Title: Black hole evaporation in random matrix theory (RMT) and statistical CFTs

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Collection/Series: Quantum Information

Subject: Quantum Information

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Abstract:

In this talk, with two parts, I will first show how to capture both Hawking's non-unitary entropy curve and density matrix-connecting contributions that restore unitarity, in a toy RMT quantum system modelling black hole evaporation. The motivation is to find the simplest possible dynamical model that captures this aspect of gravitational physics. In the model, there is a dynamical phase transition in the averaging that connects the density matrices in a replica wormhole-like manner and restores unitarity in the entropy curve. In the second half of the talk, I will discuss ongoing follow-up work describing black hole evaporation and unitarity restoration in statistical descriptions of holographic CFTs.

Black hole evaporation in random matrix theory and statistical CFTs

w/ Jan de Boer & Jildou Hollander

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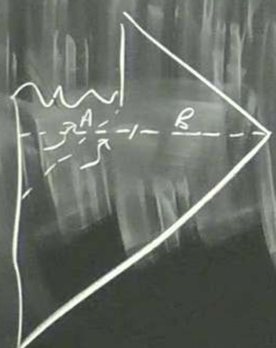
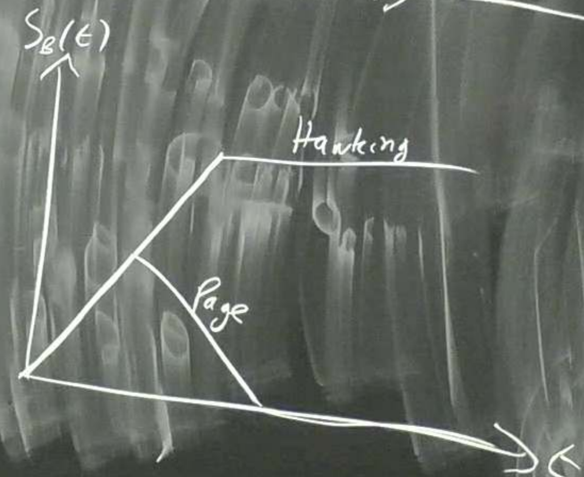
Intro



$\rho_B = \rho_{\text{therm.}}$

Radiation entropy curves



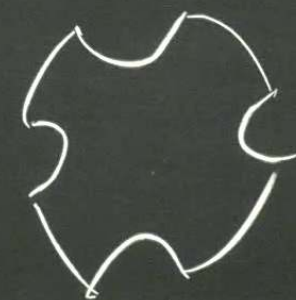
Unitarity $\Rightarrow$ Page $\Leftrightarrow$?

