Title: Quantization of the universal centralizer and central D-modules

Speakers: Tom Gannon

Collection/Series: Mathematical Physics

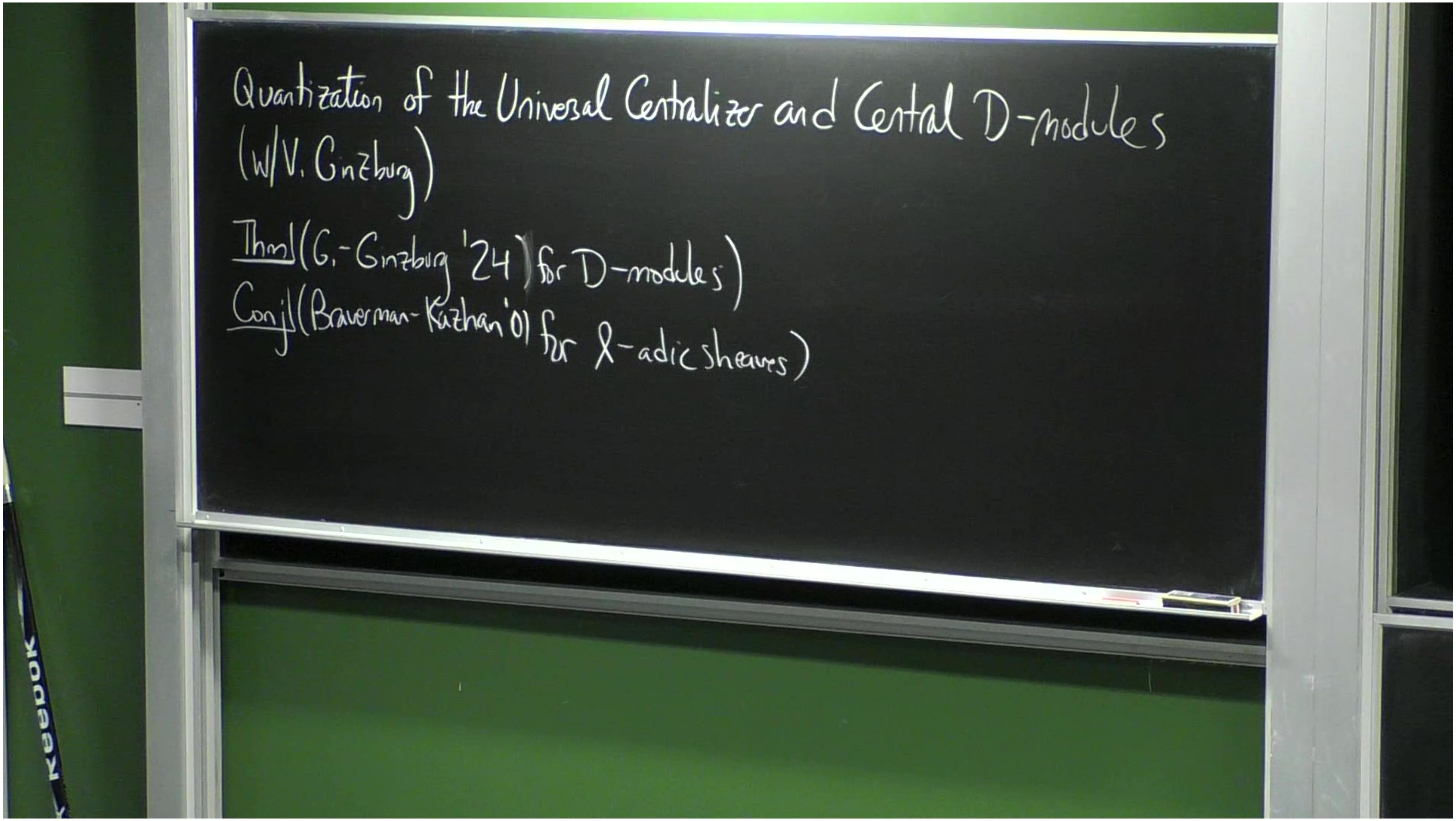
Subject: Mathematical physics

Date: November 25, 2024 - 1:00 PM

URL: https://pirsa.org/24110086

Abstract:

We will discuss some aspects of my recent preprint, joint with Victor Ginzburg, on Kostant-Whittaker reduction, a (deformation) quantization of restriction to a Kostant slice. We will explain how this functor can be used to prove conjectures of Ben-Zvi and Gunningham on parabolic induction, as well as a convolution exactness conjecture of Braverman and Kazhdan in the D-module setting. While this talk will occasionally reference facts from a talk I gave at Perimeter on other aspects of this preprint, the overlap and references will be minimal.


