

Title: Microscopic Roadmap to a Yao-Lee Spin-Orbital Liquid

Speakers: Hae-Young Kee

Collection/Series: Quantum Matter

Subject: Condensed Matter

Date: November 12, 2024 - 3:30 PM

URL: <https://pirsa.org/24110069>

Abstract:

The exactly solvable spin-1/2 Kitaev model on a honeycomb lattice has drawn significant interest, as it offers a pathway to realizing the long-sought after quantum spin liquid. Building upon the Kitaev model, Yao and Lee introduced another exactly solvable model on an unusual star lattice featuring non-abelian spinons. The additional pseudospin degrees of freedom in this model could provide greater stability against perturbations, making this model appealing. However, a mechanism to realize such an interaction in a standard honeycomb lattice remains unknown. I will present a microscopic theory to obtain the Yao-Lee model on a honeycomb lattice by utilizing strong spin-orbit coupling of anions edge-shared between two eg ions in the exchange processes. This mechanism leads to the desired bond-dependent interaction among spins rather than orbitals, unique to our model, implying that the orbitals fractionalize into gapless Majorana fermions and fermionic octupolar excitations emerge. Since the conventional Kugel-Khomskii interaction also appears, the phase diagram including these interactions using classical Monte Carlo simulations and exact diagonalization techniques will be presented. Several open questions will be also discussed.

Microscopic Roadmap to Yao-Lee Spin-Orbital Liquid & Open Questions

Hae-Young Kee
University of Toronto



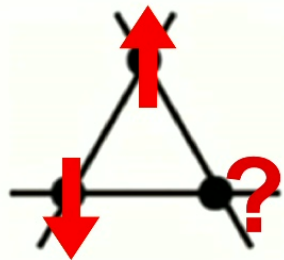
Canadian Institute for
Advanced Research



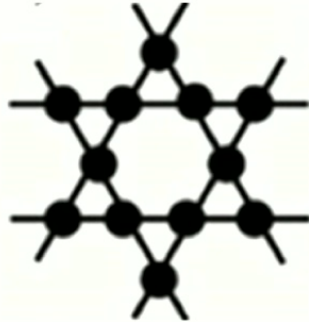
Perimeter Institute, Waterloo, Nov. 12, 2024

Quantum Spin Liquids

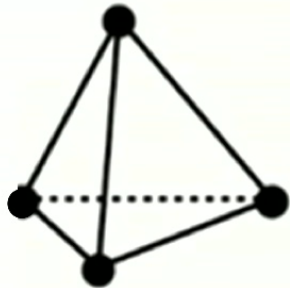
Geometrical frustration



triangle



kagome



tetrahedron

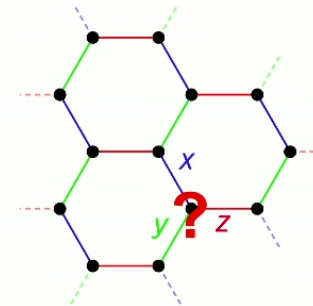
Exchange Interaction frustration

example of bond-dependent interactions

Kitaev Exchange

$$K \sum_{\langle ij \rangle \in \gamma} S_i^\gamma S_j^\gamma$$

where $\gamma = x, y, z$



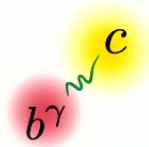
Exactly solvable: Z2 spin liquid ground state

$$S^\gamma = \frac{i}{2} b^\gamma c$$

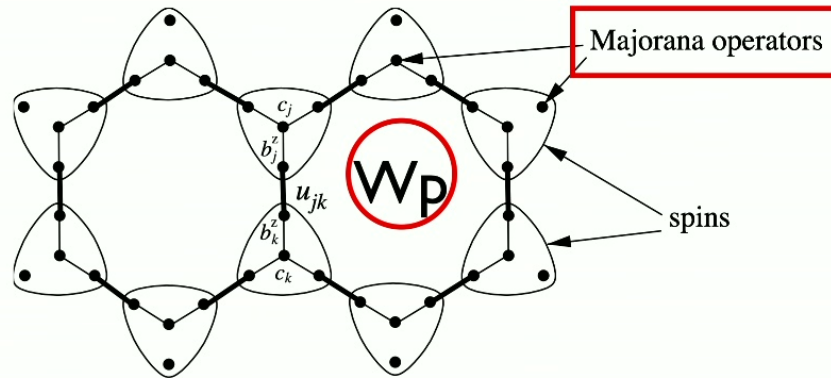
A. Kitaev, Annals of Physics 321, 2 (2006)

$$S^\gamma = \frac{i}{2} b^\gamma c$$

where $\gamma = x, y, z$



graphical representation of H



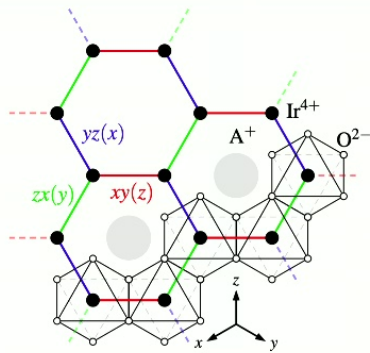
$|\xi\rangle \in \mathcal{M}$ if and only if $D|\xi\rangle = |\xi\rangle$, where $D = b^x b^y b^z c$. : physical subspace

$$H = \frac{i}{4} \sum_{\langle i, j \rangle \in \gamma} \hat{u}_{ij}^\gamma c_i c_j \quad \text{where } \hat{u}_{ij}^\gamma = i b_i^\gamma b_j^\gamma$$

