Title: Open Quantum Dynamics with Nonlinearly Realized Symmetries.

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Abstract:

In the framework of Non-Equilibrium Field Theory, I will construct the effective influence functional — generator of non-equilibrium correlation functions — for a mechanical system with degrees of freedom living on a group (e.g. rigid body) interacting with a thermal bath at high temperature. I will derive the constraints on the influence functional following from the group symmetry structure and the DKMS symmetry — generalization of the fluctuation-dissipation theorem. At the linear response level, group symmetry turns out to impose more constraints compared to DKMS. I will illustrate the general formalism with the diffusion in a Fermi gas and exhibit the large-N suppression of the non-linear response. Finally, I will introduce the Universal Bath — the generalization of the Caldeira-Leggett model. It is a dual field theory defined in one extra dimension that reproduces the classical non-equilibrium dynamics of the mechanical system. I will show that in the limit of Ohmic dissipation, when the temperature becomes the only relevant scale at play, the Universal Bath also reproduces the quantum corrections.

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Open quantum dynamics with nonlinearly realized symmetries

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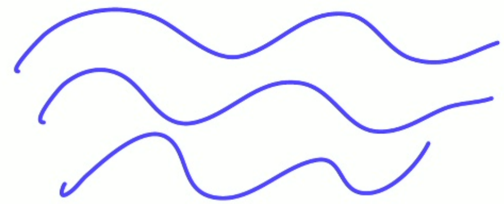
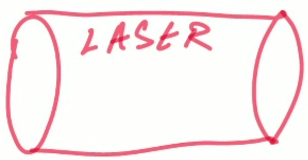
Jury Radkovski
Particle Physics Seminar

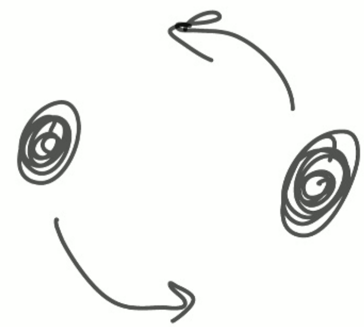
Context



A few areas where effective description of dissipation is used

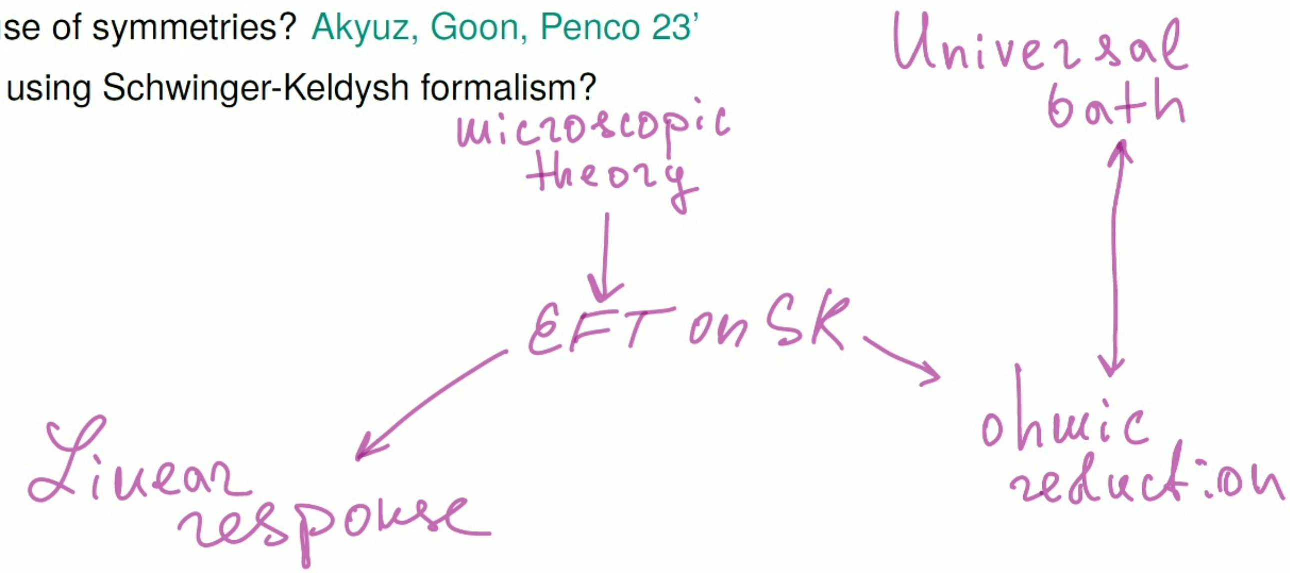
- Cosmology Salcedo, Colas, Pajer 24', Burgess et al. 24'
- Black Holes Goldberger, Rothstein 06', Ivanov, Zhou 22', Jakobsen et al 22'
- Hydrodynamics Liu, Glorioso 18'
- Driven-dissipative systems Kamenev 24', Sieberer 18'





