

Title: How Lensed Gravitational Waves Can Illuminate Dark Matter

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How Lensed Gravitational Waves Can Illuminate Dark Matter*

Mesut Çalışkan^{1,α},

Neha Anil Kumar, Lingyuan Ji, Marc Kamionkowski, Emanuele Berti et al.

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Perimeter Institute – 25 Nov 2024

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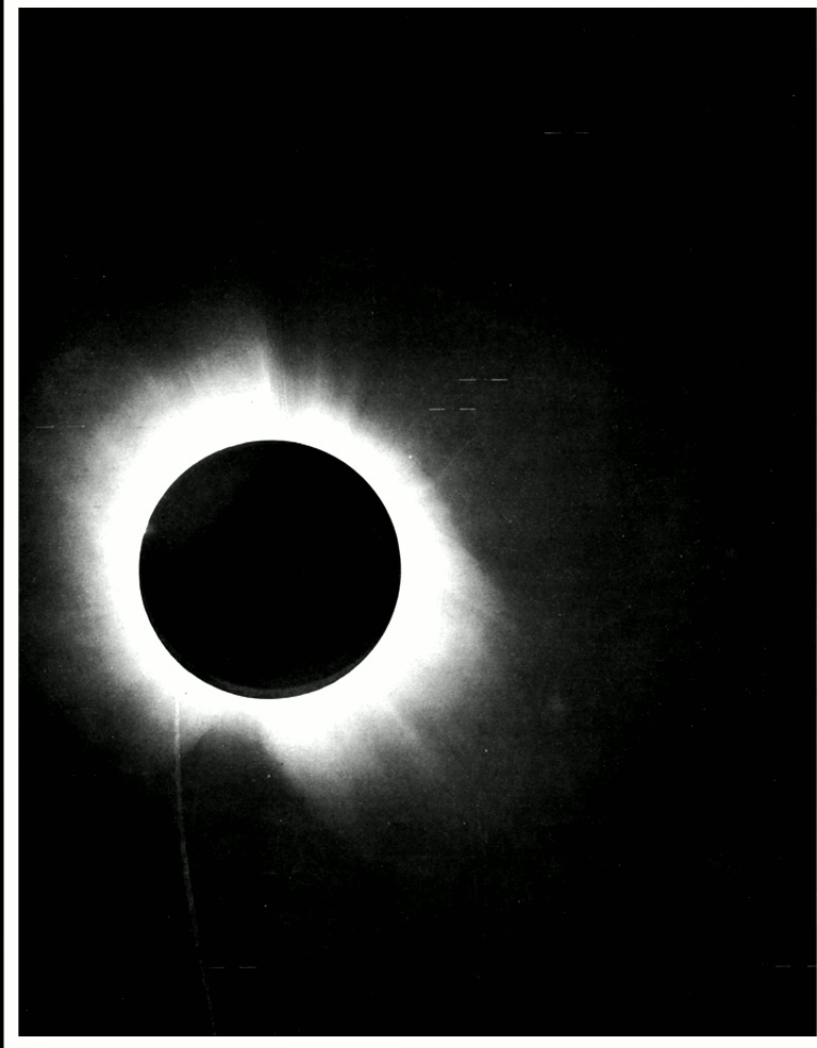
*[arXiv: 2201.04619](https://arxiv.org/abs/2201.04619), [2206.02803](https://arxiv.org/abs/2206.02803), [2307.06990](https://arxiv.org/abs/2307.06990)

Dark Matter

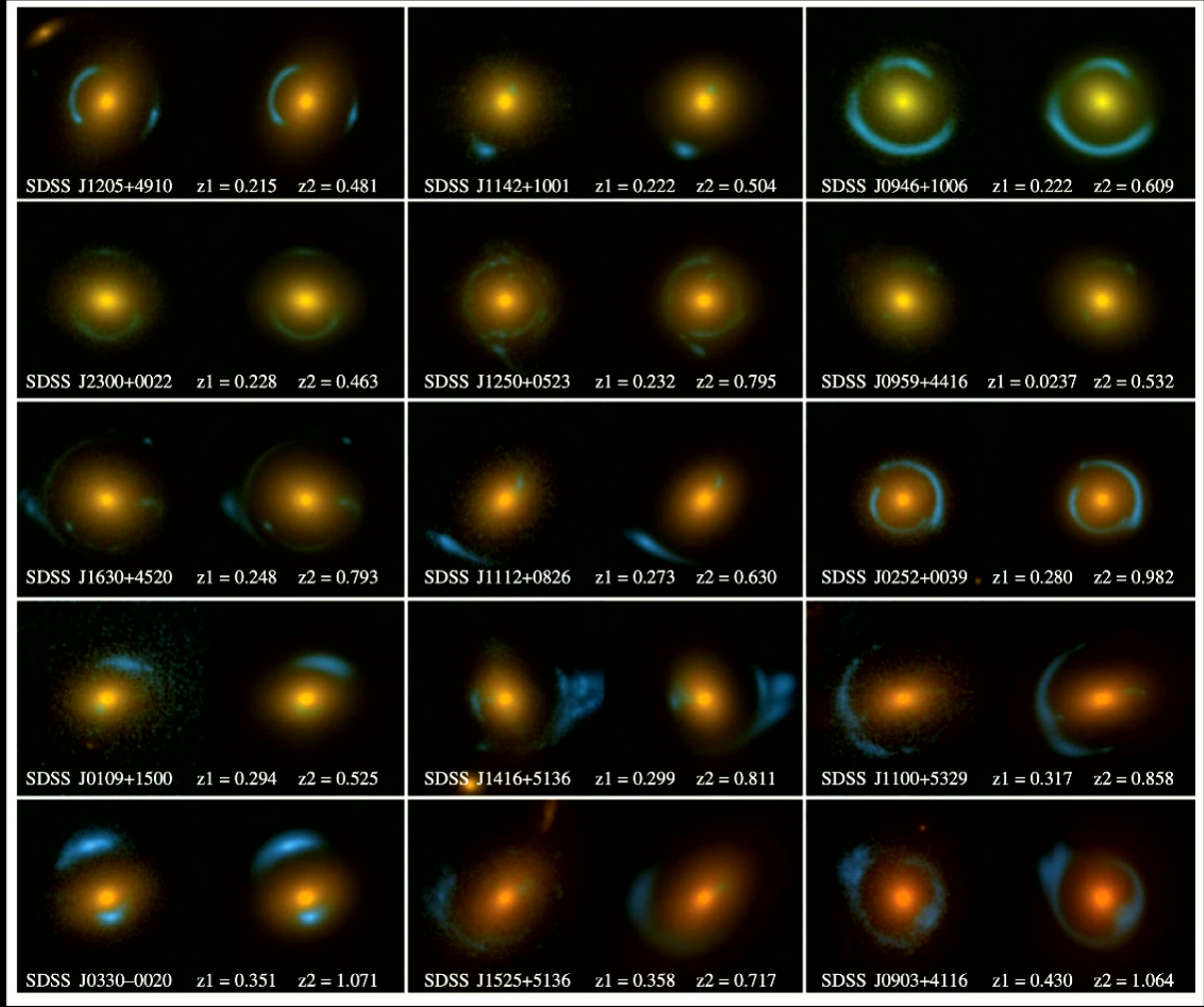
Gravitational Waves

“Do not Bodies act upon Light at a distance, and
by their action bend its rays; and is not this action
strongest at the least distance?”

Isaac Newton – *Opticks* (1704)



F. W. Dyson, A. S. Eddington, and C. Davidson (1920)



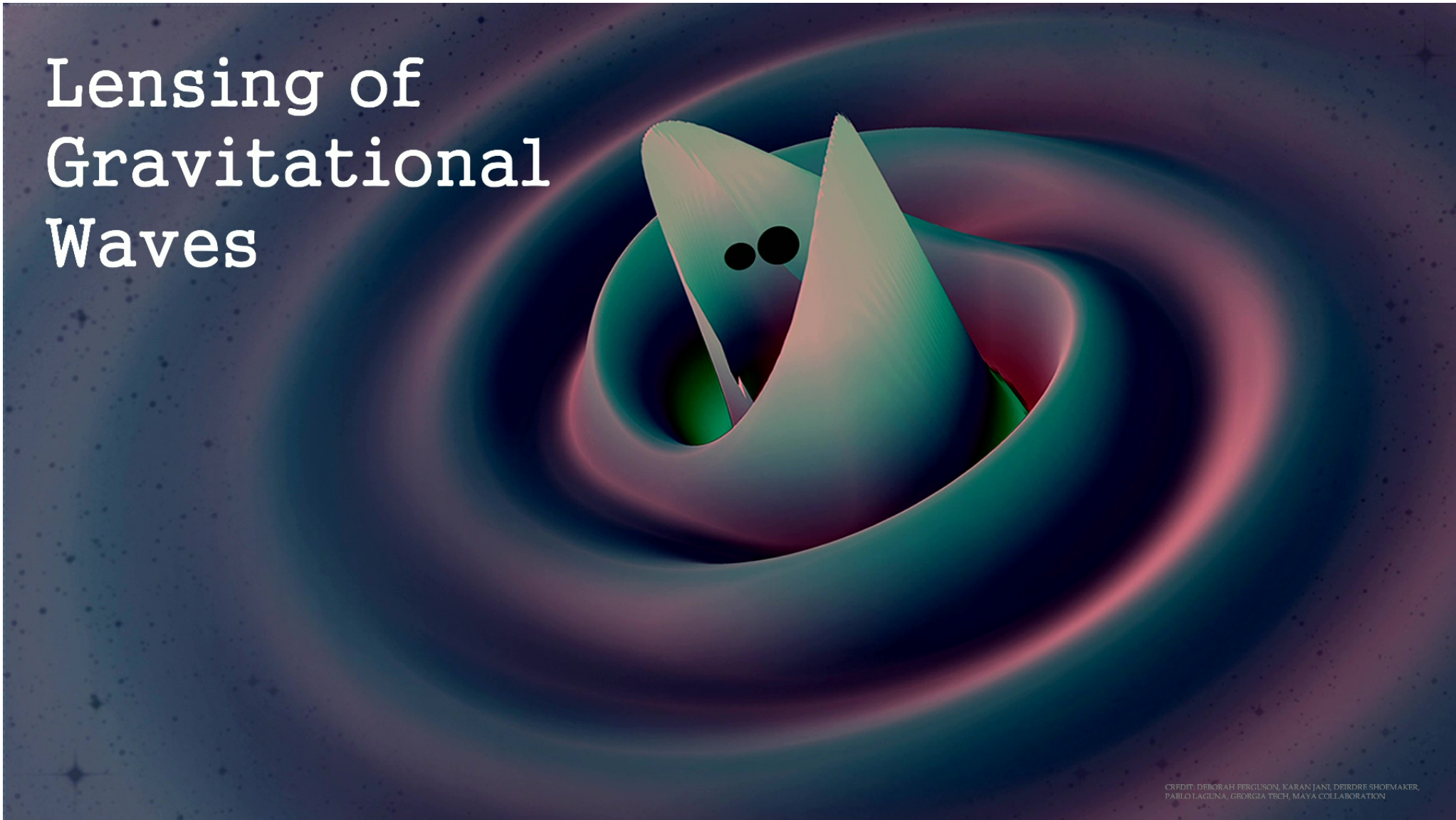
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Probing the substructures within DM halos remains challenging because of their **faint luminosity** and low masses, which lead to much lower detection probabilities.

