Title: Lecture - QFT II, PHYS 603

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Collection/Series: Quantum Field Theory II (Core), PHYS 603, November 12 - December 11, 2024

Subject: Condensed Matter, Particle Physics, Quantum Fields and Strings

Date: December 02, 2024 - 9:00 AM

URL: https://pirsa.org/24120001

Renormalization

perturbation theory 1 Loop

massless ϕ^4 theory

UV singularities QFT

& divergences $\longrightarrow$ ∞ ties

Propagator $G(k) = \overline{} = \dfrac{1}{p^2+m^2} \simeq |x-y|^{2-d}$

$d \geq 2$

$|p| \nearrow \infty$ or $|x-y| \searrow 0$

$d=1 \rightarrow$ Random walk

$d \geq 2 \rightarrow$ Distribution

wild configurations

Functional Integral Quantization $\Rightarrow$ typical $\phi(x)$

quantum random variable

S

theory 1 Loop

theory

$d=1 \rightarrow$ Random walk

$2-d$ $\quad d \geq 2$ — Distribution

1

wild configurations

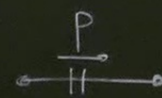
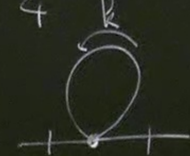
2

$\phi(x)$

random variable

$$S[\phi] = \int d^dx \; \frac{1}{2}(\partial_\mu \phi)^2 + \frac{M^2}{2}\phi^2 + \frac{g}{4!}\phi^4$$

Euclidean $(++++)$ $\quad d = 4$

$\Gamma^{(2)}(p)$  $+\, g\,\frac{1}{2}$ 

"Tadpole"

$p^2 + M^2$

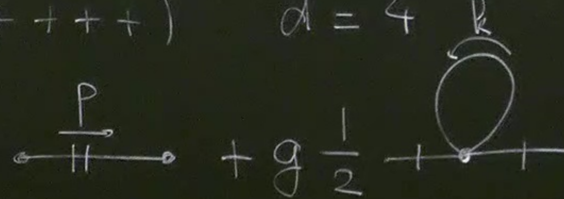
$$\langle \phi^2(0) \rangle_0 = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2+M^2} = T(p,M)$$

$|k| \nearrow \infty$

$$S[\phi] = \int d^dx \; \frac{1}{2}(\partial_\mu \phi)^2 + \frac{M^2}{2}\phi^2 + \frac{g}{4!}\phi^4$$

Euclidean $(++++)$ $\quad d = 4$

$\Gamma^{(2)}(p)$



"Tadpole"

$p^2 + M^2$

$$\langle \phi^2(0) \rangle_0 = \int_{|k|<\Lambda} \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + M^2} = T(p, M)$$

$|k| \nearrow \infty$

$$T(p, m) = \Lambda^2 + \cdots$$

Regularize the th

- cut-off (Regul
- Lattice  a
- dimensional reg

$d = 4 \longrightarrow d$

poles $\frac{1}{d-4}$

$p^2 = \sum_\mu 4 \sin^2\left(\frac{k_\mu}{\dots}\right.$

$\ldots^2 + \frac{g}{4!}\phi^4$

$= \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2+M^2} = T(p,M)$

Regularize the theory Sharp ①

- cut-off (Regulator) $\boxed{|k| < \Lambda}$

momentum regulator

- Lattice  $|x| > a$

$a \ll \frac{1}{M_{phys}}$

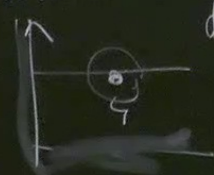
$a \to \infty$

$\Lambda \gg M_{physical}$

$\Lambda \to \infty$?