Questions

- Full EFT on the Schwinger-Keldysh contour is very general. What are the possible reductions?
- How to make use of symmetries? Akyuz, Goon, Penco 23'
- Can one avoid using Schwinger-Keldysh formalism?



Outline

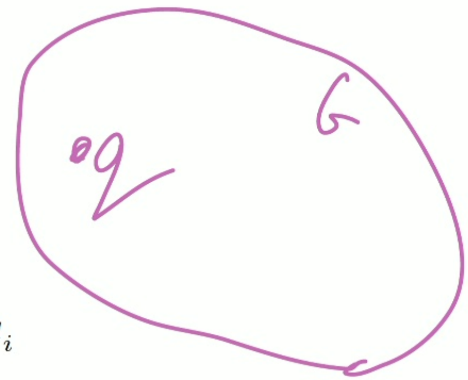
Geometric setup

- Symmetry of choosing the coordinates on the (compact) group G

$$[G_i, G_j] = C^k_{\ ij} G_k$$

- The element of the group and the *Cartan form*

$$g(q) = \mathrm{e}^{q^i G_i} \,, \qquad \Omega = g^{-1} dg = \Omega^i_j(q) dq^j G_i$$



- Invariant dynamics of the system with *covariant derivatives*

$$g(q) \mapsto g(\tilde{q}) = g_L \, g(q) \quad \Longrightarrow \quad S_{\mathsf{sys}} = S_{\mathsf{sys}}[D_t q^i] \equiv S_{\mathsf{sys}}[\Omega^i_j(q)\dot{q}^j]$$

Bringing in the bath

- Total action for the body+bath

$$S_{\rm tot}(q,\chi) = S_{\rm sys}(q) + S_{\rm bath}(\chi) + V(q,\chi)$$

- Interaction breaks the symmetry

$$G \times G \to G \implies V(q,\chi) = U(q)\, V_\chi\, U^\dagger(q)$$

- Peter–Weyl theorem

$$V(q,\chi) = \sum_{raa'} U^r_{aa'}(q)\mathcal{O}^{\dagger r}_{a'a}(\chi)$$

(basically a Fourier transformation)

Case study

- Group $ISO(2)$ with two translation generators P_X, P_Y and a rotation generator J:

$$[P_X, P_Y] = 0\,, \quad [P_X, J] = -P_Y\,, \quad [P_Y, J] = P_X$$

- Parametrization of the form

$$g(X, Y, \Theta) = \mathrm{e}^{XP_X + YP_Y}\,\mathrm{e}^{\Theta J}\,, \quad g(q) \neq e^{q^i G_i}$$

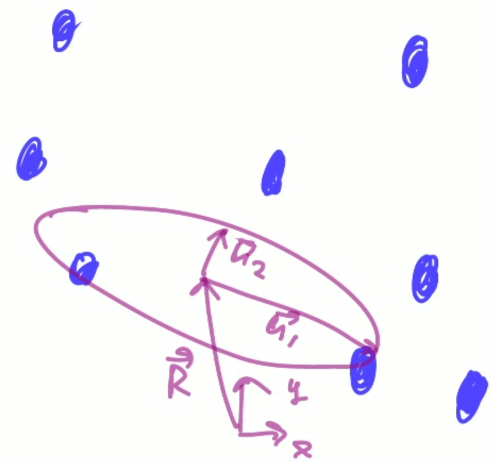
- Covariant derivatives

$$D_t\bar{X} = \dot{X}\cos\Theta + \dot{Y}\sin\Theta\,, \quad D_t\bar{Y} = -\dot{X}\sin\Theta + \dot{Y}\cos\Theta\,, \quad D_t\Theta = \dot{\Theta}$$

