Title: Lecture - QFT II, PHYS 603

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Collection/Series: Quantum Field Theory II (Core), PHYS 603, November 12 - December 11, 2024

Subject: Condensed Matter, Particle Physics, Quantum Fields and Strings

Date: December 03, 2024 - 9:00 AM

URL: https://pirsa.org/24120002

ϕ^4 theory massless $d=4$ 1 loop

$|k| < \Lambda$ cut-off in momenta

renormalization scale (momentum) μ

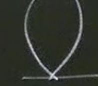
renormalized coupling g_R

$g_R :=$ 1PI or S-matrix $2 \to 2$ at reference momenta $|p| \sim \mu$

1PI - function $\to \Gamma_R$ effective action $\to \langle \phi \cdots \phi \rangle_R$

op

$$\int D[\phi]\, \exp(-S_R[\phi])$$

$\mathcal{B} \leftarrow$  (8.1) Notes

$\mathcal{C} \leftarrow$  (8.2)

$S_R[\phi]$ renormalized action

$$S_R[\phi] = \int d^4x \, \frac{\mathcal{A}}{2}(\partial_\mu\phi)^2 + \frac{\mathcal{B}}{2}\phi^2 + \frac{\mathcal{C}}{4!}\phi^4$$

0

$$\mathcal{A} = 1$$

$$\mathcal{C} = g_R + g_R^2 \frac{3}{2}\frac{1}{(4\pi)^2}\log\left(\frac{\Lambda^2}{\mu^2}\right)$$

UV finite $\mathcal{B} = 0 - g_R \frac{1}{2}\frac{1}{(4\pi)^2}\Lambda^2$

$\Lambda \to \infty$, g_R finite

$S_R[\phi]$)

...zed action

$(\partial_\mu\phi)^2 + \frac{B}{2}\phi^2 + \frac{C}{4!}\phi^4$

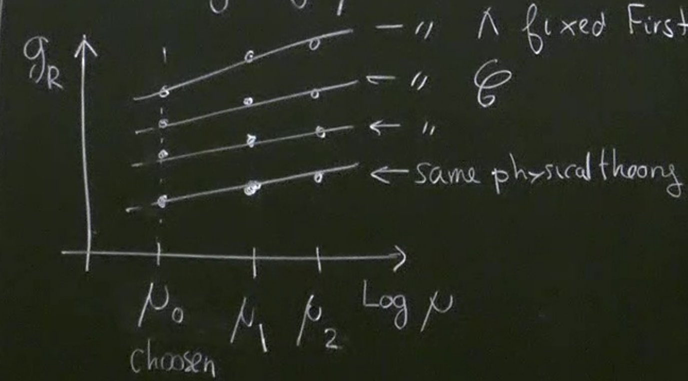
$\frac{1}{(4\pi)^2}\Lambda^2$

$B \leftarrow$ (8.1) Notes

$C \leftarrow$ (8.2)

$$C = g_R + g_R^2 \frac{3}{2}\frac{1}{(4\pi)^2}\log\left(\frac{\Lambda^2}{\mu^2}\right)$$

Meaning of μ ?



$g_R(\mu)$ depends on μ Λ, C fixed

$\Lambda \to \infty$ $g_R(\mu_0)$ Fixed

Coupling constants $g_R(\mu)$ are "running" with the energy/momentum scale

μ_0 vs μ_1

$g_R = g_0 \qquad g_1$

$$g_0 + g_0^2 * \left(\text{Log}\frac{\Lambda^2}{\mu^2}\right) = g_1 + g_1^2 * \text{Log}\frac{\Lambda^2}{\mu^2}$$

$$\boxed{g_1 - g_1^2 \frac{3}{2}\frac{1}{(4\pi)^2}\log\left(\frac{\mu_1}{\mu_0}\right)^2 + \dots = g_0}$$

