Title: Late-time signals from binary black hole mergers

Speakers: Marina de Amicis

Collection/Series: Strong Gravity

Subject: Strong Gravity

Date: November 28, 2024 - 1:00 PM

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Abstract:

Late-time tails emitted by binary black holes mergers contain invaluable information on the spacetime’s asymptotic structure. Perturbative numerical simulations of extreme mass-ratio mergers have revealed that these tails are enhanced by several orders of magnitude with the progenitors’ binary eccentricity. This amplification has the potential to bring tails within the realm of observation and shows that this effect carries significant astrophysical implications, other than fundamental physics content.

I will present an analytical perturbative model that accurately predicts the numerically observed tail and explains its enhancement with the progenitors' binary eccentricity. The model is an integral over the system's entire history, showing how the post-ringdown tail is inherited from the non-circular inspiral in a non-local fashion. I will prove the tail to be a superposition of many power-laws, with each term's excitation coefficient depending on the specific inspiral history. A single power law is recovered only in the limit of asymptotically late times, consistent with Price's results and the classical soft-graviton theorem. Finally, I will introduce a robust framework for extracting tails in fully non-linear simulations of equal masses mergers. I will present results for late-time tails emitted by these systems and discuss their phenomenology.

A fairy-TAIL story

Marina De Amicis,
Simone Albanesi, Gregorio Carullo

[*Phys.Rev.D* 110 (2024)]

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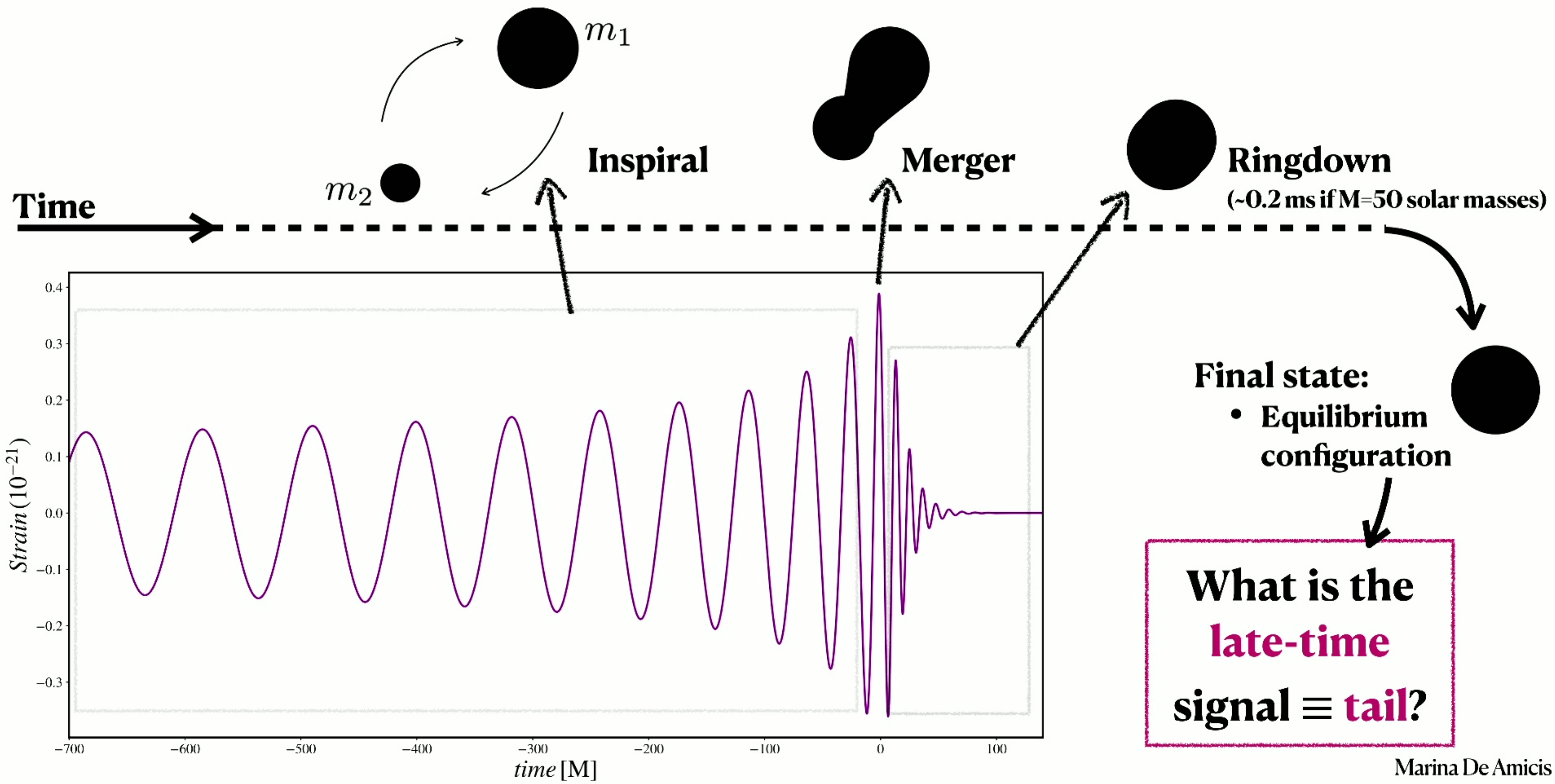
Question...



Final state:
- **Equilibrium configuration**

What is the late-time signal ≡ tail?

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Summary

- Brief introduction on tails in perturbation theory
- Tails phenomenology and model in extreme mass-ratio mergers
- Extracting tails in fully non-linear numerical relativity

Why it matters?

Foundational problem in GR

- Classical soft graviton theorems predicting new tails [Sen, 2408.08851]
- Tails related to peeling properties (/ lack of) [Gajic and Kehrberger, 2202.04093]

Probe of spacetime asymptotic structure

Astrophysical implications

- Enhancement with eccentricity makes the tail **potentially observable**
    - Plenty possible channels of highly eccentric mergers
    - Could give constraints on eccentric inspiral parameters

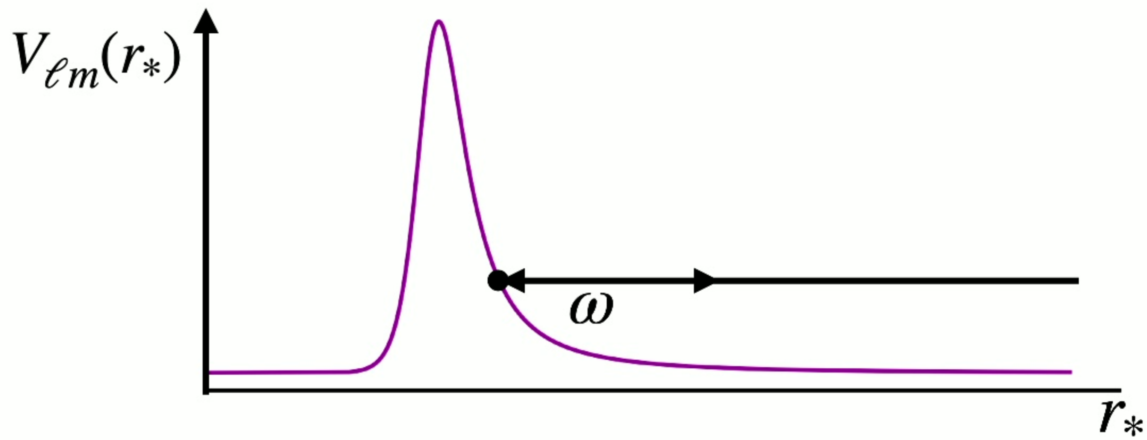
Relevant for binary formation and population studies

Initial-data driven tails

- find tail propagator,
  in (complex) ω domain

$$\left[\partial_{r_*}^2 + \omega^2 - V_{\ell m}(r_*)\right] \tilde{G}_{\ell m}(\omega; r'_*, r_*) = \delta(r_* - r'_*)$$

Originated from backscattering of small ω signal from long-range potential



[Price, Phys. Rev. D 5, 2419] , [Leaver, Phys. Rev. D 34, 384]

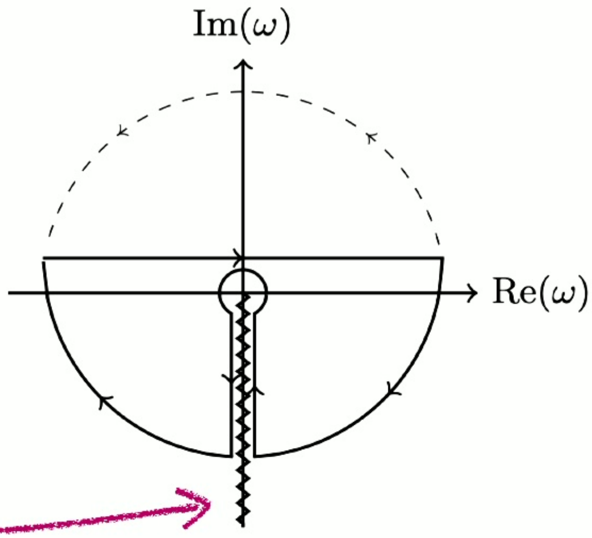
Initial-data driven tails

Expansion in large r...

$$\left[\partial_r^2 + \omega^2 \left(1 - \frac{4M}{r^2}\right) - \frac{\ell(\ell+1)}{r^2}\right] \tilde{G}_{\ell m}(\omega; r', r) = \delta(r - r')$$

...and small ω

$$G_\ell(t - t'; r, r') \propto \int_\Gamma d\omega \; e^{-i\omega(t-r-t')} \; (\omega r')^{\ell+1} \; \left[\log(2\omega) + \ldots\right]$$



[Price, Phys. Rev. D 5, 2419] , [Leaver, Phys. Rev. D 34, 384]

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Initial-data driven tails

Prediction at $\mathscr{I}^+$:

- $\Psi_{\ell m} = \dfrac{A_{\text{tail}}}{\tau^{\ell+2}}$
- $A_{\text{tail}}(\psi_0, \zeta_0)$ constant

$\tau \equiv t - r_*$

$r_* \equiv r + 2M \ln(r - 2M)$

Price's law

1) Suppressed
2) Not clear what is the information content

[Price, Phys. Rev. D 5, 2419] , [Leaver, Phys. Rev. D 34, 384]

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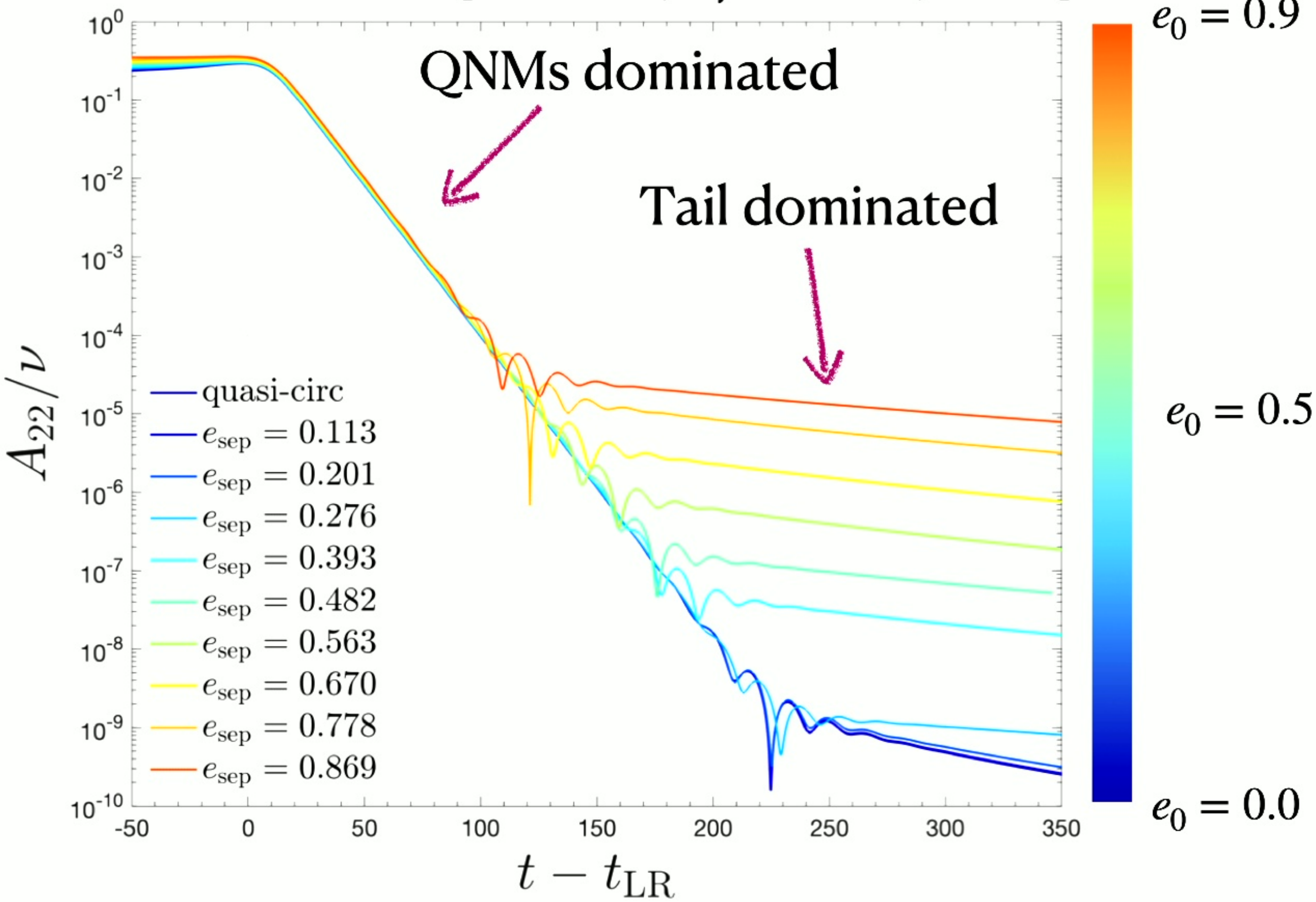
An exciting phenomenology

[Albanesi et al, Phys. Rev. D 108, 084037]



