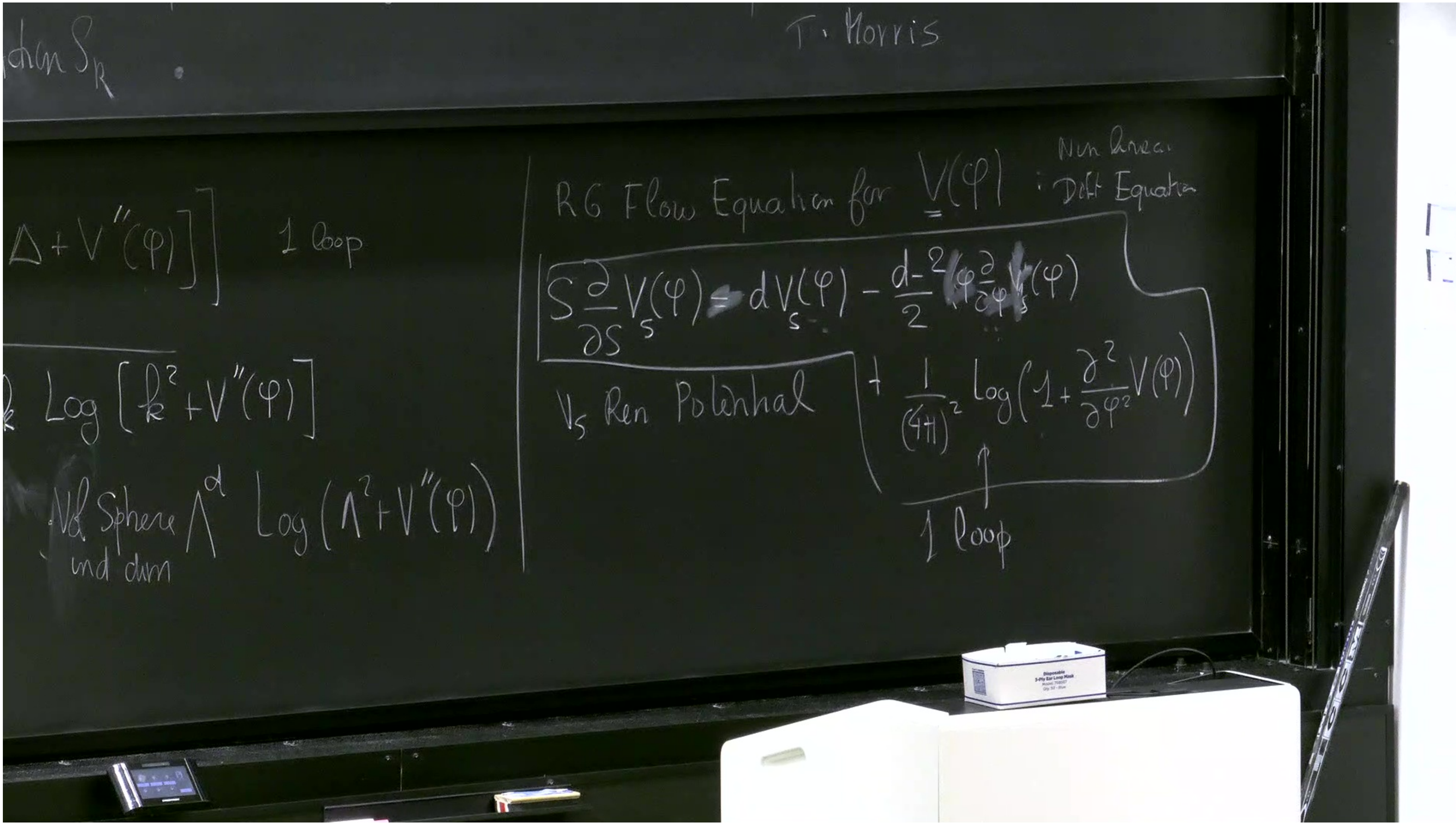


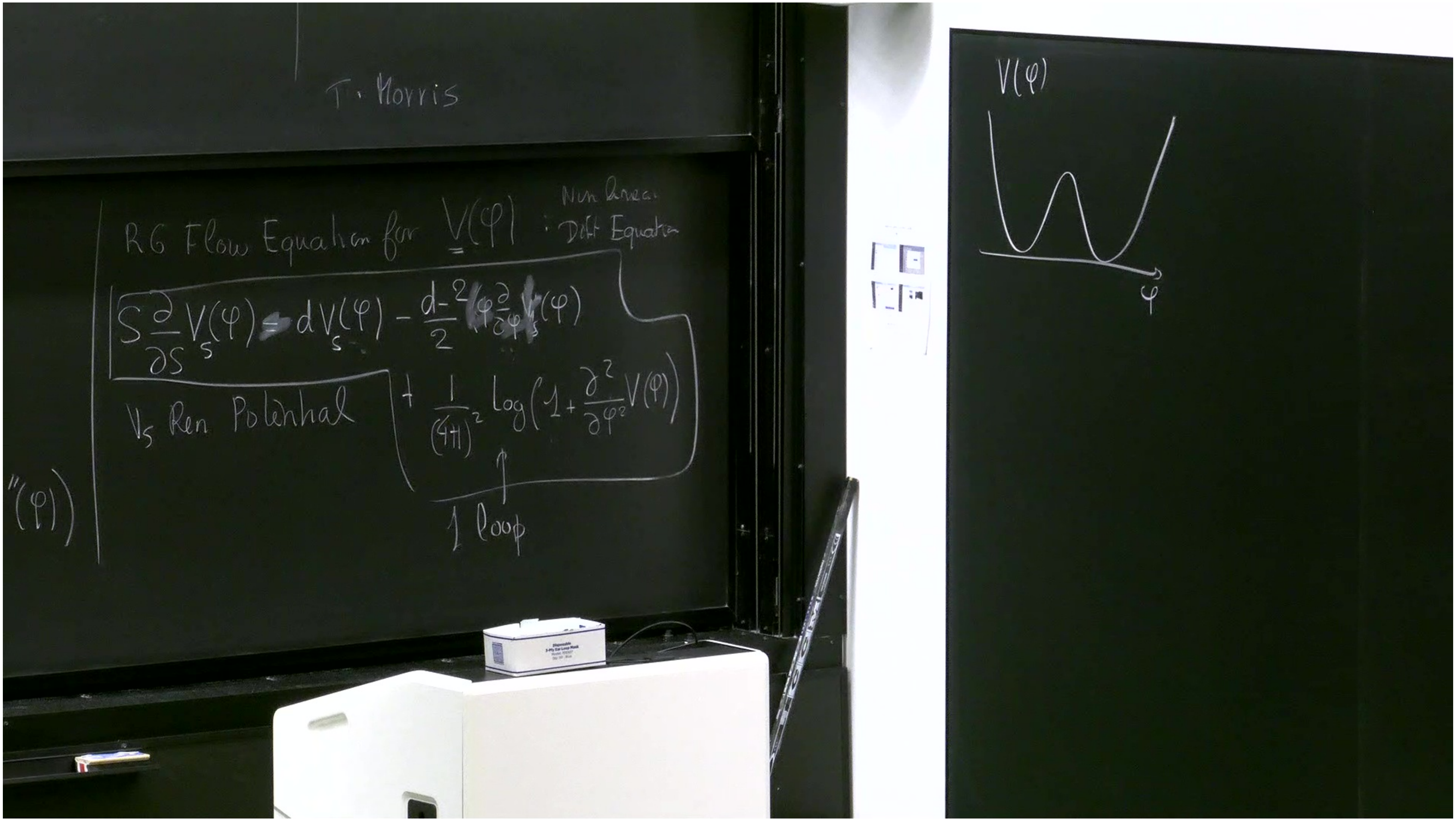
k space

RG transf

$\Lambda(1-\epsilon) < |k| < \Lambda$

$$\epsilon \; \mathrm{Vol\ Sphere}_{\text{unit dim}} \, \Lambda^d \, \mathrm{Log}\left(\Lambda^2 + V''(\varphi)\right)$$


T. Morris
Δ + V''(φ)]] 1 loop
Log [k² + V''(φ)]
Vol Sphere unit dim Λ^d Log(Λ² + V''(φ))
RG Flow Equation for V(φ)
Non linear Diff Equation
S ∂/∂S V_S(φ) = d V_S(φ) − (d−2)/2 φ ∂/∂φ V_S(φ)
+ 1/(4π)² Log(1 + ∂²/∂φ² V(φ))
V_S Ren Potential
1 loop


T. Morris
RG Flow Equation for V(φ)
Non linear
Diff Equation
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V_s Ren Potential
1 loop
V(φ)
φ