Replica wormholes

$$S(\rho) = -\partial_n \mathrm{Tr}(\rho^n)\big|_{n=1}$$

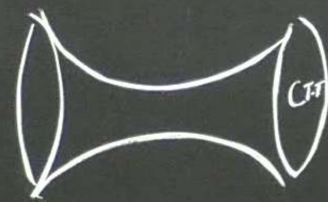
$$\mathrm{Tr}(\rho^n) =$$

$+$



RW

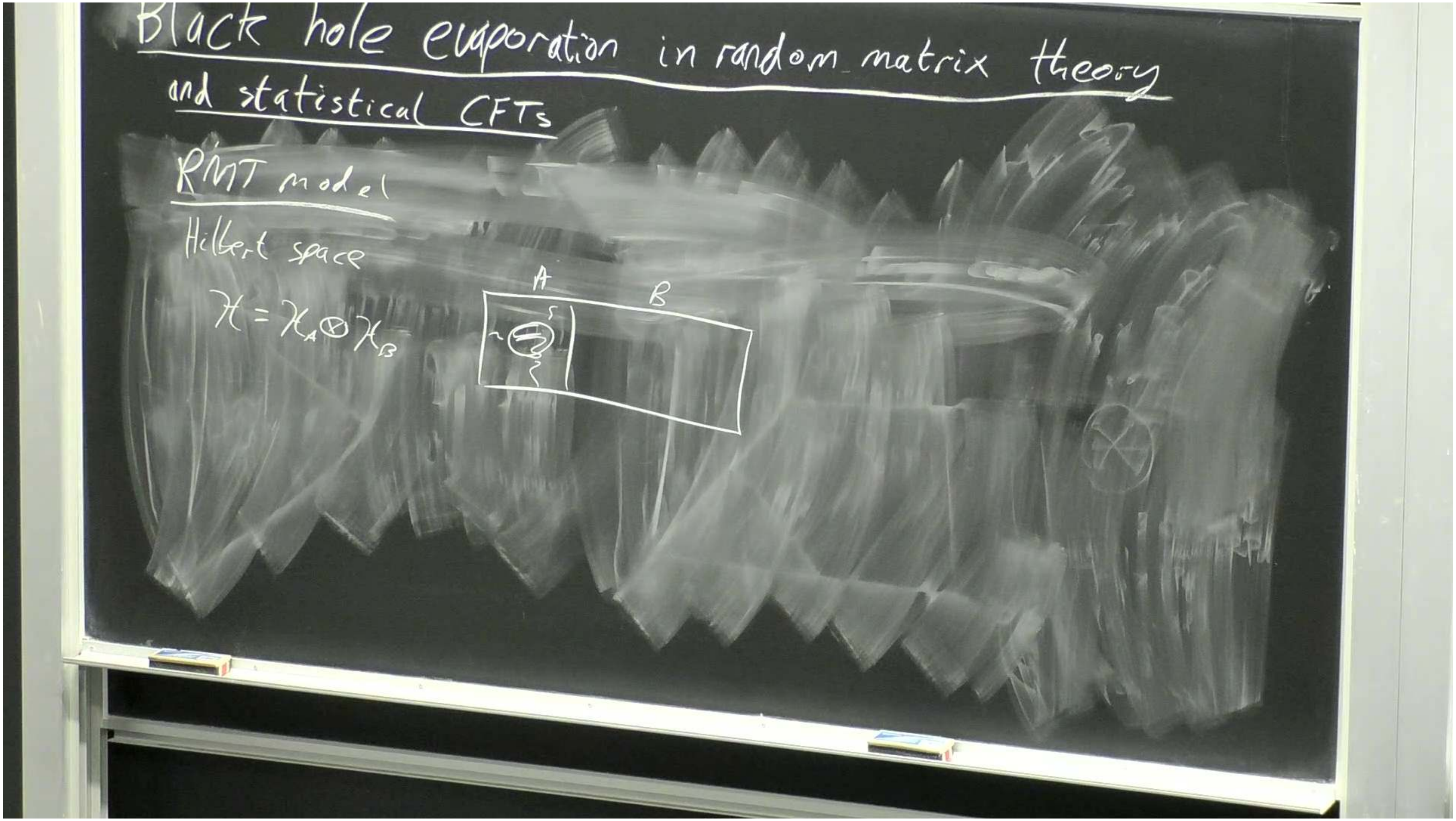
Ensemble gravity

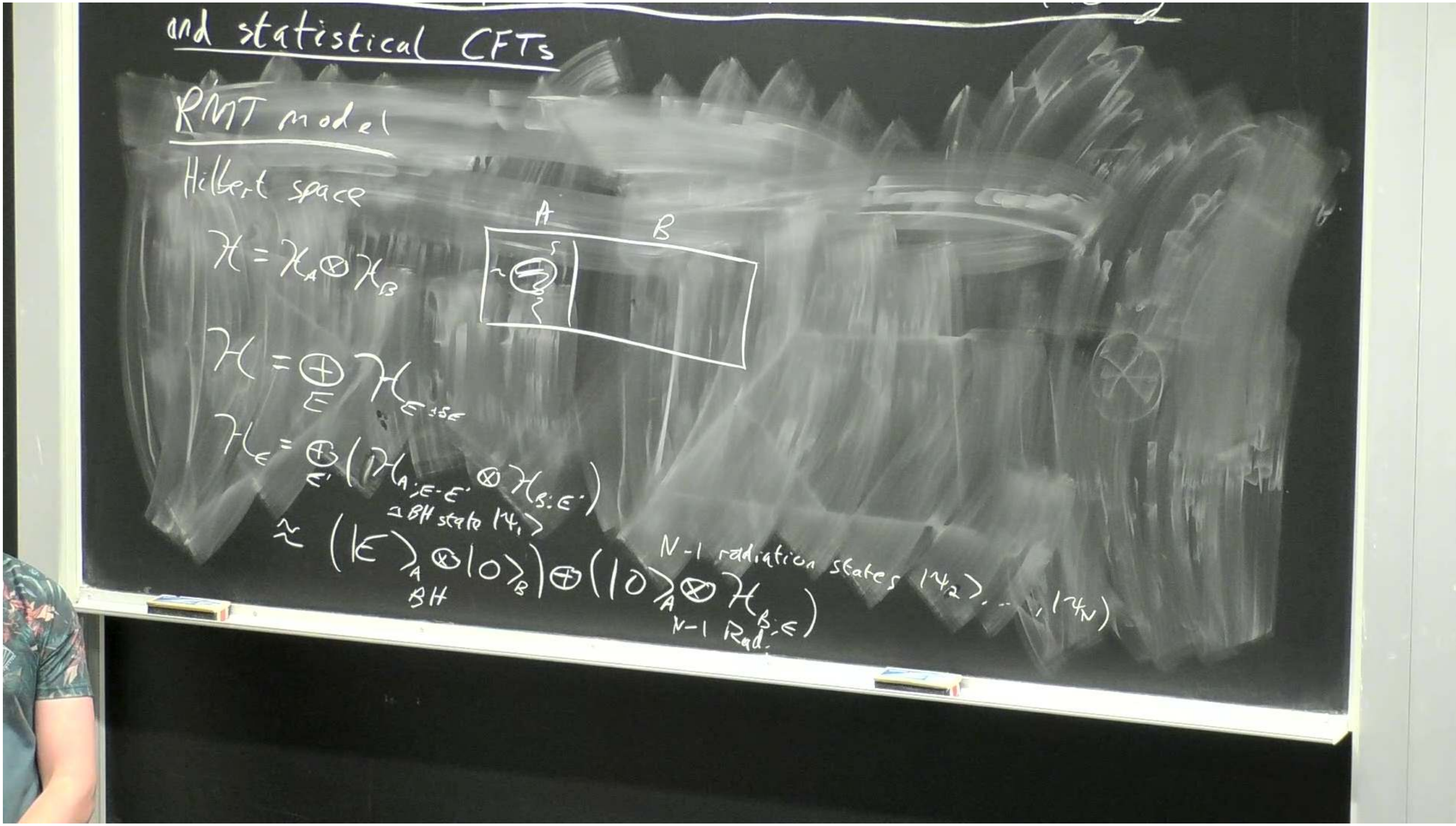
$Z(\beta)^2_{conn.} =$ 

$\Rightarrow \mathrm{Var}(Z) \neq 0$

Messages

1) Ensembles → 2 entropy
2) Haar averaging → Replica wormholes
3) Semiclassical gravity = Statistical CFT


Black hole evaporation in random matrix theory
and statistical CFTs
RMT model
Hilbert space
$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$
A
B


and statistical CFTs
RMT model
Hilbert space
A
B
$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$
$\mathcal{H} = \bigoplus_E \mathcal{H}_{E \pm \delta E}$
$\mathcal{H}_E = \bigoplus_{E'} \left(\mathcal{H}_{A;E-E'} \otimes \mathcal{H}_{B;E'} \right)$