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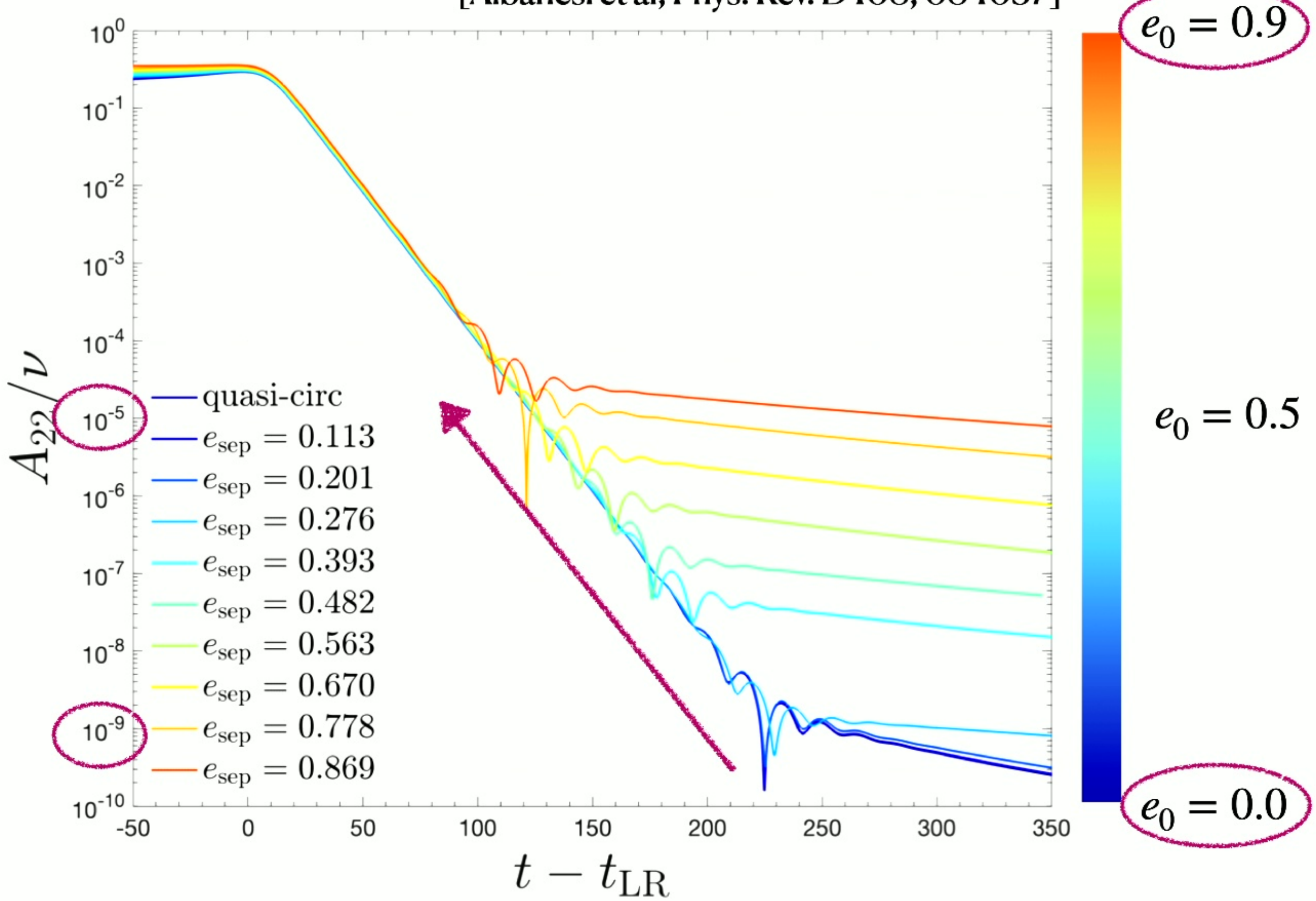

Amplitude
enhanced of several orders of magnitude by eccentricity

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An exciting phenomenology

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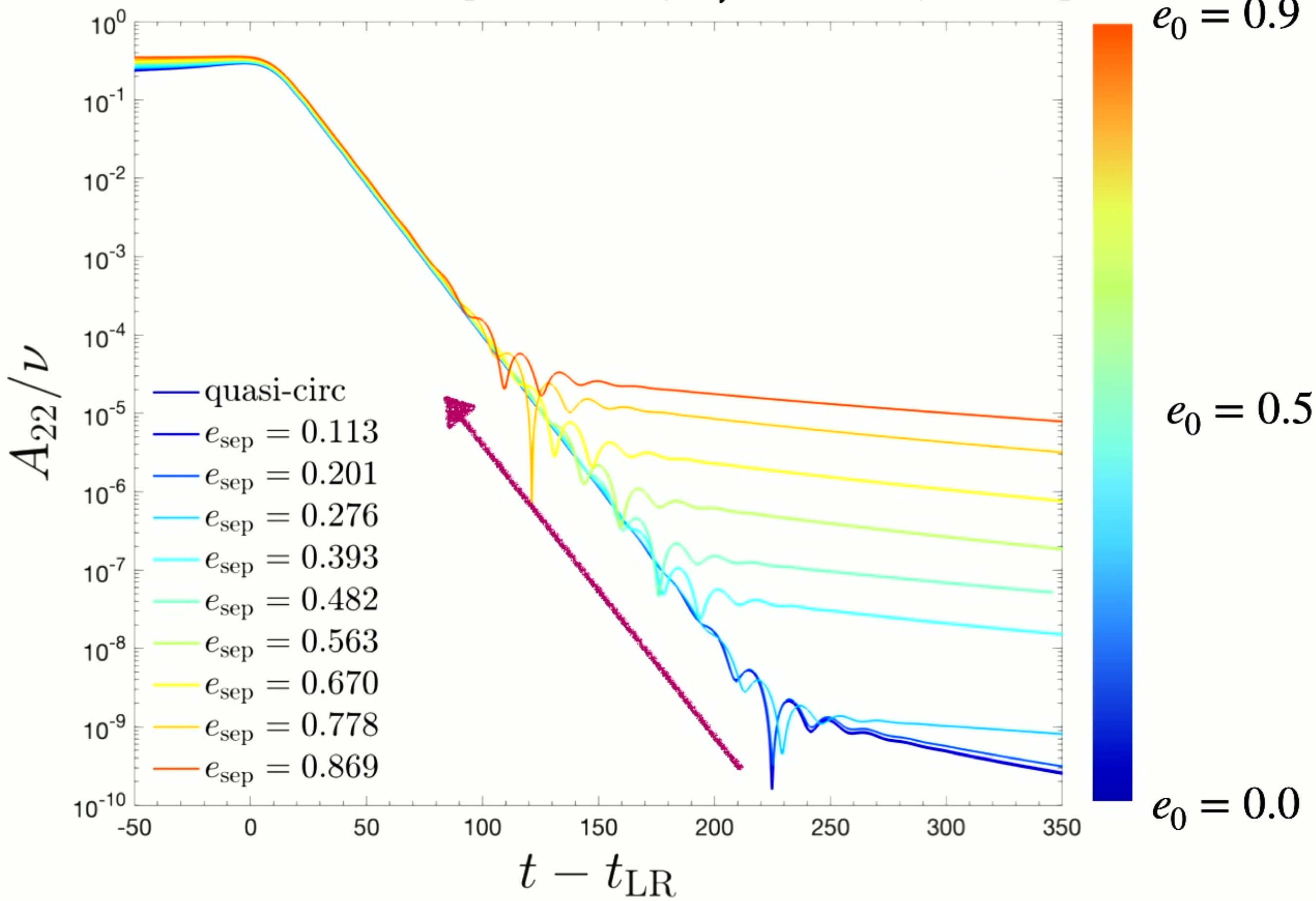

Amplitude enhanced of several orders of magnitude by eccentricity

What happens for comparable masses

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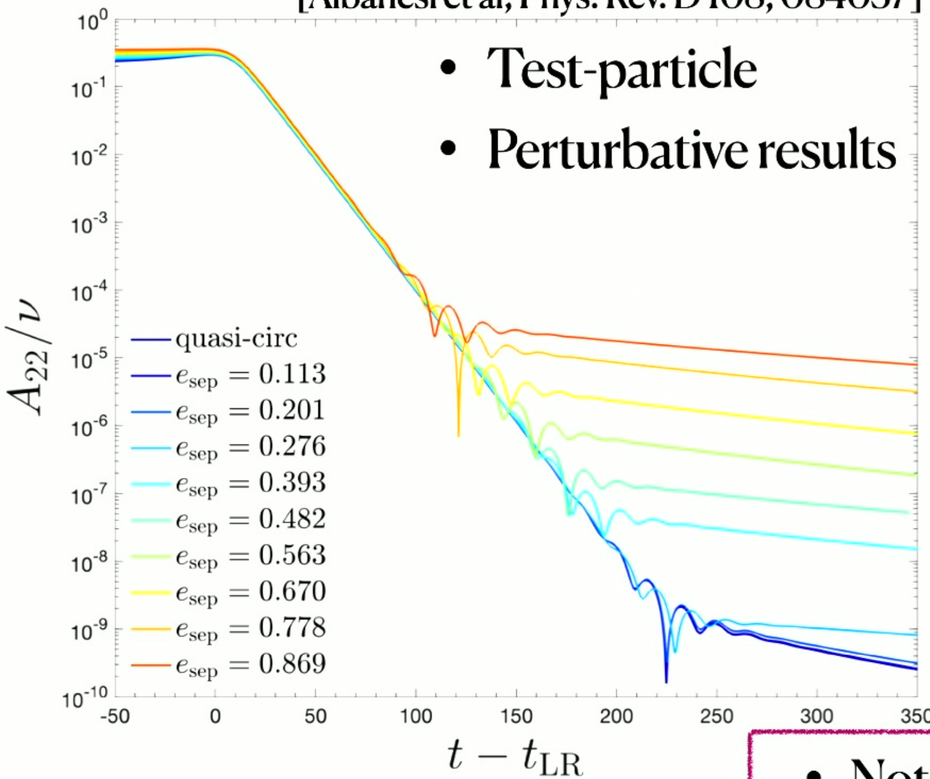
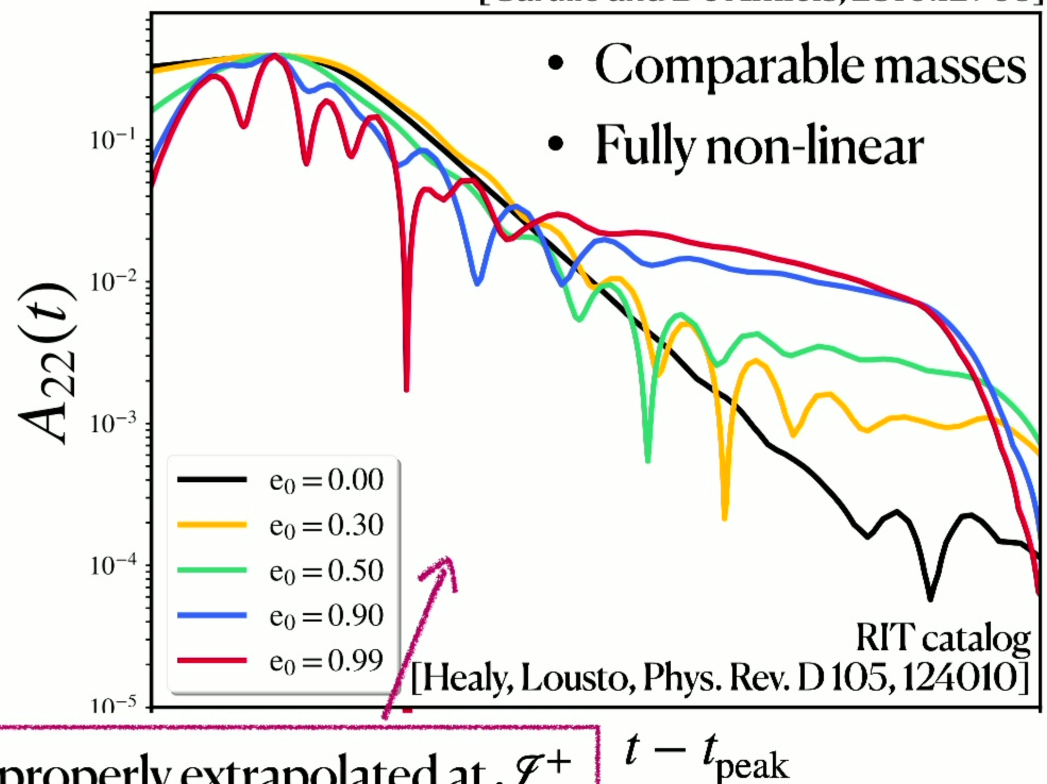
An exciting journey: EMR vs comparable masses

[Albanesi et al, Phys. Rev. D 108, 084037]



[Carullo and **De Amicis**, 2310.12968]



- Not-properly extrapolated at $\mathscr{I}^+$
- Is enhancement physical?