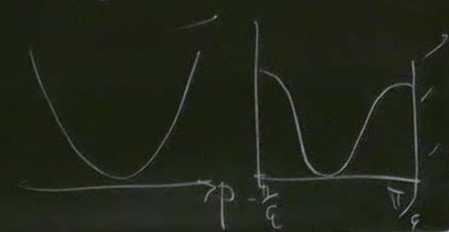
- dimensional regularization

$d = 4 \longrightarrow d$ complex



poles $\frac{1}{d-4}$

$p^2 = \sum_\mu 4\sin^2\left(\frac{k_\mu a}{2}\right)$



② Massless theory $\boxed{1\,\text{loop}}$ $\boxed{M_{phys}=0}$ 2 loop

$-\!\Sigma\!-$ $\quad p^2 + \underbrace{\left(M^2 + \frac{g}{2}T(\Lambda, M)\right)}_{M^2_{phys}} + O(g^2)$ terms

pole $\frac{1}{p^2+M^2_{phys}}$ in the K.L representation

$E^2 = \vec{k}^2 + M^2_{phys}$ $\quad M =$ "bare mass"

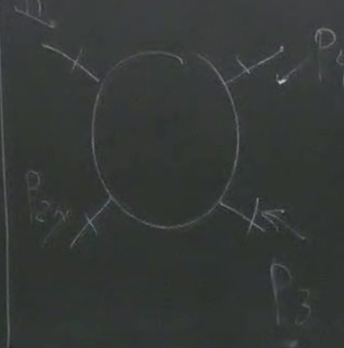
Mass Renormalization fine tune

$M_{phys} = 0 \Leftarrow M^2 = -\frac{g}{2}\left(\frac{1}{4\pi}\Lambda^2 + \dots\right)$

$M_{phys} = 0 \Leftrightarrow$ forget the

$\Gamma^{(2)}(p) = p^2 + 0 + 0$

$\Gamma^{(4)}$ 4 pt function



$= (2\pi)^4\delta(p_1 +$

$\Gamma = -W + J\phi$

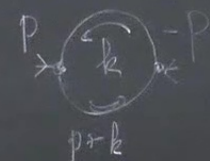
oop

terms

$M_{phys} = 0 \Leftrightarrow$ forget the

$\Gamma^{(2)}(p) = p^2 + 0 + O(g^2)$

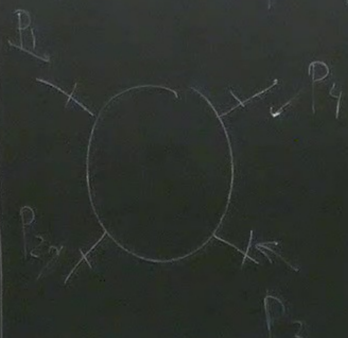
$\Gamma^{(4)}$ 4 pt function

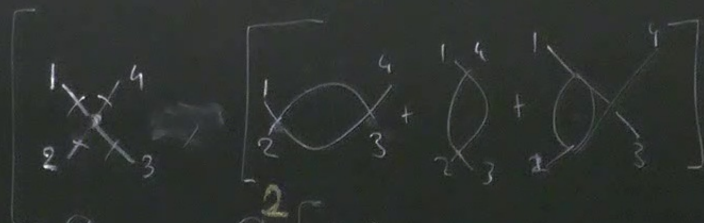
$B(p) =$  $= \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \frac{1}{(p+k)^2}$

"bubble" diagram
Feynman Integral
massless theory

cut off Λ if $|p| \ll \Lambda$

$B(p) = \int_{|k|<\Lambda} \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2)^2} + \dots$



$= (2\pi)^4 \delta(p_1 + \dots + p_4)$  $+ \dots$

$+g - \frac{g^2}{2}\left[B(p_1+p_2) + B(p_1+p_4) + B(p_1+p_3)\right]$

$\Gamma = -W + J\phi$

$B(p) = \dots$

the

$O + O(g^2)$

$B(p) = $ "bubble" diagram $= \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \frac{1}{(p+k)^2}$

Log singularity $|k| \nearrow \infty$

UV-divergence

"bubble" diagram

Feynman Integral

massless theory

cut off Λ if $|p| \ll \Lambda$ $|p| \ll k \sim \Lambda$

integral

$$B(p) = \int_{|k|<\Lambda} \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2)^2} + \dots \simeq \frac{1}{(4\pi)^2} \int_{|p|^2<|k|^2<\Lambda^2} \frac{d(k^2)}{|k|^2} \simeq \frac{1}{(4\pi)^2} \log\left(\frac{\Lambda^2}{p^2}\right) + \dots$$

