Title: Towards realistic tensor network holography using loop gravity

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Abstract:

In order to understand many Quantum information aspects of the Ads/CFT correspondence, tensor network toy models of holography have been a useful and concrete tool. However, these models traditionally lack many features of their continuum counterparts, limiting their applicability in arguments about gravity. In this talk, I present a natural extension of the tensor network holography paradigm which rectifies some of these issues. Its direct inspiration originates in Loop Quantum Gravity, which allows not only lifting existing limitations of tensor networks, but also firmly grounds the models in the context of nonperturbative canonical quantum gravity.

Towards realistic tensor network holography using loop gravity

Quantum Gravity Seminar, Perimeter Institute
28th of November

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based on ArXiv: 2207.07625 , 2404.16004

Idea of holography

BH Thermodynamics, AdS/CFT top-down, corner symmetries:
Equivalent encoding of data into lower dimensions!

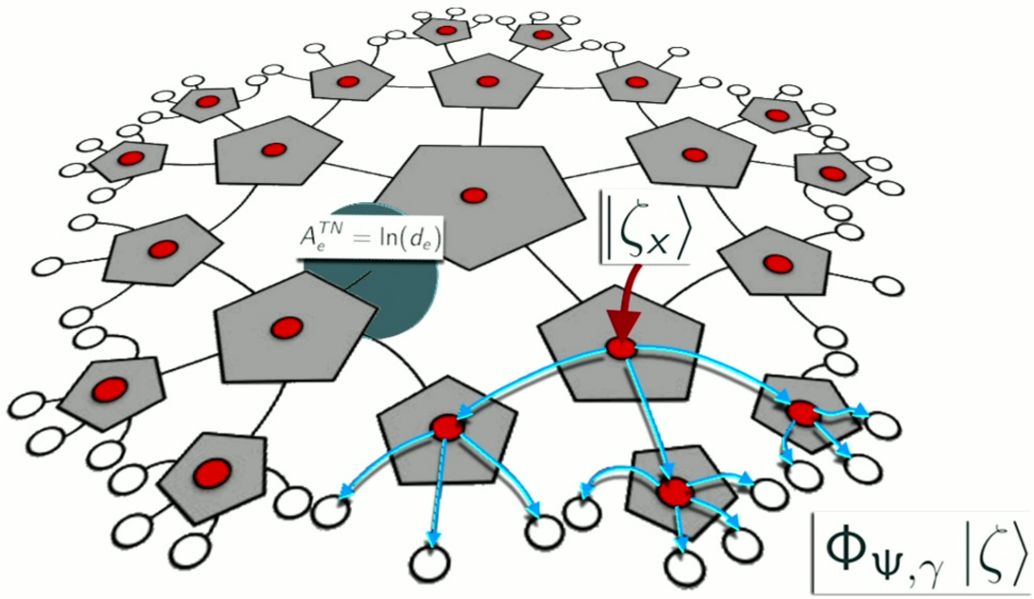
- Bounds and relations for entropies
- Matching and reconstruction of operators
- Encoding of bulk geometry into boundary
- Error correcting properties

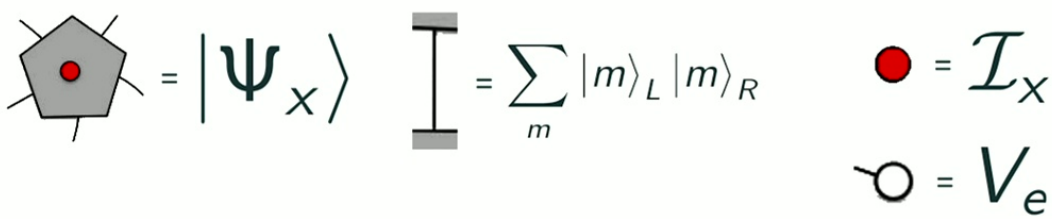
$\Longrightarrow$ **Is gravity generally holographic?**

An argument for/against holography

1. Find concrete setting where technical steps are controllable
2. Use formalism for Quantum Gravity which can be applied nonperturbatively