Testbed for the Nature of Dark Matter

Lensing of Gravitational Waves



CREDIT: DEBORAH FERGLSON, KARAN JANI, DEIRDRE SHOEMAKER,
PABLO LAGUNA, GEORGIA TECH, MAYA COLLABORATION

Mass

Profile

Number Density

Strong Lensing of GWs

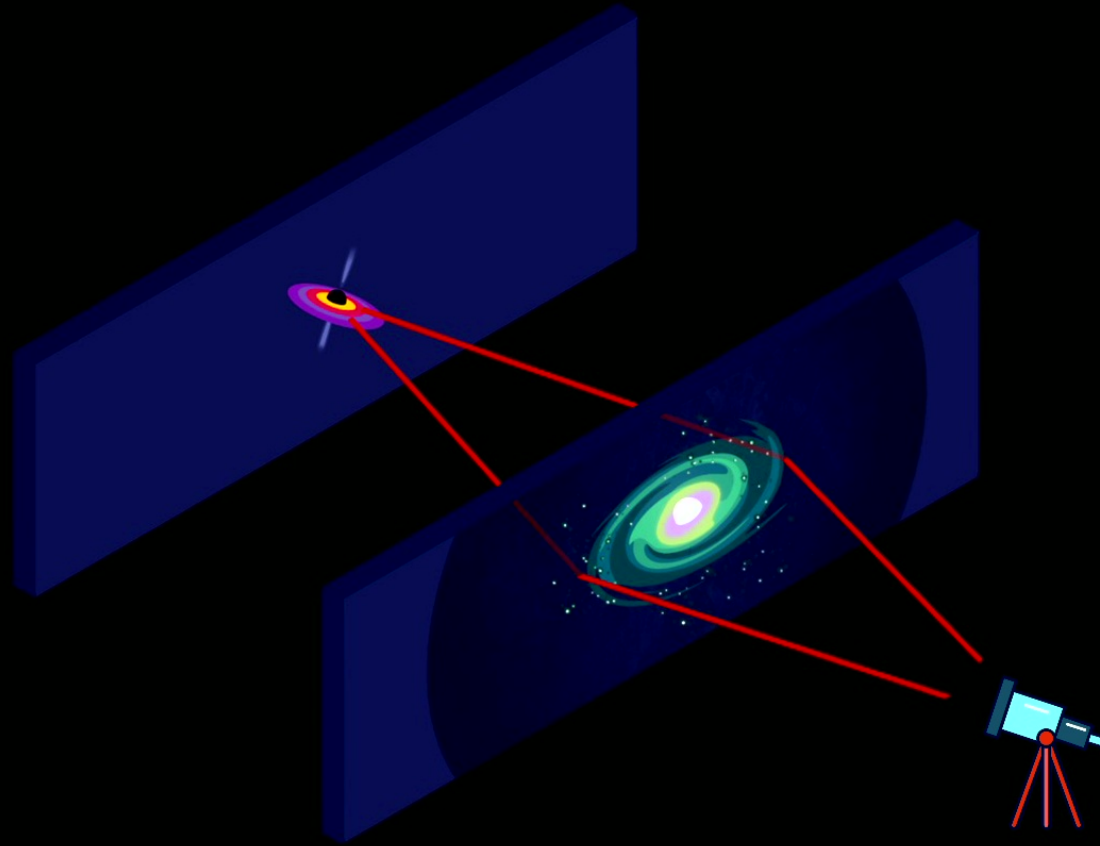


Illustration by Neha Anil Kumar

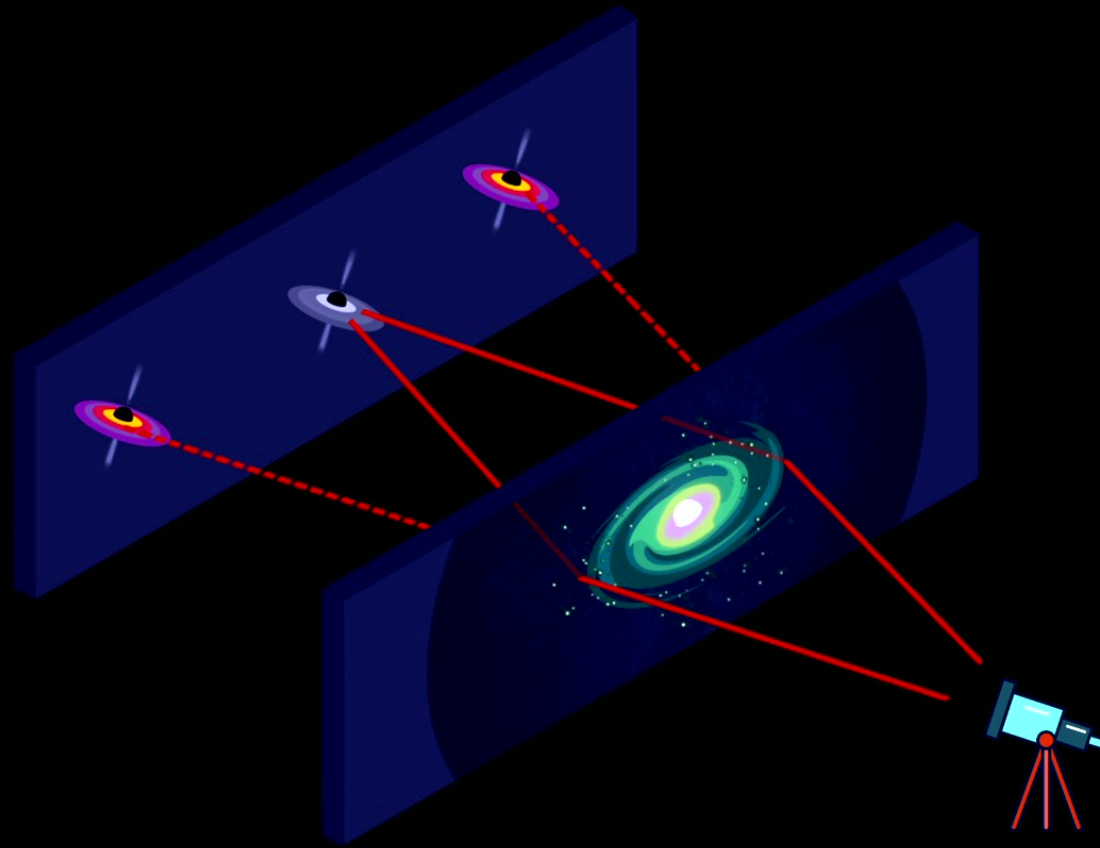


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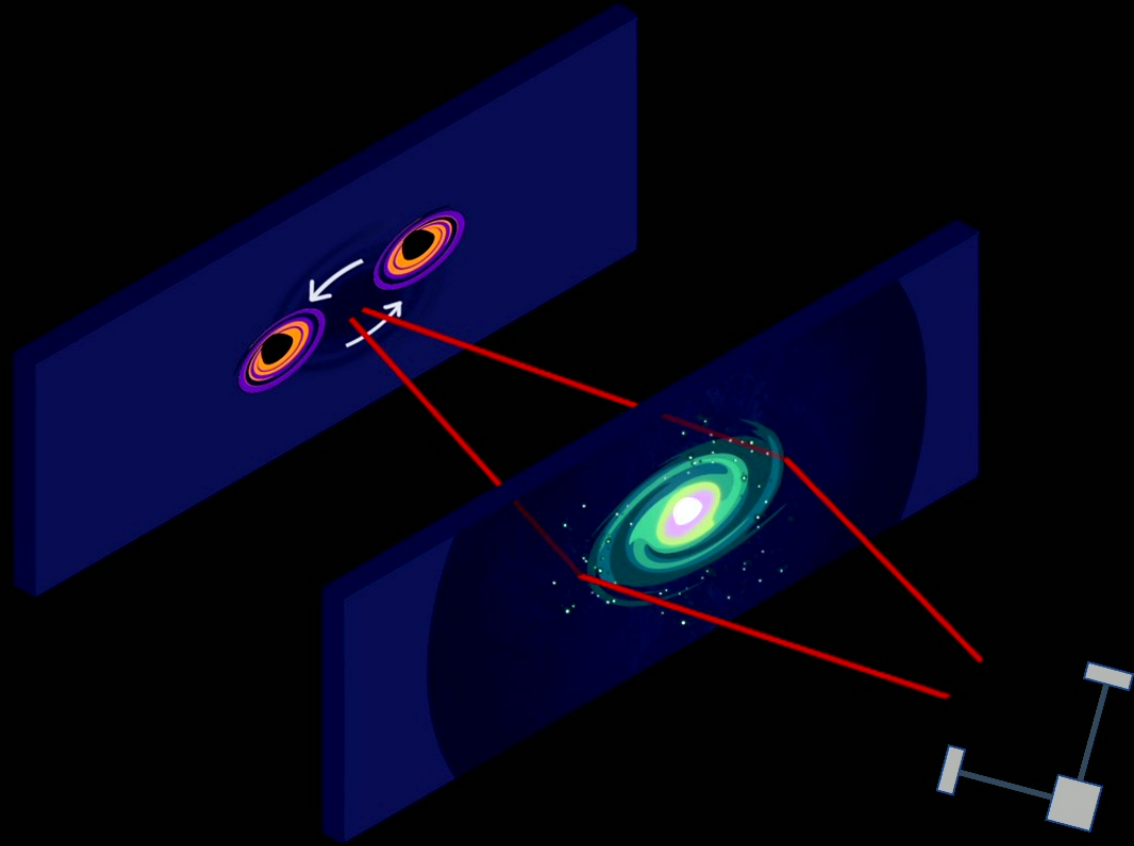
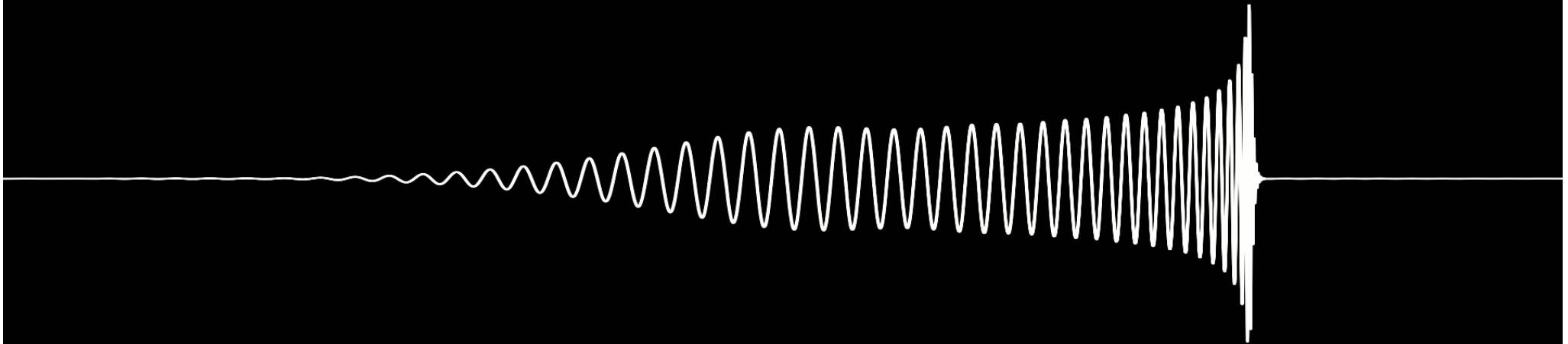


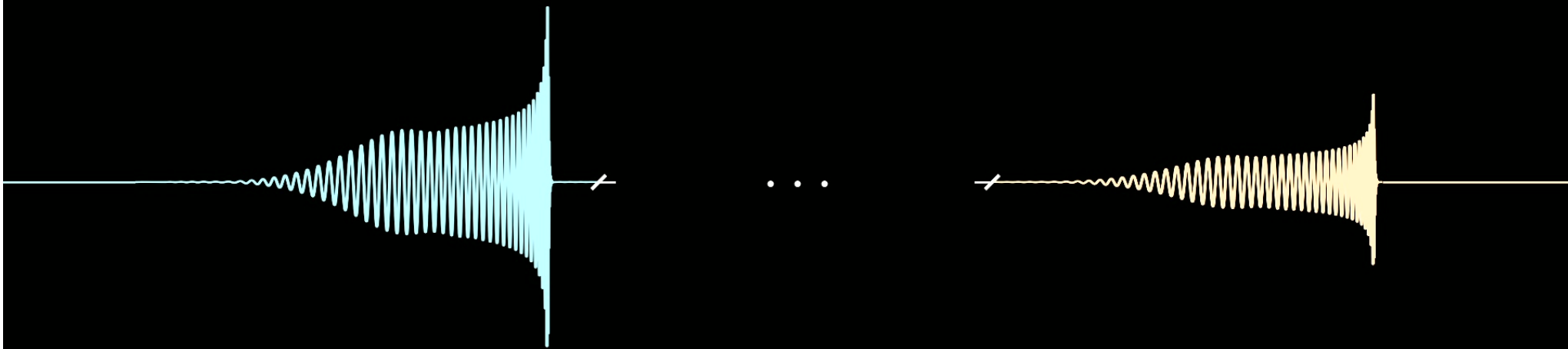
Illustration by Neha Anil Kumar

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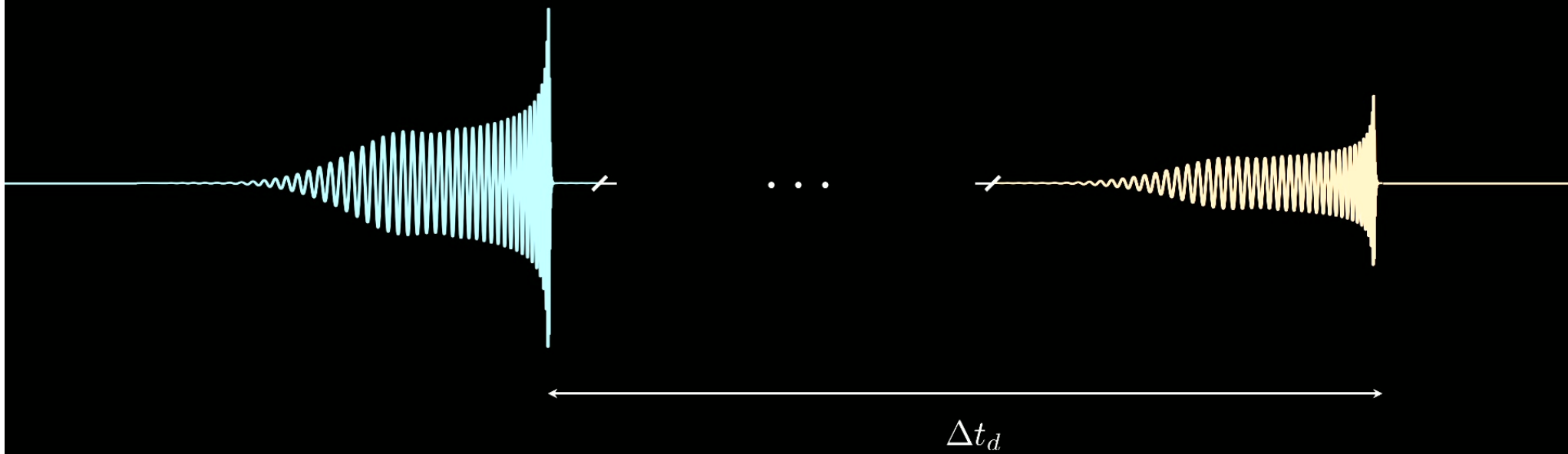
Unlensed Waveform



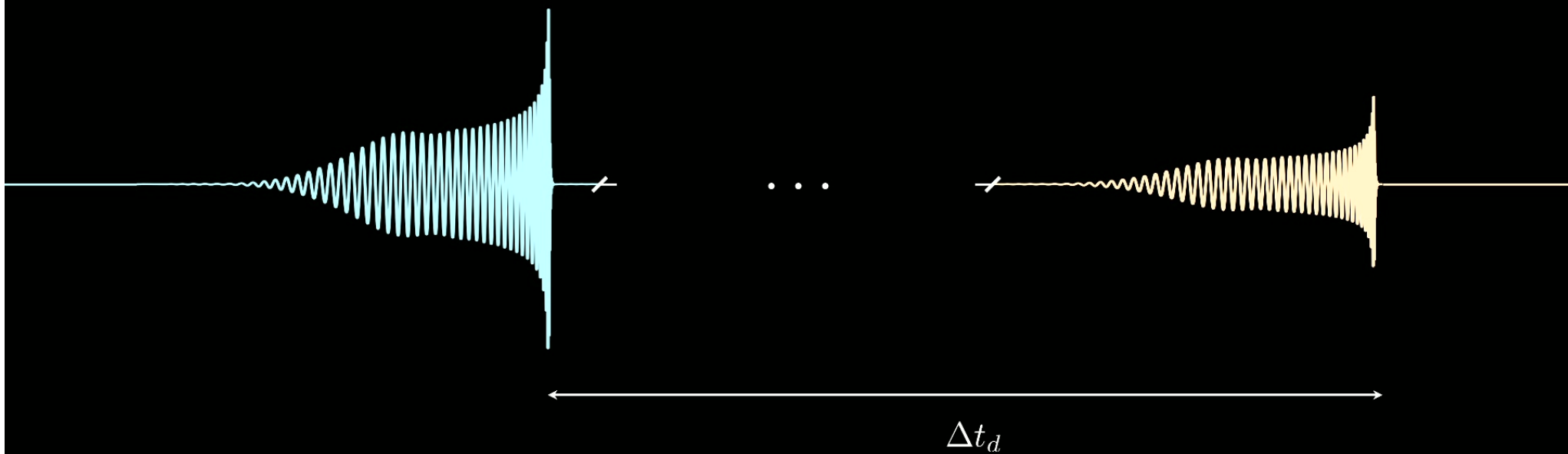
Strongly Lensed Waveforms



Strongly Lensed Waveforms

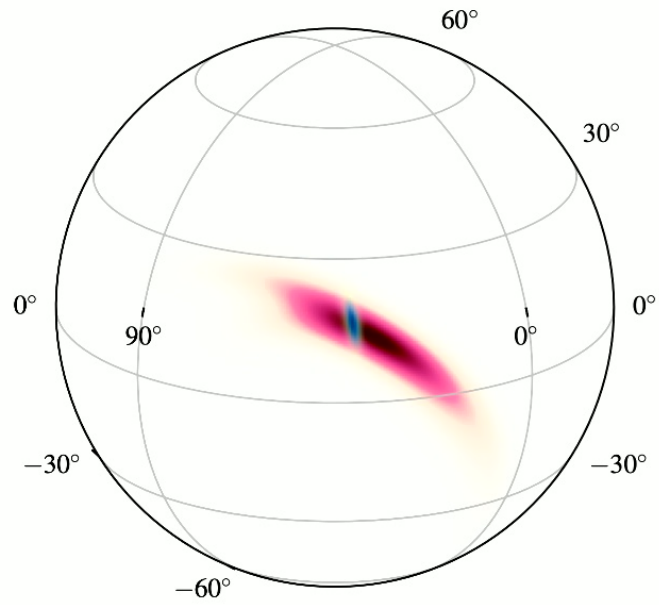


Strongly Lensed Waveforms



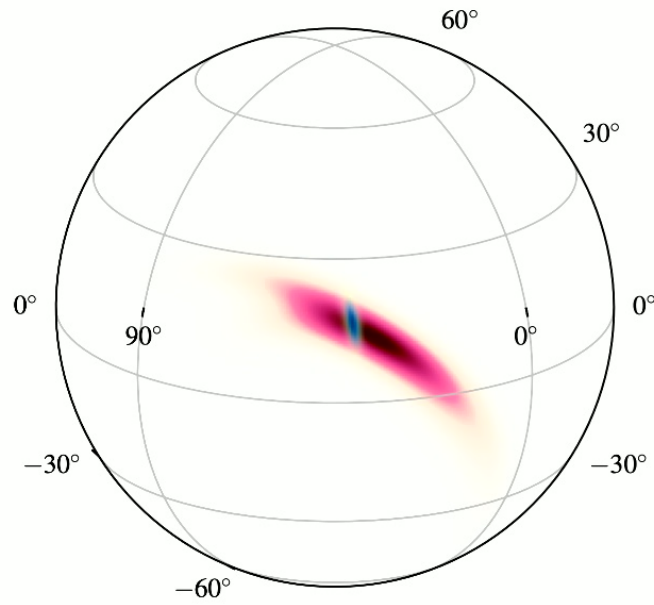
- Candidate: Dai et al. (2020)

■ Lensed Image 1
■ Lensed Image 2



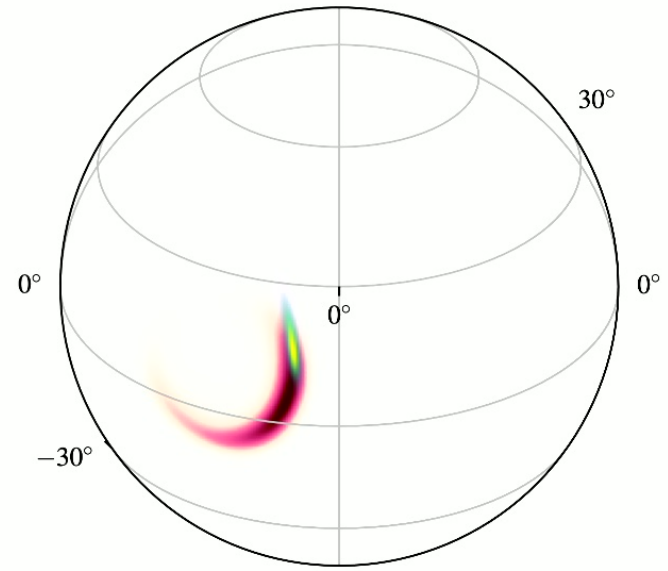
(a) lensed

■ Lensed Image 1
■ Lensed Image 2



(a) lensed

■ Event 12
■ Event 89



(b) not lensed

[2] Çalışkan et al. (2022)

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False Alarm Probability

$$\sim \frac{1}{10^4}$$

Strong Lensing Rate?

$$\sim \frac{1}{10^3}$$

$$N_{\text{lensed}} \propto N_{\text{events}}$$

$$N_{\text{lensed}} \propto N_{\text{events}}$$
$$N_{\text{false alarms}} \propto N_{\text{events}}^2$$

What if the lens is smaller?