Quantization of the Universal Centralizer and Central D-modules
(w/ V. Ginzburg)
Thm (G.-Ginzburg '24 for D-modules)
Conj (Braverman-Kazhdan '01 for ℓ-adic sheaves)
Reebok

Thm (G. Ginzburg '24 for D-modules, [illegible] with the gamma

Conj (Braverman-Kazhdan '01 for ℓ-adic sheaves) [illegible] $\Phi_\rho \star (-)$ is t-exact

i.e. it induces an exact functor

on the category of perverse sheaves on a reductive group.

Def An algebra object in a monoidal category $\mathcal{C}$ is an object $A \in \mathcal{C}$ with maps $A \otimes A \xrightarrow{m} A$ and a unit map $\mathbb{1}_{\mathcal{C}} \to A$ satisfying the algebra axioms.

<u>Def</u>] An <u>algebra object</u> in a monoidal category $\mathcal{C}$ is an object $A \in \mathcal{C}$ with maps $A \otimes A \xrightarrow{m} A$ and a unit map $\mathbb{1}_{\mathcal{C}} \to A$ satisfying the algebra axioms.

<u>Ex</u>] An algebra object in the category $\mathrm{Vect}_{\mathbb{C}}, \otimes_{\mathbb{C}}$ is a $\mathbb{C}$-alg, more generally, an algebra object in the category of R-mod, $\otimes_R$ for R a comm. ring is an R-alg.

<u>Rmk</u>] In any braided monoidal category ($\Rightarrow$ have a swap map $X \otimes Y \xrightarrow{\sim} Y \otimes X$), an alg. object is <u>commutative</u> if $m \circ (\text{swap}) = m$.

Ex] Comm. alg obj in R-mod $\longleftrightarrow$ Affine Scheme over $\operatorname{Spec} R \longleftarrow \operatorname{Spec} A$

Ex] If $\mathcal{C} = \operatorname{Sym}\mathfrak{g}\text{-mod}(\operatorname{Rep} G) = \operatorname{QCoh}^G(\mathfrak{g}^*) = \operatorname{QCoh}(\mathfrak{g}^*/G)$ for G a complex reductive group (GL_n, SL_n, SO_n). A comm. alg. object of $\operatorname{Sym}\mathfrak{g}\text{-mod}(\operatorname{Rep} G)$ is $G \curvearrowright \operatorname{Spec} A \xrightarrow{\mu} \mathfrak{g}^*$ st. μ G-equiv. (Eg $G \curvearrowright \operatorname{Spec} A_0$, $T^*(\operatorname{Spec} A_0)$ gives an alg obj in this category.)

... G-equiv. (Eg $G \curvearrowright \operatorname{Spec} A_0$, $T^*(\operatorname{Spec} A_0)$ gives an alg obj in this category.)

<u>Thm</u> (Braverman–Finkelberg–Nakajima '17) There exists, for any reductive group G and rep'n N, an algebra object $\mathcal{R}_{G,N} \in D_{G_\mathcal{O}}(Gr_G) =$

..., an algebra object $K_{G,N} \in D_{G_\mathcal{O}}(Gr_G) =$

$D_{G(\mathbb{C}[[t]])}(G(\mathbb{C}((t)))/G(\mathbb{C}[[t]]))$ whose coh'ly

$H^*_{G_\mathcal{O}} : D_{G_\mathcal{O}}(Gr_G) \longrightarrow H^*_{G_\mathcal{O}}(pt)\text{-mod} \cong H^*_G(pt)\text{-mod}$ gives the

