**Title:** Black hole evaporation in random matrix theory (RMT) and statistical CFTs

**Speakers:** Andrew Rolph

**Collection/Series:** Quantum Information

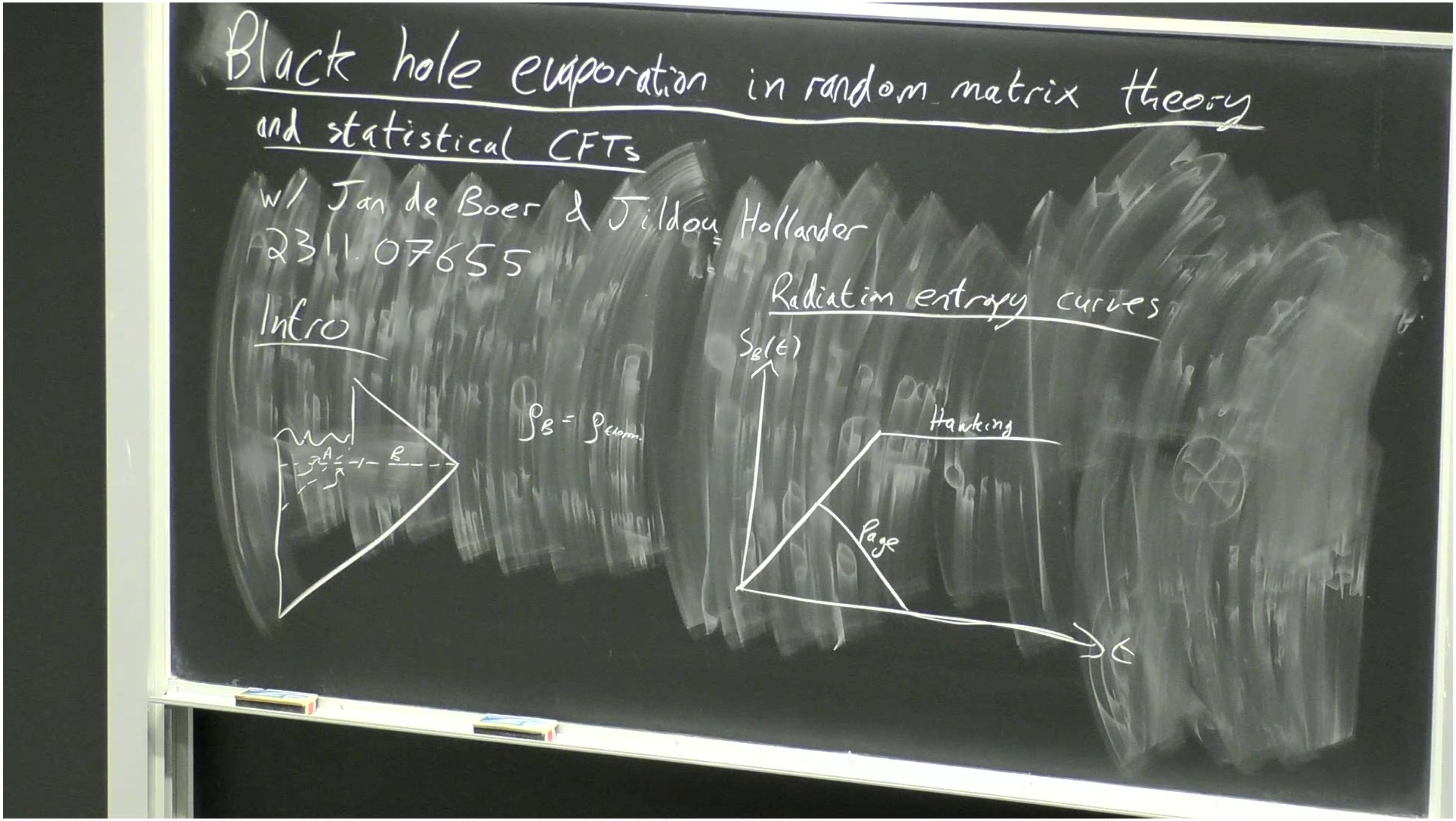
**Subject:** Quantum Information

**Date:** November 20, 2024 - 3:30 PM

**URL:** https://pirsa.org/24110077

**Abstract:**

In this talk, with two parts, I will first show how to capture both Hawking's non-unitary entropy curve and density matrix-connecting contributions that restore unitarity, in a toy RMT quantum system modelling black hole evaporation. The motivation is to find the simplest possible dynamical model that captures this aspect of gravitational physics. In the model, there is a dynamical phase transition in the averaging that connects the density matrices in a replica wormhole-like manner and restores unitarity in the entropy curve. In