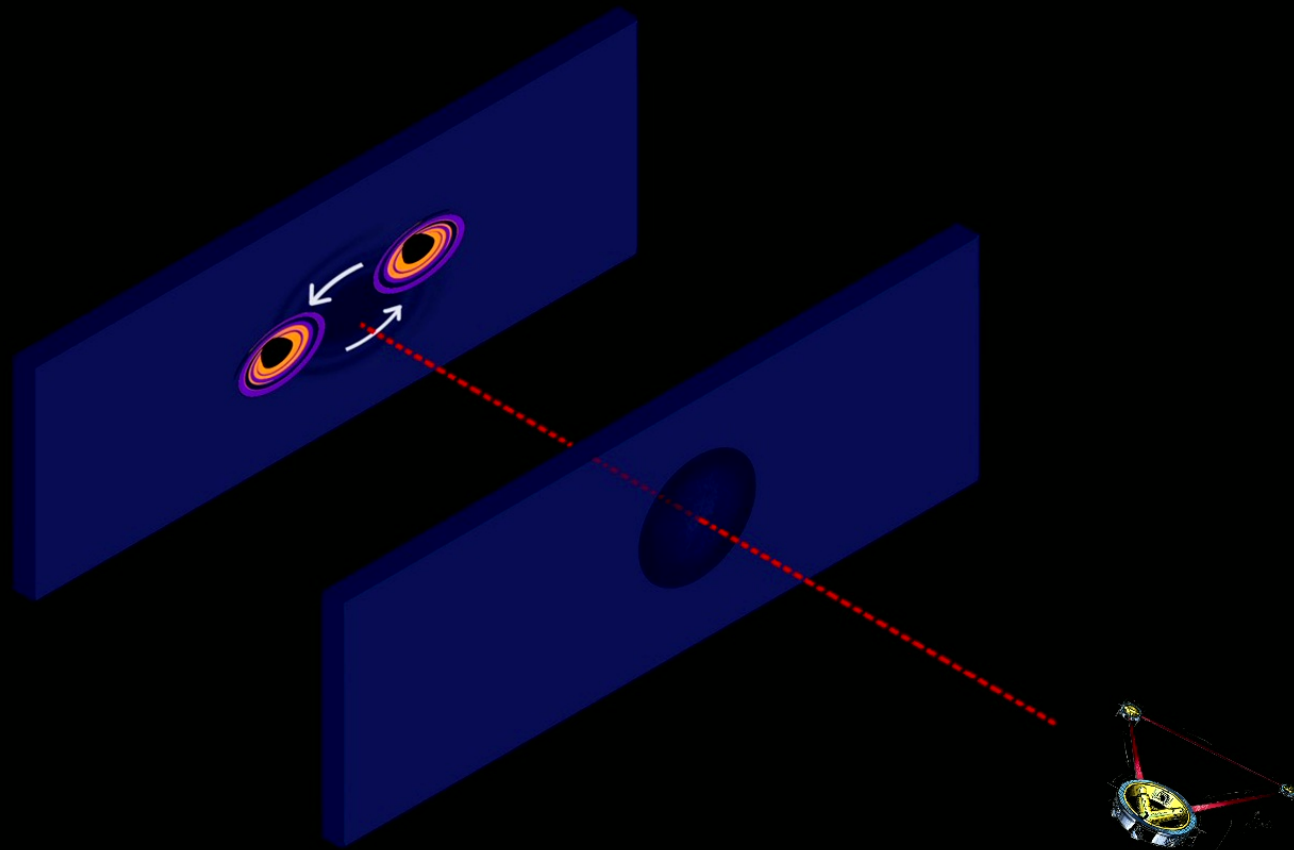


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Wave-Optics Effects

Wave-Optics Effects

$$M_L \lesssim 10^5 M_{\odot} \left(\frac{f}{\text{Hz}} \right)^{-1}$$

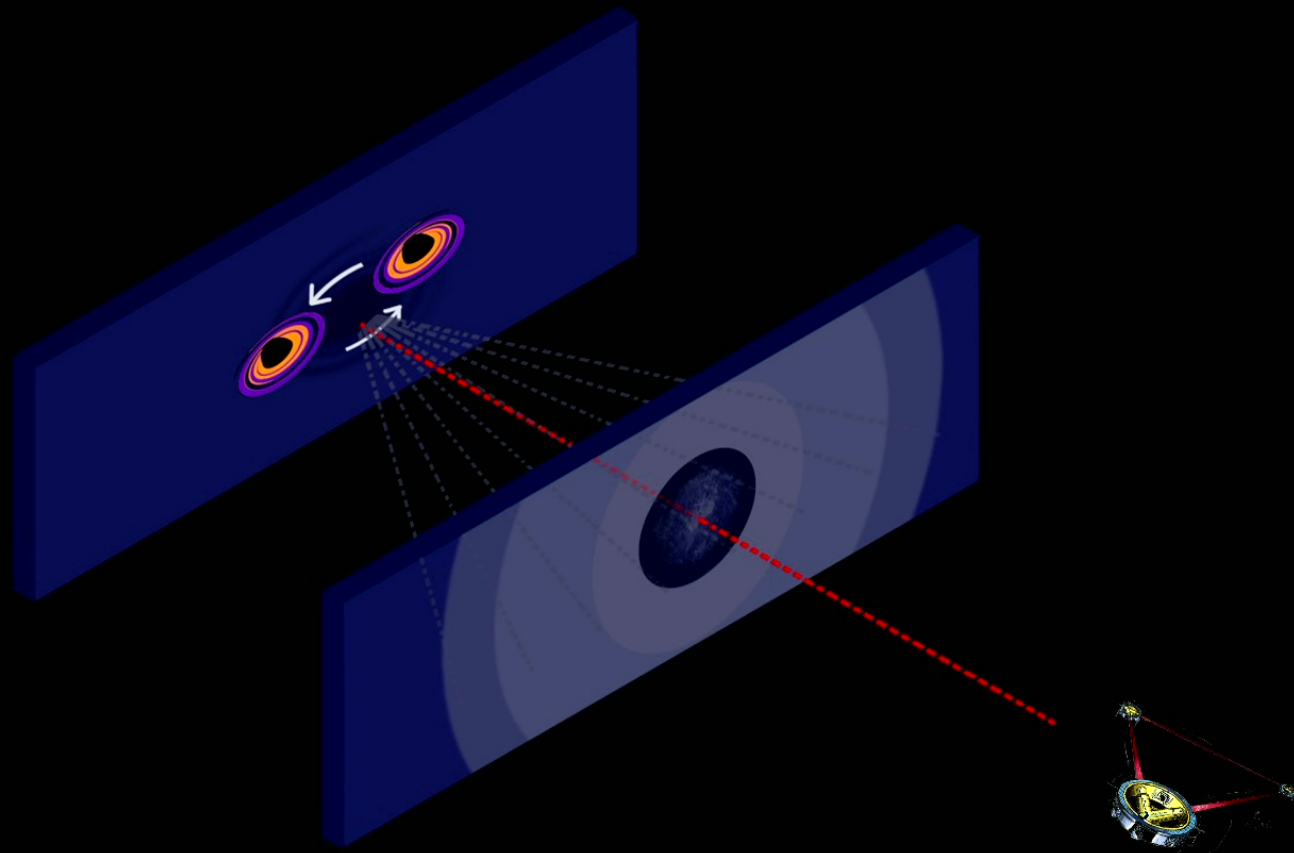


Illustration by Neha Anil Kumar

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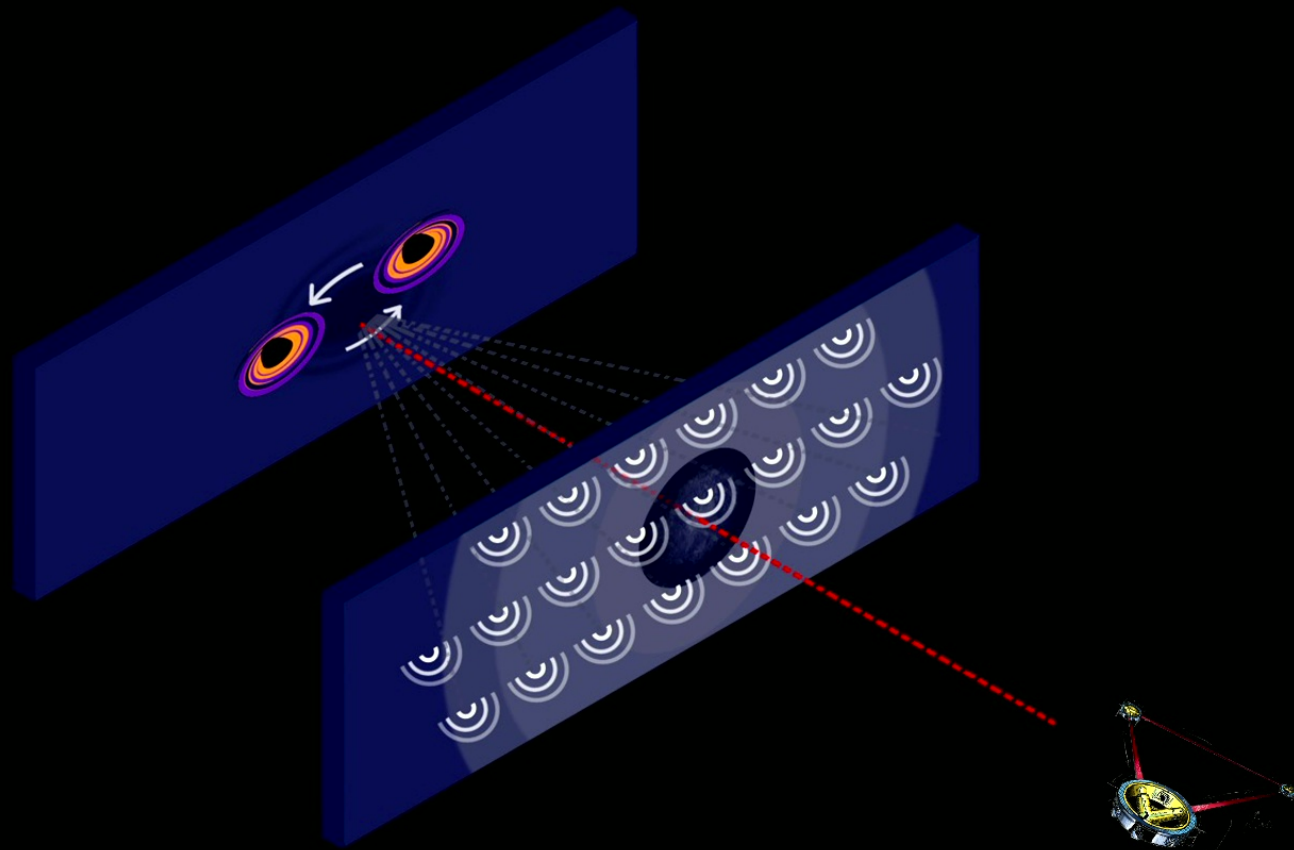
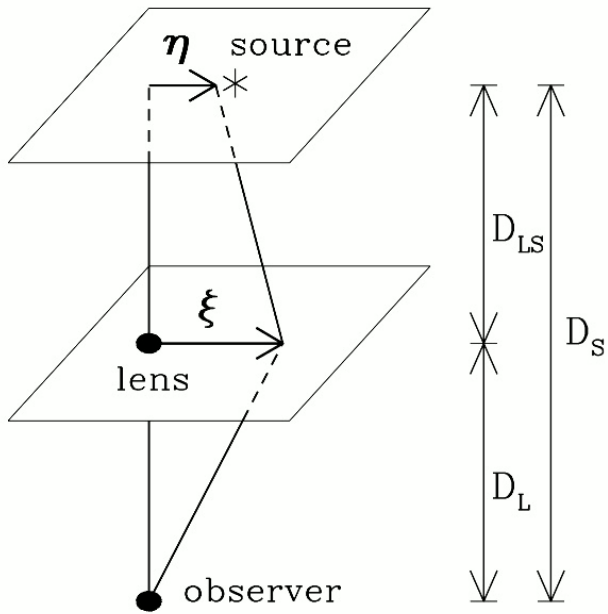


Illustration by Neha Anil Kumar

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Propagation of GWs Around a Lens and Wave Optics Effects

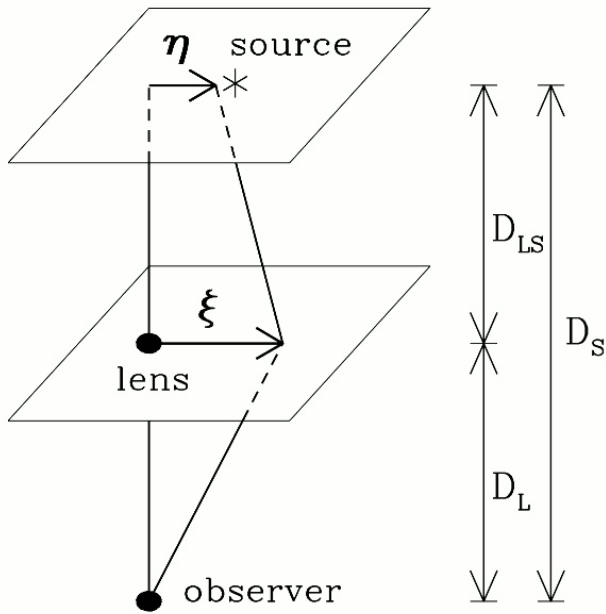


$$x \equiv \frac{\xi}{\xi_0} \quad \text{and} \quad y \equiv \eta \frac{D_L}{\xi_0 D_S}$$

↓
Impact Parameter

[1] Takahashi and Nakamura (2003)

Propagation of GWs Around a Lens and Wave Optics Effects



$$x \equiv \frac{\xi}{\xi_0} \quad \text{and} \quad y \equiv \eta \frac{D_L}{\xi_0 D_S}$$