$\ni$ BH state $|\psi_1\rangle$
$\approx \left(|E\rangle_A \otimes |0\rangle_B \right) \oplus \left(|0\rangle_A \otimes \mathcal{H}_{B;E} \right)$
BH
N-1 radiation states $|\psi_2\rangle, \dots, |\psi_N\rangle$
N-1 Rad.

Hamiltonian

$$H = H_A + H_B + \lambda H_{int}$$

$$|\psi(t)\rangle = e^{iHt}|\psi_i\rangle$$

<u>Ensemble averaged state → Mixed Hawking radiation</u>

$$\frac{d}{dt}\mathrm{Tr}\left(\rho^2(t)\right)$$

Ensemble

$$H = U^{\dagger}\Lambda U$$

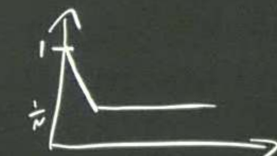
$$P(H) = P_{Haar}(U)\, P(\Lambda)$$

Averaged density matrix

$$\overline{\rho_B}(t) = \int dH\, P(H)\, \mathrm{Tr}_A(\rho(t))$$

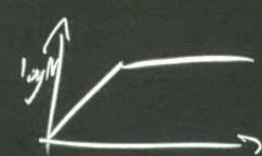
$$= \frac{1+\bar{g}(t) N}{N+1} |0\rangle\langle 0|_B + \frac{N-\bar{g}(t) N}{N^2-1} \sum_{i=2}^{N} |i\rangle\langle i|_B$$