the second half of the talk, I will discuss ongoing follow-up work describing black hole evaporation and unitarity restoration in statistical descriptions of holographic CFTs.

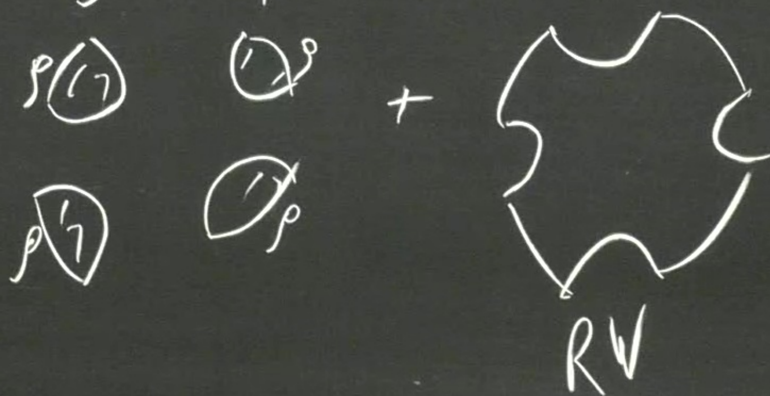

Black hole evaporation in random matrix theory
and statistical CFTs
w/ Jan de Boer & Jildou Hollander
2311.07655
Intro
A
B
Radiation entropy curves
S_B(t)
Hawking
Page
t

Unitarity $\Rightarrow$ Page $\Leftrightarrow$ ?

<u>Replica wormholes</u>

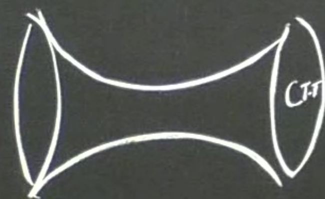
$$S(\rho) = -\partial_n \operatorname{Tr}(\rho^n)\big|_{n=1}$$

$$\operatorname{Tr}(\rho^n) =$$



# Ensemble gravity

$$Z(\beta)^2_{\text{conn.}} =$$




$$\Rightarrow \mathrm{Var}(Z) \neq 0$$

## Messages

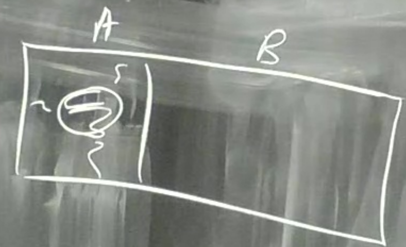
1) Ensembles → 2 entropy
2) Haar averaging → Replica wormholes
3) Semiclassical gravity = Statistical CFT