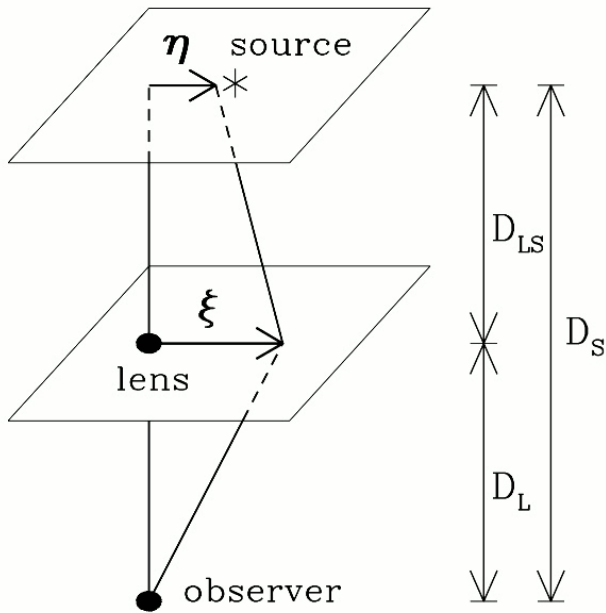
Impact Parameter

- $F(f, \mathbf{y}) = \frac{D_S(1+z_L)\xi_0^2 f}{D_L D_{LS}} \int d^2\mathbf{x} \exp[2\pi i f t_d(\mathbf{x}, \mathbf{y})]$
 - $t_d(\mathbf{x}, \mathbf{y}) = \frac{D_S \xi_0^2}{D_L D_{LS}} (1+z_L) \left[\frac{1}{2} |\mathbf{x} - \mathbf{y}|^2 - \psi(\mathbf{x}) + \phi(\mathbf{y}) \right]$
 - $w = 8\pi M_{Lz} f$
- Deflection Potential
(Based on the Matter Distribution of the Lens)

Lensed Waveform in the Frequency Domain: $\tilde{h}^L(f; w, \mathbf{y}) = F(w, \mathbf{y}) \tilde{h}(f)$

[1] Takahashi and Nakamura (2003)

Propagation of GWs Around a Lens and Wave Optics Effects



$$\mathbf{x} \equiv \frac{\boldsymbol{\xi}}{\xi_0} \quad \text{and} \quad \mathbf{y} \equiv \boldsymbol{\eta} \frac{D_L}{\xi_0 D_S}$$

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Geometrical optics limit: frequency-independent magnification, time delay, and phase shift

Wave optics effects: frequency-dependent modulations in the amplitude and phase of the waveform (directly dependent on the profile and mass of the lens)

[1] Takahashi and Nakamura (2003)

Lensing of Gravitational Waves

- The wavelength of GWs can be comparable to the size of the lenses, leading to *frequency-dependent modulations in the waveform phase and amplitude* due to wave-optics (WO) effects.
- At least in principle, this allows for the possibility of **infer the lens properties** (such as mass and profile) from a single GW observation.

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- The wavelength of GWs can be comparable to the size of the lenses, leading to *frequency-dependent modulations in the waveform phase and amplitude* due to wave-optics (WO) effects.
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[1] Takahashi and Nakamura (2003)

[5] Tambalo et al. (2022)

[8] Ji and Dai (2024)

Also:

[13] Jow and Pen (2022)

[3] **Çalışkan** et al. (2023a)

[6] Fairbairn et al. (2022)

[9] Leung et al. (2023)

[11] Feldbrugge, Pen, and Turok (2023)

[11] Jow and Pen (2024)

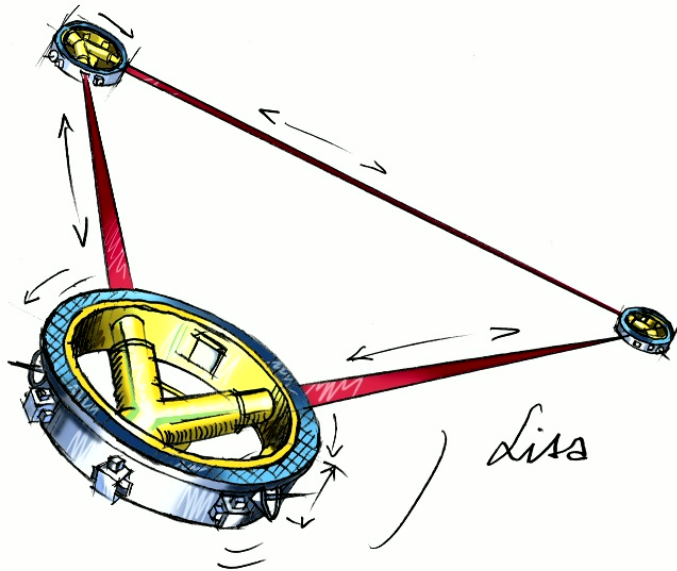
[4] **Çalışkan**, Anil Kumar, et al. (2023b)

[7] Savastano et al. (2023)

[10] Dai et al. (2018)

[12] Feldbrugge and Turok (2020)

[12] Villarrubia-Rojo et al. (2024)



Laser Interferometer Space Antenna (LISA)

- LISA will be the first dedicated space-based gravitational-wave observatory.
- LISA will detect gravitational waves (GWs) emitted by massive black hole binaries (MBHBs) in the low-frequency (\sim mHz) band.
- These MBHBs ($M_{\text{Total}} \approx 10^4 - 10^8 M_{\odot}$) will have high SNRs and large redshifts.
- Low-mass lenses, such as dark-matter (DM) subhalos, have sizes comparable to the wavelength of these GWs.

As such, MBHBs are excellent candidates for observing WO effects.



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Johns Hopkins University

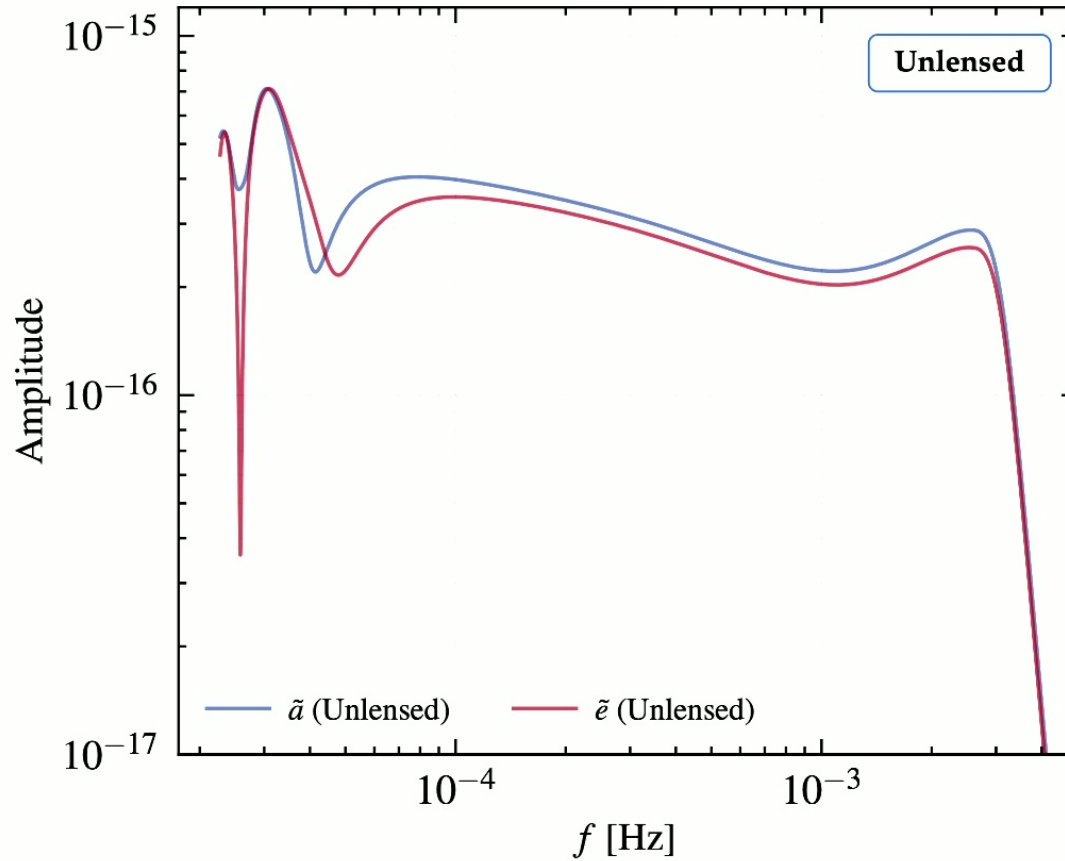


Lingyuan Ji

Berkeley / Now @ Citadel

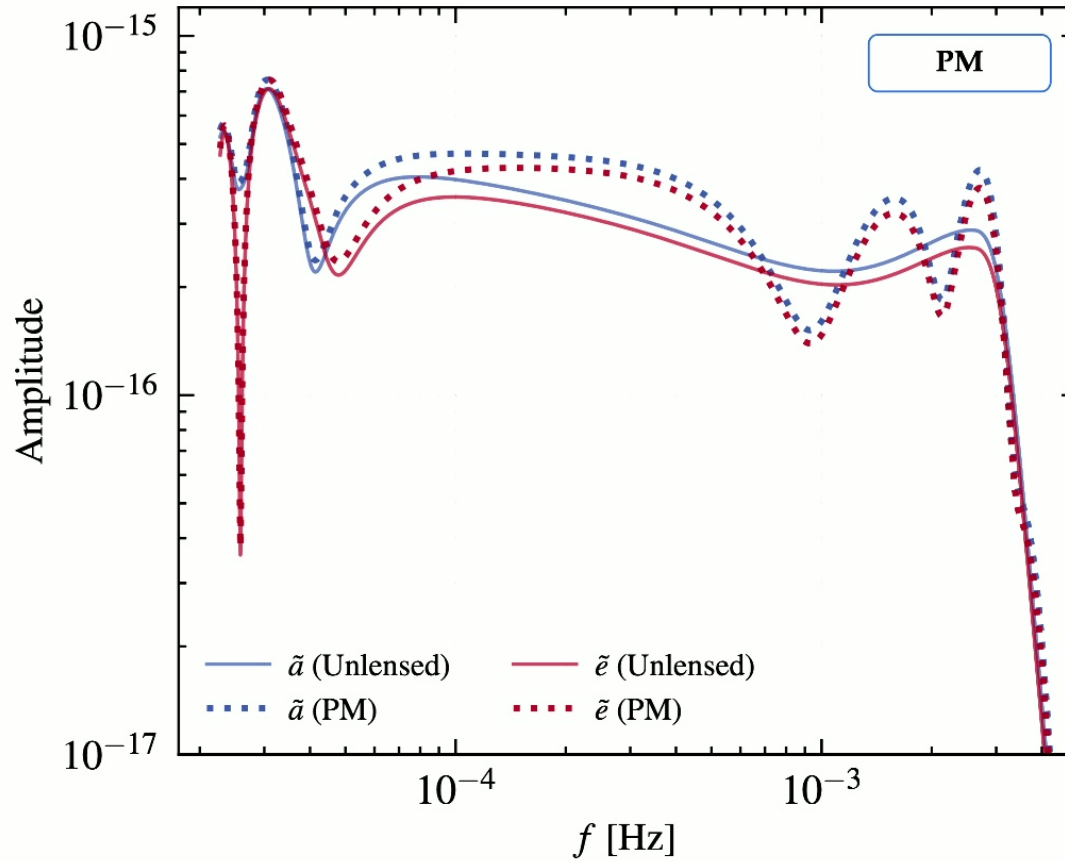
Lensed and Unlensed Waveforms

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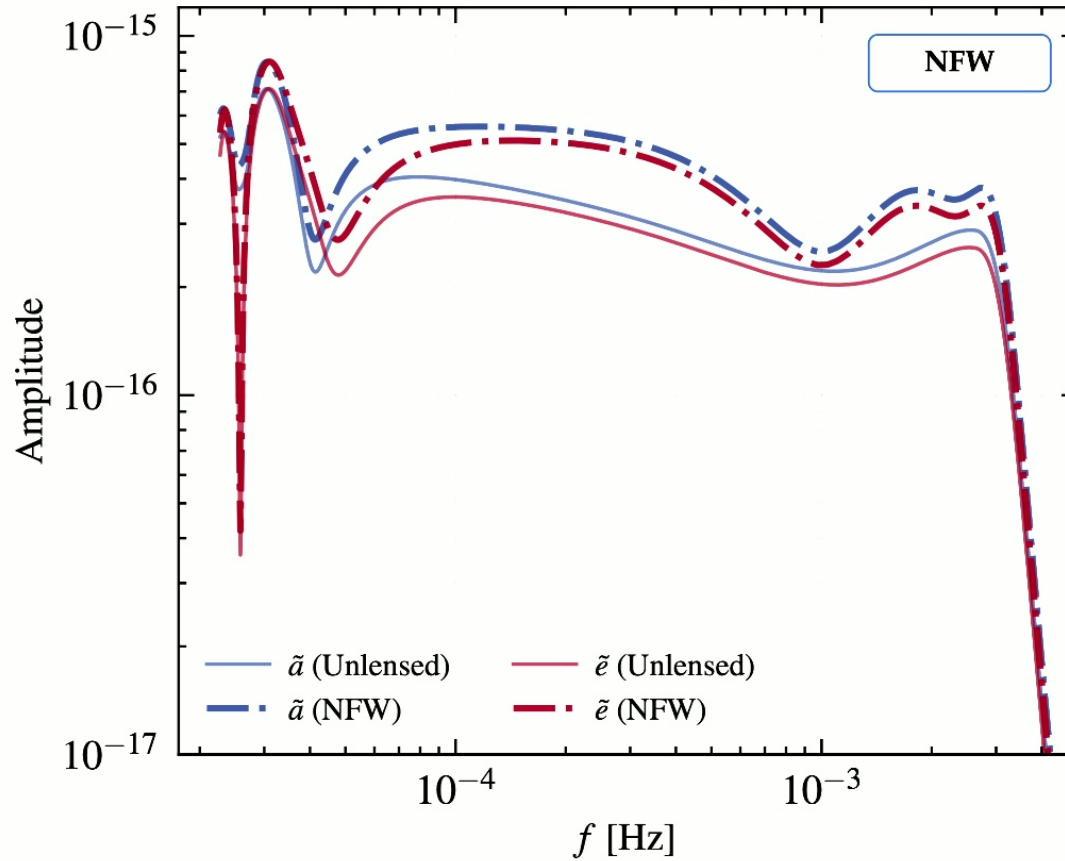


[3] Çalışkan et al. (2023a)

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Lensed and Unlensed Waveforms