Better variab

Better to consider variations of μ

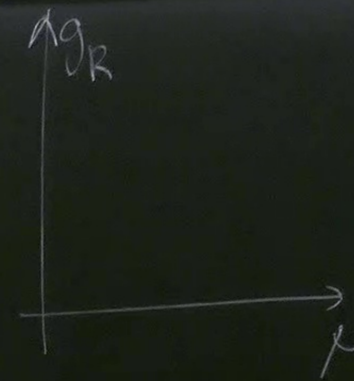
$$\mu_1 = \mu_0 + \delta\mu_0$$

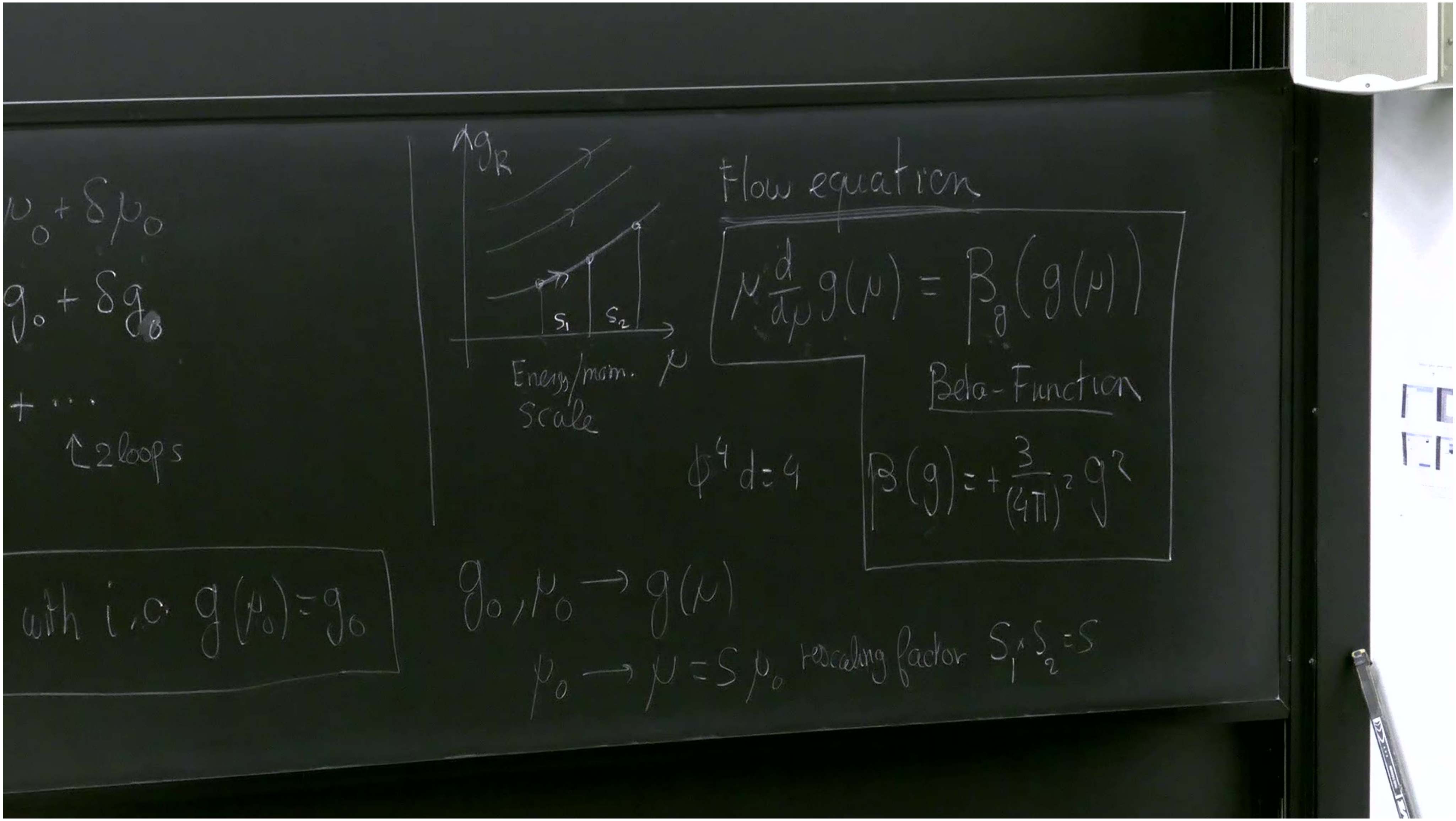
$$g_1 = g_0 + \delta g_0$$

$$\delta g_0 = \frac{\delta\mu}{\mu}\,\frac{3}{(4\pi)^2}\,g_0^2 + \underset{\uparrow\ 2\text{ loops}}{\cdots}$$

$$g_1 = g(\mu_1; g_0, \mu_0)$$

$$\boxed{\mu\frac{d}{d\mu}g(\mu) = \frac{3}{(4\pi)^2}g(\mu)^2 + \dots \quad \text{with i.c. } g(\mu_0) = g_0}$$