$$W_p = \prod_{\langle i, j \rangle_\gamma \in p} \hat{u}_{ij}^\gamma$$

Kitaev quantum spin liquid: emergent particles - Majorana fermion and vortices

Generic Spin Model in 2D honeycomb



nearest neighbour:
ideal honeycomb

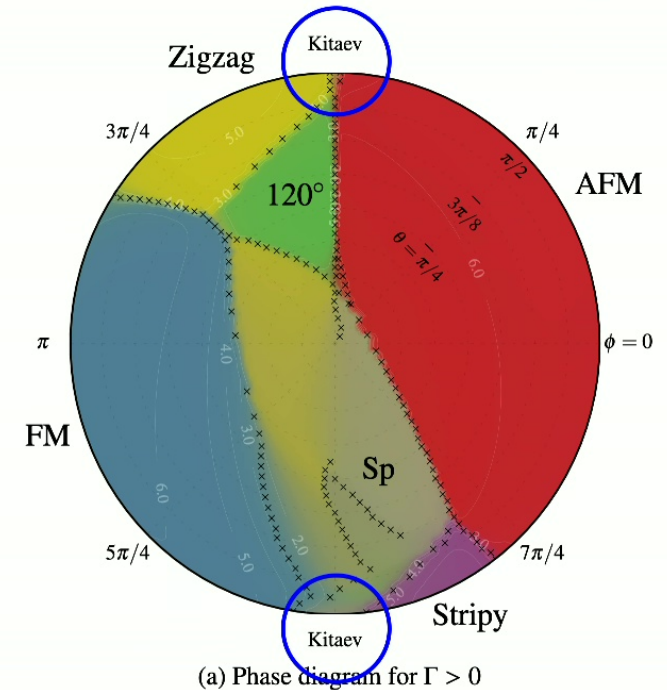
$$H = \sum_{\gamma \in x, y, z} H^\gamma,$$

bond-dep. interactions

$$H^z = \sum_{\langle ij \rangle \in z\text{-bond}} [K_z S_i^z S_j^z + \Gamma_z (S_i^x S_j^y + S_i^y S_j^x)] + JS_i \cdot S_j$$

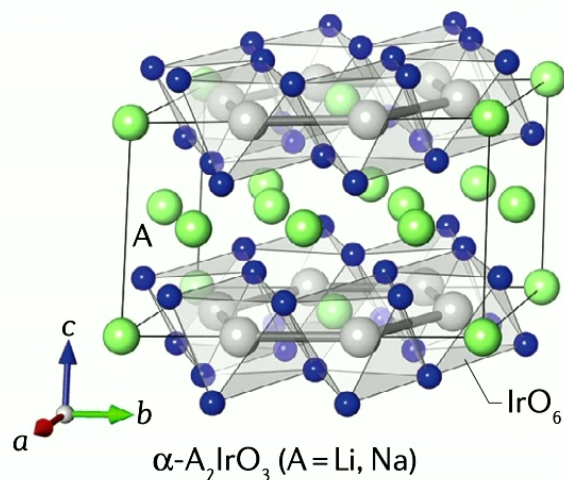
$$H^x = H^z (x \rightarrow y \rightarrow z \rightarrow x)$$

G. Jackeli & G. Khaliulin, PRL (2009); J. Rau, E. Lee, HYK, PRL (2014)



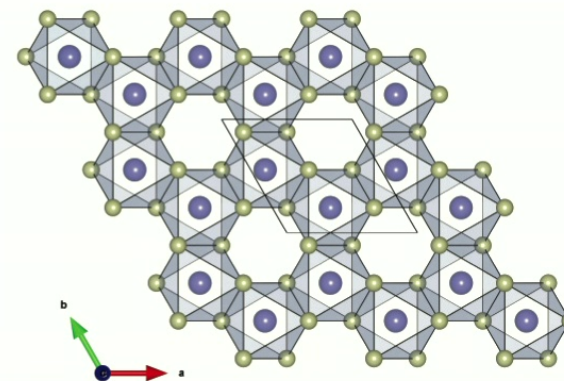
Kitaev spin liquid is fragile under other interactions

Candidates: layered quasi-2D honeycomb with SOC



Iridium oxides: A_2IrO_3

Y. Singh, et al, PRB 82, 064412 (2010); PRL 108, 127203 (2012);....



alpha- RuCl_3

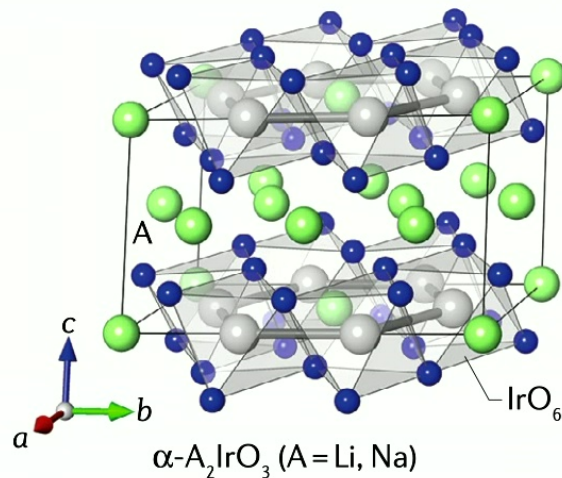
K. Plumb,... HYK, Y.-J. Kim, PRB 90 041112(R) (2014); ...