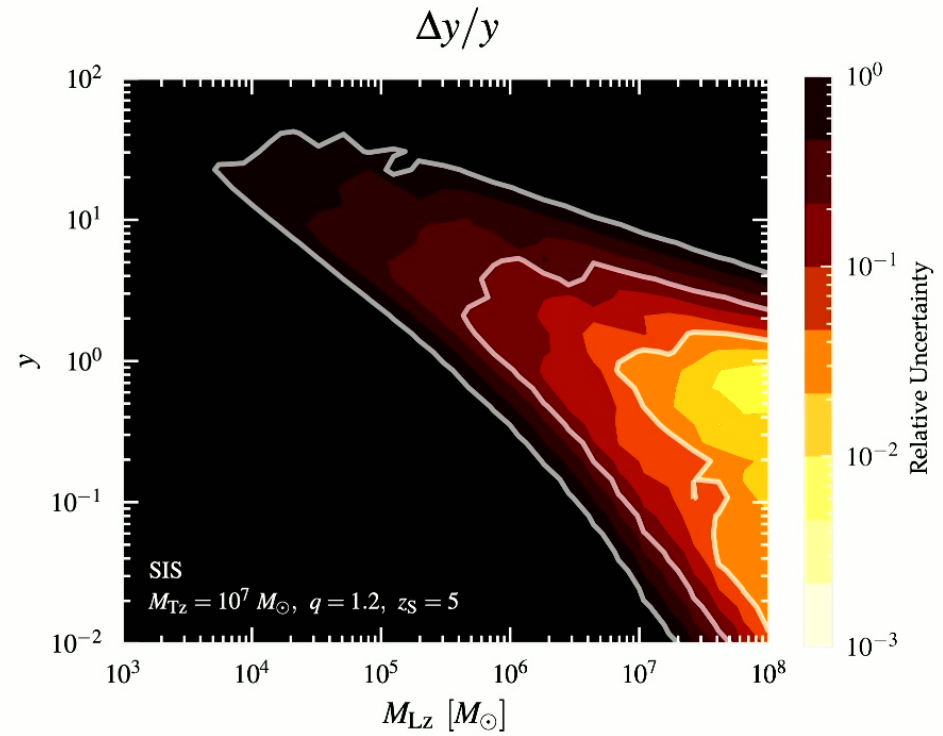
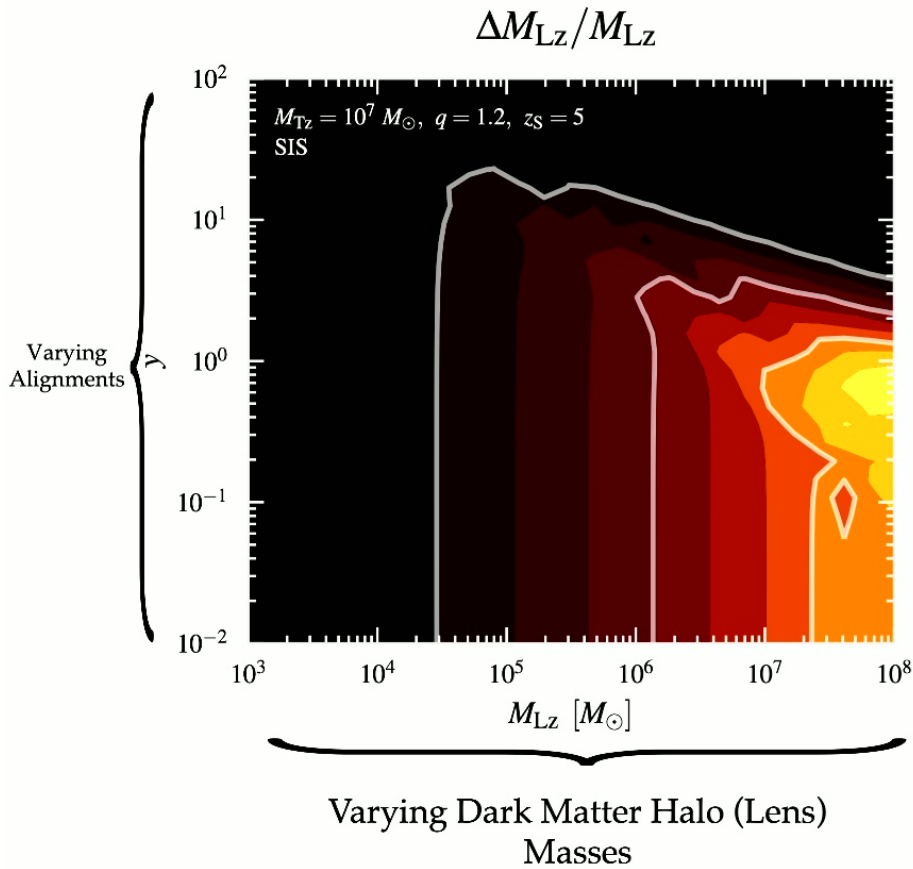
$$\tilde{h}^L(f; w, y) = F(w, y)\tilde{h}(f)$$



Measurability of the Lens Parameters

Measurability of Lens Mass (M_{Lz})

Measurability of Alignment (y)

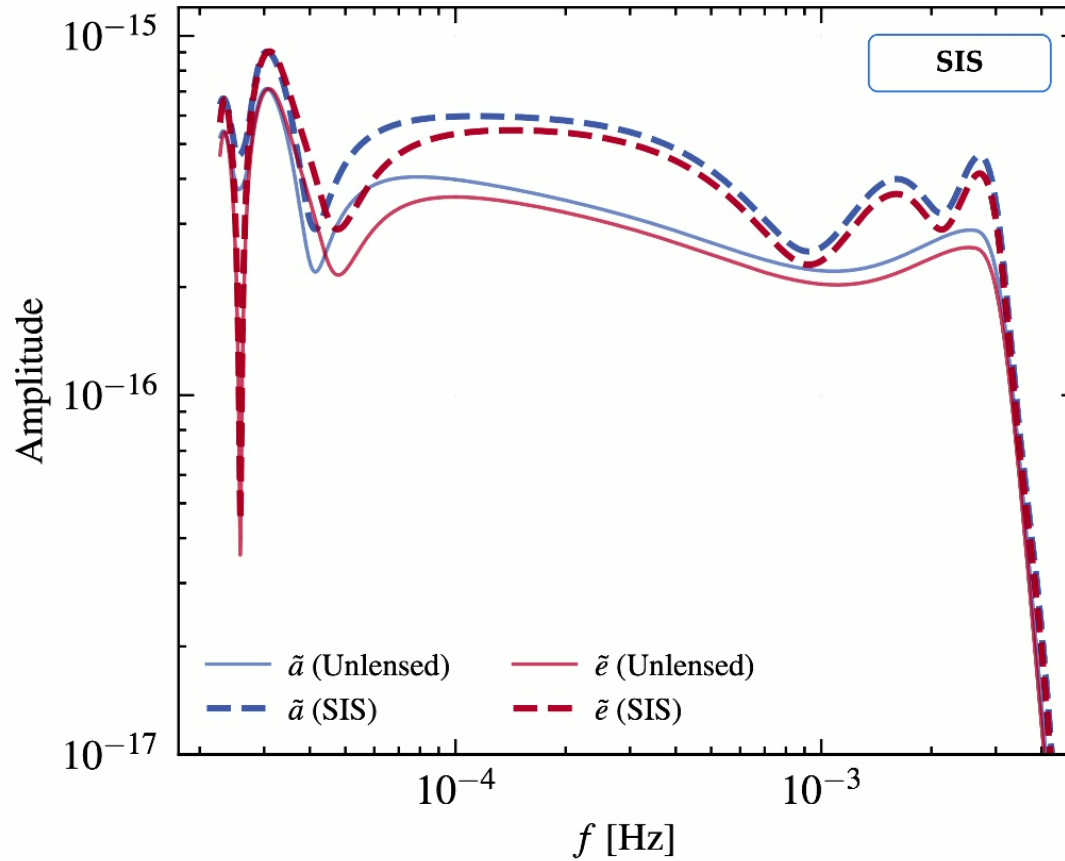


[3] Çalışkan, et al. (2023a)

[4] Çalışkan, Anil Kumar, et al. (2023b)

Lensed and Unlensed Waveforms

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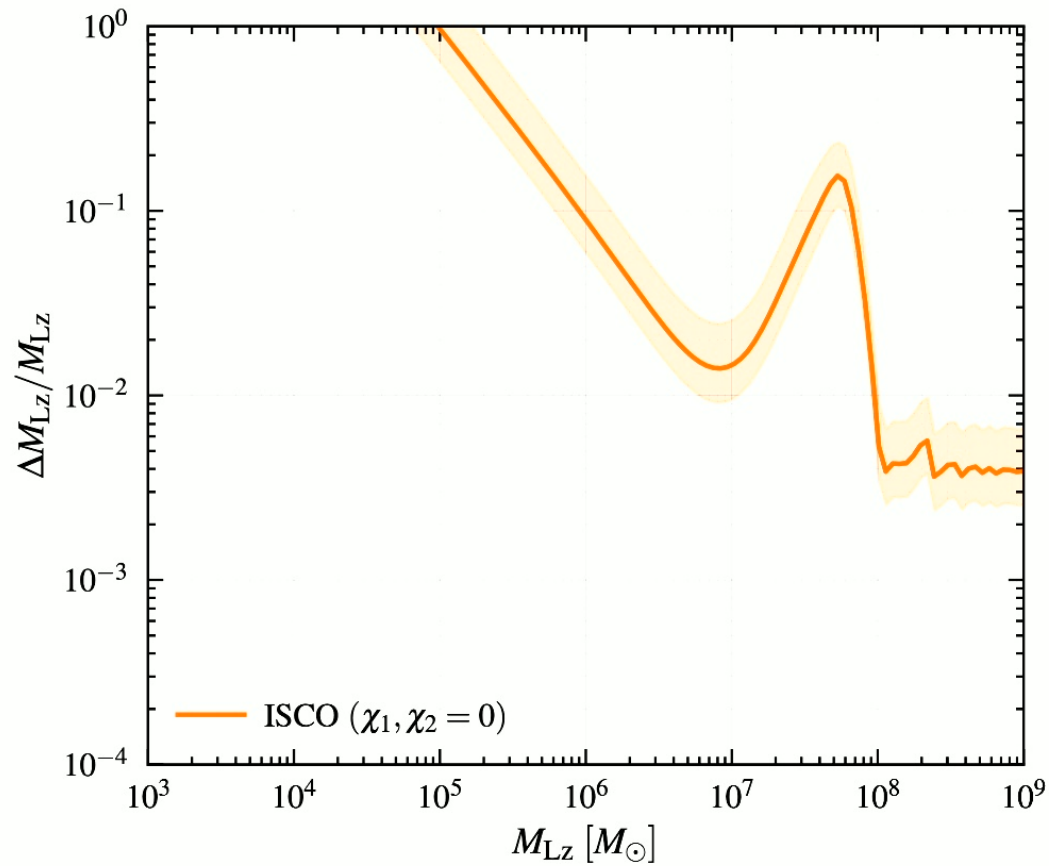
[3] Çalışkan et al. (2023a)

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What Did We (the Field)
Miss Before?

Inspiral-Only

- $M_{\text{Tz}} = 2 \times 10^6 M_{\odot}$
- $q = 1$
- $z_{\text{S}} = 1$
- $\chi_1 = \chi_2 = 0$
- PM Lens
- $y = 0.1$

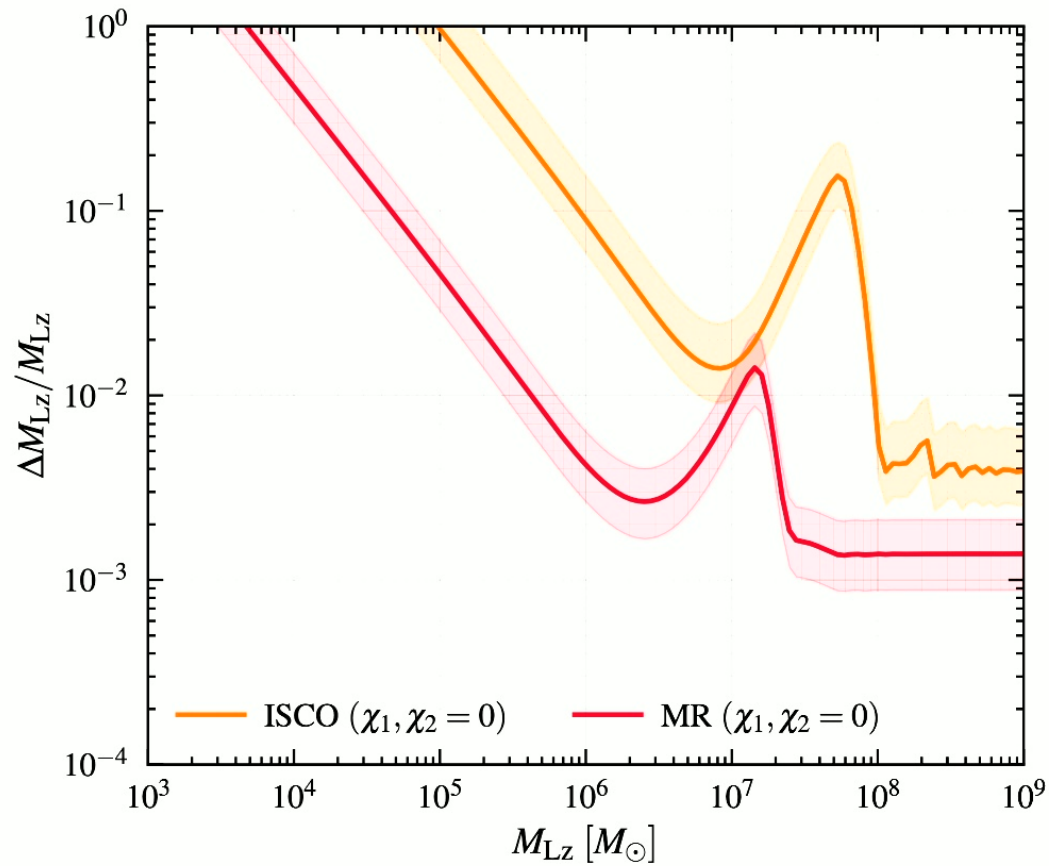


[3] Çalışkan et al. (2023a)

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Merger!

- $M_{\text{Tz}} = 2 \times 10^6 M_{\odot}$
- $q = 1$
- $z_{\text{S}} = 1$
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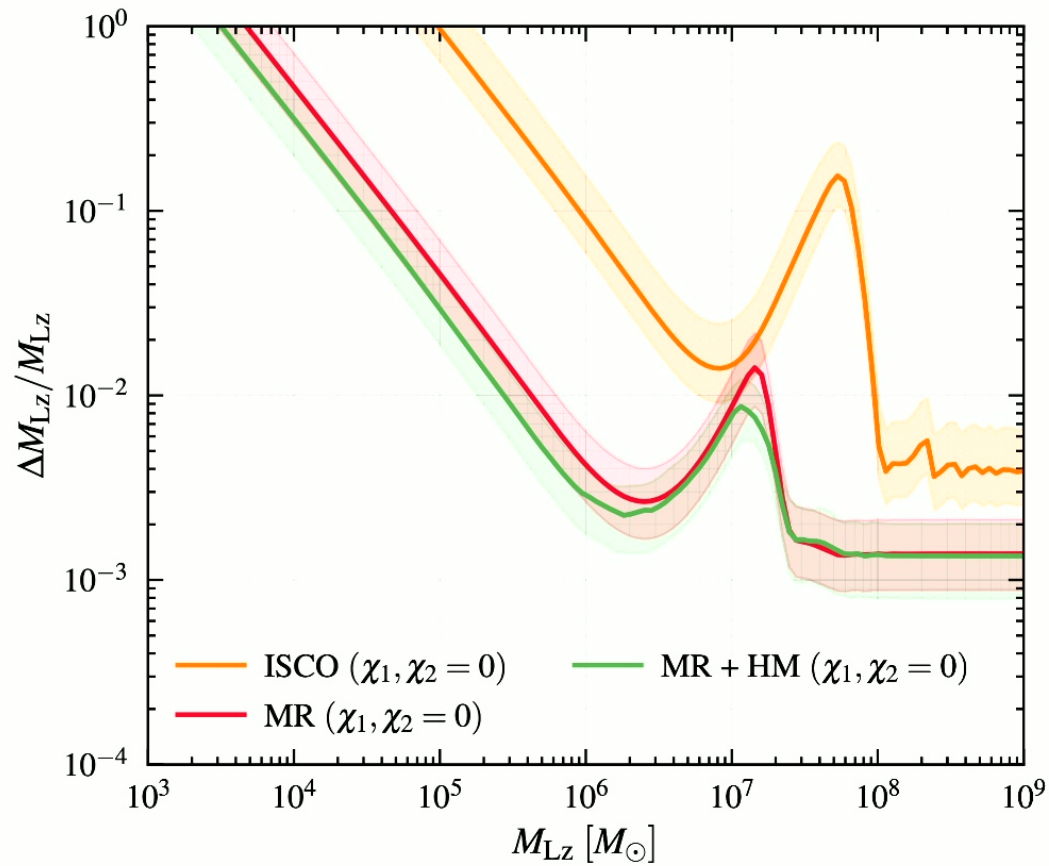


[3] Çalışkan et al. (2023a)

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Higher-Order Modes?