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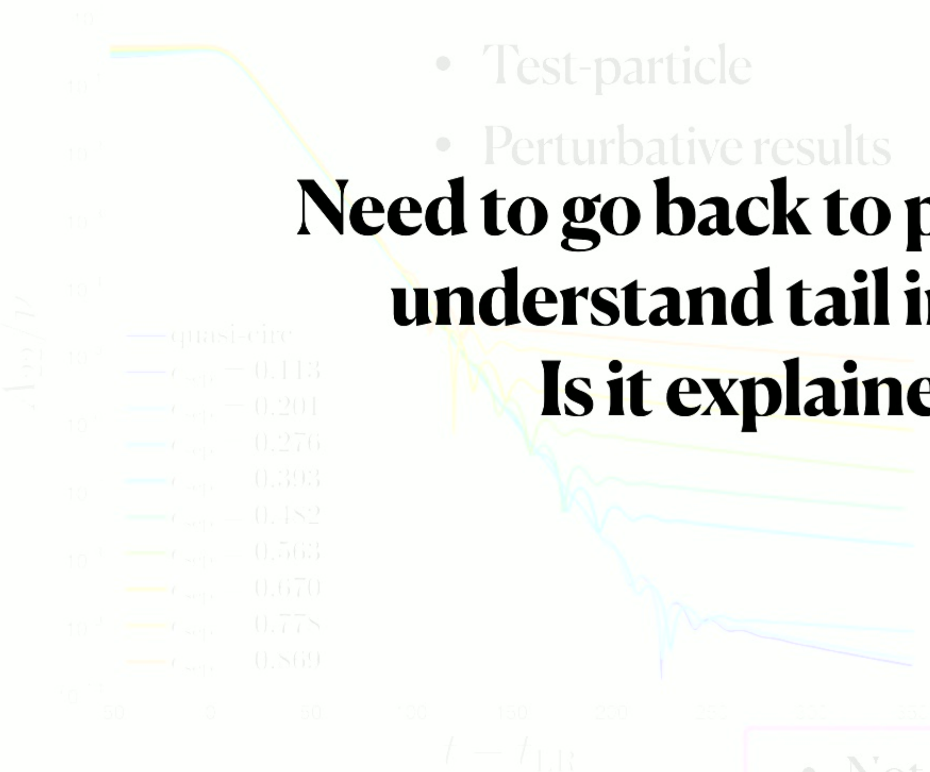
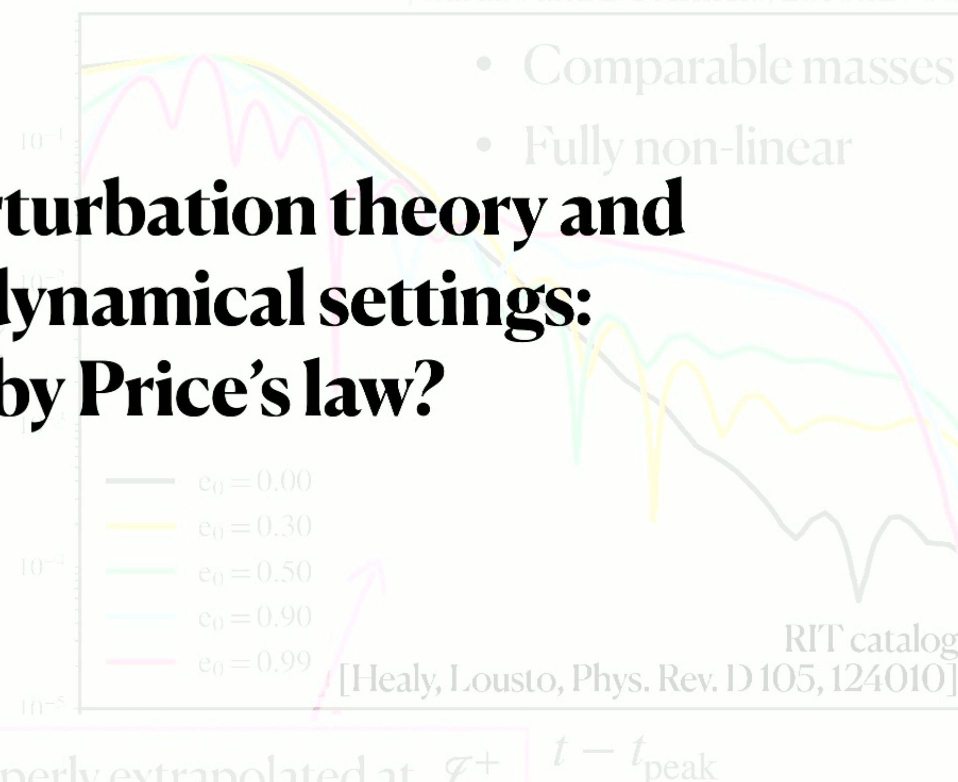
An exciting journey: EMR vs comparable masses

- Test-particle
- Perturbative results



[Carullo and De Amicis, 2510.12968]

- Comparable masses
- Fully non-linear



- Not-properly extrapolated at $\mathscr{I}^+$
- Is enhancement physical?

Need to go back to perturbation theory and understand tail in dynamical settings: Is it explained by Price's law?

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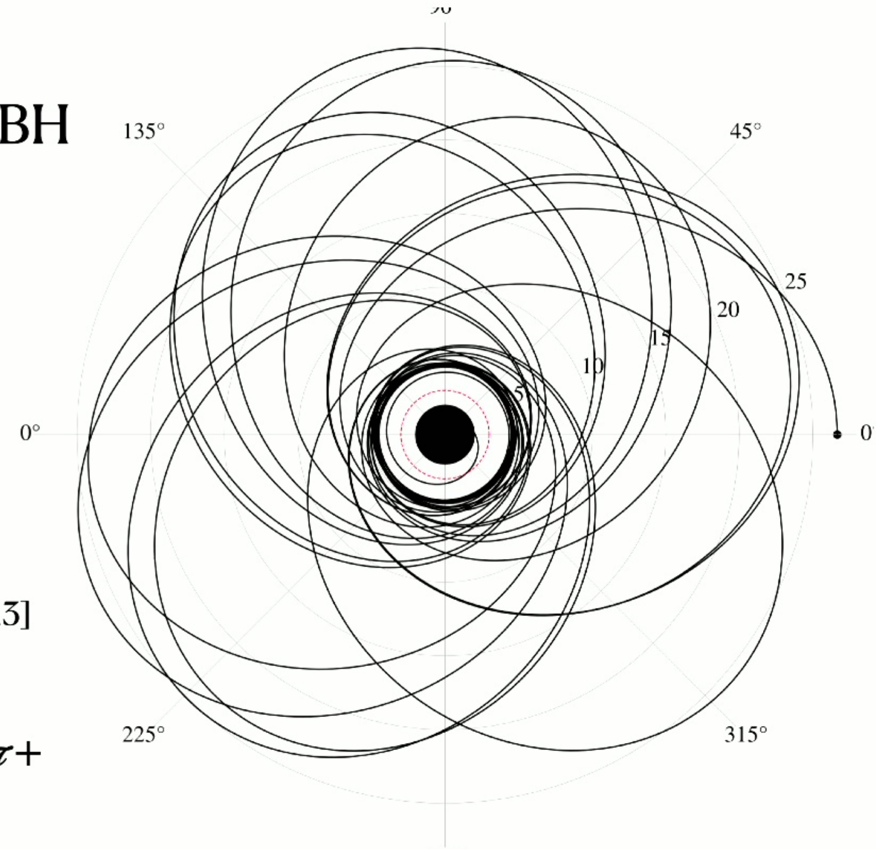
Framework

- Test particle μ infalling in a Schwarzschild BH
- Perturbation theory
- Signal computed at scri+ (null infinity)
  - As observed by real detectors [Zenginoglu, Class.Quant.Grav. 25 (2008) 175013]
  - Price's law:
    - $\Psi_{\ell m} \propto \frac{1}{\tau^{\ell+2}}, \ \ \tau \equiv t - r_* \ \text{ at } \mathscr{I}^+$
    - $\Psi_{\ell m} \propto \frac{1}{t^{2\ell+3}}$ at finite distance ⟶ Suppressed!



[Price, Phys. Rev. D 5, 2419]
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Numerical evolutions

$$\left[\partial_t^2 - \partial_{r_*}^2 + V_{\ell m}^{(e/o)}(r_*)\right] \Psi_{\ell m}^{(e/o)}(t, r_*) = S_{\ell m}^{(e/o)}(t, r)$$

$$\Psi_{\ell m}^{(e/o)}(t=0,r) = \partial_t \Psi_{\ell m}^{(e/o)}(t=0,r) = 0$$

+ **Hamiltonian equations of motion** for the trajectory, driven **radiation-reaction**

[Chiaramello and Nagar, Phys. Rev. D 101, 101501 (2020)]
[Albanesi, Nagar, Bernuzzi, Phys. Rev. D 104, 024067 (2021)]

- **Analytically** computed through Post-Newtonian expansions of fluxes extracted at infinity
- Allow to evolve a **generic orbit** up to merger

- **Hyperboloidal layer** over which r_* is **compatified**
- Compute the radiative signal at $\mathscr{I}^+$

RWZhyp code:
[Bernuzzi and Nagar, Phys. Rev. D 81, 084056(2010)]
[Bernuzzi, Nagar and Zenginoglu, Phys. Rev. D 84, 084026(2011)]

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Analytical model

$$S_{\ell m}(t,r) = f_{\ell m}(t,r)\delta(r - r(t)) + g_{\ell m}(t,r)\partial_r\delta(r - r(t))$$

Regge-Wheeler/
Zerilli equations:

$$\left[\partial_t^2 - \partial_{r_*}^2 + V_{\ell m}(r_*)\right]\Psi_{\ell m}(t, r_*) = S_{\ell m}(t,r)$$

$$\Psi_{\ell m}(t=0,r) = \partial_t \Psi_{\ell m}(t=0,r) = 0$$

Most general solution:

$$\Psi_{\ell m}(\tau,\rho_+) = \int_{T_{in}}^{\tau-\rho_+} dt' \int dr' S_{\ell m}(t',r') G_\ell(\tau,t';r',\rho_+)$$

$\rho_+ \equiv$ location of $\mathcal{J}^+$ in the compactified coordinate

Price's law propagator

$$\Psi_{\ell m}(\tau,\rho_+) = \frac{(-1)^\ell 2^{\ell+1}\ell!(\ell+1)!}{(2\ell+1)!}\int_{T_{\rm in}}^{\tau-\rho_+} dt' \frac{r^\ell(t')\left[r\left(f_{\ell m}(t') - \partial_r g_{\ell m}(t')\right) - \left(\ell+1\right) g_{\ell m}(t')\right]_{r=r(t')}}{\left(\tau-\rho_+-t'\right)^{\ell+2}}$$

[De Amicis, Albanesi and Carullo, 2406.17018]

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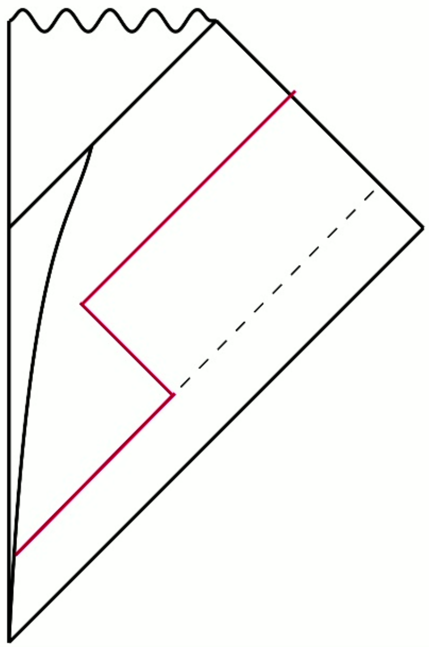
Analytical model

Analytical integral form of the tail:

$$\Psi_{\ell m}(\tau,\rho_+) = \frac{(-1)^\ell 2^{\ell+1}\ell!(\ell+1)!}{(2\ell+1)!}\int_{T_{\rm in}}^{\tau-\rho_+} dt' \frac{r^\ell(t')\left[r\left(f_{\ell m}(t') - \partial_r g_{\ell m}(t')\right) - (\ell+1)\, g_{\ell m}(t')\right]_{r=r(t')}}{\left(\tau-\rho_+-t'\right)^{\ell+2}}$$

- Tail as an **hereditary effect**

  [Blanchet-Damour ,*Phys.Rev.D* 46 (1992)]
  [Poisson, Phys.Rev.D 66 (2002)]



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- **Not** an **exact power-law** behavior

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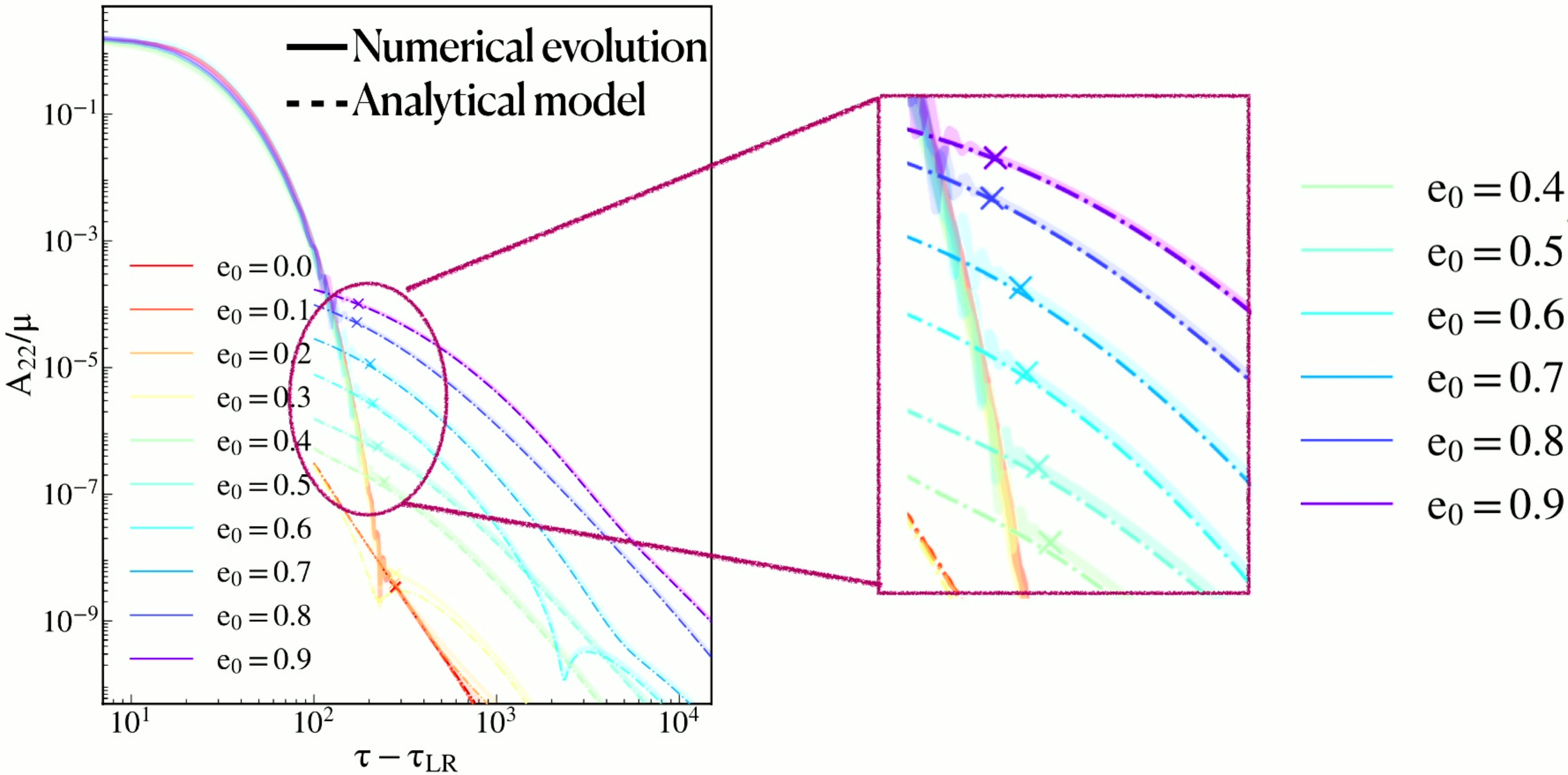
Model vs numerical evolutions: eccentric orbits