All candidates: Magnetic ordering at low T

non-Kitaev interactions are present

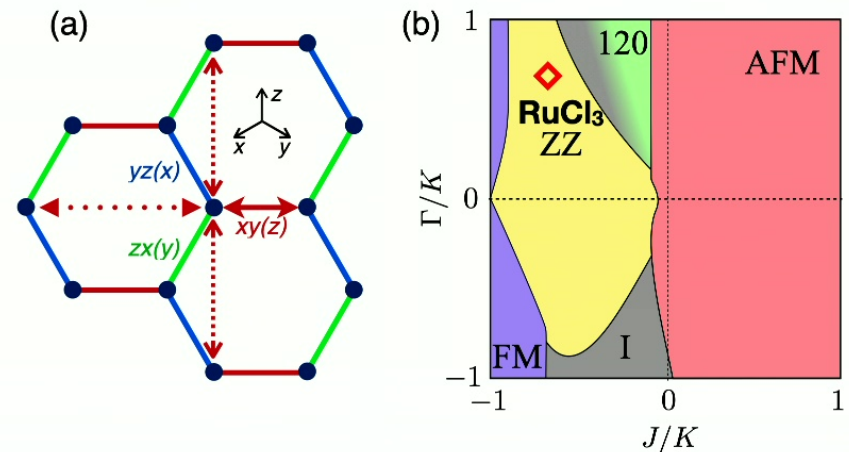
I. Rousochatzakis, N. Perkins, Q. Luo, HYK, Reports on Progress in Physics (2024)

Candidates: layered quasi-2D honeycomb with SOC



Iridium oxides: $A_2\text{IrO}_3$

Y. Singh, et al, PRB 82, 064412 (2010); PRL 108, 127203 (2012);....



H. S. Kim, V. Shankar, A. Catuneanu, HYK, PRB (2015)

alpha- RuCl_3

K. Plumb, ... HYK, Y.-J. Kim, PRB 90 041112(R) (2014); ...

All candidates: Magnetic ordering at low T

**Kitaev materials: Kitaev interaction is dominant!
but small other interactions move it away from the Kitaev spin liquid**

I. Rousochatzakis, N. Perkins, Q. Luo, HYK, Reports on Progress in Physics (2024)

more stable Quantum Spin Liquids?

Another exactly solvable model: Flavored Kitaev; Yao-Lee model

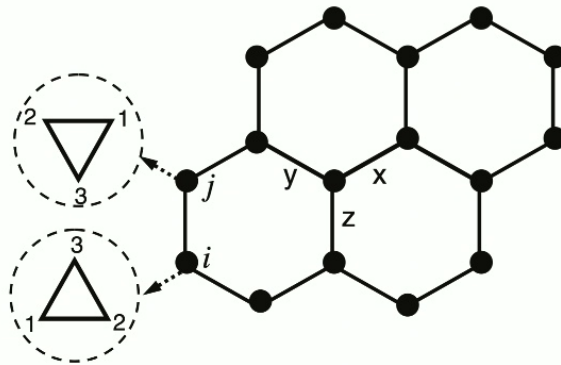
PRL **107**, 087205 (2011)

PHYSICAL REVIEW LETTERS

week ending
19 AUGUST 2011

Fermionic Magnons, Non-Abelian Spinons, and the Spin Quantum Hall Effect from an Exactly Solvable Spin-1/2 Kitaev Model with SU(2) Symmetry

Hong Yao and Dung-Hai Lee



Star lattice (decorated honeycomb lattice)

$$H = J \sum_i \mathbf{S}_i^2 + \sum_{\lambda\text{-link}\langle ij \rangle} J_\lambda [\tau_i^\lambda \tau_j^\lambda] [\mathbf{S}_i \cdot \mathbf{S}_j],$$

$$J \gg J_\lambda$$

$$H = \frac{1}{4} \sum_{\lambda\text{-link}\langle ij \rangle} J_\lambda [\tau_i^\lambda \tau_j^\lambda] [\vec{\sigma}_i \cdot \vec{\sigma}_j],$$

\uparrow \uparrow
Pseudospin & spin

$$H = J \sum_i \mathbf{S}_i^2 + \sum_{\lambda\text{-link}\langle ij \rangle} J_\lambda [\tau_i^\lambda \tau_j^\lambda] [\mathbf{S}_i \cdot \mathbf{S}_j],$$

Intratriangle

Intertriangle

$$\tau_i^x = 2(\mathbf{S}_{i,1} \cdot \mathbf{S}_{i,2} + 1/4)$$

$$\tau_i^y = 2(\mathbf{S}_{i,1} \cdot \mathbf{S}_{i,3} - \mathbf{S}_{i,2} \cdot \mathbf{S}_{i,3})/\sqrt{3}$$

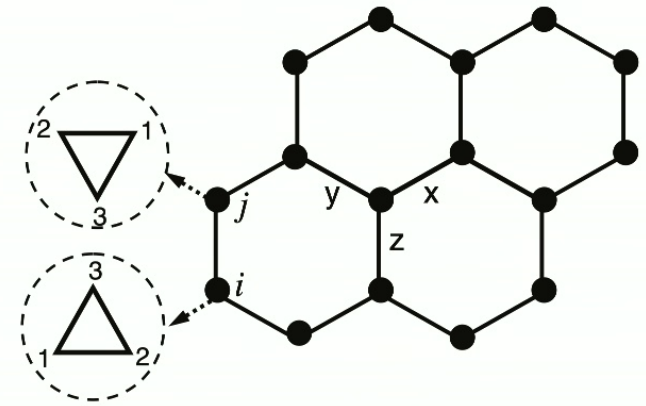
$$\tau_i^z = 4\mathbf{S}_{i,1} \cdot (\mathbf{S}_{i,2} \times \mathbf{S}_{i,3})/\sqrt{3}$$

$$[\tau_i^\alpha, \tau_i^\beta] = 2i\epsilon^{\alpha\beta\gamma}\tau_i^\gamma$$

$$\mathbf{S}_i = \mathbf{S}_{i,1} + \mathbf{S}_{i,2} + \mathbf{S}_{i,3}$$

$$[\mathbf{S}_i^2, \mathbf{S}_j] = 0$$

$$[\mathbf{S}_i^2, \tau_j^\lambda] = 0$$



Star lattice

$$J \gg J_\lambda$$

$$H = \frac{1}{4} \sum_{\lambda\text{-link}\langle ij \rangle} J_\lambda [\tau_i^\lambda \tau_j^\lambda] [\vec{\sigma}_i \cdot \vec{\sigma}_j],$$