- $M_{\text{Tz}} = 2 \times 10^6 M_{\odot}$
- $q = 1$
- $z_{\text{S}} = 1$
- $\chi_1 = \chi_2 = 0$
- PM Lens
- $y = 0.1$



[3] Çalışkan et al. (2023a)

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We Need Faster Solutions

Singular Isothermal Sphere (SIS) Lens

$$\bullet \quad \underbrace{F(w, y)}_{w = 8\pi M_{Lz}f} = \frac{w}{i} \exp \left\{ iw \left[\frac{y^2}{2} + \underbrace{\phi(y)}_{y + \frac{1}{2}} \right] \right\} \int_0^\infty x dx \exp \left\{ iw \left[\frac{x^2}{2} - \underbrace{\psi(x)}_x \right] \right\} J_0(wxy)$$

[3] Çalışkan et al. (2023a)

Singular Isothermal Sphere (SIS) Lens

- $$F(\underbrace{w}_{w = 8\pi M_{Lz}f}, y) = \frac{w}{i} \exp \left\{ iw \left[\frac{y^2}{2} + \underbrace{\phi(y)}_{y + \frac{1}{2}} \right] \right\} \int_0^\infty x dx \exp \left\{ iw \left[\frac{x^2}{2} - \underbrace{\psi(x)}_x \right] \right\} J_0(wxy)$$

- Taylor expansion of the exponential factor $\exp[-iw\psi(x)]$

- $$I_n(w, y) \equiv \int_0^\infty x^n e^{iwx^2/2} J_0(wxy) x dx = \frac{1}{2} \left(\frac{2i}{w} \right)^N \Gamma(N) \underbrace{{}_1F_1 \left(N, 1; -i \frac{wy^2}{2} \right)}_{N \equiv (n+2)/2}$$

[3] Çalışkan et al. (2023a)

Singular Isothermal Sphere (SIS) Lens

- $$F(w, y) = \frac{w}{i} \exp \left\{ iw \left[\frac{y^2}{2} + \phi(y) \right] \right\} \sum_{n=0}^{\infty} \Psi_n(w) I_n(w, y)$$

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- $$\Psi(w, x) \equiv e^{-iw\psi(x)} = \sum_{n=0}^{\infty} \Psi_n(w) x^n$$

$\Psi_n(w) = (-iw)^n / n!$

$N \equiv (n+2)/2$



PM Lens

SIS Lens

- $F(w, y) = \exp \left\{ \frac{\pi w}{4} + i \frac{w}{2} \left[\ln \frac{w}{2} - 2\phi(y) \right] \right\} \Gamma \left(1 - \frac{w}{2} i \right) {}_1F_1 \left(\frac{w}{2} i, 1; \frac{w y^2}{2} i \right)$

- $F(w, y) = \frac{w}{i} \exp \left\{ i w \left[\frac{y^2}{2} + \phi(y) \right] \right\} \sum_{n=0}^{\infty} \Psi_n(w) I_n(w, y)$

- $\Gamma(z) = \left\{ z e^{cz} \prod_{r=1}^{\infty} \left[\left(1 + \frac{z}{r} \right) e^{-z/r} \right] \right\}^{-1}$
- $\Gamma'(z) = \Gamma(z) \Psi(z)$

$$\left. \begin{array}{l} \bullet \frac{\partial \Gamma \left(1 - \frac{w}{2} i \right)}{\partial w} = -\frac{1}{2} i \Gamma \left(1 - \frac{w}{2} i \right) \Psi \left(1 - \frac{w}{2} i \right) \end{array} \right\}$$

- ${}_1F_1(a, b; z) = 1 + \frac{a}{b} \frac{z}{1!} + \frac{a(a+1)}{b(b+1)} \frac{z^2}{2!} + \dots$

$$\left. \begin{array}{l} \bullet \frac{\partial {}_1F_1(a, b; z)}{\partial z} = \frac{a}{b} {}_1F_1(a+1, b+1; z) \\ \bullet \frac{\partial {}_1F_1(a, b; z)}{\partial a} = \sum_{k=0}^{\infty} \frac{(a)_k \Psi(a+k) z^k}{k!(b)_k} - \Psi(a) {}_1F_1(a, b; z) \end{array} \right\}$$

- $\frac{\partial {}_1F_1(a, b; z)}{\partial w} = \frac{\partial {}_1F_1(a, b; z)}{\partial a} \cdot \frac{\partial a}{\partial w} + \frac{\partial {}_1F_1(a, b; z)}{\partial z} \cdot \frac{\partial z}{\partial w}$

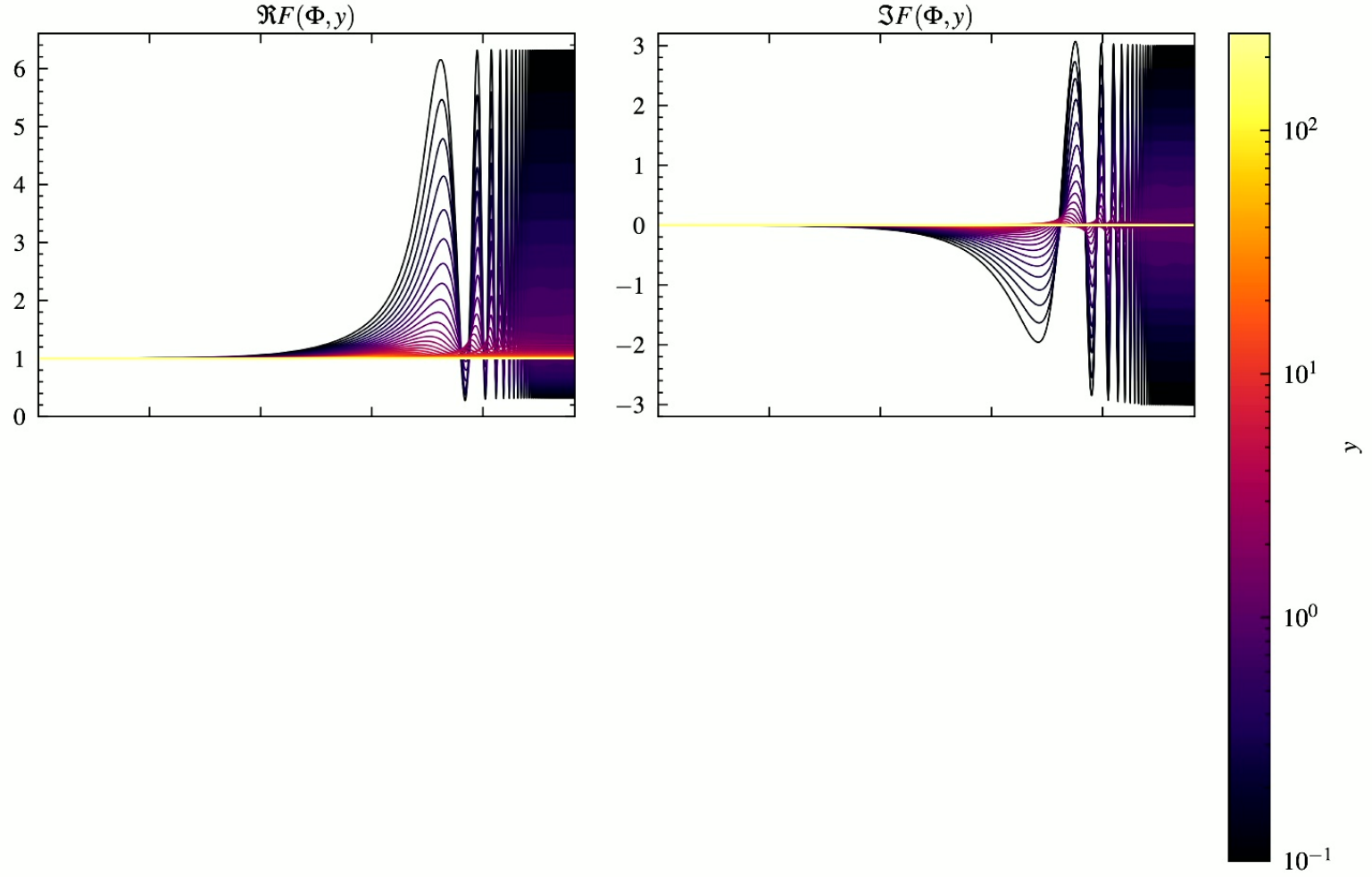
$$= \frac{i}{2} \cdot \frac{\partial {}_1F_1(a, b; z)}{\partial a} + \frac{y^2}{2} i \cdot \frac{\partial {}_1F_1(a, b; z)}{\partial z}$$

$$= \frac{i}{2} \sum_{k=0}^{\infty} \frac{(i \frac{w}{2})_k \Psi(i \frac{w}{2} + k) z^k}{k!(1)_k} - \Psi \left(i \frac{w}{2} \right) {}_1F_1 \left(i \frac{w}{2}, 1; i \frac{w y^2}{2} \right) - \frac{w y^2}{4} {}_1F_1 \left(i \frac{w}{2} + 1, 2; i \frac{w y^2}{2} \right)$$

- $\frac{\partial {}_1F_1(a, b; z)}{\partial y} = -\frac{w^2 y}{2} {}_1F_1 \left(\frac{w}{2} i + 1, 2; \frac{w y^2}{2} i \right)$

- $\frac{\partial \Psi_n(w)}{\partial w} = \frac{(-i)^n}{(n-1)!} w^{n-1}$
- $\frac{\partial I_n(w, y)}{\partial w} = \frac{N}{2} \left(\frac{2i}{w} \right)^N \Gamma(N) \left[\left(-i \frac{y^2}{2} \right) {}_1F_1 \left(N+1, 2; -i \frac{w y^2}{2} \right) - \frac{1}{w} {}_1F_1 \left(N, 1; -i \frac{w y^2}{2} \right) \right]$
- $\frac{\partial I_n(w, y)}{\partial y} = -i N \frac{w y}{2} \left(\frac{2i}{w} \right)^N \Gamma(N) {}_1F_1 \left(N+1, 2; -i \frac{w y^2}{2} \right)$
- $\frac{\partial F(w, y)}{\partial w} = \frac{\partial E(w, y)}{\partial w} \sum_{n=0}^{\infty} \Psi_n(w) I_n(w, y) + E(w, y) \sum_{n=0}^{\infty} \left[\frac{\partial \Psi_n(w)}{\partial w} I_n(w, y) + \Psi_n(w) \frac{\partial I_n(w, y)}{\partial w} \right]$
- $\frac{\partial F(w, y)}{\partial y} = \frac{\partial E(w, y)}{\partial y} \sum_{n=0}^{\infty} \Psi_n(w) I_n(w, y) + E(w, y) \sum_{n=0}^{\infty} \Psi_n(w) \frac{\partial I_n(w, y)}{\partial y}$

[3] Çalışkan et al. (2023a)

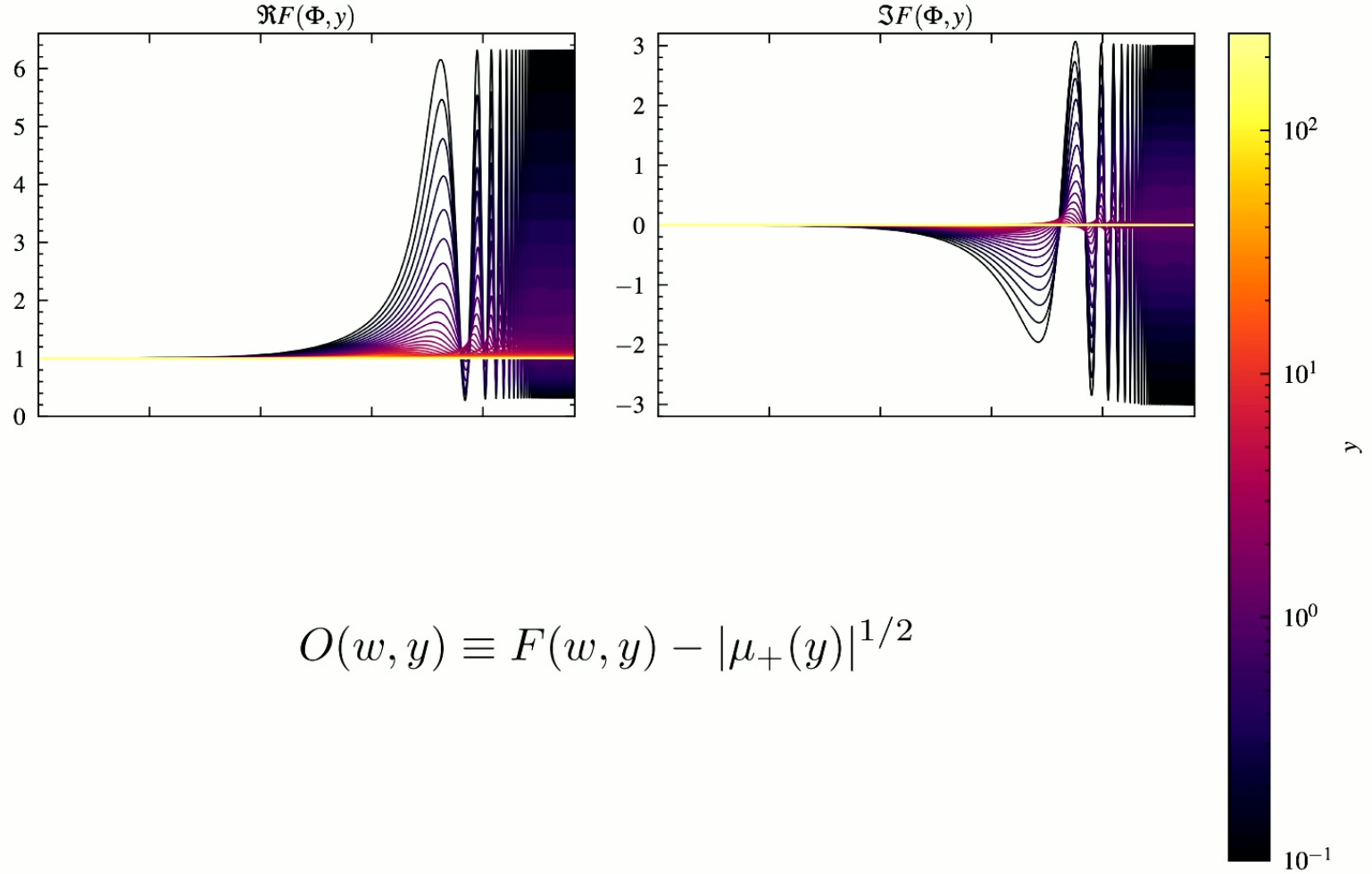


geometric phase $\Phi = 2\omega y$

geometric phase $\Phi = 2\omega y$

[4] Çalışkan, Anil Kumar, et al. (2023b)