$\mu_0 + \delta\mu_0$
$g_0 + \delta g_0$
$+ \cdots$
2 loops
with i.c. $g(\mu_0) = g_0$
g_R
s_1 s_2
μ
Energy/mom. scale
Flow equation
$\mu \frac{d}{d\mu} g(\mu) = \beta_g(g(\mu))$
Beta-Function
ϕ^4 $d=4$
$\beta(g) = + \frac{3}{(4\pi)^2} g^2$
$g_0, \mu_0 \longrightarrow g(\mu)$
$\mu_0 \longrightarrow \mu = s\,\mu_0$ rescaling factor $s_1 \times s_2 = s$

Flow equation generates a (semi)group additive in $\log \mu$

$$\mu_0 \longrightarrow \mu_1 = S_1 \mu_0 \longrightarrow \mu_2 = S_2 \mu_0$$

$$g_0 \longrightarrow g_1 = \mathcal{R}_{S_1}(g_0) \to g_2 = \mathcal{R}_{S_2}(g_1) = \mathcal{R}_{S_2 S_1}(g_0)$$

Scale transformation in the coupling

group in perturbative QFT $\longleftarrow$ Wilson Ren

relevant couplings irrelevant couplings

1PI - function $\to \Gamma_R$ effective action $\to \langle \phi \cdots \phi \rangle_R$ UV finite $B = 0 - g_R \frac{1}{2} \frac{1}{(4\pi)^2} \Lambda$

$\Lambda \to \infty$, g_R finite

3 Flow equation generates a (semi)group additive in $\log \mu$ = Renormalization Group

$\mu_0 \longrightarrow \mu_1 = S_1 \mu_0 \longrightarrow \mu_2 = S_2 \mu_0$

$g_0 \longrightarrow g_1 = R_{S_1}(g_0) \to g_2 = R_{S_2}(g_1) = R_{S_2 S_1}(g_0)$

Scale transformation in the coupling

group in perturbative QFT $\longleftarrow$ Wilson Ren

relevant couplings   irrelevant couplings

QED Gell-Man - Low '54

Stueckelberg - Petermann '53

Callan - Symanzik '70

QCD & Stat. Mech.

'54

...rmann '53

0

$$\beta(g) = g^2 \frac{3}{(4\pi)^2} \qquad \frac{dg(\mu)}{d\log\mu} = \frac{3}{4\pi^3}\, g(\mu)^2$$