Pseudospin & spin

$$H = \frac{1}{4} \sum_{\lambda\text{-link}\langle ij \rangle} J_\lambda [\tau_i^\lambda \tau_j^\lambda] [\vec{\sigma}_i \cdot \vec{\sigma}_j],$$

$$\sigma_i^\alpha \tau_i^\beta = ic_i^\alpha d_i^\beta, \quad \sigma_i^\alpha = -\frac{\epsilon^{\alpha\beta\gamma}}{2} ic_i^\beta c_i^\gamma,$$

$$\tau_i^\alpha = -\frac{\epsilon^{\alpha\beta\gamma}}{2} id_i^\beta d_i^\gamma,$$

$$D_i |\Psi\rangle_{\text{phys}} = |\Psi\rangle_{\text{phys}}, \quad \forall i,$$

$$D_i = -ic_i^x c_i^y c_i^z d_i^x d_i^y d_i^z.$$

$$\mathcal{H} = \sum_{\langle ij \rangle} J_{ij} u_{ij} [ic_i^x c_j^x + ic_i^y c_j^y + ic_i^z c_j^z], \quad J_{ij} = J_\lambda/4$$

$$u_{ij} = -id_i^\lambda d_j^\lambda$$

$$[u_{ij}, \mathcal{H}] = 0 \quad [u_{ij}, u_{i'j'}] = 0$$

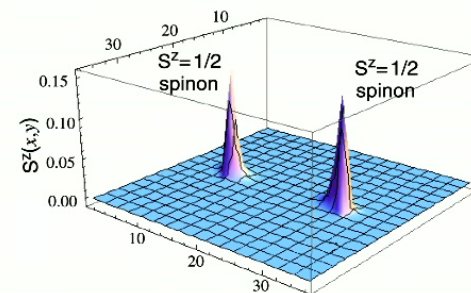
Z2 gauge transformation:

$$c_i^\alpha \rightarrow \Lambda_i c_i^\alpha \quad \text{and} \quad u_{ij} \rightarrow \Lambda_i u_{ij} \Lambda_j, \quad \Lambda_i = \pm 1.$$

GS has 0 flux

3 types of Majorana fermions couple with Z2 gauge field

When TRS is broken: two localized $S_z = 1/2$ spinons occur by creating two vortex excitations



More stable to some perturbations than original Kitaev model

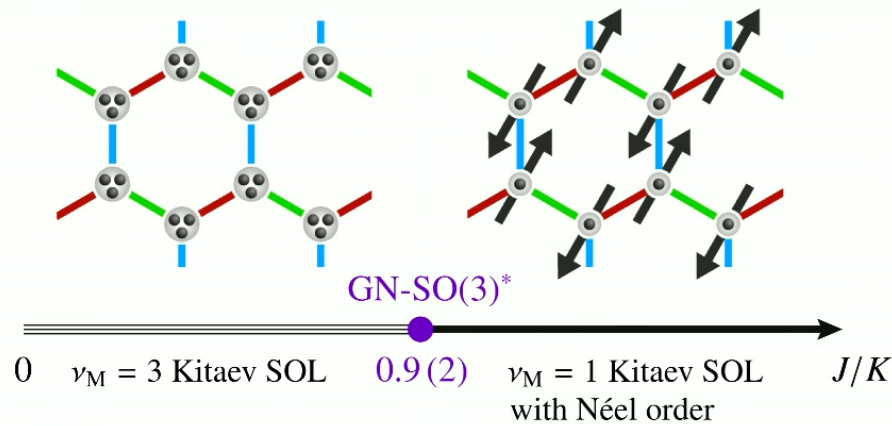
PHYSICAL REVIEW LETTERS **125**, 257202 (2020)

Fractionalized Fermionic Quantum Criticality in Spin-Orbital Mott Insulators

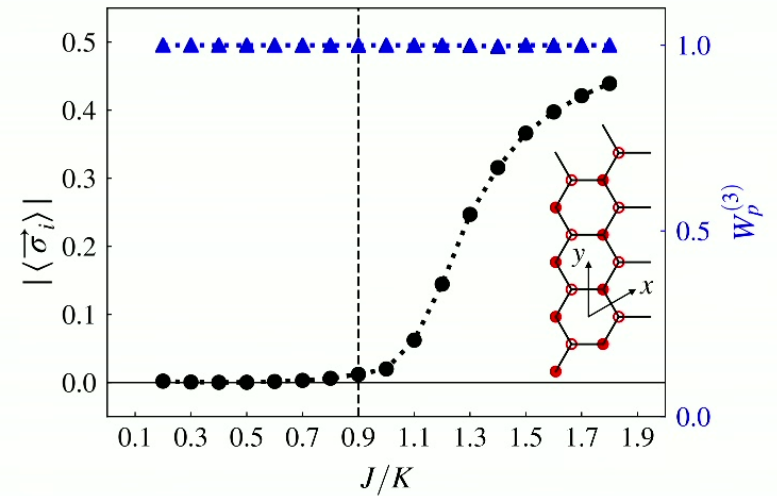
Urban F. P. Seifert,¹ Xiao-Yu Dong,² Sreejith Chulliparambil^{1,3}, Matthias Vojta,¹ Hong-Hao Tu¹, and Lukas Janssen¹

$$\mathcal{H}_K^{(3)} = -K \sum_{\langle ij \rangle_\gamma} \vec{\sigma}_i \cdot \vec{\sigma}_j \otimes \tau_i^\gamma \tau_j^\gamma, \quad K > 0,$$

$$\mathcal{H}_J^{(3)} = J \sum_{\langle ij \rangle} \vec{\sigma}_i \cdot \vec{\sigma}_j \otimes \mathbb{1}_i \mathbb{1}_j, \quad J > 0.$$



$$W_p^{(3)} = \mathbb{1} \otimes \tau_i^x \tau_j^y \tau_k^z \tau_l^x \tau_m^y \tau_n^z$$