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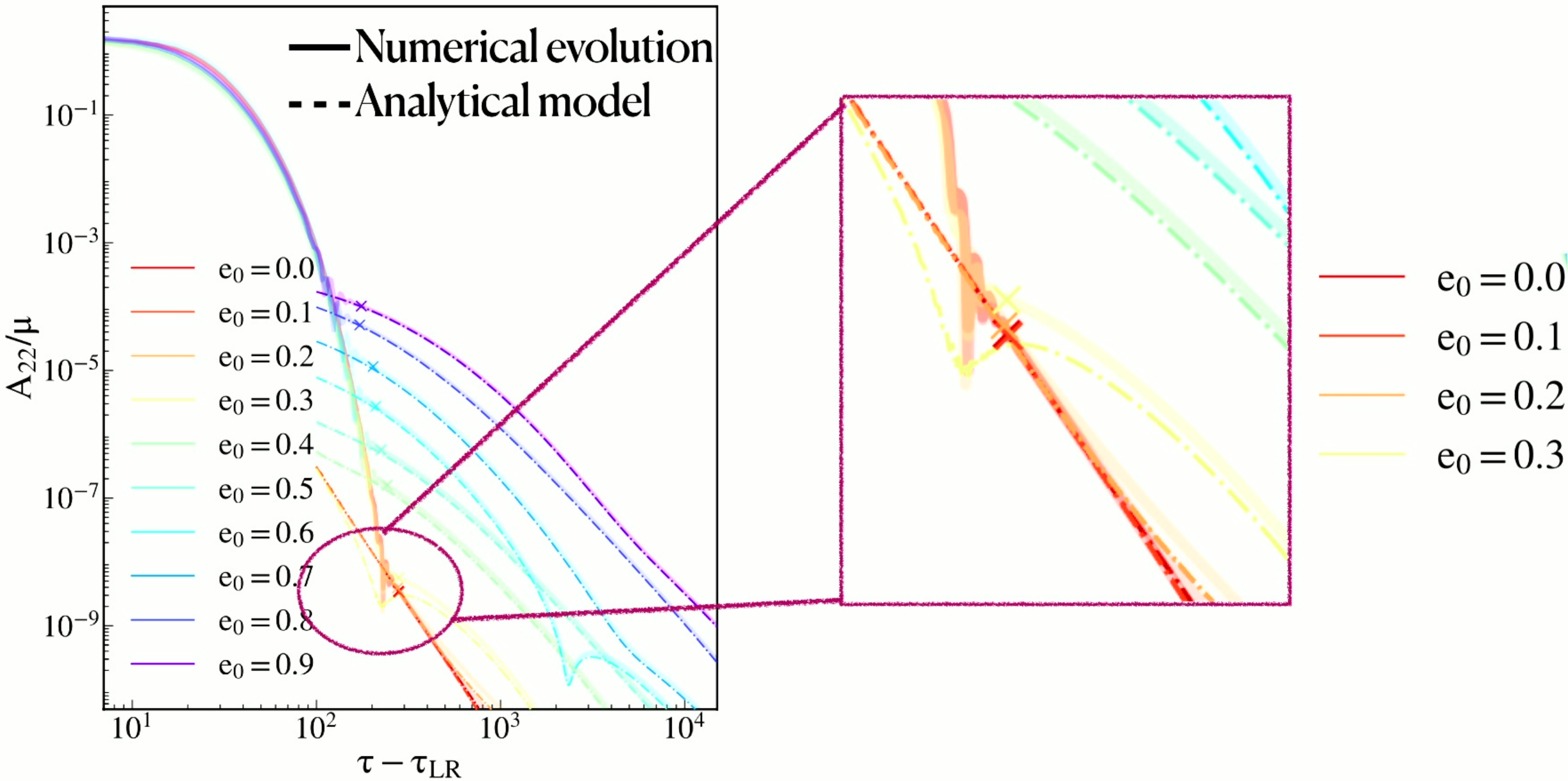

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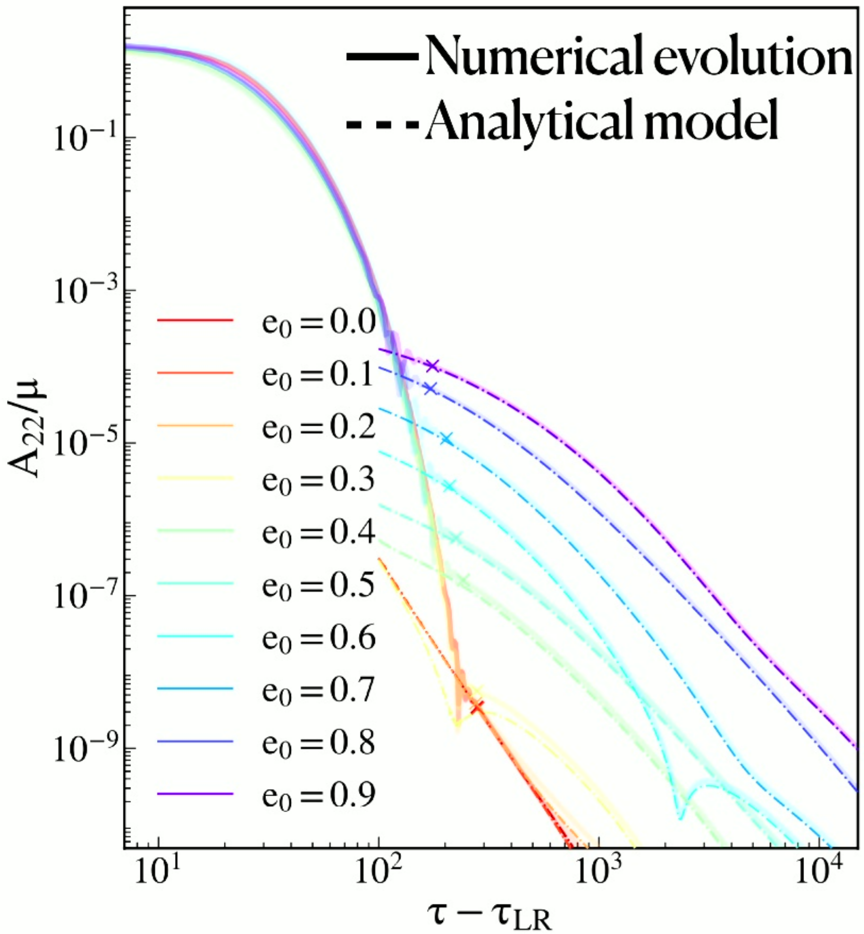

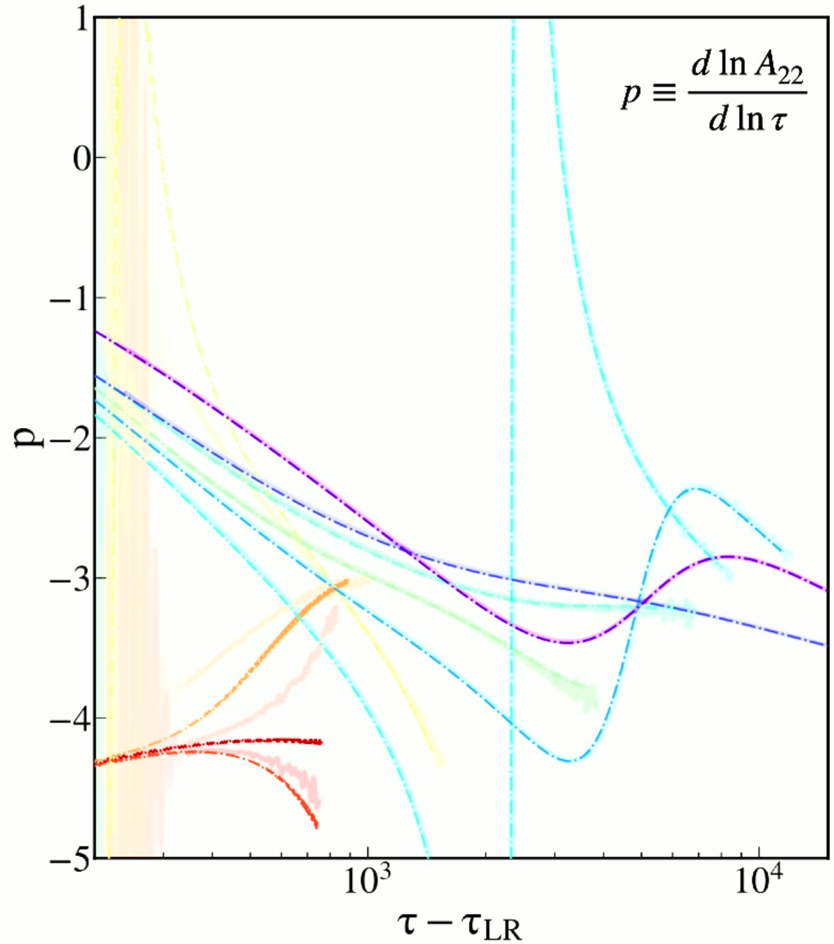

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Take a breath

- Integral model for tail in EMR, as an hereditary effect ✓
- Tail exponent is in general non monotonic
- Tail amplitude is enhanced by eccentricity

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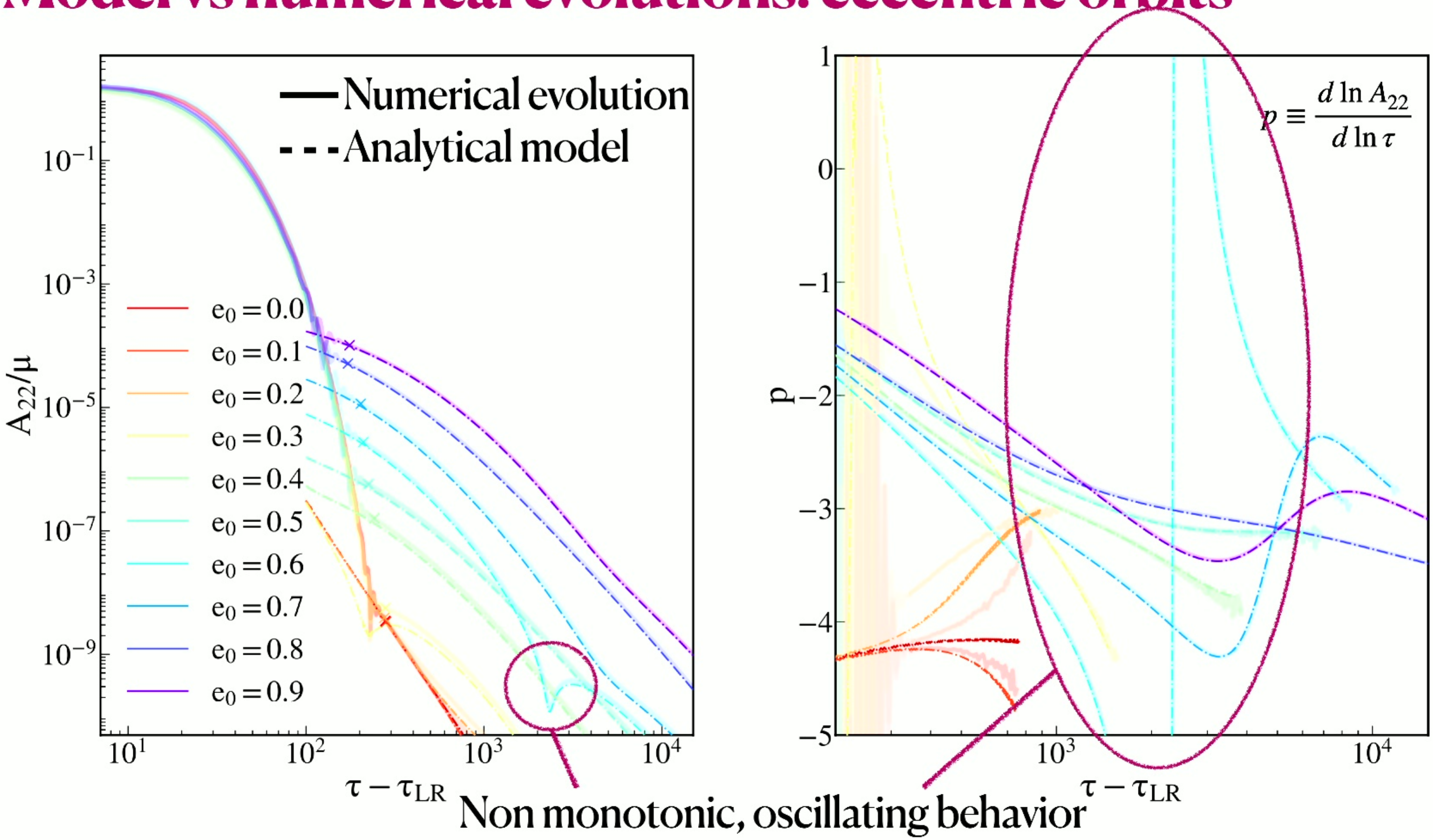
Model vs numerical evolutions: eccentric orbits



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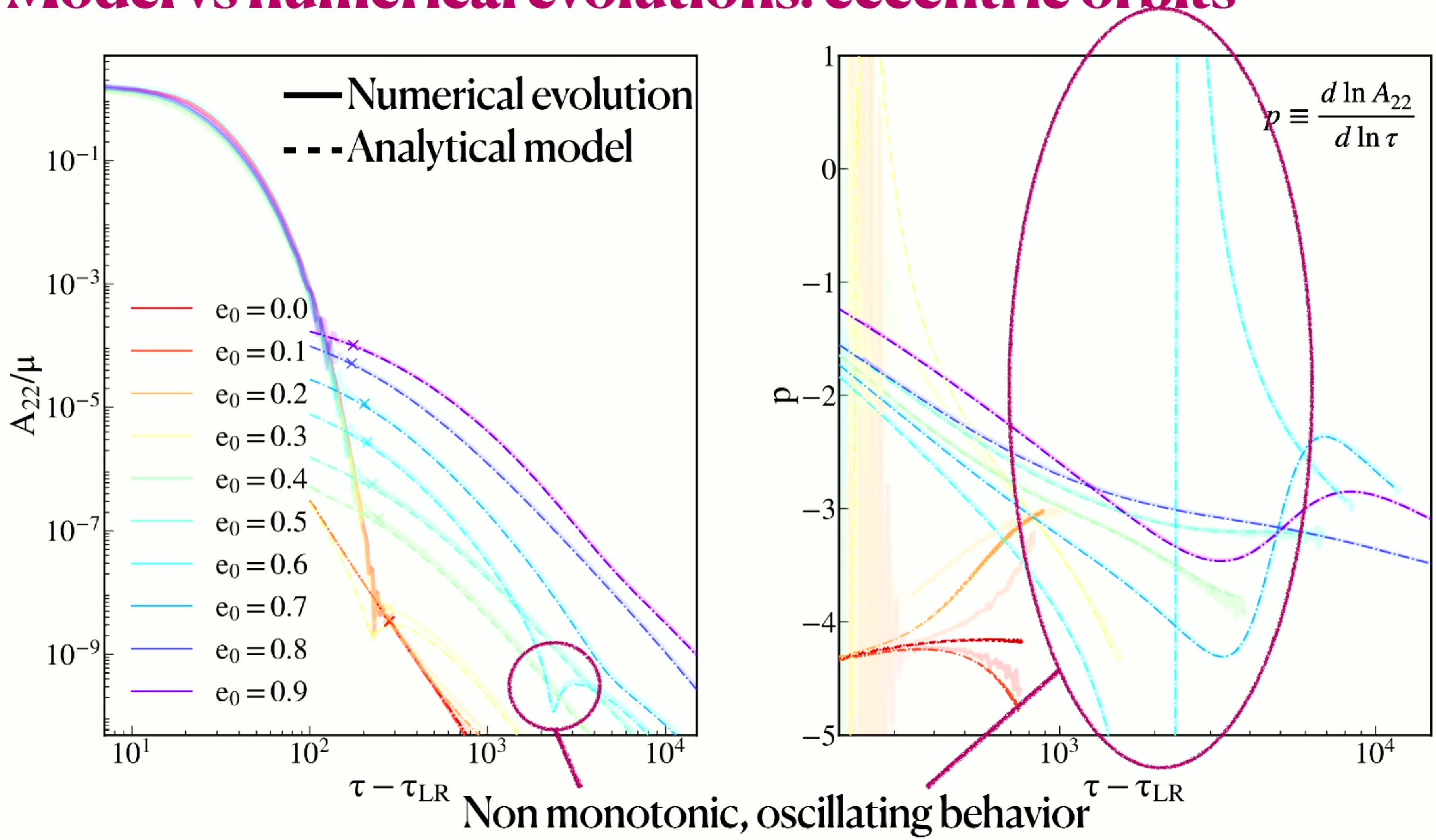