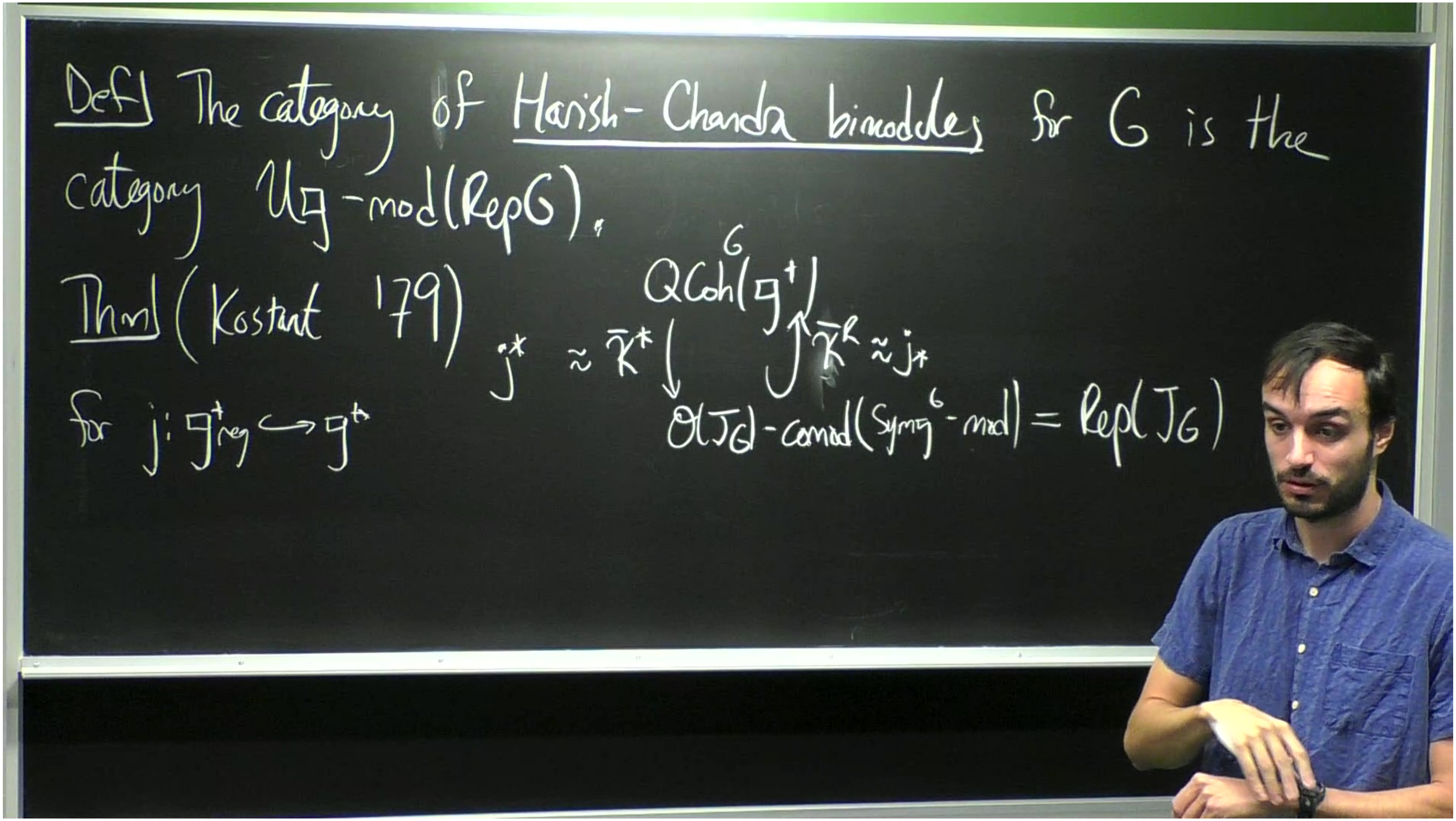
monoidal | algebra of functions on $\mathcal{M}_C(G,N)$.

<u>Thm</u> (Bezrukavnikov – Finkelberg '09)

$$\begin{array}{ccc} D_{G^\vee_\mathcal{O}}(Gr_{G^\vee}) & \overset{\text{monoidal}}{\cong} & QCoh^G(\mathfrak{g}^*) \\ \downarrow H^*_{G_\mathcal{O}}(-) & & \downarrow \kappa^* \\ H^*_G(pt)\text{-mod} & \cong & QCoh(S) \end{array}$$

Coh'l shift suppressed

such that if $S \cong Spec(Sym\,\mathfrak{g}^G) \subseteq \mathfrak{g}^*$ is the <u>Kostant slice</u>


Def] The category of Harish-Chandra bimodules for G is the category $U\mathfrak{g}$-mod(Rep G).
Thm] (Kostant '79)
$QCoh^G(\mathfrak{g}^*)$
$j^* \approx \bar{\kappa}^*$
$\bar{\kappa}^R \approx j_*$
$\mathcal{O}(J_G)$-comod(Sym $\mathfrak{g}^G$-mod) = Rep(J_G)
for $j: \mathfrak{g}^*_{reg} \hookrightarrow \mathfrak{g}^*$

for $j: \mathfrak{g}^*_{reg} \hookrightarrow \mathfrak{g}^*$ $\qquad$ $\mathcal{O}(J_G)\text{-comod}(\mathrm{Sym}\,\mathfrak{g}^G\text{-mod}) = \mathrm{Rep}(J_G)$