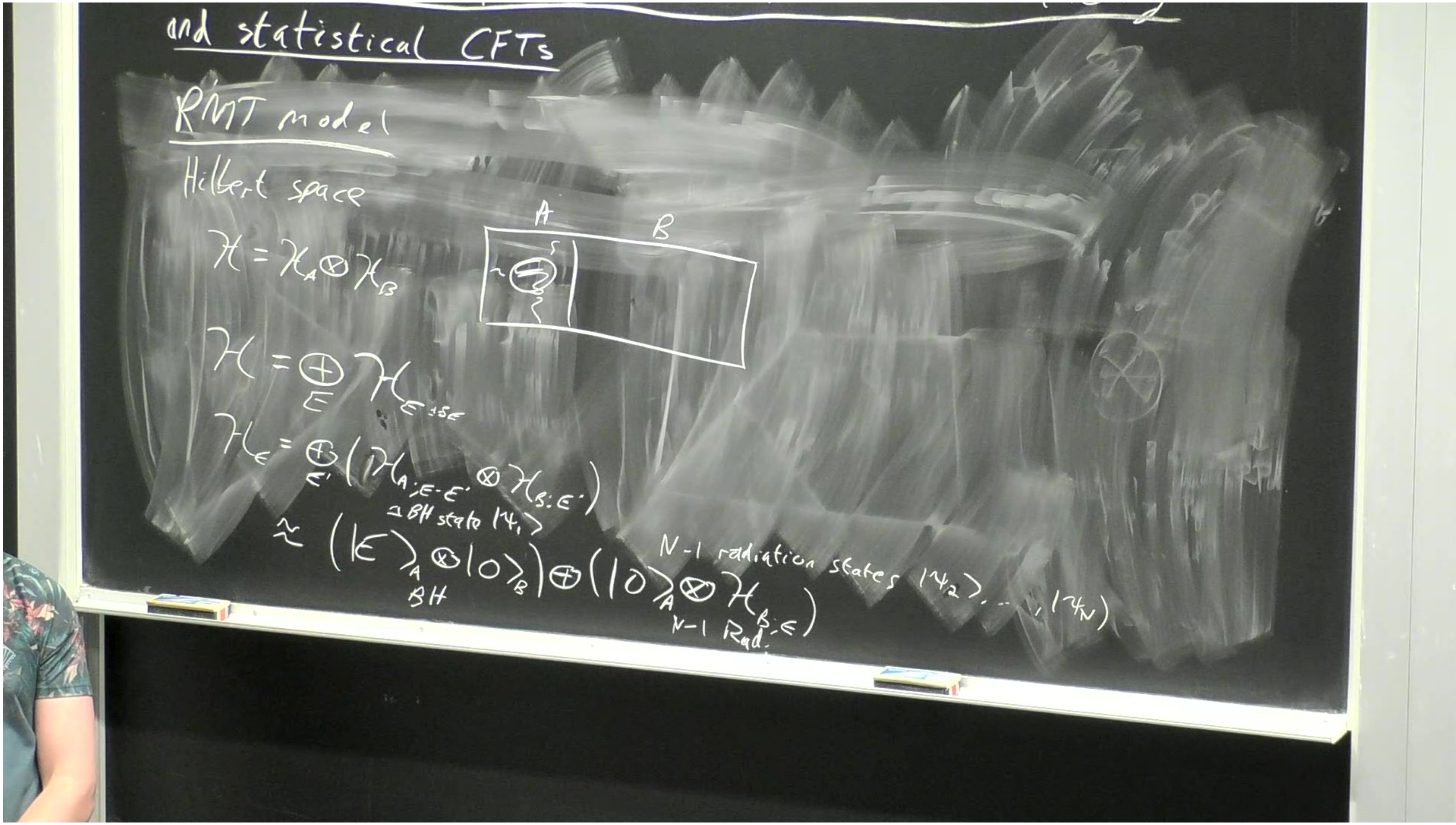
# Black hole evaporation in random matrix theory and statistical CFTs

## RMT model

Hilbert space

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$




and statistical CFTs
RMT model
Hilbert space
A
B
$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$
$\mathcal{H} = \bigoplus_E \mathcal{H}_{E \pm \delta E}$
$\mathcal{H}_E = \bigoplus_{E'} \left( \mathcal{H}_{A;E-E'} \otimes \mathcal{H}_{B;E'} \right)$
$\ni$ BH state $|\psi_1\rangle$
$\approx \left( |E\rangle_A \otimes |0\rangle_B \right) \oplus \left( |0\rangle_A \otimes \mathcal{H}_{B;E} \right)$
BH
N-1 Rad.
N-1 radiation states $|\psi_2\rangle, \ldots, |\psi_N\rangle$