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Analytical model

Analytical integral form of the tail:

$$\Psi_{\ell m}(\tau,\rho_+) = \frac{(-1)^{\ell} 2^{\ell+1} \ell!(\ell+1)!}{(2\ell+1)!} \int_{T_{\rm in}}^{\tau-\rho_+} dt' \frac{r^{\ell}(t')\left[r\left(f_{\ell m}(t') - \partial_r g_{\ell m}(t')\right) - (\ell+1)\, g_{\ell m}(t')\right]_{r=r(t')}}{\left(\tau-\rho_+-t'\right)^{\ell+2}}$$

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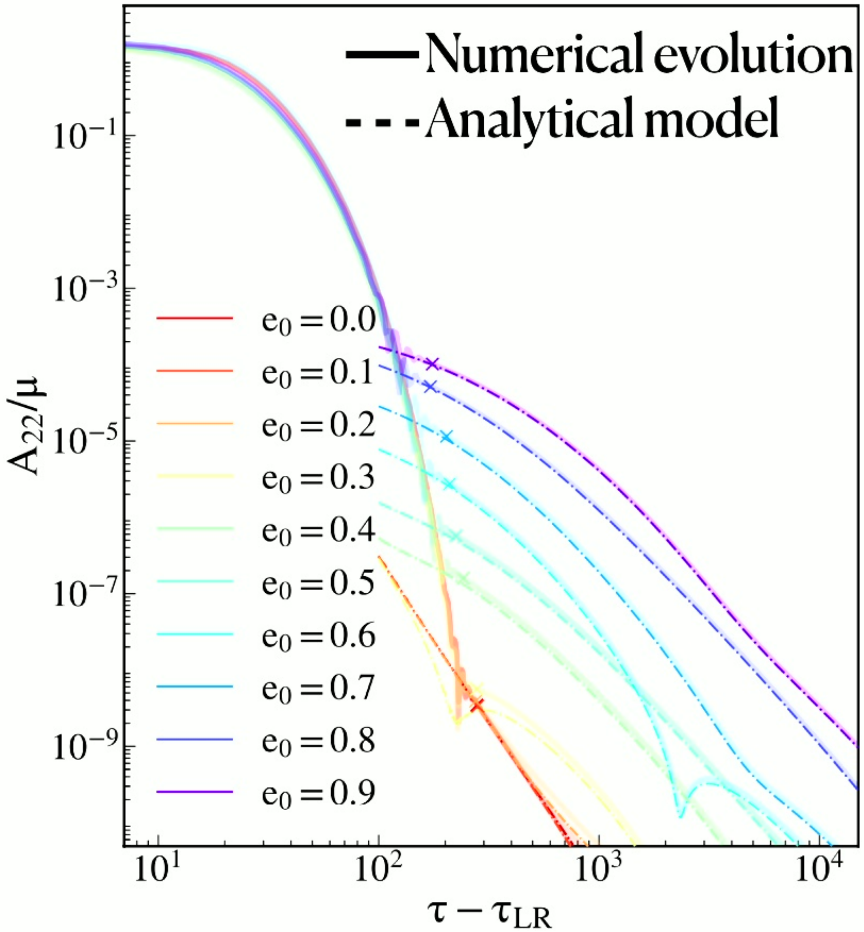

[De Amicis, Albanesi and Carullo, 2406.17018]

Tail as superposition of power-laws

$$\Psi_{\ell m}(\tau,\rho_+) = \frac{c_\ell}{\tau^{\ell+2}} \int_{t_{\rm in}}^{t_f} dt'\, S_\ell(t') \left[1 + \sum_{n=1}^{\infty} \frac{(\ell+1+n)!}{n!(\ell+1)!} \left(\frac{t'+\rho_+}{\tau}\right)^n\right]$$

$$\tau - \rho_+ \gg t_{\rm in}, t_f$$

- Superposition of power-laws
- Slowest decay is Price's law
- **Excitation coefficient** of each power-law depends on:
  - **amount of history**
  - **specific orbit**

$t_{\rm in}$ = initial time

t_f = common horizon

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Tail as superposition of power-laws

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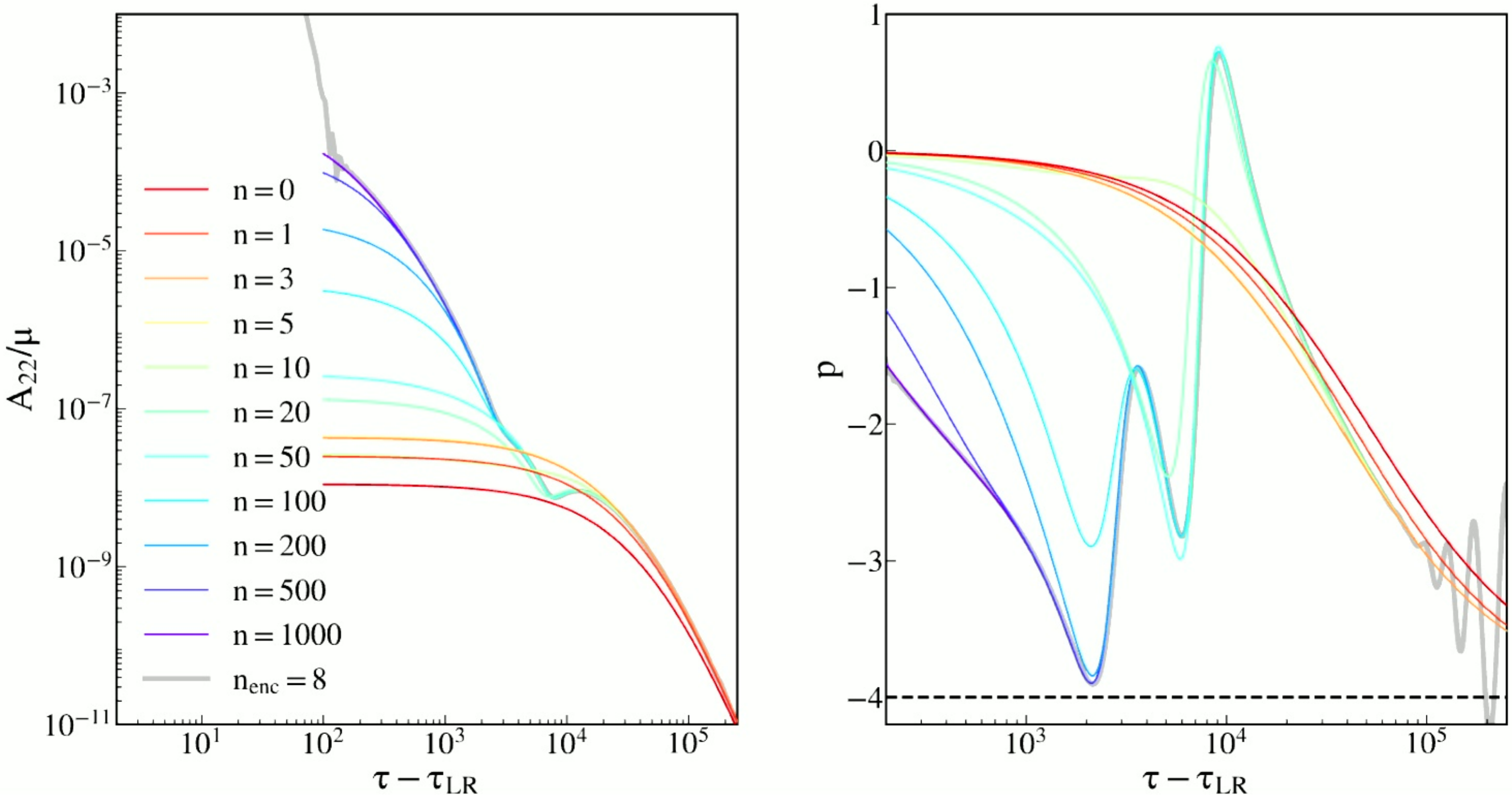
$$\tau - \rho_+ \gg t_{\rm in}, t_f$$



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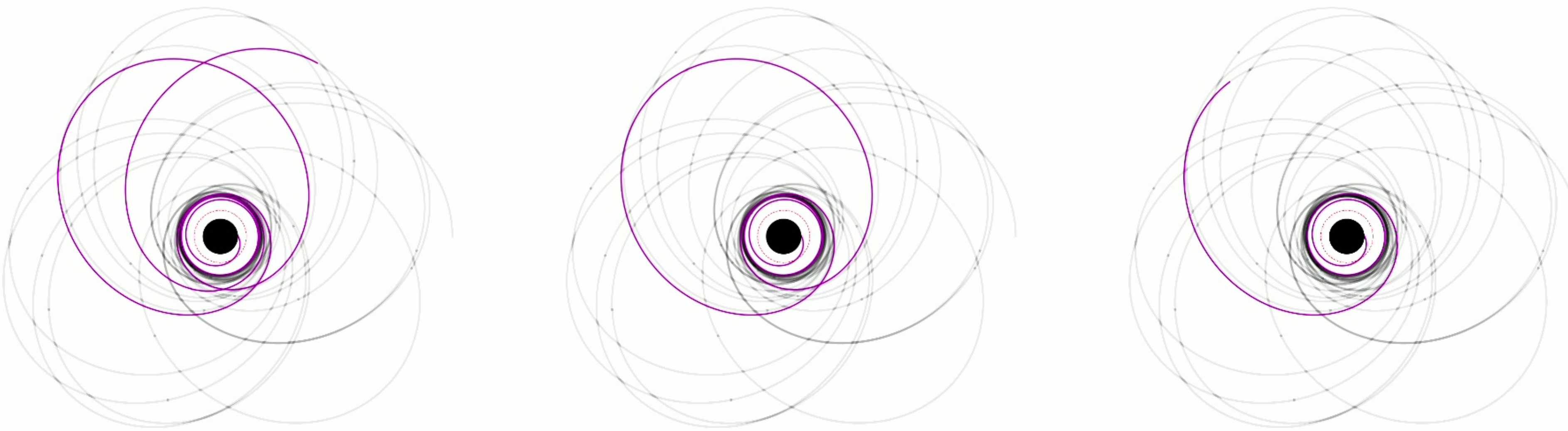
Take a breath

- Integral model for tail in EMR, as an hereditary effect
- Tail as superposition of power laws $\tau^{-\ell-2-n}$, with $n \geq 0$
- Tail amplitude is enhanced by eccentricity

Enhancement with eccentricity

$$\Psi_{\ell m}(\tau,\rho_+) = \frac{(-1)^\ell 2^{\ell+1}\ell!(\ell+1)!}{(2\ell+1)!}\int_{t_{\rm in}}^{\tau-\rho_+} dt' \frac{r^\ell(t')\left[r\left(f_{\ell m}(t') - \partial_r g_{\ell m}(t')\right) - \left(\ell+1\right) g_{\ell m}(t')\right]_{r=r(t')}}{\left(\tau-\rho_+-t'\right)^{\ell+2}}$$

Isolate the part of the trajectory which determines the amplitude at the transition from QNMs to tail



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Enhancement with eccentricity

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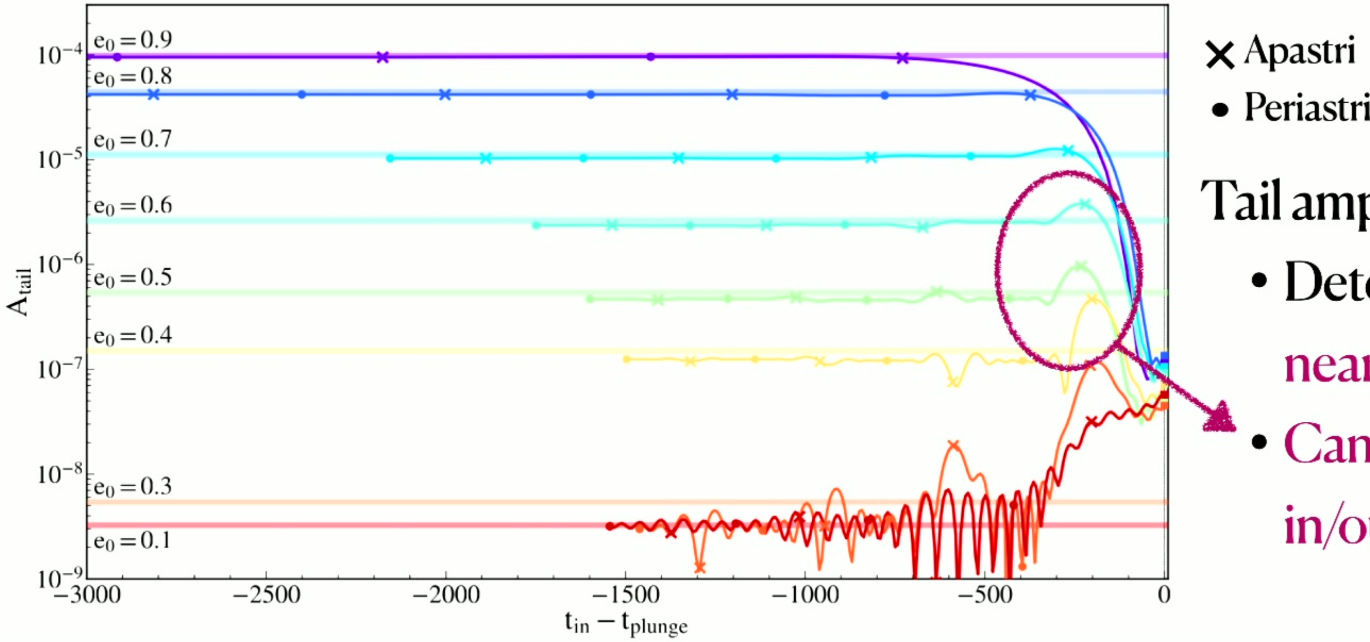