Exact deconfined gauge structures in the higher-spin Yao-Lee model:
a quantum spin-orbital liquid with spin fractionalization and non-Abelian anyons

Zhengzhi Wu,^{*} Jing-Yun Zhang,^{*} and Hong Yao[†]
Institute for Advanced Study, Tsinghua University, Beijing 100084, China
(Dated: April 12, 2024)

Topological transitions in the Yao-Lee spin-orbital model and effects of site disorder

Vladislav Poliakov,^{1,*} Wen-Han Kao,^{2,*} and Natalia B. Perkins^{2,†}
¹*Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA*
²*School of Physics and Astronomy, University of Minnesota, Minneapolis, MN 55455, USA*

ARTICLE OPEN

 Check for updates

Kitaev spin-orbital bilayers and their moiré superlattices

Emilian Marius Nica^{1,2,✉}, Muhammad Akram^{1,3}, Aayush Vijayvargia¹, Roderich Moessner⁴ and Onur Erten¹

PHYSICAL REVIEW B **102**, 201111(R) (2020)

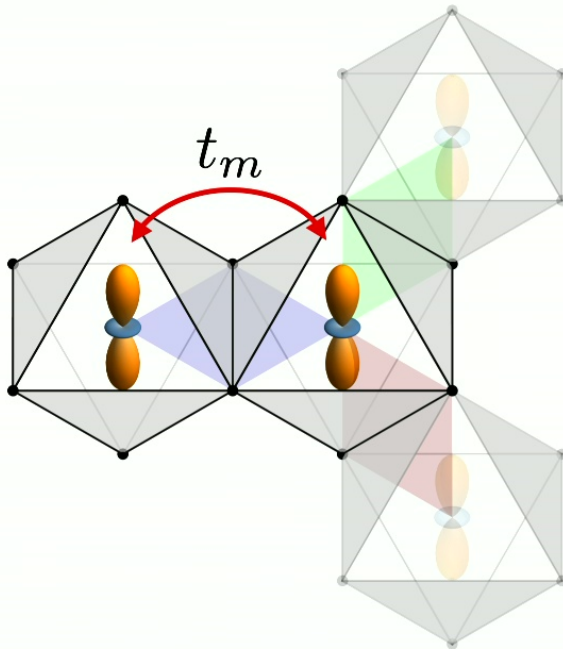
Rapid Communications

Microscopic models for Kitaev's sixteenfold way of anyon theories

Sreejith Chulliparambil^{1,2}, Urban F. P. Seifert,¹ Matthias Vojta,¹ Lukas Janssen¹ and Hong-Hao Tu^{1,*}
¹*Institut für Theoretische Physik and Würzburg-Dresden Cluster of Excellence ct.qmat,
Technische Universität Dresden, 01062 Dresden, Germany*
²*Max-Planck-Institut für Physik komplexer Systeme, Nöthnitzer Straße 38, 01187 Dresden, Germany*

Maybe Yao-Lee model from spin & orbital degree of freedom?

Typically, we have the Kugel-Khomskii Model



$$t_{ij}^{\text{direct}} = \sum_{m,\sigma} t_m c_{i,m\sigma}^\dagger c_{j,m\sigma} + \text{H.c.},$$

t_m : intraorbital hopping

$$H_{\text{KK}} = J_{\text{kk}} \sum_{\langle ij \rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j + \frac{1}{4} \right) \otimes \left(\mathbf{T}_i \cdot \mathbf{T}_j + \frac{1}{4} \right).$$

$$J_{\text{kk}} = \frac{8t_m^2}{U}$$

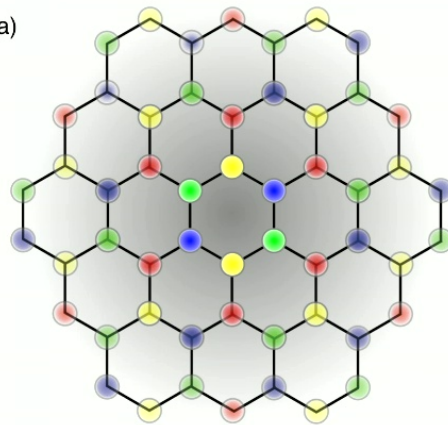
There is no angular momentum change nor spin change during the exchange processes:
no bond-dependence

Kugel-Khomskii SU(4) model on honeycomb lattice

$$\mathcal{H} = \sum_{\langle i,j \rangle} \left(2\mathbf{S}_i \cdot \mathbf{S}_j + \frac{1}{2} \right) \left(2\mathbf{T}_i \cdot \mathbf{T}_j + \frac{1}{2} \right),$$

$$|\bullet\rangle = |\uparrow a\rangle, |\color{green}\bullet\rangle = |\downarrow a\rangle, |\color{blue}\bullet\rangle = |\uparrow b\rangle, |\color{yellow}\bullet\rangle = |\downarrow b\rangle$$