Schematic for TN holography





Conventional TN holography

1. Select tensor network state $|\Psi_\gamma\rangle \in \mathbb{H}_\gamma = \bigotimes_x \mathcal{I}_x \otimes \bigotimes_{e\in\partial\gamma} V_e$
2. Select bulk input state $|\zeta\rangle \in \mathbb{H}_b := \bigotimes_x \mathcal{I}_x$

$$\Phi_\Psi\,|\zeta\rangle := \langle\zeta\,|\Psi_\gamma\rangle \in \mathbb{H}_\partial := \bigotimes_{e\in\partial\gamma} V_e \tag{1}$$

3. Φ_Ψ defines state/geometry dependent bulk-to-boundary mapping

Holography through information transport

We call the state $|\Psi\rangle$ *holographic* if $\Phi_\gamma : \mathbb{H}_b \to \mathbb{H}_\partial$ is **isometric**,

$$\Phi_\gamma^\dagger \Phi_\gamma = \mathbb{I}_b \tag{2}$$

Conventional isometry criterion

In RTNs: Require large *bond dimensions* $d_e = \dim(V_e) >> 1$ (concentration of measure)

Requirement on bulk/boundary dimensions for all local regions Ω:

$$\frac{D_{b,\Omega}}{D_{\partial,\Omega}} < 1 \tag{3}$$

If $D_{b,\Omega}$ interpreted as measure for volume, $D_{\partial,\Omega}$ for area: 'local negative curvature'.

Conventional RTN result

Generic holographic behaviour for states with large bond dimensions and 'hyperbolic' geometry.

Restrictive?

Superposed tensor networks from LQG

Issues of conventional TN states [1811.05382]

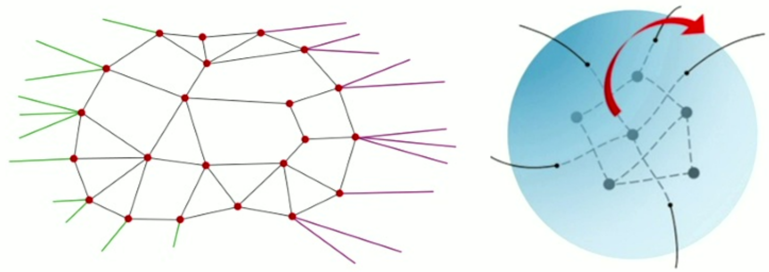
Cool results, but:

1. Entanglement too simple to match with CFTs: ('fixed-area states only')

$$\langle S_n^{TN}(R)\rangle_U \approx \log(D_{\Sigma_R}) \quad S_n^{CFT}(|0\rangle_l) = (1+\frac{1}{n})\frac{c}{6}\log(l) \tag{4}$$

2. Lack of true geometric interpretation and operators
3. No obvious action of diffeomorphisms
4. Nondynamical: No way to find actual causal wedges or study true horizons

LQG spin network space



$$\mathbb{H}_\gamma := \bigotimes_e L^2(SU(2)_e) / \prod_x SU(2)_x \qquad (5)$$

Carries action of gravitational constraints, geometric operators, i.p.

$$\hat{A}_e = \sum_{j_e} a^{LQG}_{j_e} \mathbb{I}_{j_e} \quad a^{LQG}_{j_e} \simeq \sqrt{j(j+1)} \sim j \qquad (6)$$

Superposed PEPS

Full Hilbert space has **direct sum structure**:

$$\mathbb{H}_\gamma \cong \bigoplus_{\vec{\mathbf{j}}} \mathbb{H}_{\gamma,\vec{\mathbf{j}}} \cong \bigoplus_{j_\partial} \mathcal{I}_{j_\partial} \otimes \bigotimes_{e\in\partial\gamma} V_{e,j_e} \quad (7)$$

Represents possible *superpositions of bulk geometry*

Define analogous PEPS-like states

Superposed Projected entangled pair states

$$|\Psi_\gamma\rangle = \bigotimes_{e\in\Gamma} \langle e| \bigotimes_x |\psi_x\rangle \quad |\psi_x\rangle \in \mathbb{H}_x = \bigoplus_{\mathbf{j}^x} \mathcal{I}_{\mathbf{j}^x} \otimes V_{\mathbf{j}^x} \quad (8)$$

Extended formulation of holography

Generalised subsystems

$$\mathbb{H}_\gamma \cong \bigoplus_E \mathcal{I}_E \otimes V_E \tag{9}$$

Algebraic notion of subsystems:

$$(\mathcal{A}_b)' = \mathcal{A}_\partial \qquad (\mathcal{A}_\partial)' = \mathcal{A}_b \tag{10}$$