Tail amplitude:

- Determined by motion near last apastron
- Cancellation among in/outgoing motion

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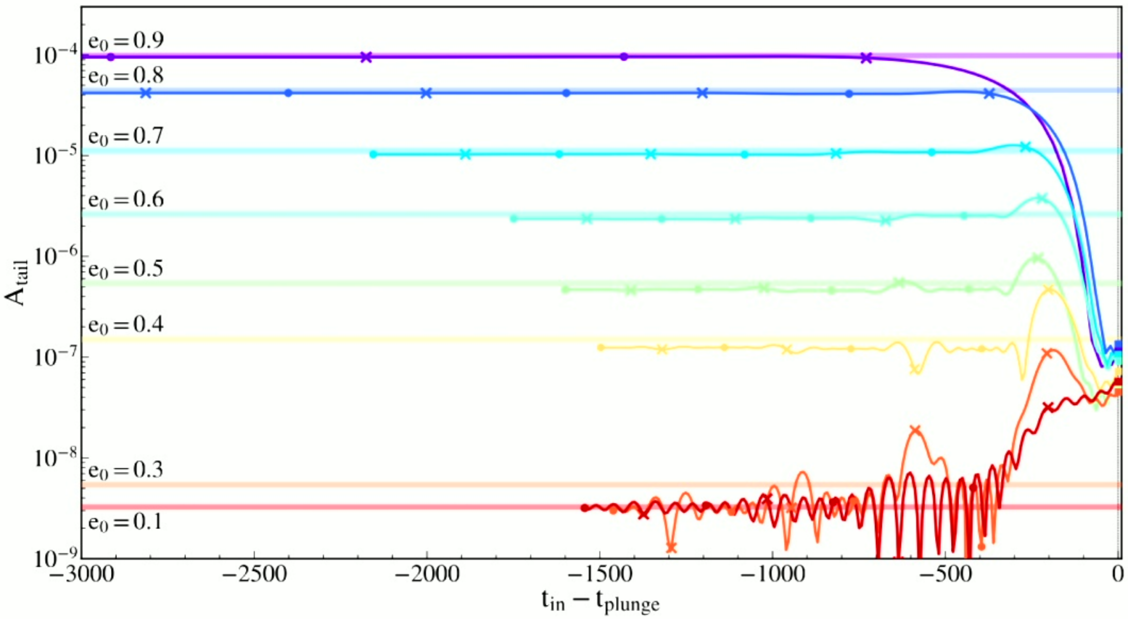
Enhancement with eccentricity

Expand in large r and small p_φ/r: $\Psi_{\ell m}(\tau,\rho_+) = \int_{t_{in}}^{t_f} dt' \frac{r^\ell(t') e^{-im\varphi(t')} P_{\ell m}(\cos\theta_0)}{(\tau - t' - \rho_+)^{\ell+2}} \cdot \left[a_1 - \frac{a_1}{2}\dot{r}^2 + a_2 \dot{r}\frac{p_\varphi}{r} + \left(a_3 + \frac{a_1}{2}\right)\frac{p_\varphi^2}{r^2}\right]$

× Apastri

• Periastri



Oscillanting contribution can induce destructive interference

- Tail maximied for radial infall!
- Tail enhanced for $m = 0$ modes, even for quasi-circular binaries

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Enhancement with eccentricity

Expand in large r and small p_φ/r: $\Psi_{\ell m}(\tau,\rho_+) = \int_{t_{in}}^{t_f} dt' \frac{r^\ell(t')e^{-im\varphi(t')}P_{\ell m}(\cos\theta_0)}{(\tau - t' - \rho_+)^{\ell+2}} \cdot \left[a_1 - \frac{a_1}{2}\dot{r}^2 + a_2\dot{r}\frac{p_\varphi}{r} + \left(a_3 + \frac{a_1}{2}\right)\frac{p_\varphi^2}{r^2}\right]$

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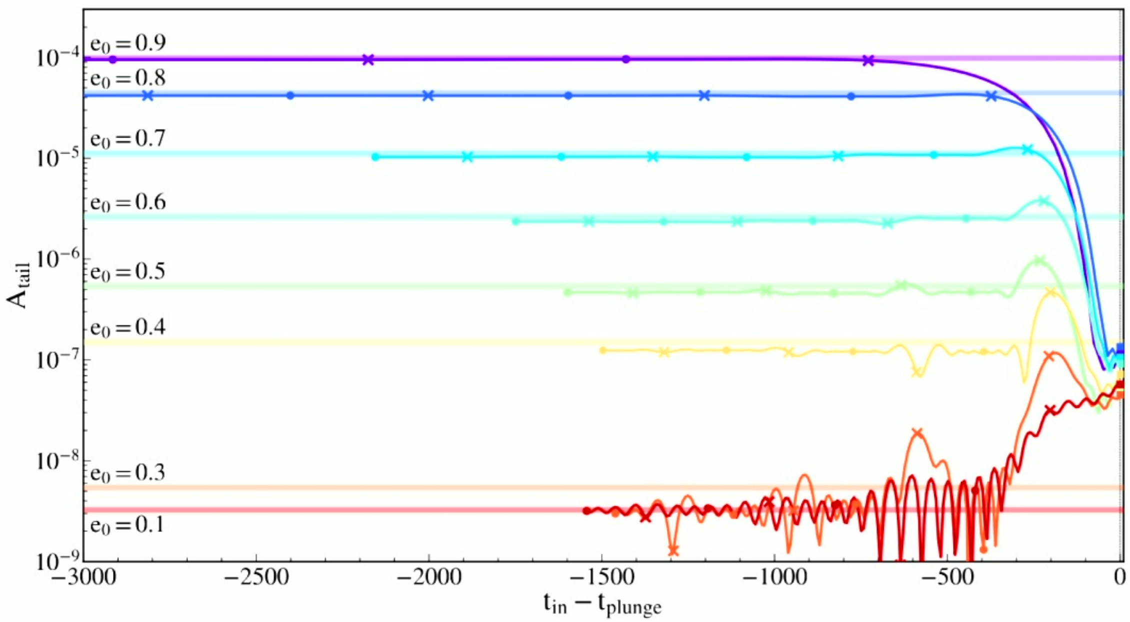

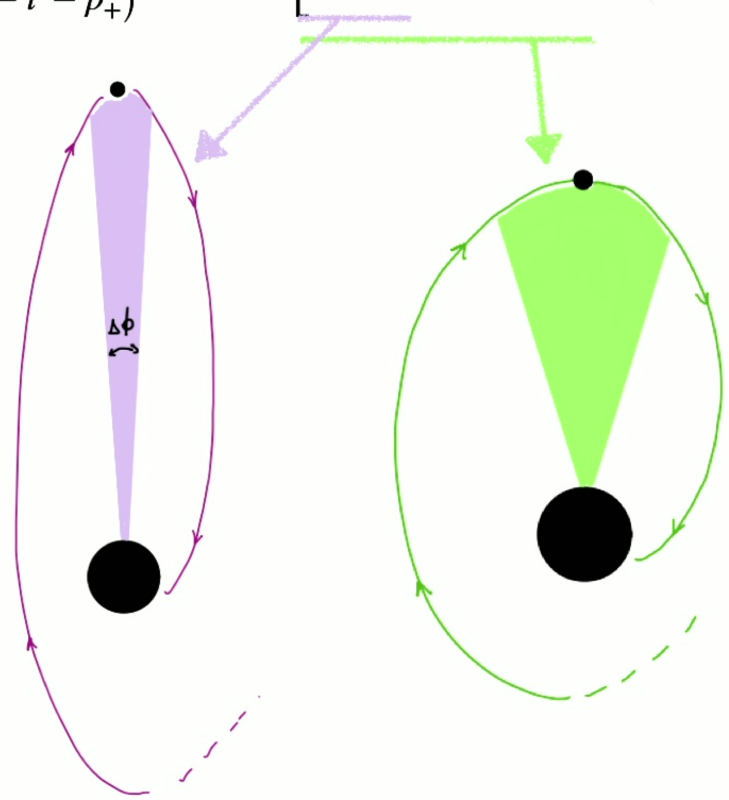

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Where do we go from here?

- Integral model for tail in EMR, as an hereditary effect ✓
- Tail as superposition of power laws $\tau^{-\ell-2-n}$, with $n \geq 0$ ✓
- Tail emission enhanced for motion at large distances $r \gg$ M, with small tangential velocity. Hence, emission is maximized at apastra ✓
- Tails in a comparable masses merger, in fully non-linear setting

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What was wrong with previous analysis

- RIT solves for Ψ^4 ⟶ suppressed tail
- Extract h: 2 integrations in t-domain

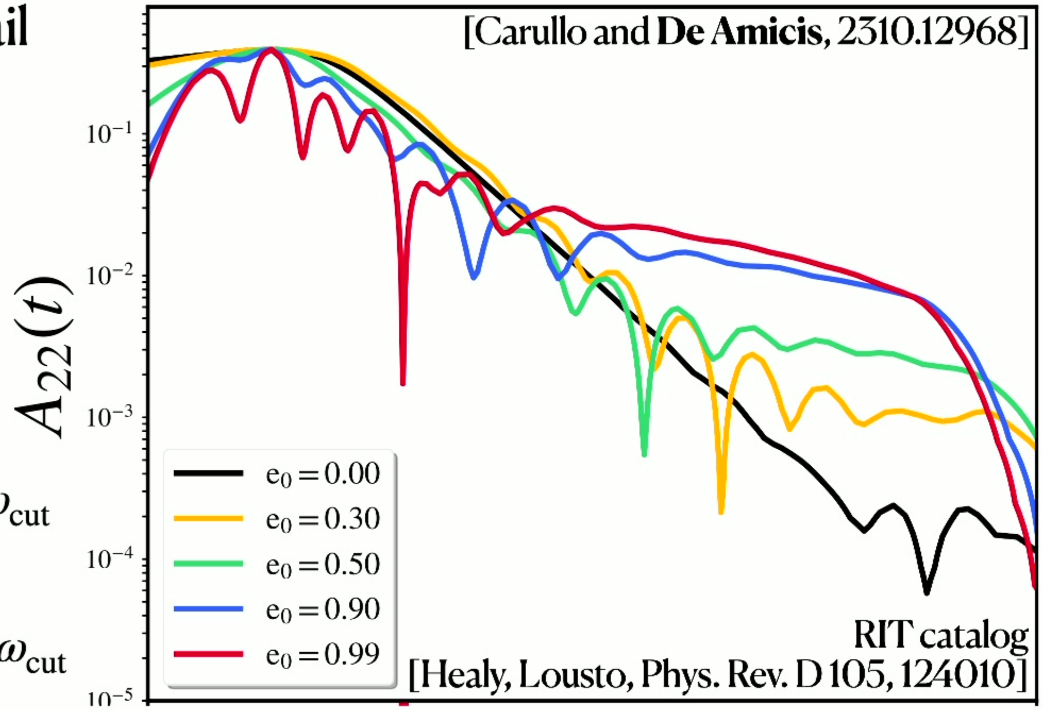
  ✗ Enhances noise!

- Extract h: 2 integrations in ω-domain

$$\int_{-\infty}^{t} dt' \; H(t') = \mathcal{F}^{-1}\left[\frac{i}{\omega}\tilde{H}(\omega)\right] \quad , \quad \omega > \omega_{\rm cut}$$

$$= \mathcal{F}^{-1}\left[\frac{i}{\omega_{\rm cut}}\tilde{H}(\omega)\right] \quad , \quad \omega \leq \omega_{\rm cut}$$

✗ ω cutoff induce spurious tails!



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What was wrong with our analysis

- RIT solves for the signal at finite distances ⟶ extrapolate to $\mathscr{I}^+$

  If not under control, can induce systematics!

- Compute waveform at N_r finite radii $\{r_j\}_{N_r}$

- Build $u(t, r)$ as null coordinate for $r \to \infty$ and compute $h(u, r_j)$

- $\forall u_i$ fit with template: $h(u_i, \{r_j\}) = \dfrac{a_1}{r} + \dfrac{a_2}{r^2} + \ldots + \dfrac{a_N}{r^N}\ , \quad N \leq N_r - 1$

  $\lim_{r \to \infty} rh(u, r) = h(u)$ as expected for asymptotically flat spacetimes

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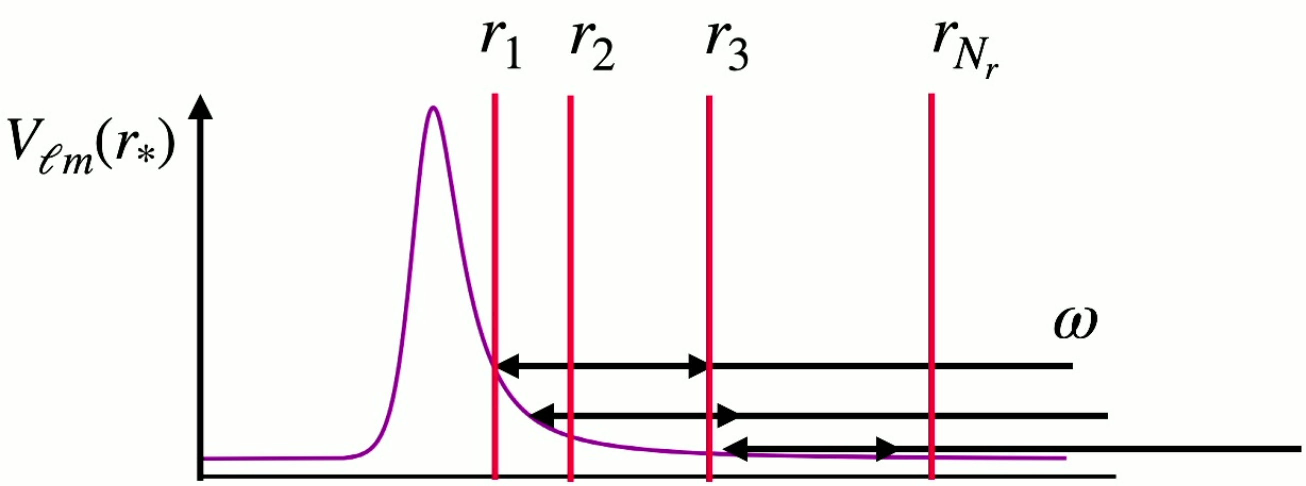
What was wrong with our analysis

- RIT solves for the signal at finite distances ⟶ extrapolate to $\mathscr{I}^+$

If $\{r_j\}_{N_r}$ are too close to the BH ⟶ we are cutting tail contribution!