- Dynamics of the particle

$$S_{\text{particle}} = \int_t \frac{M}{2}(\dot{X}^2 + \dot{Y}^2) + \frac{\mathcal{I}}{2}\dot{\Theta}^2 = \int_t \frac{M}{2}\left[(D_tX)^2 + (D_tY)^2\right] + \frac{\mathcal{I}}{2}(D_t\Theta)^2$$

Case study



- Bath of free (spinless) fermions

$$S_{\text{bath}} = \int_{t,\mathbf{r}} \psi^\dagger \left(i\partial_t - \frac{\Delta}{2m} \right) \psi$$

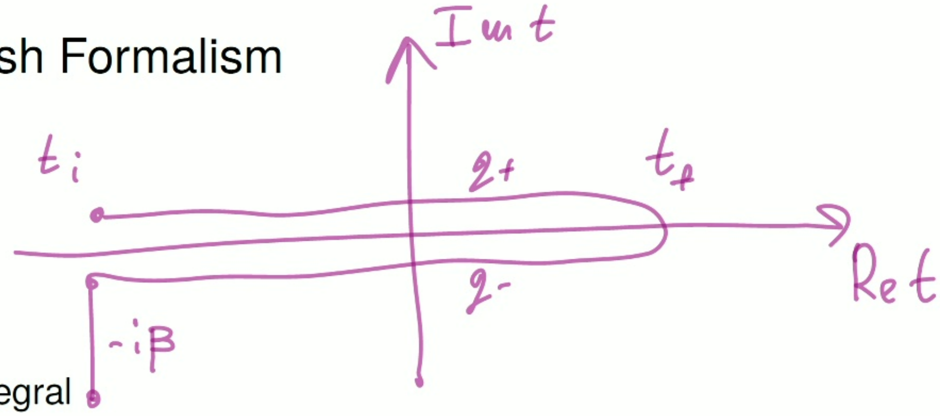
- Interaction of the denisty $\rho \equiv \psi^\dagger \psi$ with a "fluffy" particle

$$V = \int_{t,\mathbf{r}} \rho \mathcal{V} \,, \qquad \mathcal{V} = \frac{\lambda}{l_1 l_2} \int_{-\infty}^{\infty} d\xi_1 d\xi_2 \, e^{-\frac{\xi_1^2}{2l_1^2} - \frac{\xi_2^2}{2l_2^2}} \delta\Big(\mathbf{r} - \mathbf{R} - \xi_i \mathbf{n}_i\Big)$$

- Peter-Weyl theorem:

$$V = \sum_{\mathbf{k}} V_{\mathbf{k}}\big(q(t)\big)\, \rho_{-\mathbf{k}}(t)$$

Schwinger-Keldysh Formalism



- Ordinary path integral

$$Z[J] = \int Dq e^{iS[q]+iJq}, \quad \langle Tq(t_1)q(t_2)\dots\rangle = \frac{\delta}{i\delta J(t_1)}\frac{\delta}{i\delta J(t_2)}\dots Z[J]$$

- Double-time Schwinger 60',61', Keldysh 64'

$$Z[J_\pm] = \int_\rho Dq_+ Dq_- e^{iS[q_+]-iS[q_-]+iJ_+q_+-iJ_-q_-}, \quad \langle q(t_1)q(t_2)\dots\rangle = \frac{\delta}{i\delta J_\pm(t_1)}\frac{\delta}{i\delta J_\pm(t_2)}\dots Z[J_\pm]$$

- Keldysh rotation $q_+, q_- \to \bar{q}, \hat{q}$

$$\bar{q} \sim \text{"}q_+ + q_-\text{"}$$
$$\hat{q} \sim \text{"}q_+ - q_-\text{"}$$

Localization

- High temperature makes excitations gapped:

$$\beta \to 0 \implies \langle \mathcal{O}(t)\mathcal{O}(0)\rangle_{\text{ret}} < e^{-\Lambda t} \quad \Lambda^{-1} - \text{relaxation time}$$

- For Fermi gas exponential decay only after integrating with the potential

$$\int_{\mathbf{k}} e^{-k^2 l^2} \Im \langle \rho_{\mathbf{k}}(t)\rho_{\mathbf{k}}(0)\rangle_{\text{ret}} \sim e^{-t^2 v_F^2/l^2} \implies \Lambda \sim \frac{v_F}{l}$$