$|p|$ Λ

$$\boxed{B(p) = \frac{1}{(4\pi)^2} \log\left(\frac{\Lambda^2}{p^2}\right) + cst}$$

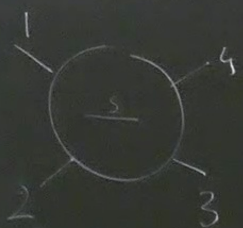
depends on regularization

$\pi)^4 \delta(p_1 + \dots + p_4) \Big[\quad + \dots \Big)$

$+g - \frac{g^2}{2} \left[B(p_1+p_2) + B(p_1+p_4) + B(p_1+p_3) \right]$

$+ J\phi$

$$\Gamma^{(4)}(p_1,\dots,p_4) = g - g^2 \frac{1}{2}\frac{1}{(4\pi)^2}\left[\log\frac{\Lambda^2}{(p_1+p_2)^2} + \text{''} + \text{''}\right] + \dots$$



"bare mass"

"physical coupling": g_{phys} but at which momentum/energy scale?

need to choose a renormalization scale

renormalized coupling

Summary: Logic of renormalization theory

$S_R[\phi] =$

choose a ren. scale μ,

renormalized coupling g_R

UV finite

consistent QFT

$$\langle \phi \cdots \phi \rangle_R \Leftarrow \lim_{\Lambda \to \infty} \int D[\phi] \exp\left(- S_R[\phi]\right)$$

finite as an expansion in g_R (1 loop)

Renormalized action

theory | $S_R[\phi] = \int d^4x \, \frac{A}{2}(\partial^\mu \phi)^2 + \frac{B}{2}\phi^2 + \frac{C}{4!}\phi^4$ Massless

A, B, C are functions of g_R, μ, Λ | why

$A = 1$ — 1 loop counterterms | R.G

$\exp\left(-S_R[\phi]\right)$

Renormalized action

$B = -g_R \frac{1}{2}\frac{1}{(4\pi)^2}\Lambda^2$

$C = g_R + g_R \frac{3}{2}\frac{1}{(4\pi)^2}\log\frac{\Lambda^2}{\mu^2}$

$(\partial\phi)^2 + \frac{B}{2}\phi^2 + \frac{\mathcal{E}_1}{4!}\phi^4$

Massless. Ren. ϕ^4

functions of g_R, μ, Λ

1 loop counterterms

$\frac{1}{(4\pi)^2}\Lambda^2$

$\frac{3}{2}\frac{1}{(4\pi)^2}\log\frac{\Lambda^2}{\mu^2}$

why μ is not a new parameter

$\downarrow$

R.G equations & β functions

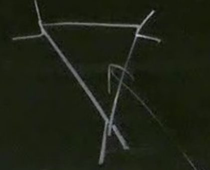
$\Gamma^{(4)}_R(\{p_i\}; g_R)$ is UV finite at $\underline{1 \text{ loop}}$

Ren. 4 pt function $\boxed{\text{Sect } 8.2}$

$g - g^2 c \left(\text{Log} \frac{\Lambda^2}{(p_1+p_2)^2} + '' + '' \right) + \dots$

$g = g_R + g_R^2\, c\, 3 \,\text{Log}\left(\frac{\Lambda^2}{\mu^2}\right) + \dots$

No need of renormalization at 1 loop for $N \geq 6$ Pt functions



g_R^3 just replace $g \to g_R$

UV finite integral

$\Lambda = M_{Planck}$ or M_{GUT}

μ = Mev, kev, ev,

$\mu \ll \Lambda$ $\quad g_R$ small but finite

$$g - g^2 C\left(\log\frac{\Lambda^2}{(p_1+p_2)^2} + '' + ''\right) + '' = g_R + g_R^2\left[3C\log\left(\frac{\Lambda^2}{\mu^2}\right) - C\left[\log\frac{\Lambda^2}{(p_1+p_2)^2} + '' + ''\right]\right] + \cdots$$

$$g = g_R + g_R^2\, C\, 3\log\left(\frac{\Lambda^2}{\mu^2}\right) + \cdots$$

$\Lambda = M_{Planck}$ or M_{GUT}

μ = MeV, keV, eV,

$\mu \ll \Lambda$ g_R small but finite