Hamiltonian

$$H = H_A + H_B + \lambda H_{int}$$

$$|\psi(t)\rangle = e^{-iHt}|\psi_i\rangle$$

Ensemble averaged state → Mixed Hawking radiation

$$\frac{d}{dt}\mathrm{Tr}(\rho^2(t))$$

Ensemble

$$H = U^{\dagger}\Lambda U$$

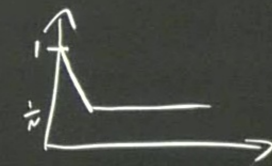
$$P(H) = P_{Haar}(U)\, P(\Lambda)$$

## Averaged density matrix

$$\overline{\rho_B(t)} = \int dH\, P(H)\, \mathrm{Tr}_A(\rho(t))$$

$$= \frac{1+\bar{g}(t) N}{N+1} |0\rangle\langle 0|_B + \frac{N-\bar{g}(t) N}{N^2-1}\sum_{i=2}^{N} |i\rangle\langle i|_B$$

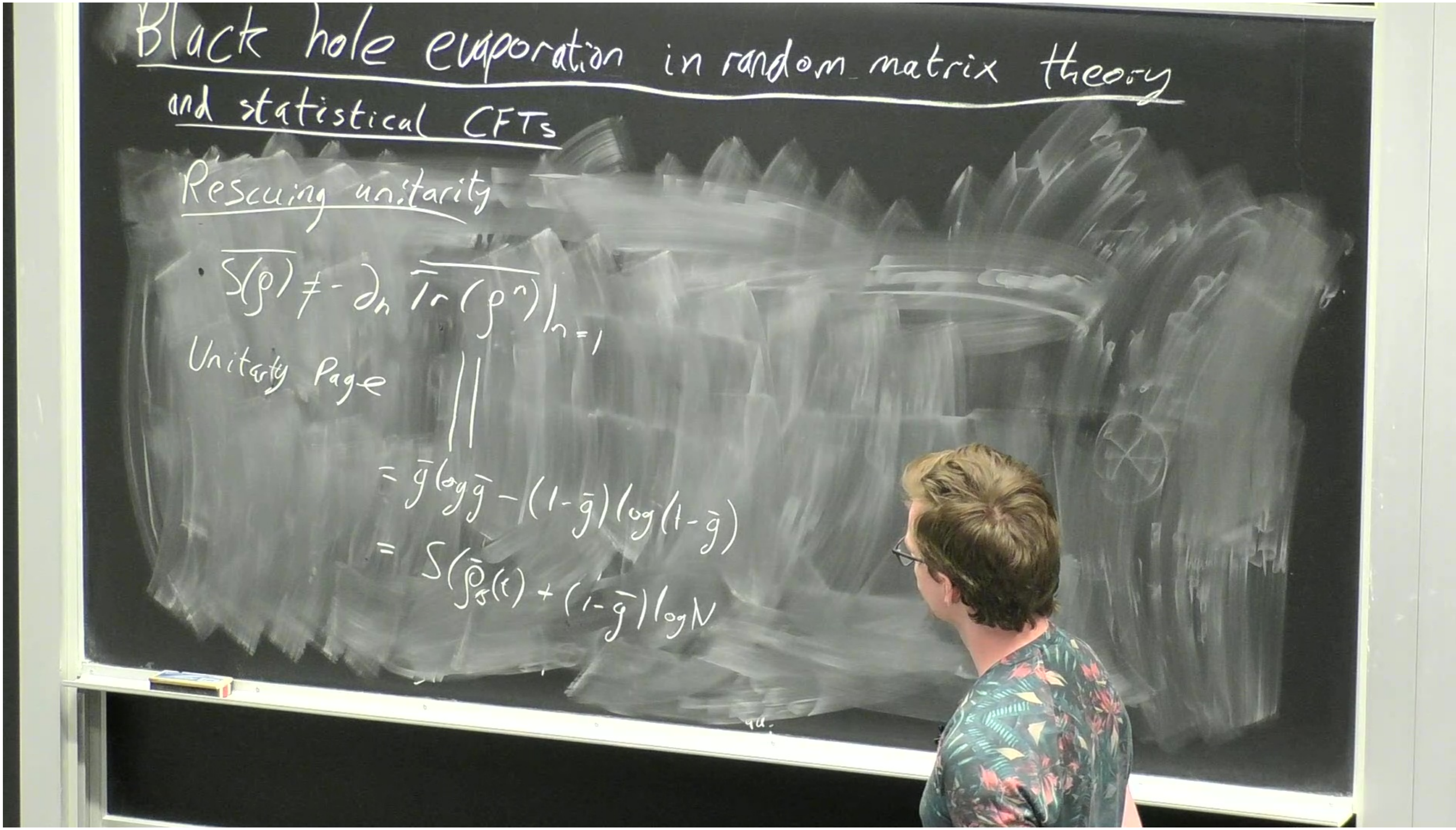
$$\overline{g(t)} := \int dH\, \rho(H)\, \frac{|\mathrm{tr}\, e^{iHt}|^2}{N^2}$$