- Derivative expansion Nicolis '11

$$\Im(\langle \mathcal{O}(\omega)\mathcal{O}(0)\rangle_{\text{ret}})^r_{ab} \sim \omega\left(\varrho^r_{0,ab} + \frac{\omega^2}{\Lambda^2}\varrho^r_{2,ab} + O\left((\omega/\Lambda)^4\right)\right)$$

Integrating out the bath

- In the partition function integrate over the bath degrees of freedom ($V \sim \sum U(q)\mathcal{O}(\chi)$)

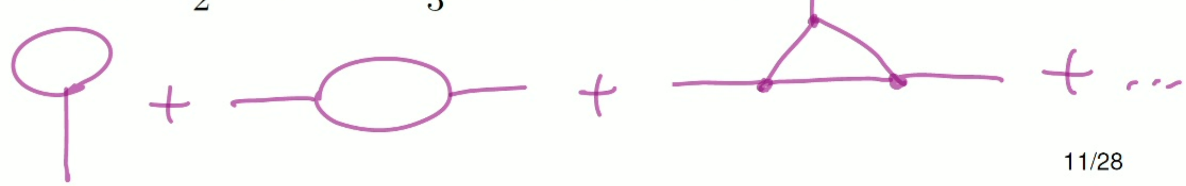
$$e^{i\mathcal{I}[q_+,q_-]} \sim \int_{\rho_\chi} D\chi_+ D\chi_- e^{iS[\chi_+]+iV[q_+,\chi_+]-(+\to-)} \sim 1 + \frac{1}{2}\int_{t_+,t_-}\sum_{\pm} U_\pm U_\pm \langle \mathcal{O}_\pm \mathcal{O}_\pm \rangle_{\rho_\chi} + \dots$$

- For the particle in Fermi gas (Kamenev '24)

$$\mathrm{e}^{i\mathcal{I}[q_+,q_-]} = \int_{\rho_\beta} D\psi D\bar{\psi}\, e^{iS[\psi_+]+iV[q_+,\psi_+]-(+\to-)} \propto \det\left(\mathbf{G}^{-1} - \mathbf{V}\right)$$

- Perturbative expansion

$$i\mathcal{I} = \mathrm{tr}\,\mathbf{GV} - \frac{1}{2}\mathrm{tr}\,\mathbf{GVGV} + \frac{1}{3}\mathrm{tr}\,\mathbf{GVGVGV} + \dots$$



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Influence functional

- Derivative expansion in ω translates into ultralocal expansion in time. Schematically

$$\langle \mathcal{O}_\pm \mathcal{O}_\pm \rangle_{\rho_\chi}(t) \sim \frac{\#}{\beta} \varrho_0 \, \delta(t) + \# \varrho_0 \delta'(t) + \# \varrho_0 \theta(t) \delta'(t) + \ldots$$

- Influence functional

$$\mathcal{I}[q_+, q_-] \sim \frac{1}{2} \int_{t_+, t_-} \sum_\pm U(q_\pm) U(q_\pm) \langle \mathcal{O}_\pm \mathcal{O}_\pm \rangle_{\rho_\chi}$$

- Covariant classical-quantum split

$$\mathrm{e}^{q_\pm G} = \mathrm{e}^{\bar{q} G} \mathrm{e}^{\pm \hat{q} G} \implies \mathcal{I} = \mathcal{I}(\hat{q}, \dot{\hat{q}}, D_t \bar{q})$$

Gaussian Order

- Influence functional at Gaussian order

$$\mathcal{I}[\bar{q},\hat{q}]\Big|_{\text{Gauss}} = \int dt\left(\frac{4i}{\beta}\gamma_{ij}\hat{q}^i\hat{q}^j - 2\gamma_{ij}\hat{q}^i D_t\bar{q}^j\right)$$

- Hubbard-Stratonovich transformation

$$e^{-4\int_t \frac{\gamma}{\beta}\hat{q}^2} = \int D\xi\, e^{-\int_t \frac{\beta}{4\gamma}\xi^2 - 2i\xi\hat{q}}$$

- Langevin equation (for simplicity, for one d.o.f.)