Largest natural choice:

$$\mathcal{A}_b = \bigoplus_E \mathbb{B}(\mathcal{I}_E) \quad \mathcal{A}_b = \bigoplus_E \mathbb{B}(V_E) \tag{11}$$

These have natural partial trace and extension maps

Qubit example

$$(X \otimes \mathbb{I}_B) |\Phi^{\pm}\rangle = (\mathbb{I}_A \otimes \pm X^t) |\Phi^{\pm}\rangle \tag{12}$$

Example of a *Transport superoperator* $\mathcal{T} : \mathcal{A}_A \rightarrow \mathcal{A}_B$

Can rewrite this through a *Choi* mapping for $\rho = |\Phi^{\pm}\rangle \langle\Phi^{\pm}|$

$$\mathcal{T}_\rho(X) := \mathrm{Tr}_A[(X \otimes \mathbb{I}_B)\rho^{t_A}] = \pm X^t \tag{13}$$

Transport superoperators characterise seperability

ρ max. entangled $\Longleftrightarrow$ $\mathcal{T}_\rho$ *isometric* (ρ pure)

Isometry refers to the Hilbert-Schmidt inner product on operators

$$\langle \mathcal{T}_{\Phi^\pm}(X), \mathcal{T}_{\Phi^\pm}(Y) \rangle_{HS,B} = \langle X, Y \rangle_{HS,A} \tag{14}$$

Holography for direct sum structures

1. Select tensor network state $|\Psi_\gamma\rangle \in \mathbb{H}_\gamma$
2. Select bulk input operator $X \in \mathcal{A}_b$
3. Choi mapping $\mathcal{T}_\rho$ for $\rho = |\Psi_\gamma\rangle \langle\Psi_\gamma|$ defines bulk/geometry bulk-to-boundary mapping

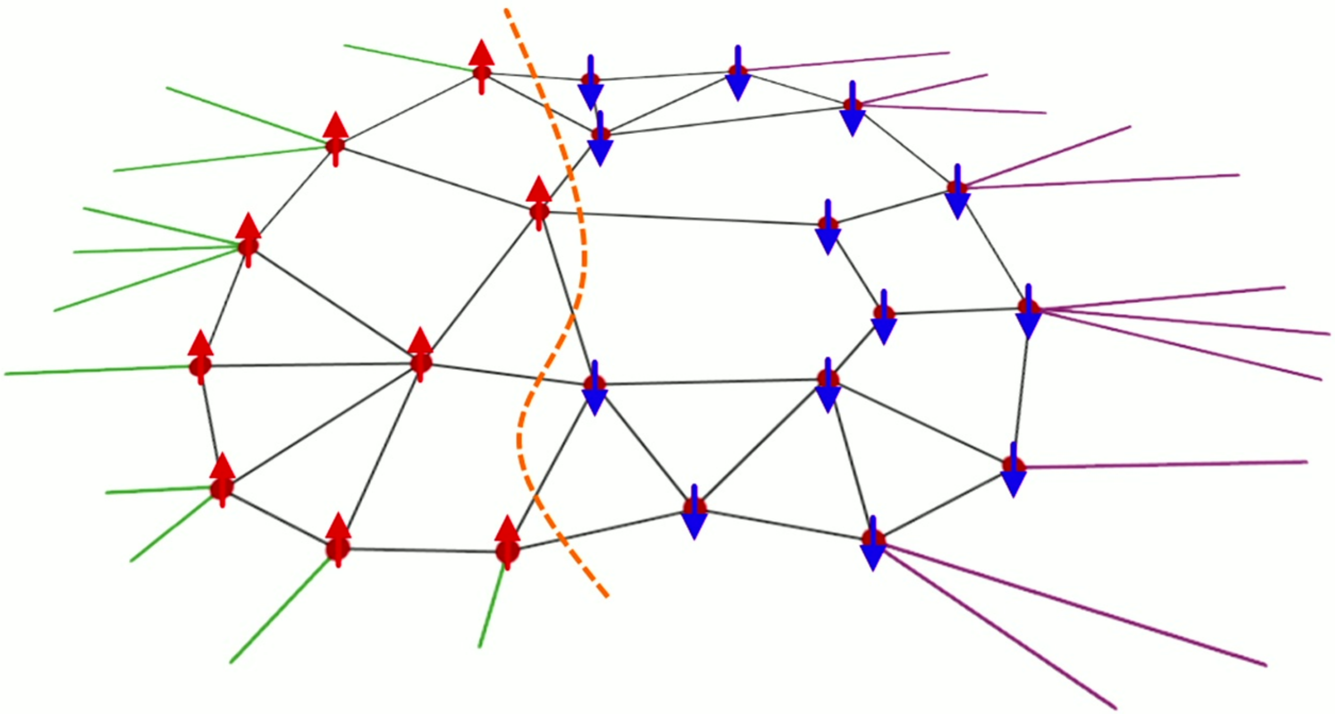
Holography for geometry in superposition