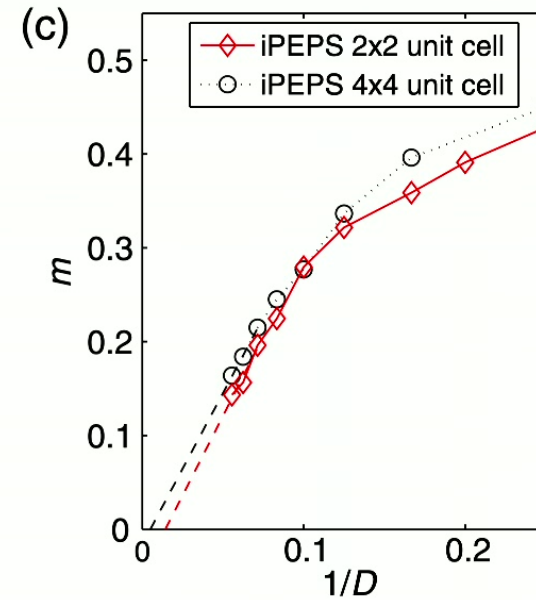
(a)



local color order

P. Corboz, et al, PRX 2, 041013 (2012)

Algebraic spin-orbital (SO) liquids



$$m = \sqrt{\frac{4}{3} \sum_{\alpha, \beta} (\langle S_{\alpha}^{\beta} \rangle - \frac{\delta_{\alpha\beta}}{4})^2},$$

$S_{\alpha}^{\beta} = |\alpha\rangle\langle\beta|$ are the generators of SU(4)

Focus : d^5 systems with strong SOC

$$\mathcal{H} = \sum_i \mathcal{H}_i + \sum_{i,j} \mathcal{H}_{ij}$$

on-site H

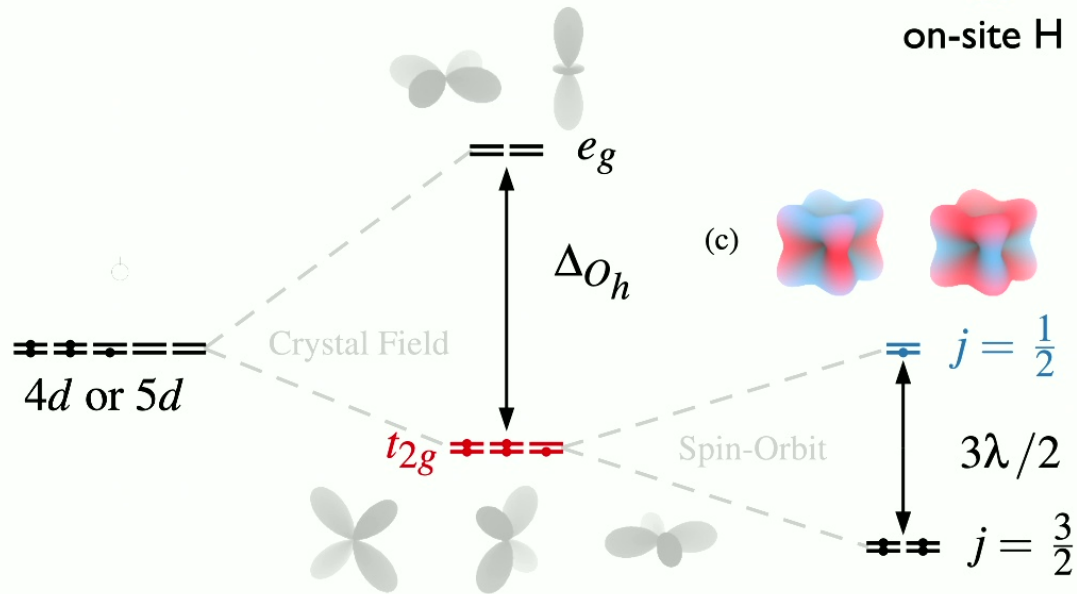


Figure credit: J. Rau, E. Lee, HYK, ARCOMP (2016)