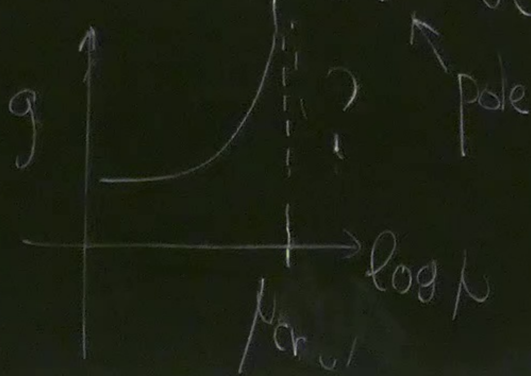
$$\Downarrow$$

start from $\mu_0, g_0, \to g(\mu)$

$$\frac{dg^{-1}}{d\log\mu} = -\frac{3}{4\pi^3}$$

$$g(\mu) = \frac{g_0}{1 - \frac{3}{(4\pi)^3}\, g_0 \log(\mu/\mu_0)}$$

pole



$$\mu_{crit} = \mu_0 \exp\left(\frac{(4\pi)^3}{3}\frac{1}{g_0}\right)$$

$$g(\mu) = \infty$$

Landau Pole $\Rightarrow$ ghost in the spectrum

...ation Group

Gell-Mann - Low '54

Stueckelberg - Petermann '53

Symanzik '70

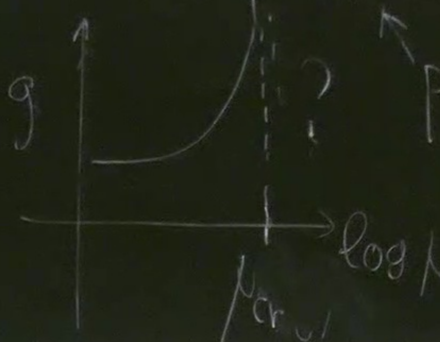
...t. Mech

$$\beta(g) = g^2 \frac{3}{(4\pi)^2}$$

start from $\mu_0, g_0 \to g(\mu)$

$$g(\mu) = \frac{g_0}{1 - \frac{3}{(4\pi)^2} g_0 \log(\mu/\mu_0)}$$

pole



$$\frac{dg(\mu)}{d\log\mu} = \frac{3}{(4\pi)^2} g(\mu)^2$$

$$\Downarrow$$

$$\frac{dg^{-1}}{d\log\mu} = -\frac{3}{(4\pi)^2}$$

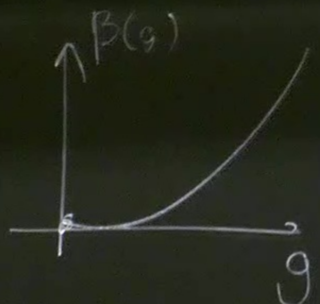
$$\mu_{crit} = \mu_0 \exp\left(\frac{(4\pi)^2}{3}\frac{1}{g_0}\right)$$

$$g(\mu) = \infty$$

Landau Pole $\Rightarrow$ ghost in the spectrum

This Problem

$\beta(g) > 0$



UV $\quad g \nearrow$

IR $\quad g \searrow 0$

$$\mu \searrow 0 \quad g(\mu) \simeq \frac{(4\pi)^2}{3} \frac{1}{\log\left(\frac{1}{\mu}\right)}$$

ϕ^4 weakly coupled IR

(too) strongly coupled UV

Consistency of ϕ^4 theory in HEP

But this occurs at energies

$$E_{Landau} \simeq \exp\left(\frac{\circ}{g}\right) = \infty \text{ large}$$

in perturbation theory

non perturbative

mptotic "slavery"