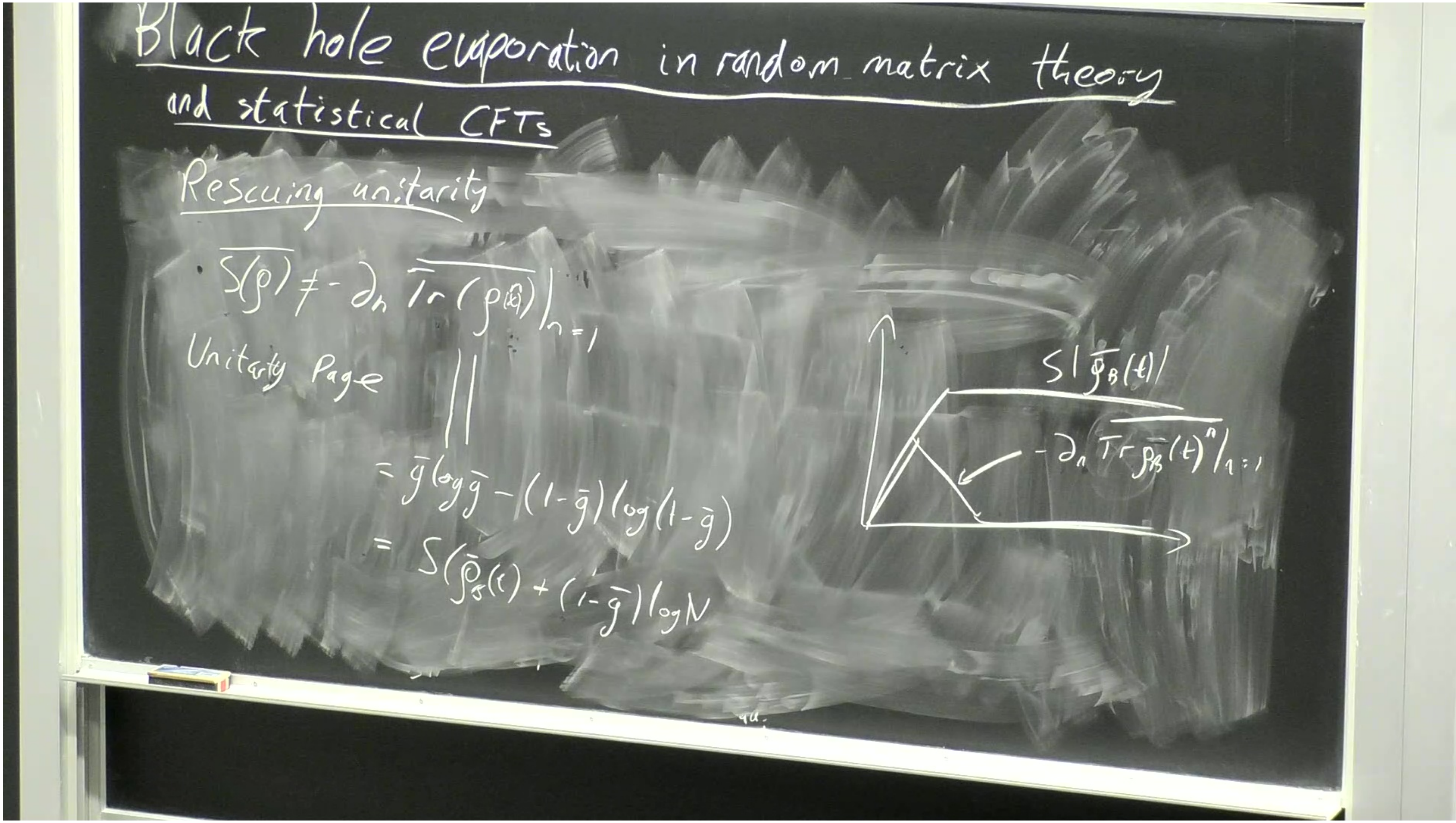



$$= |0\rangle_B\langle 0| \quad \text{at } t=0$$

$$= \frac{1}{N-1}\sum_{i=2}^{N} |i\rangle\langle i| \quad \text{as } t\to\infty$$

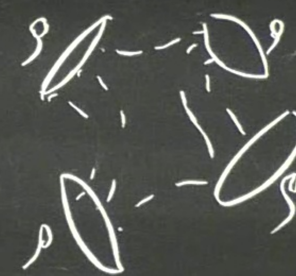

$$S(\bar{\rho}_B(t)) = -\bar{g} \log \bar{g} - (1-\bar{g})\log\left(\frac{1-\bar{g}}{N}\right)$$


Black hole evaporation in random matrix theory
and statistical CFTs
Rescuing unitarity
$\overline{S(\rho)} \neq -\partial_n \overline{\mathrm{Tr}(\rho^n)}\big|_{n=1}$
Unitarity Page
$= \bar{g}\log\bar{g} - (1-\bar{g})\log(1-\bar{g})$
$= S(\bar{\rho}_g(t)) + (1-\bar{g})\log N$


Black hole evaporation in random matrix theory
and statistical CFTs
Rescuing unitarity
$\overline{S(\rho)} \neq -\partial_n \overline{\mathrm{Tr}(\rho^n)}\big|_{n=1}$
Unitarity Page
$= \bar{g}\log\bar{g} - (1-\bar{g})\log(1-\bar{g})$
$= S(\bar{\rho}_B(t)) + (1-\bar{g})\log N$