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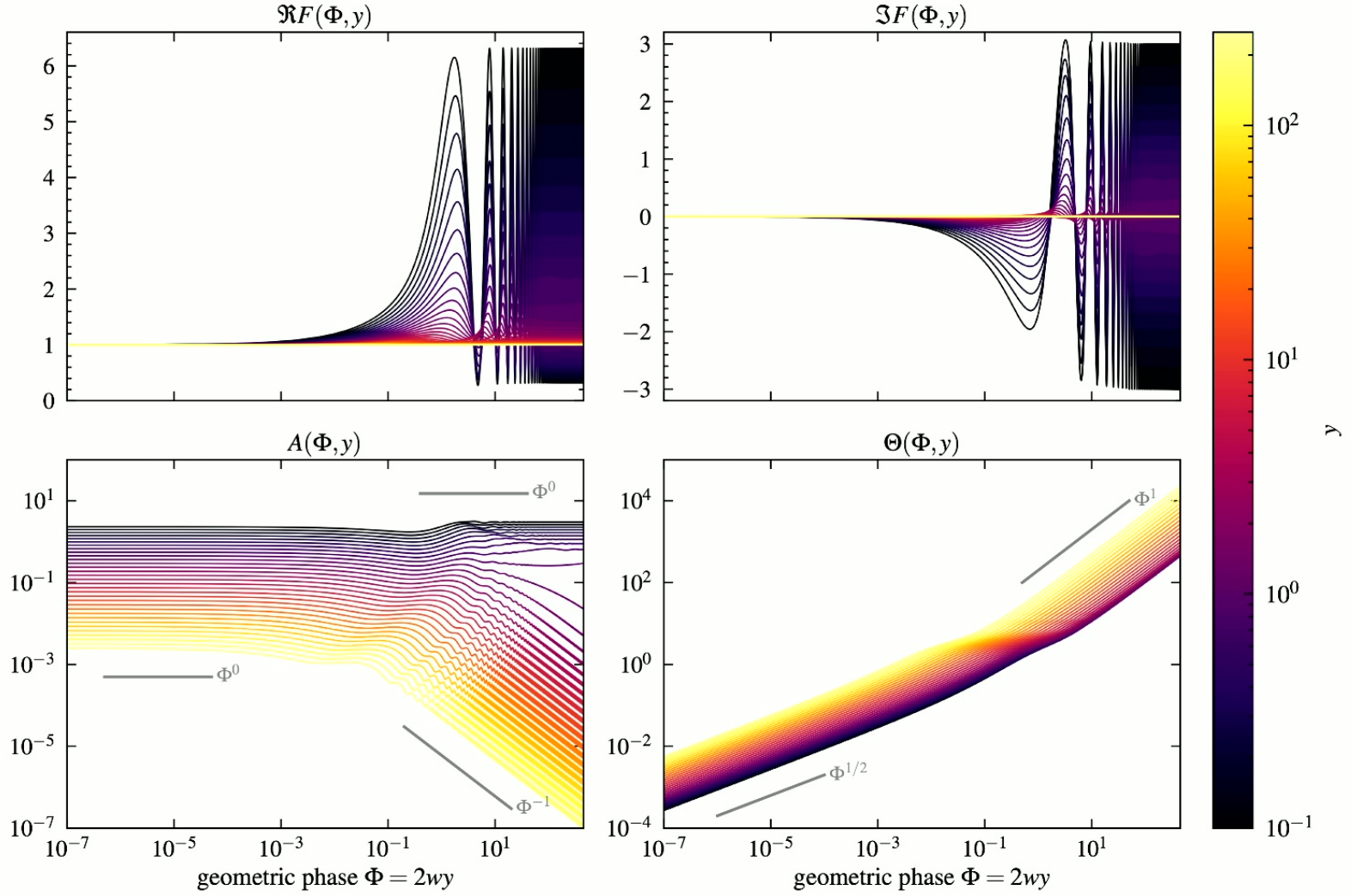


$$O(w, y) \equiv F(w, y) - |\mu_+(y)|^{1/2}$$

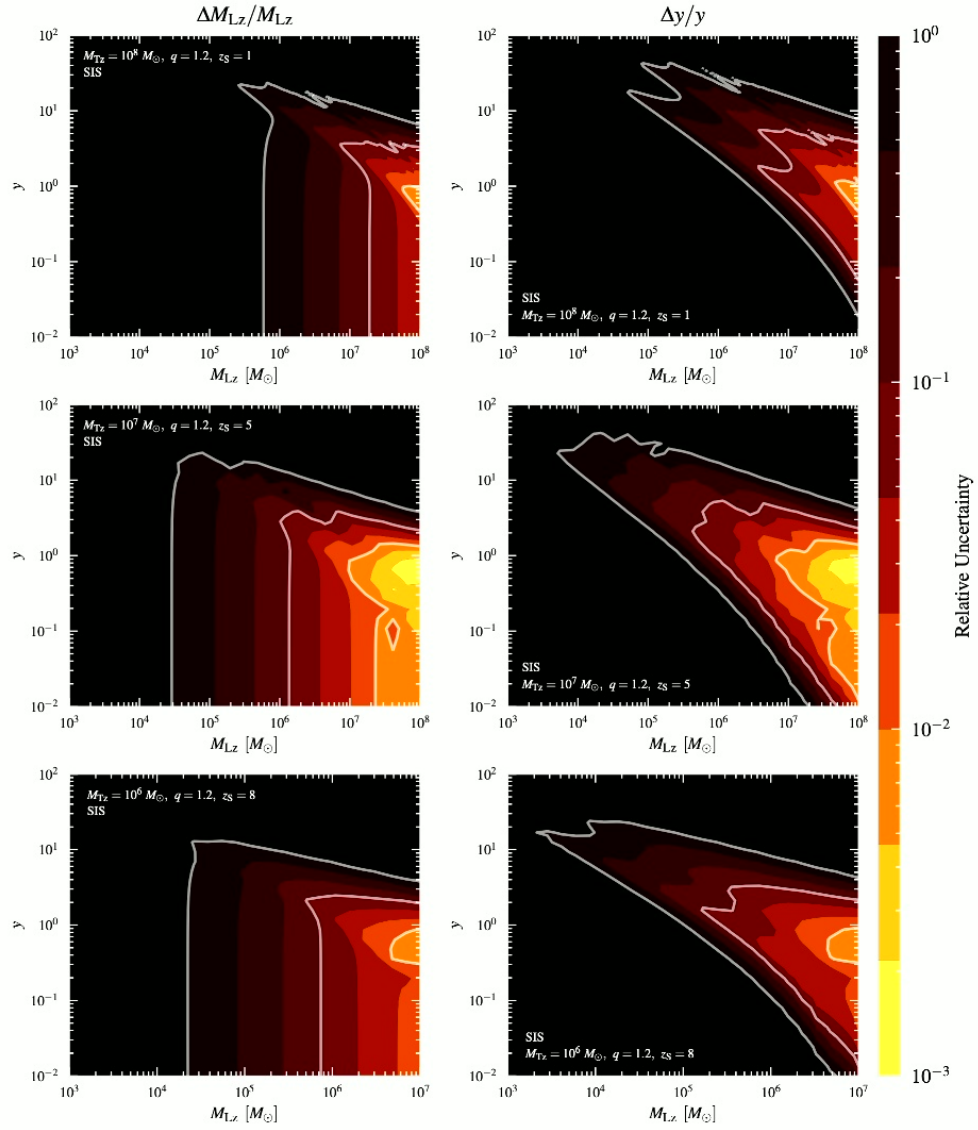
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[4] Çalışkan, Anil Kumar, et al. (2023b)



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- [3] Çalışkan, et al. (2023a)
 [4] Çalışkan, Anil Kumar, et al. (2023b)

Optical Depth and Detection Probability

Optical Depth and Probability

$$\tau(\boldsymbol{\theta}^S = \{z_S, \dots\}) = \int_0^{z_S} dz_L \int_{M_L^{\min}}^{M_L^{\max}} dM_L \frac{4\pi M_L D_{LS}}{D_L D_S} y_{\text{cr}}^2(M_L | \boldsymbol{\theta}^S) n[M_{200}, z_L, z_S] \chi^2(z_L) \frac{d\chi(z_L)}{dz_L} \frac{dM_{200}}{dM_L}$$

$$P = 1 - e^{-\tau(\boldsymbol{\theta}^S = \{z_S, \dots\})}$$

[4] Çalışkan, Anil Kumar, et al. (2023b)
 [2] Schneider et al. (1992)

Optical Depth and Probability

$$\tau(\boldsymbol{\theta}^S = \{z_S, \dots\}) = \int_0^{z_S} dz_L \int_{M_L^{\min}}^{M_L^{\max}} dM_L \frac{4\pi M_L D_{LS}}{D_L D_S} \left[y_{cr}^2(M_L | \boldsymbol{\theta}^S) \right] n[M_{200}, z_L, z_S] \chi^2(z_L) \frac{d\chi(z_L)}{dz_L} \frac{dM_{200}}{dM_L}$$

The equation is annotated with labels and arrows:

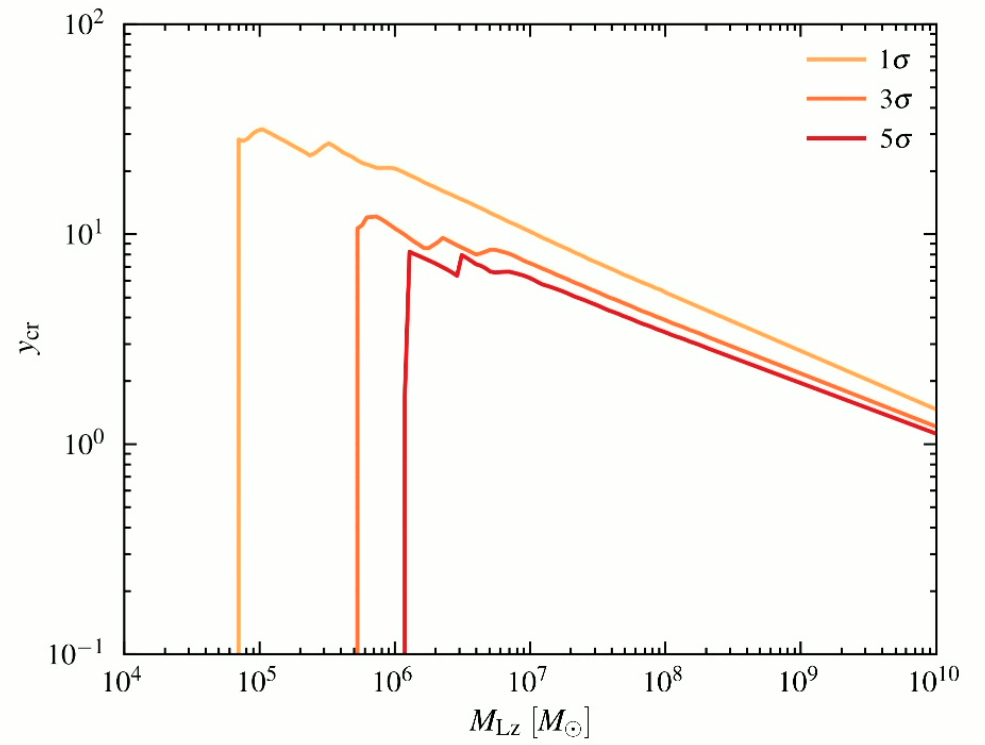
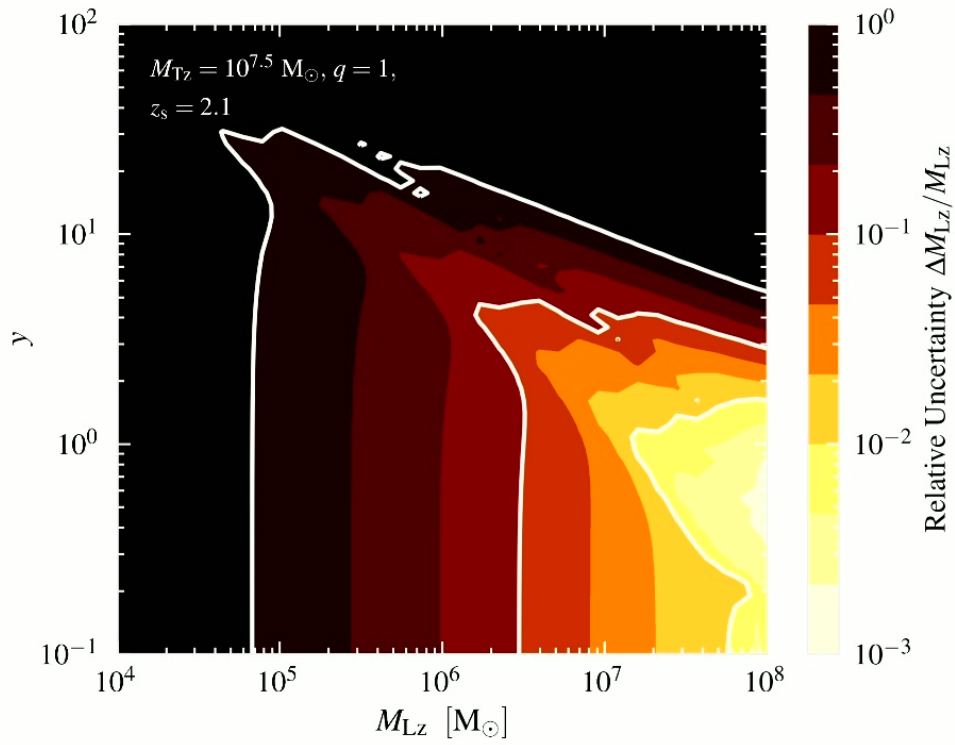
- Set of Source Parameters** points to $\boldsymbol{\theta}^S = \{z_S, \dots\}$.
- Source Redshift** points to z_S .
- Lens Redshift** points to dz_L .
- Lens Mass** points to dM_L .
- Critical Impact Parameter** points to $y_{cr}^2(M_L | \boldsymbol{\theta}^S)$.
- Comoving Number Density of Halos** points to $n[M_{200}, z_L, z_S]$.
- Comoving Distance** points to $\chi^2(z_L)$.
- Halo Virial Mass** points to $\frac{dM_{200}}{dM_L}$.

Strong Lensing: $y_{cr}^{SL} = 1$

Wave-Optics: $y_{cr}^{WO} \in [10, 100]$

[4] Çalışkan, Anil Kumar, et al. (2023b)

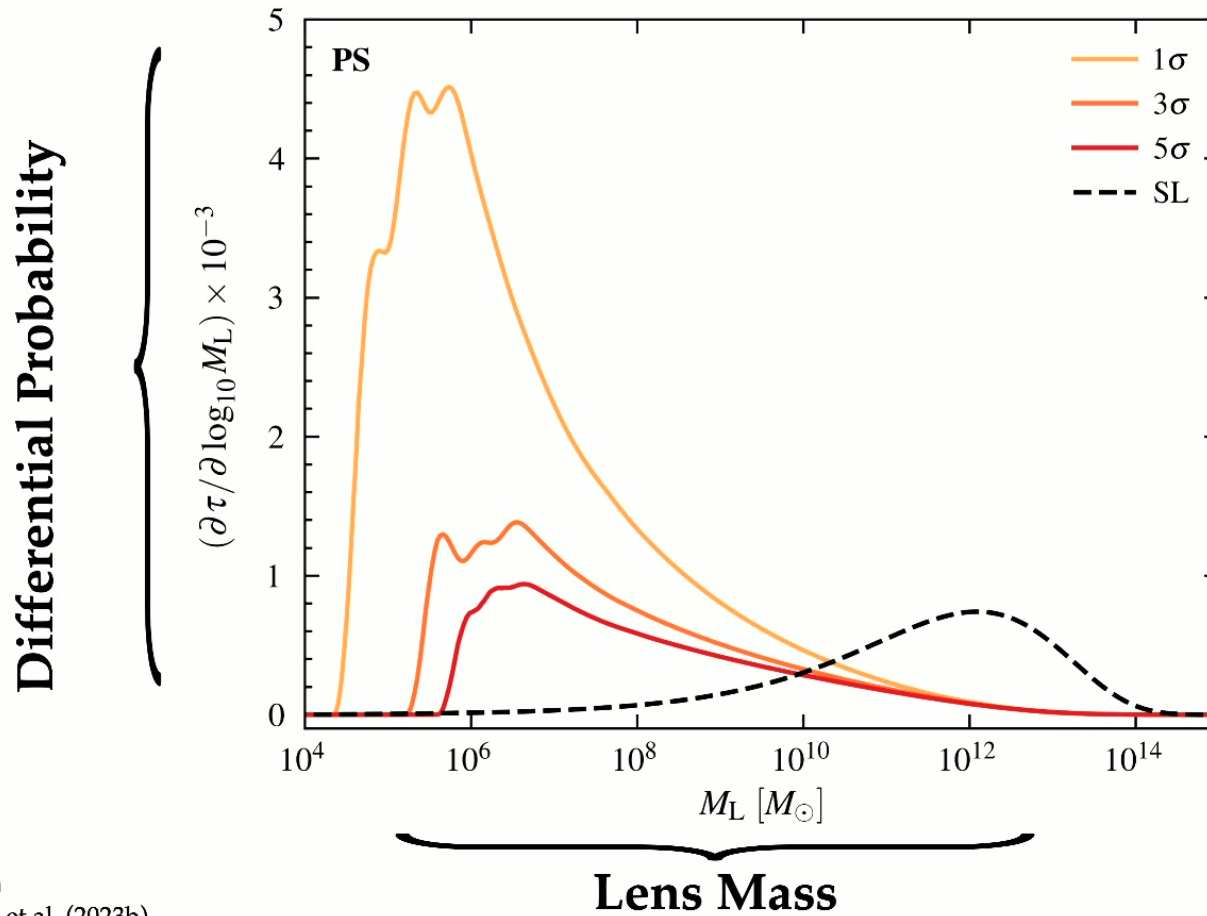
[2] Schneider et al. (1992)