We call the state $|\Psi_\gamma\rangle$ *holographic* if $\mathcal{T}_\rho$ is **isometric**

This turns out to be equivalent to asking $\mathcal{T}$ to be a **channel**, so

$$\mathrm{Tr}[\mathcal{T}(X)] = \mathrm{Tr}[X] \tag{15}$$

Sketch: Calculation of entropies



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First isometry condition

Using localisation of measure + expansion of Rényi entropy:

$$D_{\partial,E} = \prod_{e \in \partial\gamma} d_{j_e} \stackrel{!}{=} D_\partial \quad \forall E \tag{16}$$

Introduces an overall *scale*: Total boundary area!

First constraint for holography

One may only superpose geometries within a superselection sector of fixed total boundary area.

Edge mode interpretation: *Area density* w:

$$\oint_{\partial\Sigma} w Y_0^0 \text{ fixed}, \quad \oint_{\partial\Sigma} w Y_m^l \text{ free} \tag{17}$$

Second isometry condition

With scale D_∂ fixed, find further:

$$\langle S_2(\rho_{b,E}) \rangle_U \stackrel{!}{=} \ln(D_{b,E}) \text{ maximal} \tag{18}$$

Isometry in each boundary sector E:
Slightly weaker than isometry in each bulk geometry

Depends on ground state energy $E_{min;E}$ and degeneracy g_E of Ising models

Second constraint for holography

For each boundary sector $E = \{j_e : e \in \partial\gamma\}$: Maximal entropy

Possibly asking for 'average negative curvature'

Nontrivial area operator

Fixed bond dimensions: Ryu-Takayanagi formula through *bulk minimal surface* Σ_R

$$\langle S_2(R_\partial)\rangle_U \approx \sum_{e\in\Sigma_R} \ln(d_{j_e}) = \langle \Psi_\gamma | \hat{A}_{\Sigma_R} | \Psi_\gamma \rangle \tag{19}$$

$$\hat{A}_{\Sigma_R} := \sum_{e\in\Sigma_R} \hat{A}_e^{TN} \quad \hat{A}_e^{TN} = \ln(d_{j_e})\mathbb{I}_{j_e} \tag{20}$$

Relevant operator in superposition case instead

$$\hat{A}_e^{TN} = \sum_{j_e} \ln(d_{j_e})\mathbb{I}_{j_e} \tag{21}$$

Bulk geometries in genuine quantum superposition

Nonunique bulk surfaces

Superposition case: *combinatorial weighted sum over bulk geometries*

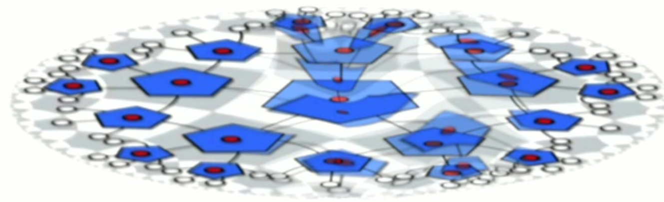
$$\langle S_2(R_\partial)\rangle_U \approx \langle\langle \Psi_\gamma | \hat{A}_{\Sigma_R} | \Psi_\gamma \rangle\rangle_p \quad (22)$$