$$\overline{g(t)} := \int dH\, \rho(H) \frac{|\mathrm{tr}\, e^{iHt}|^2}{N^2}$$

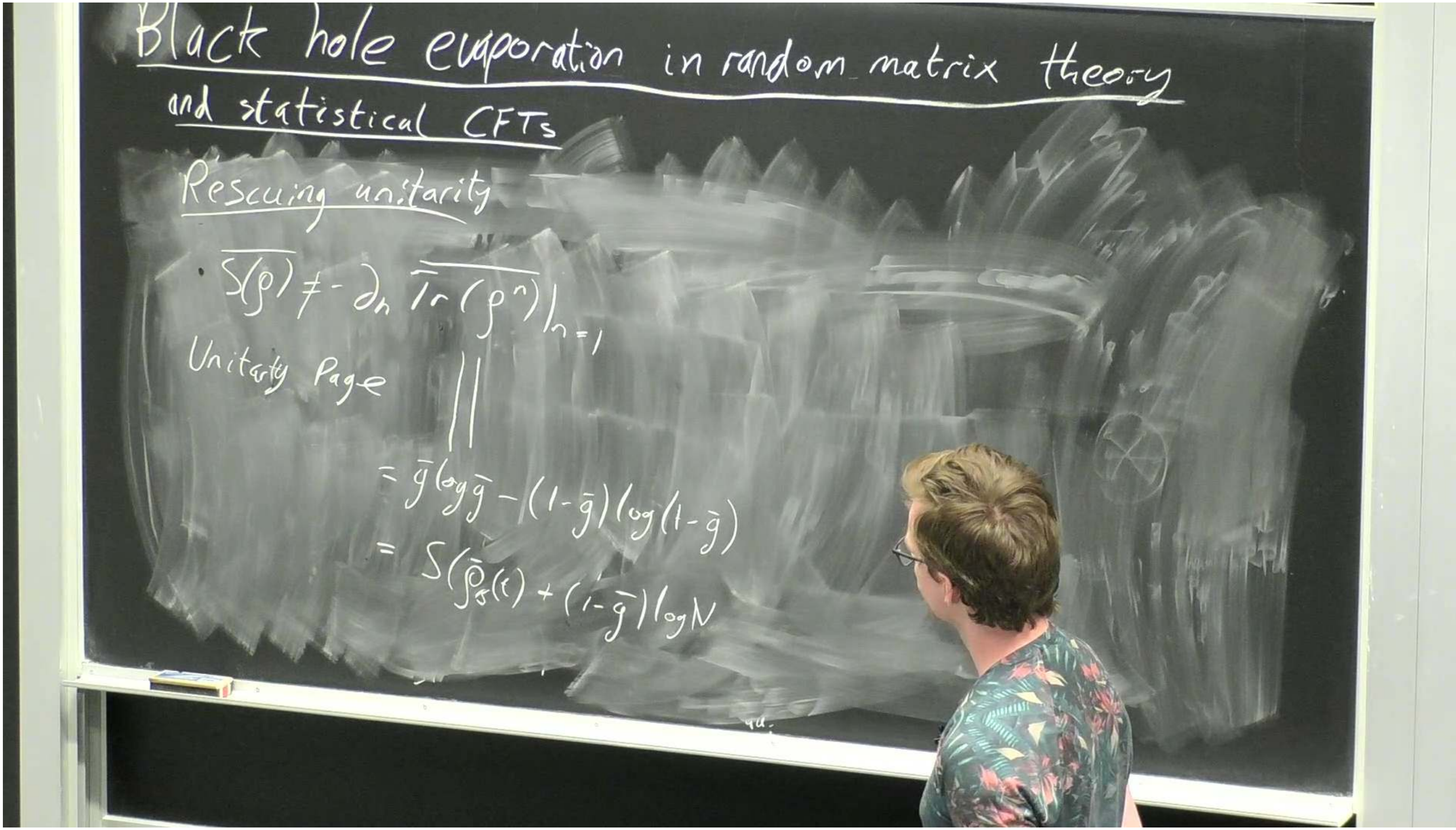


$$= |0\rangle_B\langle 0| \quad \text{at } t=0$$

$$= \frac{1}{N-1} \sum_{i=2}^{N} |i\rangle\langle i| \quad \text{as } t \to \infty$$



$$S(\bar{\rho}_B(t)) = -\bar{g} \log \bar{g} - (1-\bar{g}) \log\left(\frac{1-\bar{g}}{N}\right)$$