energy: $\uparrow$

QCD dichotomy

Same in QED $\leftarrow$ up to $\pm 10^{-15}$

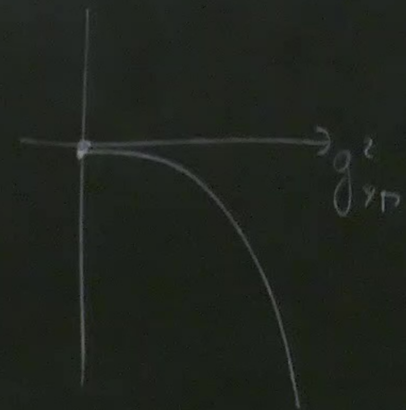
not consistent in the UV

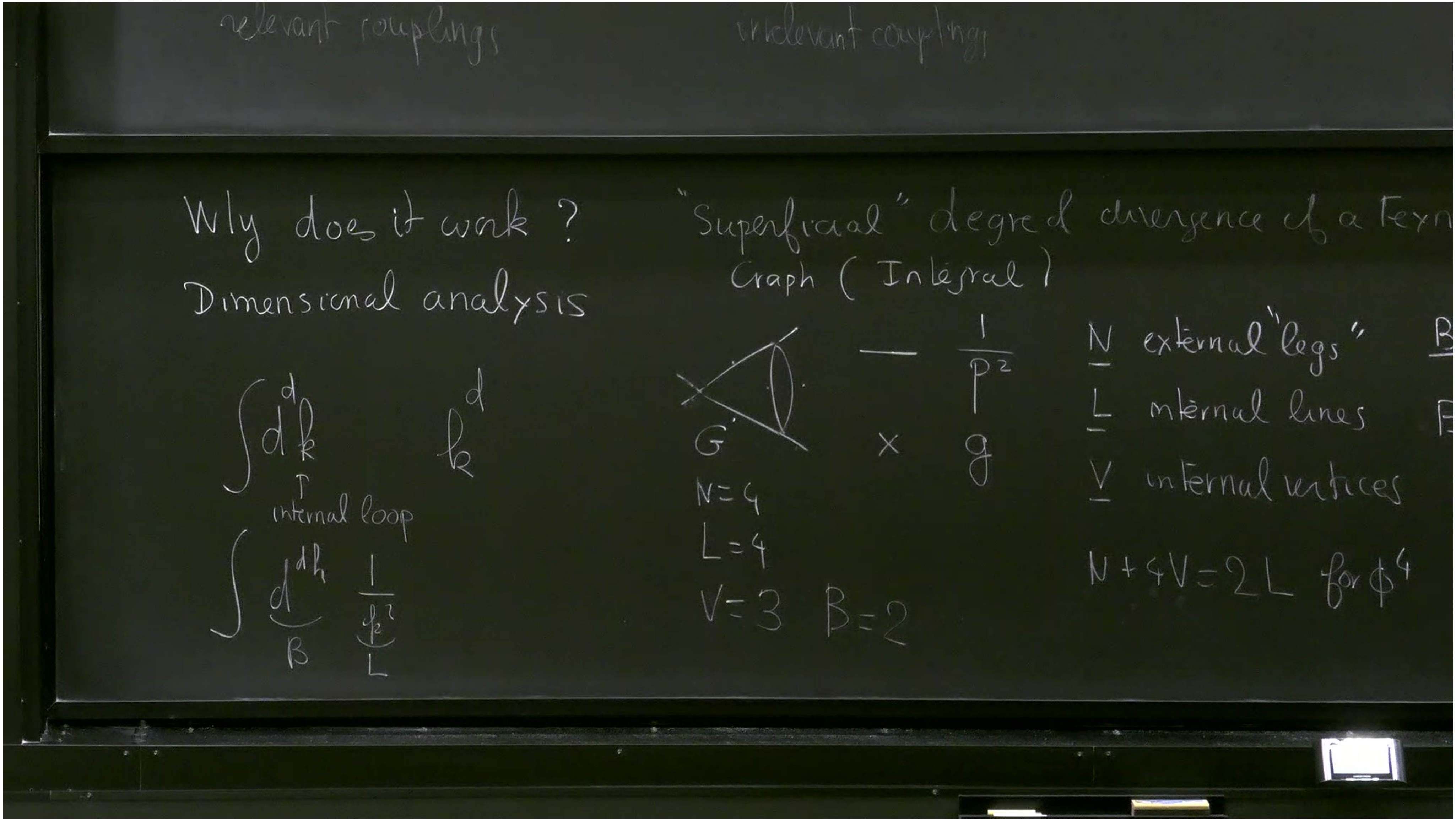
$\exp(137) \ggg 1$

Asymptotically "free"

QCD - YM

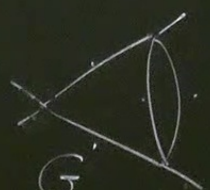
$$\beta(g^2_{YM}) = -C(g^2_{YM})^2$$




relevant couplings
irrelevant couplings
Why does it work ?
Dimensional analysis
"Superficial" degree of divergence of a Feyn
Graph (Integral)
N external "legs"
L internal lines
V internal vertices
N + 4V = 2 L for ϕ^4
N = 4
L = 4
V = 3 B = 2
internal loop

irrelevant couplings

"Superficial" degree of divergence of a Feynman Graph (Integral) $\omega(G)$



G

$—— \quad \frac{1}{p^2}$

$\times \quad g$

$N=4$

$L=4$

$V=3 \quad B=2$

$\underline{N}$ external "legs"

$\underline{L}$ internal lines

$\underline{V}$ internal vertices

$N + 4V = 2L$ for ϕ^4

$\underline{B}$ internal loops

$B = 1 + L - V$ Euler

↑connected graph

$B = 1 + L/2 - N/4$

$\omega(G)$

...ternal loops

$1 + L - V$ Euler

↑ connected graph

$= 1 + L/2 - N/4$

$$\omega(G) = dB - 2L$$

$\omega(G) < 0$ superficially convergent $O(1)$

$\omega(G) = 0$ marginally divergent $(\mathrm{Log}\,\Lambda)$

$\omega(G) > 0$ superficially divergent $\to \Lambda^{\omega(G)}$

$$\boxed{\omega(G) = (d-4)B + (4-N) \text{ external lines}}$$

dimension of space

$d = 4 \quad N=0,\ N=2 \quad \omega = 2 \quad \phi^2 \quad \Lambda^2 + \log\Lambda^* \cdot M^2 \ CT + (\partial\phi)^2 \cdot \log\Lambda^* \cdot CT$

$N = 4 \quad \omega = 0 \quad \phi^4 \quad \log\Lambda^* \cdot CT$ Renormalizable

$d > 4$ Any $N > 4$, B large enough $\downarrow$ divergences $\quad \phi^6, (\partial^2\phi)^2, \phi^8, \dots$ no closure Non-renormalizable

dimensions in $[k]$ or $[E]$ $\quad [M^2] = 2$

$S[\phi] \quad [S] = 0 \quad [\phi] = \frac{d-2}{2}$ engineering dim $\quad [g] = d - 4\frac{d-2}{2} = \boxed{4 - d = [g]}$ dim of coupling

$d^d x \, \partial_\mu \cdot \partial_\nu$