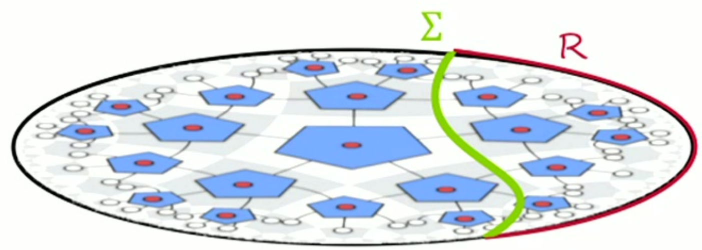
- Minimal surfaces $\Sigma_R(\{j_e\})$ now depend on bulk geometry
- All possible geometries in superposition contribute
- Possibly different number of surfaces per bulk geometry
- Weight $p(\{j_e\})$ determined entirely through combinatorial likelihood $\hat{=}$ dimensions $d_j, D_{I,j}$ but normalised

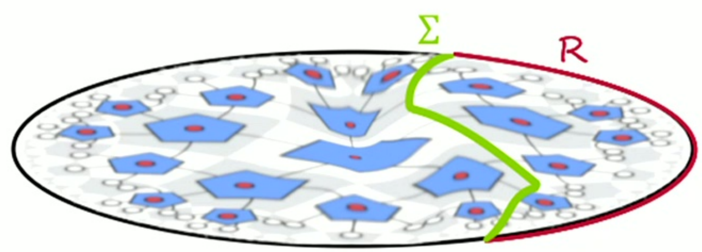
Generalised RT formula

Entropy of boundary region is weighted sum of areas of all minimal bulk surfaces.

$$|\Psi\rangle = |\Psi_g\rangle + |\Psi_{\bar{g}}\rangle$$







Nontrivial entanglement spectrum

$$\langle S_k \rangle_U \approx \sum_{m=1}^{\infty} \frac{(-1)^{m-1}}{m!} \frac{\kappa_m(X)(k)}{k-1} \tag{23}$$

- $X_{j_1,\ldots,j_k}$ a tensor depending on graph and dimensions
- $p(\{j_e\})$ same weight as before
- $\kappa_m(X)(k)$ kth cumulant of X with respect to $p^{\otimes k}$

1. X diagonal, p peaks on one geometry: **Flat spectrum**
2. X diagonal, p equal weight on two geometries: $S_k \sim \frac{1}{2^k}$
3. X *factorises*: $S_k \sim \frac{1}{k-1}$!

Sketch of entanglement spectrum

(Highly) nontrivial conditions on geometries possibly allow CFT-like scaling

Outlook

Discussion

Take-home messages

- Tensor network models can be more than just toy models
- Extensions directly motivated by nonperturbative discrete approaches
- Strong interplay between QI, LQG and AdS/CFT ideas
- Holography generically only in 'hyperbolic' geometries

Long-term perspectives

Multiple angles:

- Discrete version of AdS/CFT: Find holographic dual bottom-up?
- Define QG side of the duality fully, using LQG?
- Get better handle on LQG through tentative holographic dual?
- Find ways to relate setup to flat space holography studies?

Many things to do!

Sketch: Diffeomorphisms [gr-qc/0212001,1101.0590]

Open question: How do diffeos act on TN states?
LQG gives answer in 3D: *Shifts*.

Diffeomorphism invariance in 3D Ponzano-Regge (flat Euclidean gravity)

Shift symmetry $\Leftrightarrow$ Triangulation invariance $\Leftrightarrow$ Vertex translation symmetry

Implementing these symmetries is well understood in 3D, shows how to work with diffeos in principle
4D case has similar symmetry, but much less well understood

Sketch: Boundary dynamics [1803.02759,1710.04237]

How can one implement dynamics in TN models?

$\Longrightarrow$ Usual LQG, Spin foam models...

Main issue: Bulk dynamics is pure constraints! How to connect to AdS/CFT?

$\Longrightarrow$ Ask about boundary dynamics

Quantum boundary dynamics

In 3D gravity: Boundary conditions $\Leftrightarrow$ Statistical spin models

In principle, extremely rich class of theories

Sketch: Causal structures [2206.15442,2402.05993]

Many interesting questions in AdS/CFT rely on causal structures:
Scattering, Causal wedges, reconstruction of bulk horizons...
All absent in nondynamical models

Discrete spacetime histories from spin foams

Using the spin foam approach to discrete QG dynamics, open door to study also causal structure in TN holography

E.g. use causal extensions with timelike, spacelike, lightlike building blocks

Thank you for listening!