$$\langle O(\bar{q})\rangle = \int D\xi e^{-\frac{\beta}{4\gamma}\xi^2}\int D\bar{q}D\hat{q}O(\bar{q})e^{-2i\int_t \hat{q}(\ddot{\bar{q}}+\gamma\dot{\bar{q}}-\xi)} = \int D\xi e^{-\frac{\beta}{4\gamma}\xi^2}\int D\bar{q}D\hat{q}O(\bar{q})\delta\Big(\ddot{\bar{q}}+\gamma\dot{\bar{q}}-\xi\Big)$$

$$\ddot{\bar{q}} + \gamma\dot{\bar{q}} = \xi(t), \quad \langle\xi(t)\xi(t')\rangle = 2\gamma T\delta(t-t')$$

Gaussian order

- Influence functional at Gaussian order (once again)

$$\mathcal{I}[\bar{q},\hat{q}]\Big|_{\rm Gauss} = \int dt \left(\frac{4i}{\beta}\gamma_{ij}\hat{q}^i\hat{q}^j - 2\gamma_{ij}\hat{q}^i D_t\bar{q}^j \right)$$

- Dissipative coefficients in terms of a function on the group

$$\gamma_{ij} = -\pi\,\partial_i\partial_j\sigma_0(0)\,, \quad \sigma_0(q) \equiv \sum_{rab} U^r_{ab}(q)\varrho^r_{0,ab}$$

- Power counting for $\hat{q}$

$$\frac{T}{\hbar^2}\frac{\gamma\hat{q}^2}{\omega} \sim 1 \quad \Longrightarrow \quad \hat{q} \sim \hbar\sqrt{\frac{\omega}{\gamma T}}$$

- Power counting for $\bar{q}$ (in the overdamped regime)

$$\frac{\gamma\hat{q}\bar{q}}{\hbar} \sim 1 \quad \Longrightarrow \quad \bar{q} \sim \sqrt{\frac{T}{\gamma\omega}} \Longrightarrow \bar{q} \sim \sqrt{t}$$

Brownian motion

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NNLO corrections

- Next to next to leading order

$$\mathcal{I}[\bar{q},\hat{q}]\Big|_{\mathrm{NNLO}} = \int_t \left[i\frac{4}{3\beta}\mu_{ijkl}\hat{q}^i\hat{q}^j\hat{q}^k\hat{q}^l + \left(-\frac{4}{3}\mu_{ijkl} + \frac{1}{3}\gamma_{im}C^m_{\ jn}C^n_{\ kl}\right)\hat{q}^i\hat{q}^j\hat{q}^k D_t\bar{q}^l - \gamma_{im}C^m_{\ jk}\hat{q}^i\hat{q}^j\dot{\hat{q}}^k \right]$$

with new couplings

$$\mu_{ijkl} = -\pi\,\partial_i\partial_j\partial_k\partial_l\sigma_0(0)$$

- Power counting

$$\frac{1}{\hbar}\int dt\,\frac{\mu}{\beta}\hat{q}^4 \sim \frac{1}{\hbar}\int dt\,\mu\hat{q}^3 D_t\bar{q} \sim \frac{\mu}{\gamma}\cdot\frac{\hbar^2\omega}{\gamma T}\,, \qquad \frac{1}{\hbar}\int dt\,\gamma\hat{q}^2\dot{\hat{q}} \sim \frac{\hbar^2\omega^{3/2}}{\gamma^{1/2}T^{3/2}}$$

Higher derivative corrections to the noise

- Higher derivative corrections to the noise

$$\Delta\mathcal{I}_{\rm noise} = i\frac{4}{\beta\Lambda^2}\int_t \left\{ \left[-\check{\mu}_{2,ijkl} + \text{"}\check{\gamma}CC\text{"} \right] \hat{q}^i \hat{q}^j D_t\bar{q}^k D_t\bar{q}^l + \left[\text{"}\check{\gamma}CC\text{"} \right] \hat{q}^i \dot{\hat{q}}^j D_t\bar{q}^k + \check{\gamma}_{2,ij}\dot{\hat{q}}^i \dot{\hat{q}}^j \right\}$$

where

$$\check{\gamma}_{2,ij} = \gamma_{2,ij} + \frac{(\beta\Lambda)^2}{12}\gamma_{ij} \ , \qquad \gamma_{2,ij} = -\pi\partial_i\partial_j\sigma_2(0)$$

$$\check{\mu}_{2,ijkl} = \mu_{2,ijkl} + \frac{(\beta\Lambda)^2}{12}\mu_{ijkl} \ , \qquad \mu_{2,ijkl} = -\pi\partial_i\partial_j\partial_k\partial_l\sigma_2(0)$$