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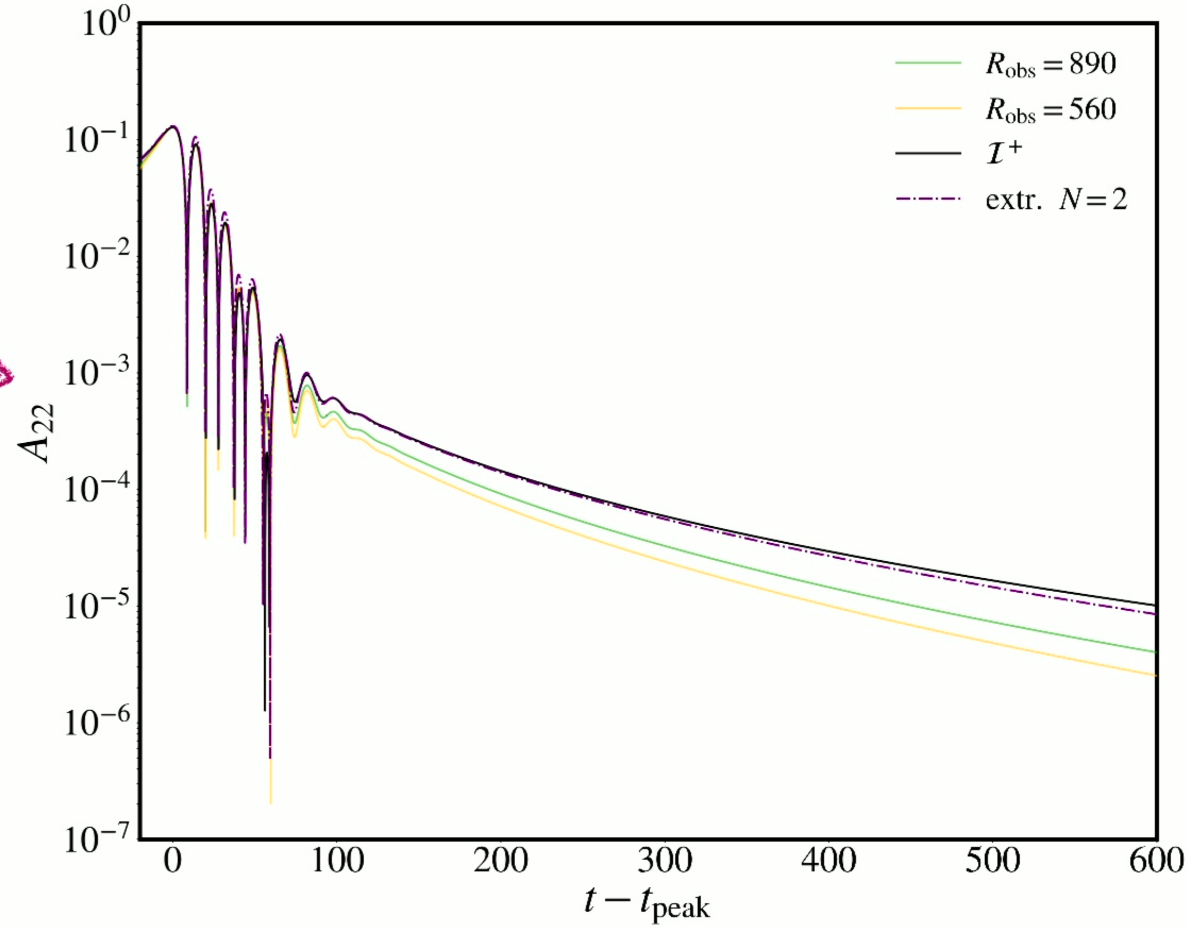
What was wrong with our analysis

- Large extrapolation radii needed

Test on perturbative results



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Extracting tails in full numerical relativity

- SXS solves for $h(t, r)$ ⟶ no integrations needed!

- $h(t, r)$ characterized by **late-times constant**: memory? gauge effects?

  ⟶ We analize $\dot{h}(t, r)$

- What is the **best configuration** for a **robust extraction of tails**?

  - Maximum enhancement
  - Comparision with perturbative results

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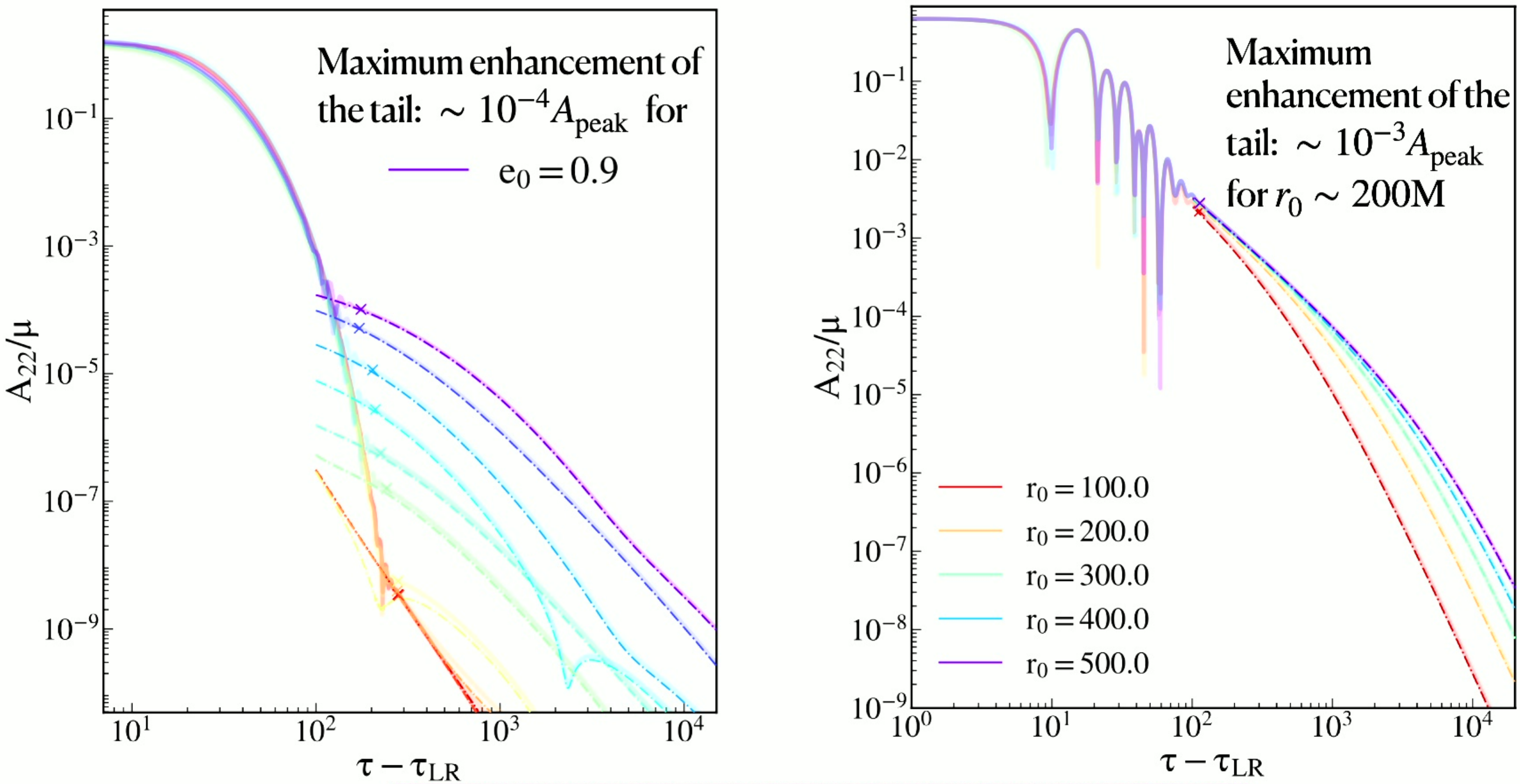
Eccentric orbits vs radial infalls (PT results)



Tail emission maximied for radial infalls from $r_0 \sim 200M$

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Extracting tails in full numerical relativity

- SXS solves for $h(t, r)$ → no integrations needed!

- $h(t, r)$ characterized by **late-times constant**: memory? gauge effects?

→ We analize $\dot{h}(t, r)$

- What is the **best configuration** for a **robust extraction of tails**?
  - Maximum enhancement
  - Comparision with perturbative results

→ Head-on collisions with different initial separations $R_0 \geq 100$

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Extracting tails in full numerical relativity

- Study head-on collisions with different initial separations $R_0 \geq 100$ ✓
- Use large extraction radii for the extrapolation ✓
- Boundary of integration can contaminate the late-times waveform!

→ Push boundary at large r, such that is it **not in causal contact with the waveform** along the whole evolution

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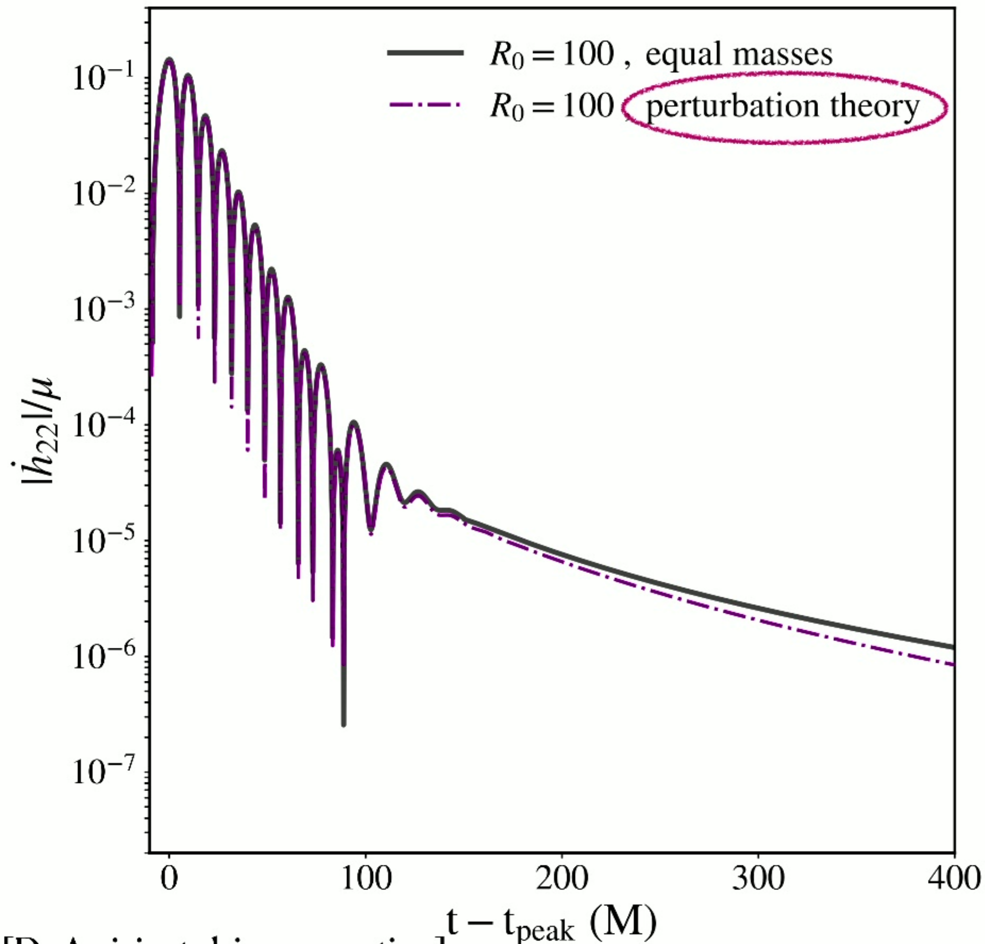
Extracting tails in full numerical relativity



[De Amicis et al, in preparation]

- Test-particle infall initialized from the same distance R_0
- Same initial ADM energy
- Waveform rescaled by $\frac{1}{\mu}$

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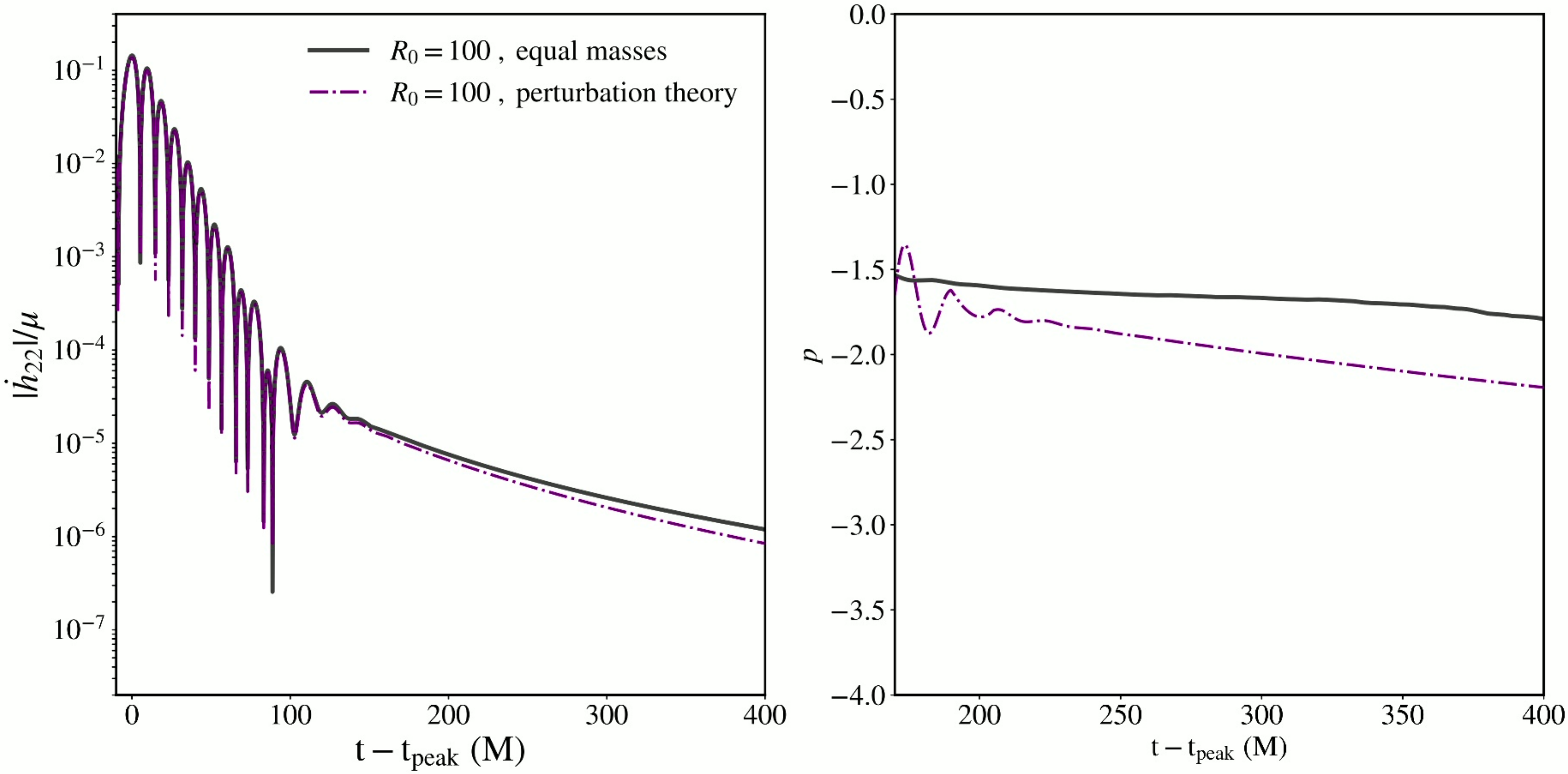
Extracting tails in full numerical relativity