Edge sharing lattice structure

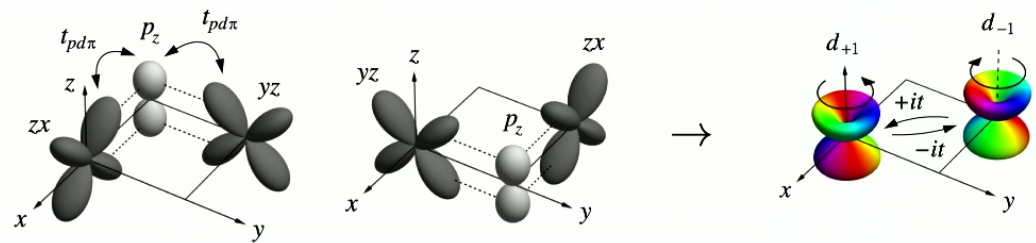
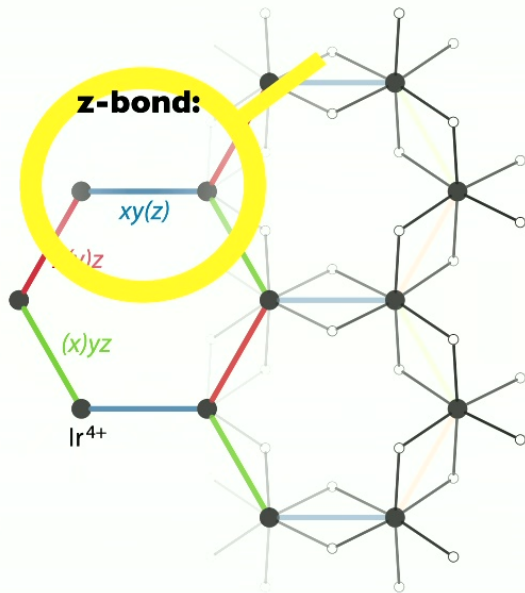
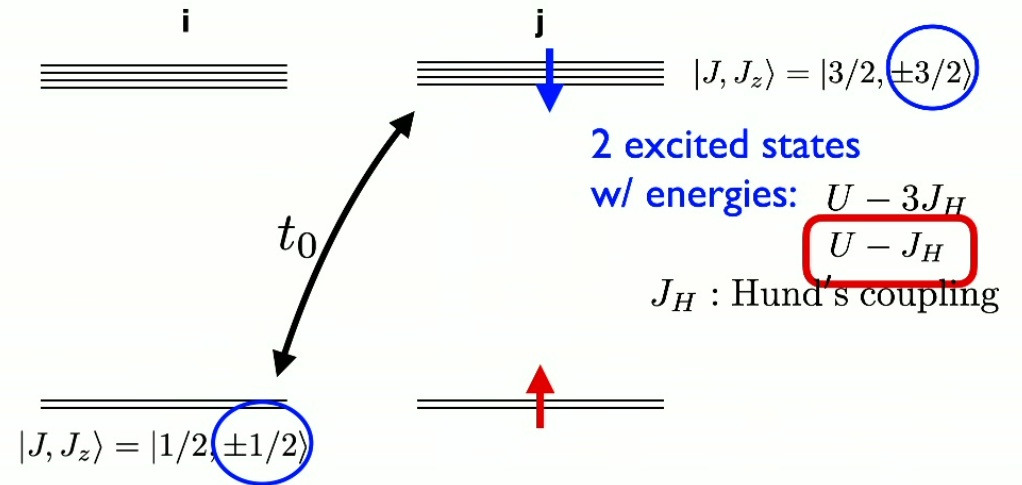
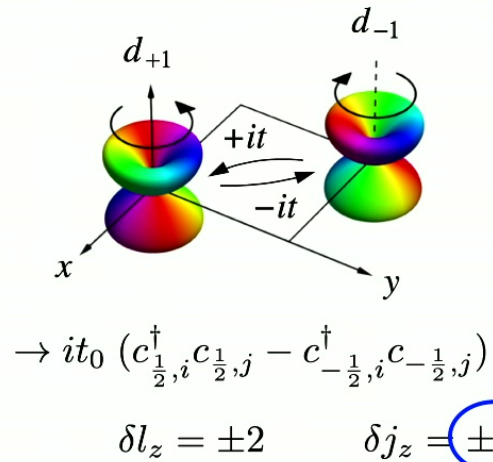


Fig. credit: T. Takayama et al, JPSJ (2021)

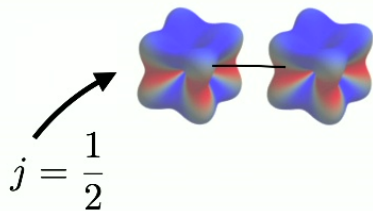
only **interorbital** hopping : $t_0(d_{xz,\sigma,i}^\dagger d_{yz,\sigma,j} + h.c.) \rightarrow it_0(d_{+1,\sigma,i}^\dagger d_{-1,\sigma,j} + h.c.) \quad \delta l_z = \pm 2$

Excited states?



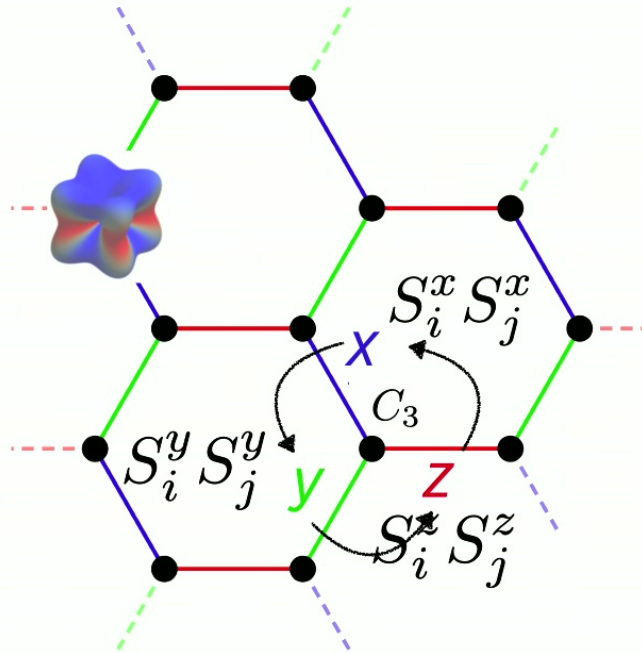
$\delta j_z = \pm 2$: should involve excited states with $J=3/2$ at n.n. site

z-bond: $H_K = K S_i^z S_j^z$



$$K \propto -\frac{t_0^2 J_H}{U^2} \propto t_0^2 \left(\frac{1}{U - J_H} - \frac{1}{U - 3J_H} \right)$$

With only p-orbital mediate (**interorbital**) hopping



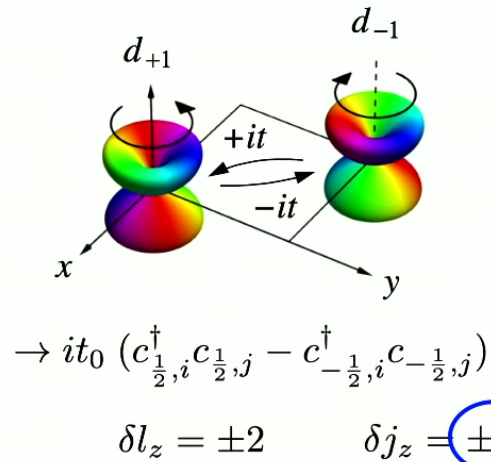
bond-dependent Ising interaction is due to **orbital that bridges pseudospin interaction via **SOC!****