Black hole evaporation in random matrix theory
and statistical CFTs
Rescuing unitarity
• $\overline{S(\rho)} \neq -\partial_n \overline{\mathrm{Tr}(\rho^n)}\big|_{n=1}$
Unitarity Page
$= \bar{g}\log\bar{g} - (1-\bar{g})\log(1-\bar{g})$
$= S(\bar{\rho}_g(t)) + (1-\bar{g})\log N$

Black hole evaporation in random matrix theory and statistical CFTs

Rescuing unitarity

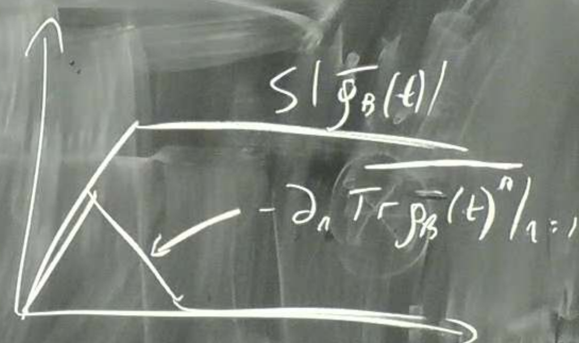
- $\overline{S(\rho)} \neq -\partial_n \overline{\mathrm{Tr}\,(\rho(t)^n)}\Big|_{n=1}$

Unitarity Page

$$\Big\|$$

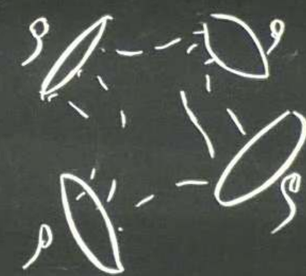
$$= \bar{g}\log\bar{g} - (1-\bar{g})\log(1-\bar{g})$$

$$= S(\bar{\rho}_B(t)) + (1-\bar{g})\log N$$



Replica wormholes from Haar averaging

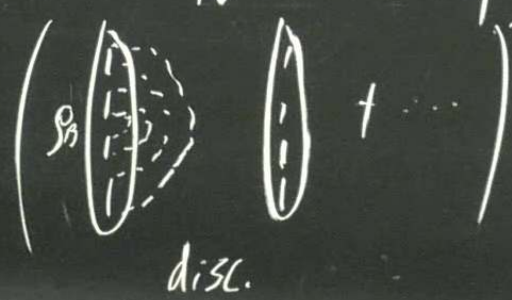
$$\rho(t) = U e^{iHt} U^\dagger |\psi_1\rangle\langle\psi_1| U e^{-iHt} U^\dagger$$



$$\overline{\mathrm{Tr}\,\rho_B^n} = \overline{\underbrace{U^\dagger U \cdots}_{4n} \cdot \, r)}$$

$$\overline{U_{ab}\, U^\dagger_{cd}} \sim \frac{\delta_{ad}\,\delta_{bc}}{N}$$

$\overline{\mathrm{Tr}(\rho_B^2)} =$



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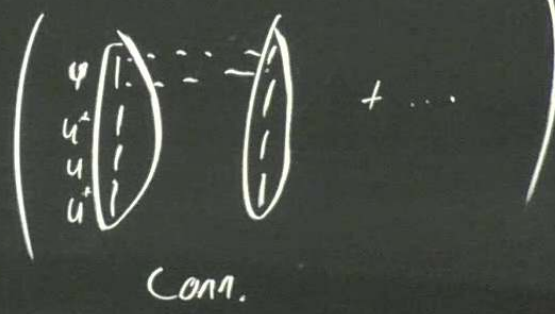


BHs in stat. holo. CFTs
|ψ(t)⟩ = e^{iHt} φ(x₁) φ(x₂)|0⟩
φ(x₂)
S(ρ̄)
∂ₙ Tr ρⁿ |ₙ₌₁
t
C²φφH = Δ_H^{2Δφ-1} / ρ(Δ_H)