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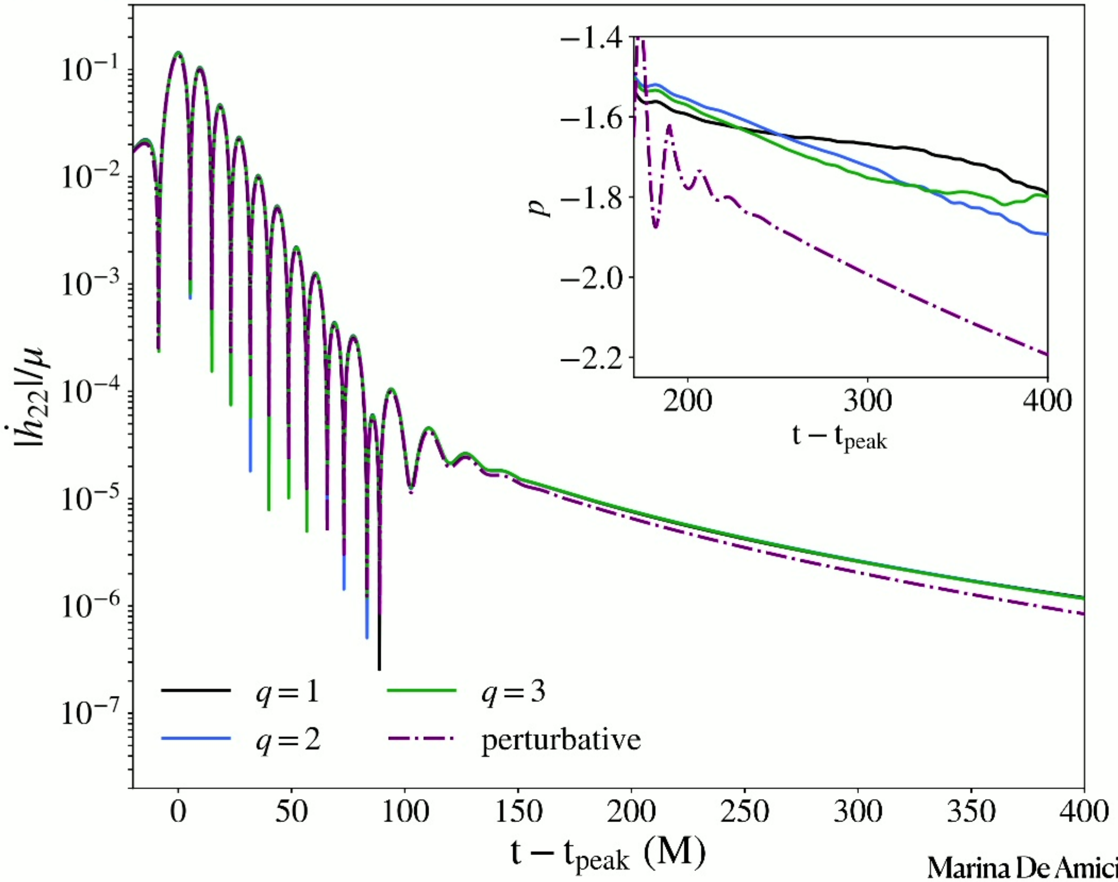
Dependence on mass-ratio

Tails are hereditary effects: can accumulate and amplify non linearities!

- $\mathcal{O}(\mu^2)$ corrections in source
- Second order tails
  [Okuzumi et al, 0803.0501]
  [Cardoso et al (De Amicis), 2405.12290]
- Dynamical background (3rd order)



[De Amicis et al, in preparation]

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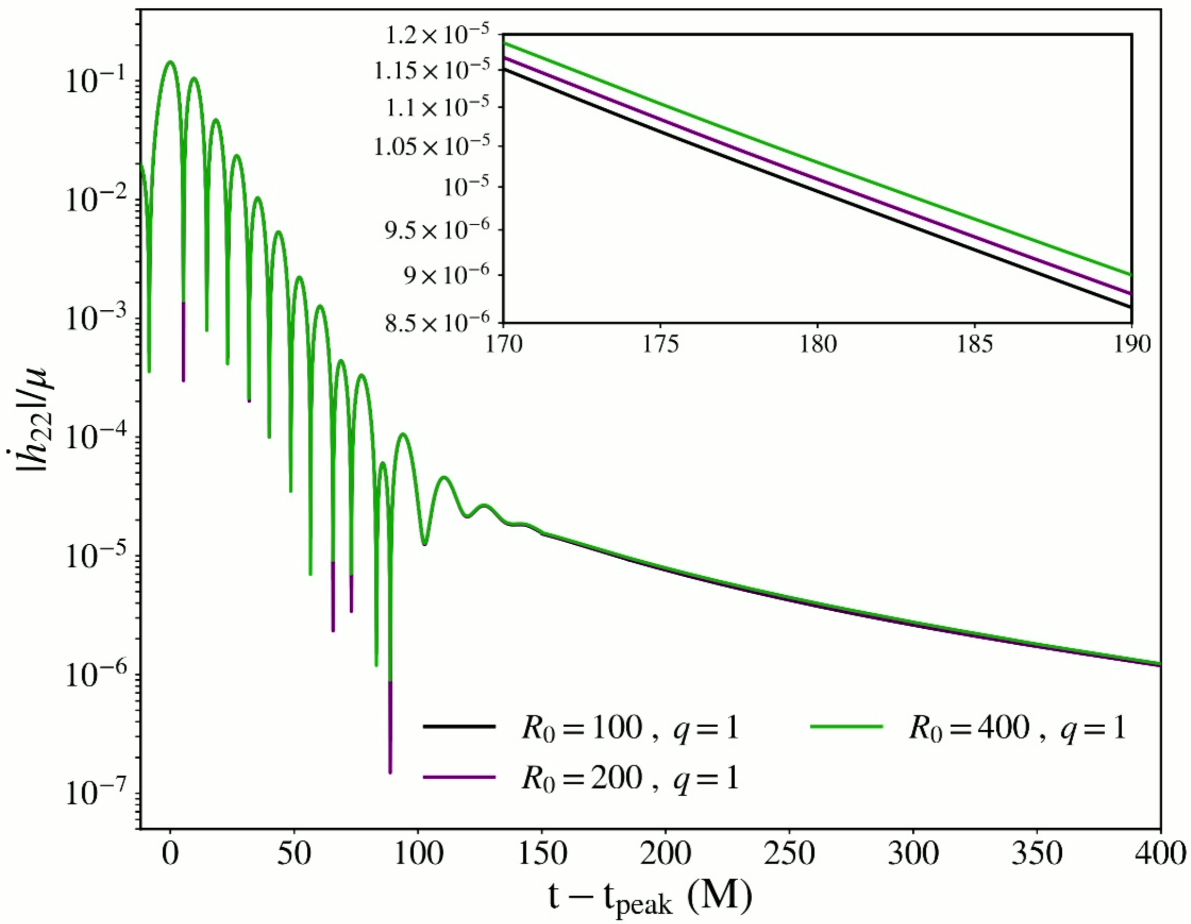
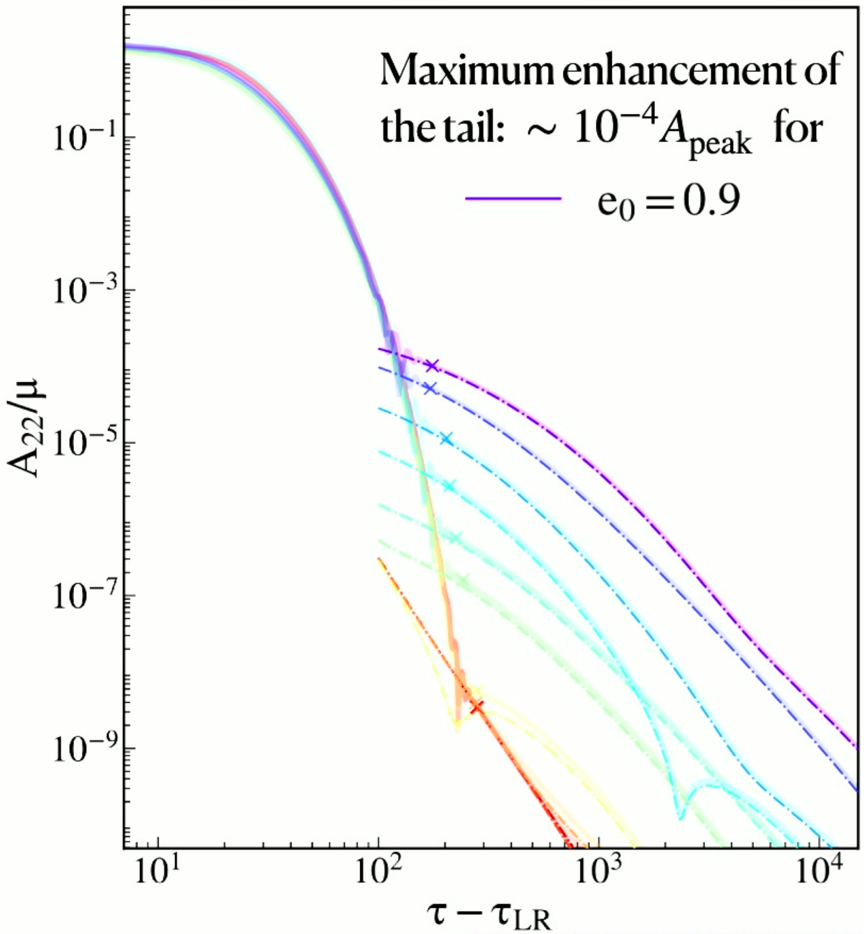
Dependence on initial separation

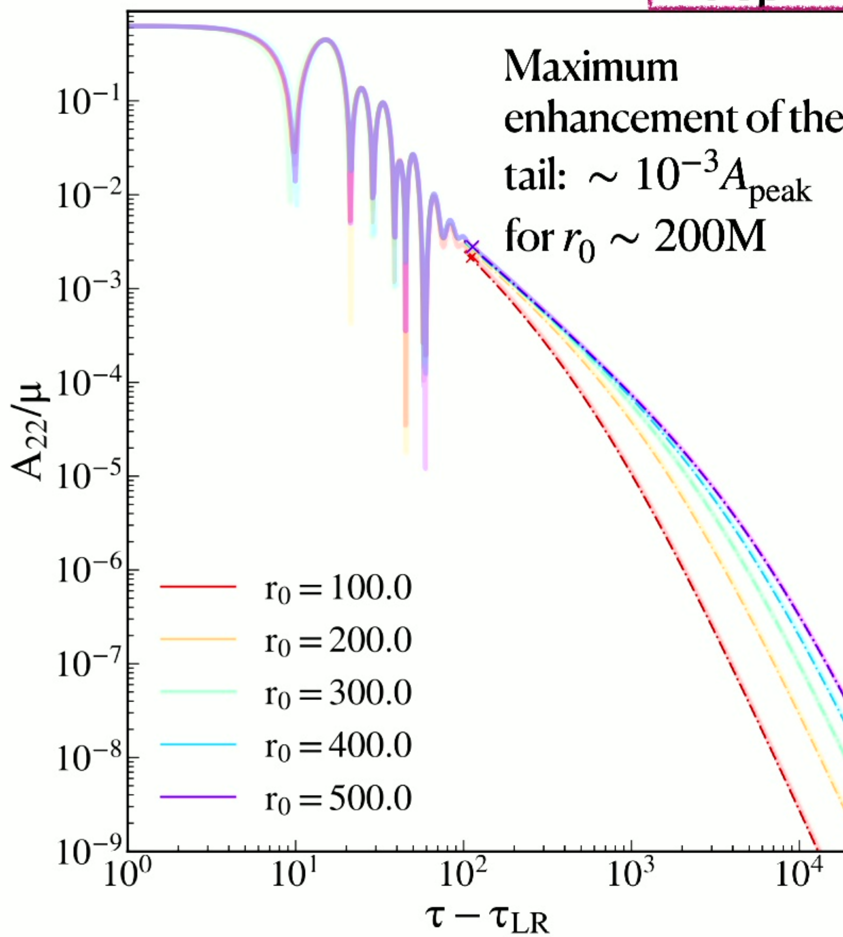


[De Amicis et al, in prep

Eccentric orbits vs radial infalls (PT results)

No spin!





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Conclusions

- Integral model for tail in EMR, as an hereditary effect ✓
- Tail as superposition of power laws $\tau^{-\ell-2-n}$, with $n \geq 0$ ✓
- Tail emission enhanced for motion at large distances $r \gg$ M, with small tangential velocity. Hence, emission is maximized at apastra ✓
- Tails in a comparable masses merger, in fully non-linear setting ✓

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Future directions

- Extend the model to Kerr
  - Long-range propagator in Kerr
  - Test for EMR against Teukode [Harms et al, CQG 31, 245004 (2014)]
- Perturbative study of tails in dynamical spacetimes…
- …and in environments
- Estimate the observability with LISA

BH spectroscopy: "small scale" information
Tails and memories: "large scale" information