- As important as the non-Gaussian contributions whenever $\beta\Lambda >> 1$, and typically larger whenever $\beta\Lambda << 1$

Higher derivative corrections to the dissipation

- Correction to the dissipation

$$\Delta\mathcal{I}_{\text{diss}} = \frac{1}{\Lambda^2}\int_t \Big\{ \big[2\mu_{2,ijkl} + \text{"}\gamma_2 CC\text{"}\big]\hat{q}^i D_t\bar{q}^j D_t\bar{q}^k D_t\bar{q}^l + \big[\text{"}\gamma_2 CC\text{"}\big]\hat{q}^i D_t\bar{q}^j \partial_t D_t\bar{q}^k + 2\gamma_{2,ij}\hat{q}^i\partial_t^2 D_t\bar{q}^j \Big\}$$

- Suppressed only by Λ
- Modifies the classical equations of motion with terms of cubic order in velocities and terms with velocity derivatives in the friction force Van Kampen 86', Plyukhin 03'

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DKMS

- DKMS symmetry Kubo 57', Martin, Schwinger 59':

$$q'^j_+(t) = q^j_+(-t + i\beta/2)\ , \qquad q'^j_-(t) = q^j_-(-t - i\beta/2)$$

- At high temperature the relations localize

$$\bar{q}'^j(t) = \bar{q}^j(t') + \frac{i\beta}{2}\dot{\hat{q}}^j(t') + \dots\Big|_{t'=-t}, \quad \hat{q}'^j(t) = \hat{q}^j(t') + \frac{i\beta}{2}\dot{\bar{q}}^j(t') + \dots\Big|_{t'=-t}$$

- Take $\mathcal{I} = \int_t \beta^{-1} a_{ij}\hat{q}^i\hat{q}^j + b_{ij}\hat{q}^i D_t\bar{q}^j + \dots$
- At Gaussian order recover the *fluctuation-dissipation theorem*:

$$\mathcal{I} = \int_t \left(\beta^{-1} 4i\gamma_{ij}\hat{q}^i\hat{q}^j - 2\gamma_{ij}\hat{q}^i D_t\bar{q}^j\right)$$

Ohmic reduction

- At higher orders

$$\mathcal{I} = \int_t \Big(\cdots + \beta^{-1} A_{ijkl} \hat{q}^i \hat{q}^j \hat{q}^k \hat{q}^l + B_{ijkl} \hat{q}^i \hat{q}^j \hat{q}^k D_t \bar{q}^l$$
$$+ \beta D_{ijkl} \hat{q}^i \hat{q}^j D_t \bar{q}^k D_t \bar{q}^l + \beta^2 E_{ijkl} \hat{q}^i D_t \bar{q}^j D_t \bar{q}^k D_t \bar{q}^l + \text{other terms linear in } \hat{q} \Big),$$

- The symmetry relates coefficients of different terms, for example

$$B_{ijkl} = i A_{ijkl} + \text{"}\gamma CC\text{"}\,, \quad E_{ijkl} = \frac{i}{2} D_{i(jkl)} + \frac{i}{8} A_{ijkl} + \text{"}\gamma CC\text{"}$$

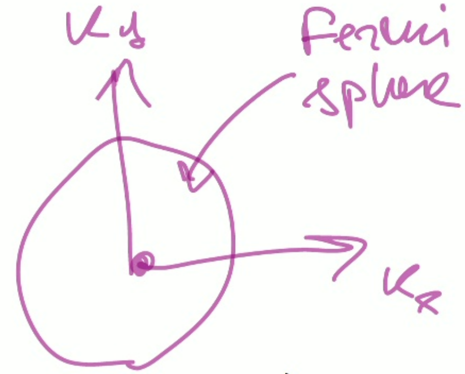
- Ohmic reduction: set to 0 higher order terms linear in $\hat{q} \implies A, B, D$ expressed in terms of one tensor ν !

$$\nu_{ijkl} = \nu_{(ij)(kl)} = \nu_{(kl)(ij)}$$

Spherical particle in the Fermi gas

- Gaussian noise

$$\mathcal{I}\Big|_{\text{Gauss, noise}} \propto N g^2 \int_t \frac{4i}{\beta}\hat{\mathbf{R}}^2\,, \quad N \sim k_F l$$