[3] Çalışkan, et al. (2023a)

[4] Çalışkan, Anil Kumar, et al. (2023b)

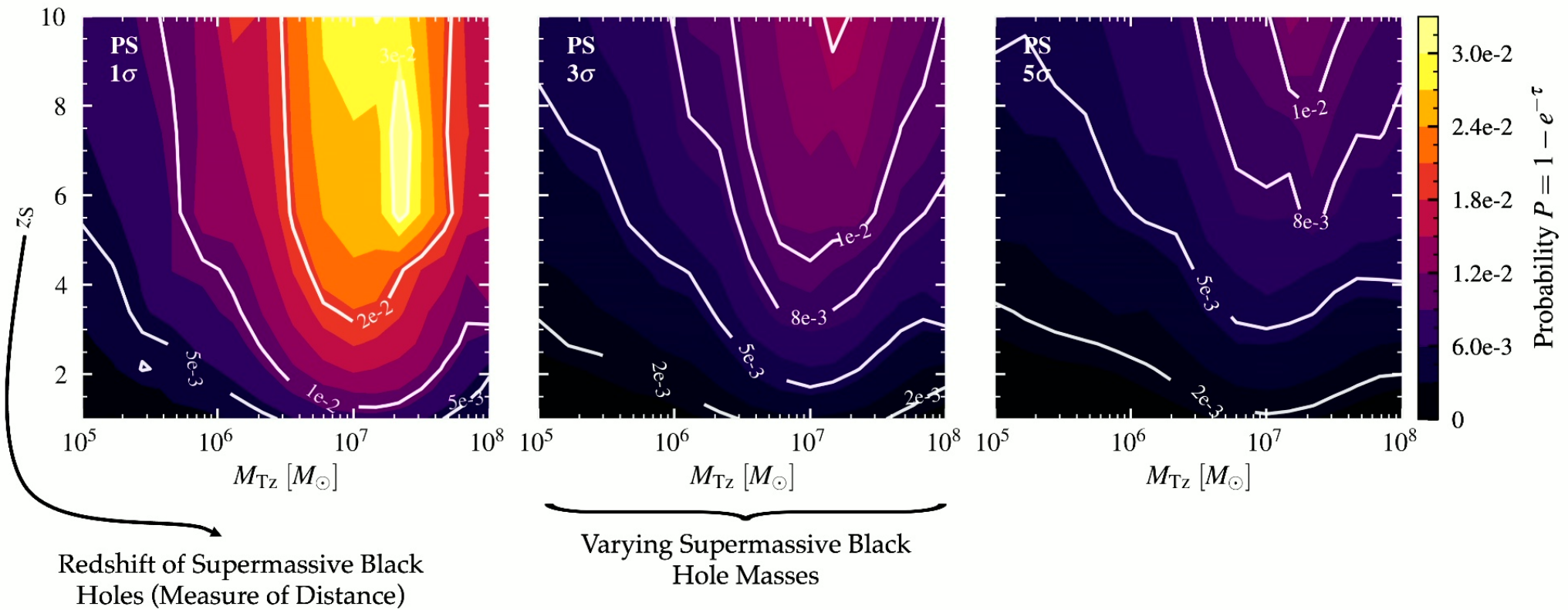
Differential Optical Depth (Probability)



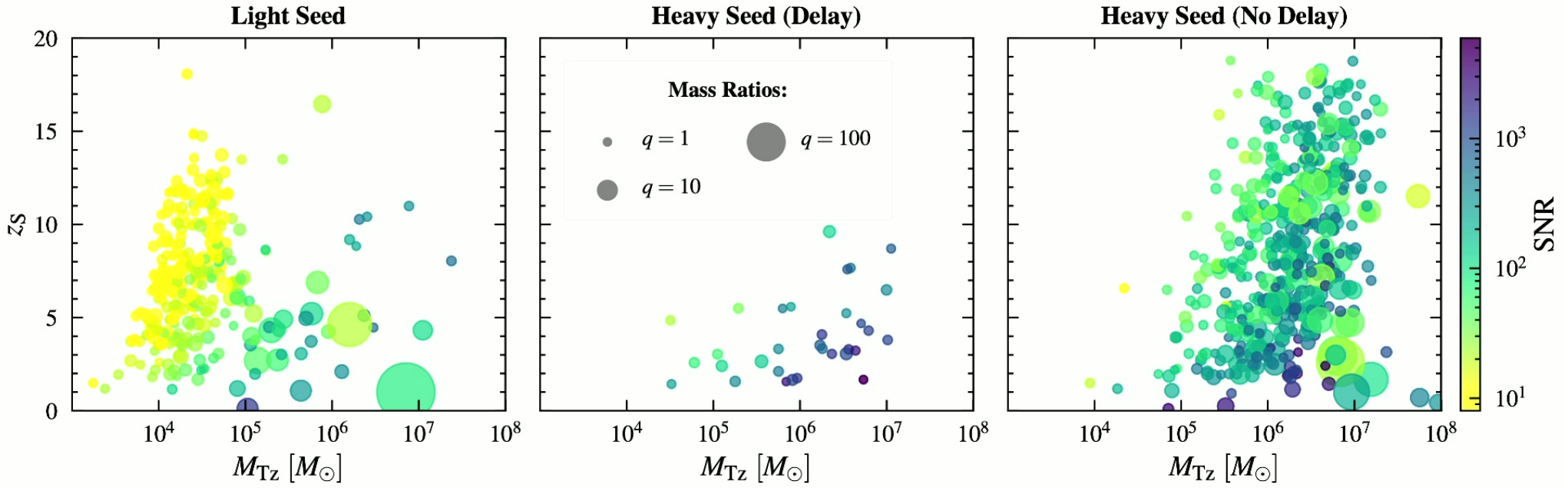
[3] Çalışkan, et al. (2023a)

[4] Çalışkan, Anil Kumar, et al. (2023b)

Detection Probability (Source Model Agnostic)



[4] Çalışkan, Anil Kumar, et al. (2023b)



[4] Çalışkan, Anil Kumar, et al. (2023b)

[11] Barausse (2012)

[12] Klein, Barausse, et al. (2016)

Total Number
of Binaries in
the Population

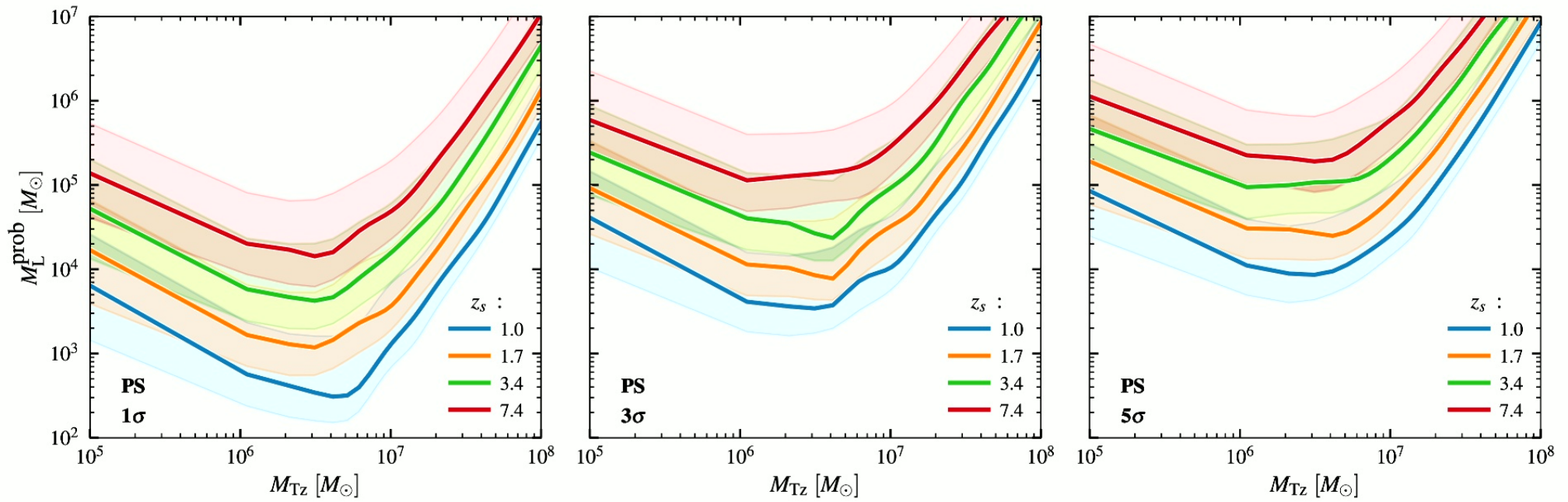
Expected Number of Lensed
Binaries with WO Effects (for
Varying Detection Thresholds)

Resulting Lensing Rate

Source Population	Lens Population	N_{detect}	$N_{\text{lensed}}^{1\sigma}$	$N_{\text{lensed}}^{3\sigma}$	$N_{\text{lensed}}^{5\sigma}$	Lensing Rate $\{1\sigma, 3\sigma, 5\sigma\}$ [%]
Heavy Seed (No Delay)	PS	474	7.96	4.13	3.24	{1.68, 0.87, 0.68}
Heavy Seed (No Delay)	MVF	474	0.42	0.36	0.35	{0.09, 0.07, 0.07}
Heavy Seed (Delay)	PS	32	0.47	0.21	0.15	{1.47, 0.65, 0.47}
Heavy Seed (Delay)	MVF	32	0.01	0.009	0.008	{0.03, 0.03, 0.03}
Light Seed	PS	282	1.51	0.57	0.37	{0.53, 0.20, 0.13}
Light Seed	MVF	282	0.02	0.01	0.01	{0.007, 0.004, 0.004}

[4] Çalışkan, Anil Kumar, et al. (2023b)

Detectable Halo Mass Range



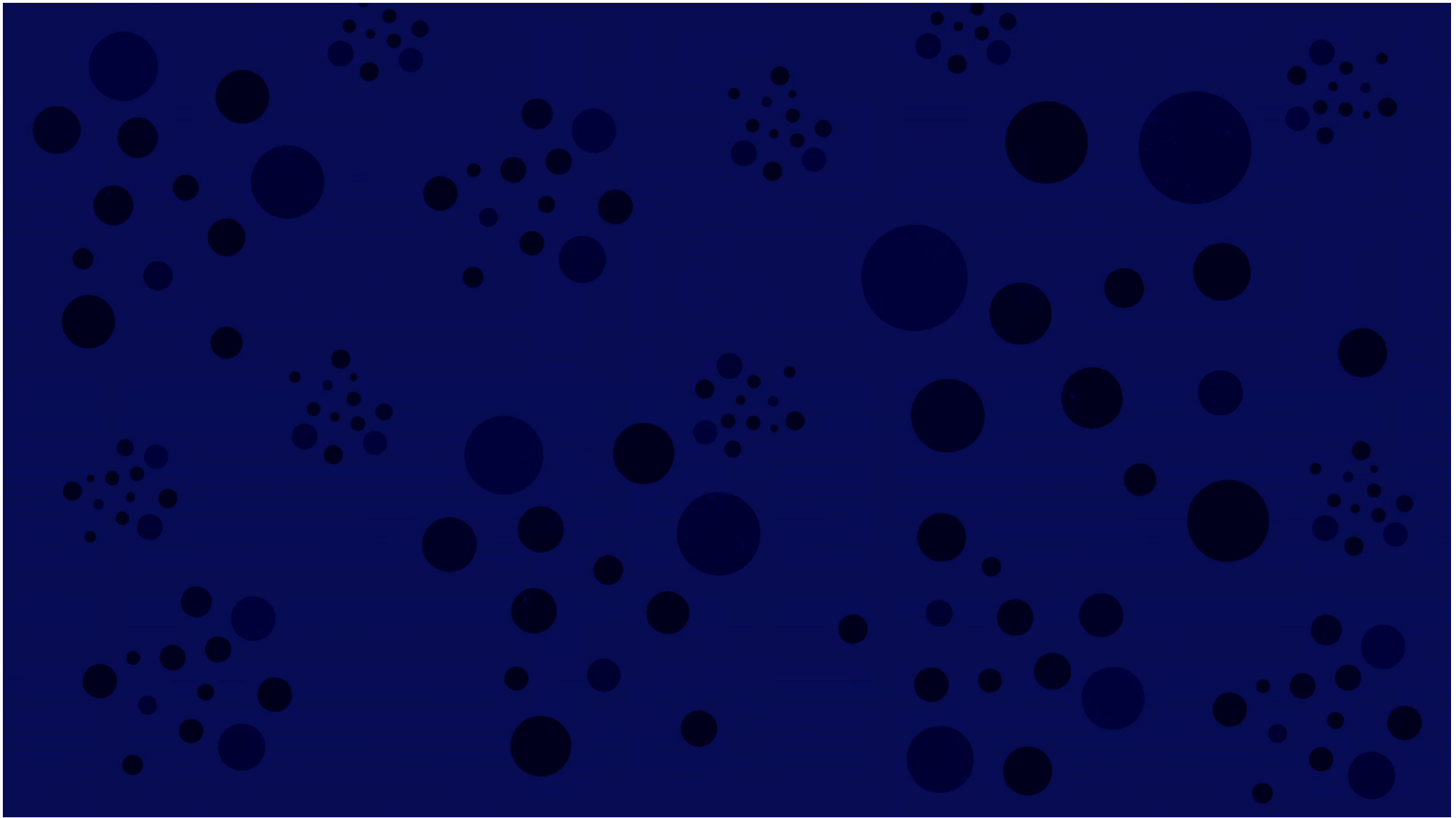
[4] Çalışkan, Anil Kumar, et al. (2023b)

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LIMITATIONS

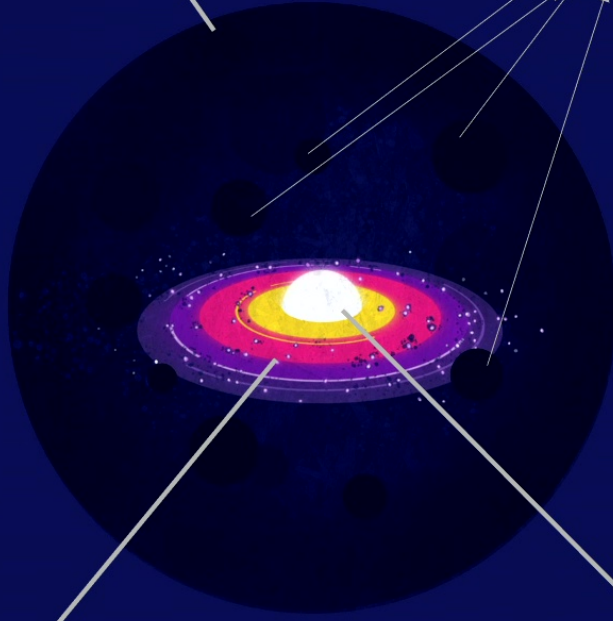
Navarro–Frenk–White?

$$P_{\text{NFW}} \lesssim \mathcal{O}(10^{-1}) P_{\text{SIS}}$$



Dark Matter
Halo

Subhalos



Galaxy

SMBH

Illustration by Neha Anil Kumar

Differing Profiles?

How Lensed Gravitational Waves Can Illuminate Dark Matter*

Mesut Çalışkan^{1,α},

Neha Anil Kumar, Lingyuan Ji, Marc Kamionkowski, Emanuele Berti et al.

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Perimeter Institute – 25 Nov 2024

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*[arXiv: 2201.04619](https://arxiv.org/abs/2201.04619), [2206.02803](https://arxiv.org/abs/2206.02803), [2307.06990](https://arxiv.org/abs/2307.06990)