Thm (G.–Ginzburg) There is a $Z\mathfrak{g} = (U\mathfrak{g})^G$-coalgebra structure on the algebra of bi-Whittaker diff'l operators

Thm (G. Ginzburg) There is a $Z\mathfrak{g} = (U\mathfrak{g})$ - coalgebra structure on the algebra of bi-Whittaker diff'l operators $\mathcal{W}$ making

$$\begin{array}{c} HC_G := U\mathfrak{g}\text{-mod}(\mathrm{Rep}\,G) \\ \text{monoidal} - \kappa \downarrow \quad \uparrow \kappa^R \\ \mathcal{W}\text{-comod}(Z\mathfrak{g}\text{-mod}) \end{array}$$

Conj (Langlands '71) Automorphic L-functions $L(s, \pi, \rho)$ admits analytic continuation and is functional in ρ.

ρ a rep'n of Langlands dual gp

(Tate's Thesis: $G = GL_1 \curvearrowright \mathbb{A}^1$
Godement - Jacquet $G = GL_n \curvearrowright \mathbb{A}^n$)

An algebra object in a monoidal category $\mathcal{C}$ is an

Braverman-Kazhdan ('99, '01) Verify analytic continuation directly using some ρ-Fourier transform, pose a certain ℓ-adic sheaf, the gamma sheaf $\Phi_\rho \in D^\ell(G_{\mathbb{F}_q})^{G_{\mathbb{F}_q}}$, and posed two conjectures on

Def (Braverman-Kazhdan 199) $\rho: G^\vee \to GL_n$,

$\Phi_\rho := \mathrm{ind}_T^G\left(\Phi_{\rho|_{T^\vee}}\right)^W$.

Defn (Braverman - Kazhdan '99) GL_n,

$$\Phi_\rho := \mathrm{ind}_T^G\left(\Phi_{\rho|_{T^\vee}}\right)^W.$$

Thm (G.-Ginzburg '24)

Conj (BZ-G. '17)

$$\begin{array}{ccc} \mathcal{W}\text{-mod} & \xrightarrow{\text{Gin, Lo, G.}} & \mathcal{D}(T)^W\text{-mod} \\ \cong & & \downarrow\wr \\ (\mathcal{D}(T) \rtimes W)\left[\frac{1-s_\alpha}{\alpha^\vee}\right]\text{-mod} & \hookrightarrow & (\mathcal{D}(T)\rtimes W)\text{-mod} \xrightarrow{\mathrm{ind}(-)^W} \mathcal{D}(G)\text{-mod} \end{array}$$

Def (Braverman - K...

$$\Phi_\rho := \mathrm{ind}_T^G\left(\Phi_{\rho|_{T^\vee}}\right)^W.$$

Thm (G.-Ginzburg '24)

Conj (BZ-G. '17)

$$\begin{tikzcd}
W\text{-mod} \arrow[r, leftrightarrow, "\substack{\text{Gin, Lo,}\\ G.}"] \arrow[d, phantom, "\cong"] \arrow[rr, bend left, "\mathcal{U}\mathfrak{g} \otimes_{Z\mathfrak{g}} (-)"] & D(T)^W\text{-mod} \arrow[d, "\wr"] & \\
(D(T) \rtimes W)\left[\frac{1-s_\alpha}{\alpha^\vee}\right]\text{-mod} \arrow[r, hook] & (D(T) \rtimes W)\text{-mod} \arrow[r, "\mathrm{ind}(-)^W"] & D(G)\text{-m}
\end{tikzcd}$$

Thm ($G = \mathbb{G}_m$)

$$\begin{tikzcd}
 & Z(HC_G) \arrow[d, shift right, "K"'] & \mathcal{D}(G)\text{-mod}^G \arrow[l, "BFO"', "\sim"] \\
W\text{-mod} \arrow[r, "\text{braided monoidal}"] \arrow[d, phantom, "\cong" rotate=90] & Z(W\text{-comod}) \arrow[u, shift right, hook, "\kappa^R"'] & \\
\mathrm{IndCoh}(\mathfrak{t}^*/\!/W^{\mathrm{aff}}) & & \\
\mathfrak{t}^*/\!/W & &
\end{tikzcd}$$