- Higher derivative correction to the noise

$$\Delta\mathcal{I}_{\text{noise}} \propto \frac{i}{\beta\Lambda^2} N g^2 \int dt \left\{ \frac{1}{2}\left(1+\frac{\beta^2\Lambda^2}{4}\right)\left(\hat{\mathbf{R}}^2\dot{\hat{\mathbf{R}}}^2 + 2(\hat{\mathbf{R}}\dot{\hat{\mathbf{R}}})^2\right) + \left(4+\frac{\beta^2\Lambda^2}{3}\right)\dot{\hat{\mathbf{R}}}^2 \right\}$$

- Symmetry of μ_{ijkl}

$$\hat{\mathbf{R}}^2\dot{\hat{\mathbf{R}}}^2 + 2(\hat{\mathbf{R}}\dot{\hat{\mathbf{R}}})^2 = \left(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right)\hat{R}^i\hat{R}^j\dot{\hat{R}}^k\dot{\hat{R}}^l$$

- Beyond linear response

$$\Delta\mathcal{I}_{\text{hd, fluct}} \propto \frac{i}{\beta\Lambda^2} g^3 \int dt \left\{ \frac{5\beta^2\Lambda^2}{18}\left(\hat{\mathbf{R}}^2\dot{\hat{\mathbf{R}}}^2 + 2(\hat{\mathbf{R}}\dot{\hat{\mathbf{R}}})^2\right) + \frac{3}{2}\left(7\hat{\mathbf{R}}^2\dot{\hat{\mathbf{R}}}^2 - 2(\hat{\mathbf{R}}\dot{\hat{\mathbf{R}}})^2\right) + \ldots \right\}$$

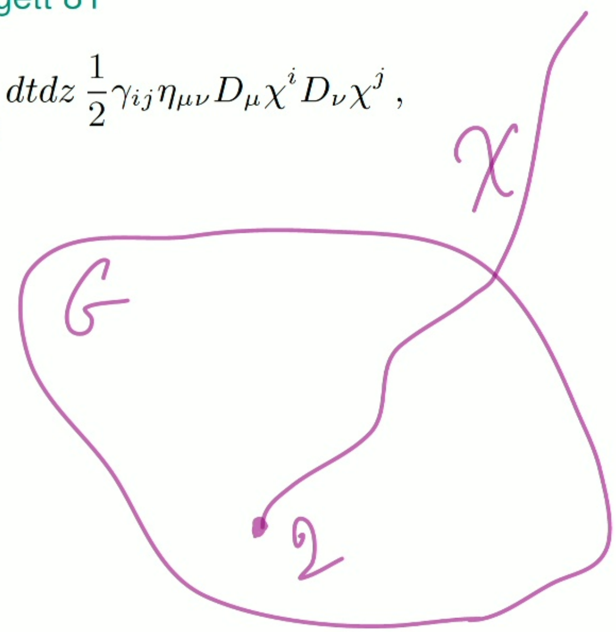
Classical Model

- Classical model Besharat, JR, Sibiryakov 23', Caldeira-Leggett 81'

$$S[q,\chi] = S_{\text{sys}}[q] + S_\sigma[\chi;q] = \int dt L(D_t q) + \int_{z>0} dt dz \, \frac{1}{2}\gamma_{ij}\eta_{\mu\nu}D_\mu\chi^i D_\nu\chi^j \,,$$

with fields χ satisfying the boundary condition

$$\chi^i\Big|_{z=0} = q^i$$



- Equations of motion in the bulk have a solution

$$\chi^i(t,z) = \chi^i(t-z)$$

- Equations of motion on the boundary

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}^i} - \frac{\partial L}{\partial q^i} = \gamma_{kl}\Omega_i^k(\chi)\Omega_n^l(\chi)\partial_z\chi^n\Big|_{z=0} = -\gamma_{kl}\Omega_i^k(q)\Omega_n^l(q)\dot{q}^n$$

Conclusions and Outlook

Conclusions

- Effective description of the dissipative dynamics for a mechanical system with symmetry G
- Linear response + $G >$ DKMS
- Large-N suppression of the beyond linear response
- Dual description for the ohmic reduction

Outlook

- Coset construction
- Zero temperature
- Field theory
- Computing observables via the bulk
- Applications (mechanics, hydrodynamics, dissipative CFTs....)