$S(\bar{\rho}_B(t))$
$-\partial_n \overline{\mathrm{Tr}\,\rho_B(t)^n}\big|_{n=1}$

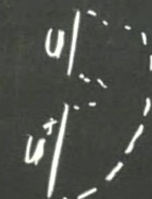
# Replica wormholes from Haar averaging

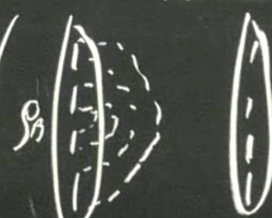
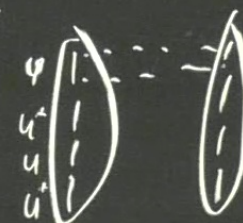
$$\rho(t) = U e^{iHt} U^\dagger |\psi_1\rangle\langle\psi_1| U e^{-iHt} U^\dagger$$




$$\overline{\mathrm{Tr}\,\rho_B^n} = \overline{\underbrace{U^\dagger U \cdots}_{4n} \cdots)}$$

$$\overline{U_{ab}\,U^\dagger_{cd}} \sim \frac{\delta_{ad}\,\delta_{bc}}{N}$$




$\overline{\mathrm{Tr}(\rho_B^2)} = \Big($   $+ \cdots \Big) + \Big($   $+ \cdots \Big)$

disc. conn.


BHs in stat. holo. CFTs
$|\psi(t)\rangle = e^{iHt}\phi(x_1)\phi(x_2)|0\rangle$
$\phi(x_1)$
$\phi(x_2)$
$C^2_{\phi\phi H} = \frac{\Delta_H^{2\Delta_\phi-1}}{\rho(\Delta_H)}$
$S(\bar{\rho})$
$\partial_n \mathrm{Tr}\,\rho^n|_{n=1}$
$t$