Kitaev Exchange

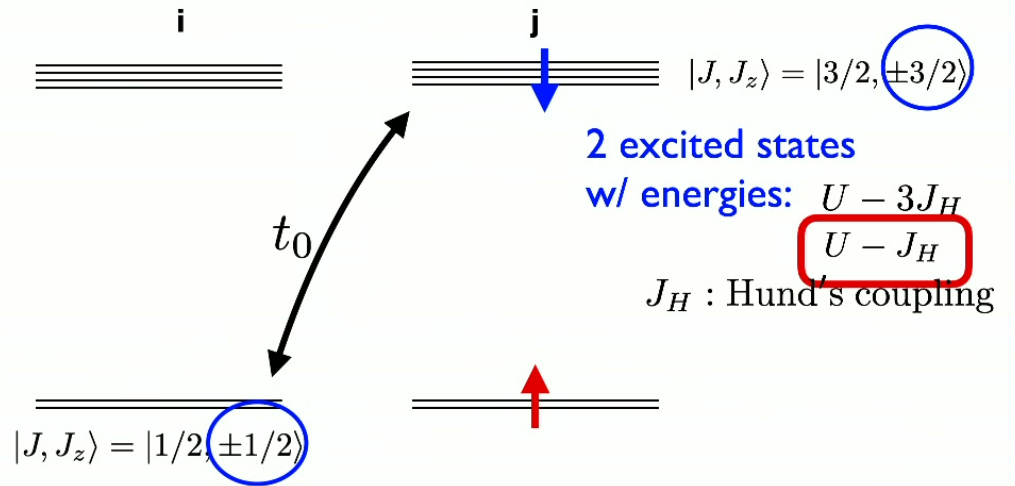
$$K \sum_{\langle ij \rangle \in \gamma} S_i^\gamma S_j^\gamma$$

G. Jackeli, G. Khaliullin, Phys. Rev. Lett. 102, 017205 (2009)

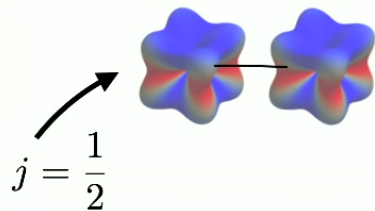
Excited states?



should involve excited states with $J=3/2$ at n.n. site



z-bond: $H_K = K S_i^z S_j^z$



$$K \propto -\frac{t_0^2 J_H}{U^2} \propto t_0^2 \left(\frac{1}{U - J_H} - \frac{1}{U - 3J_H} \right)$$

multi-orbital systems with SOC and Hund's coupling

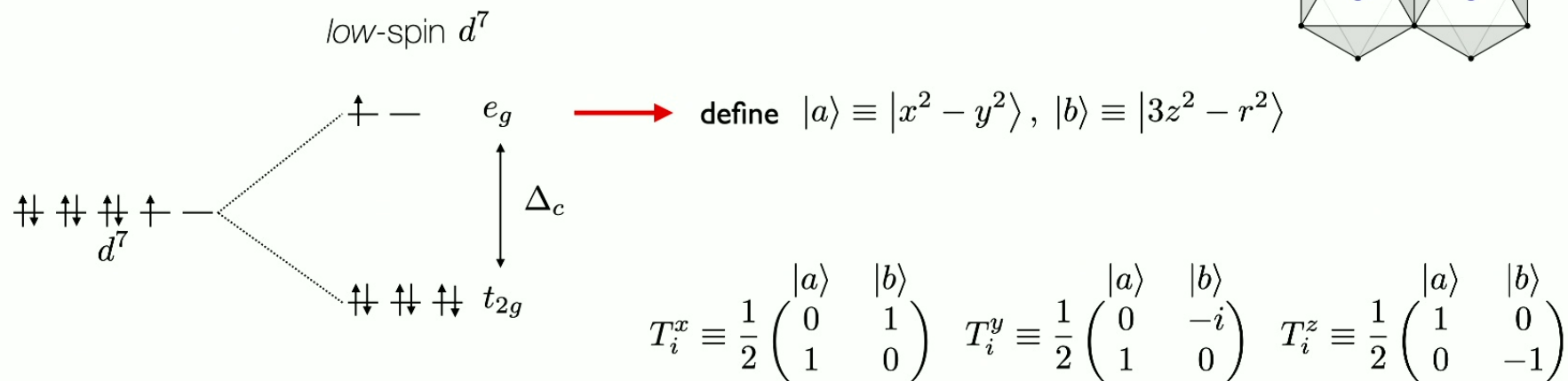
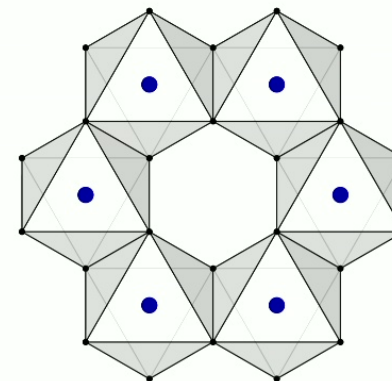
Once we have SOC to generate bond-dependent Kitaev interaction, H reduces to pseudospin interaction like the original Kitaev (or compass model of J operators) model.

Conversely, if we leave the orbital d.o.f., we are back to Kugel-Khomskii model (or compass model), as there is no SOC that bridges the spin interaction?

How do we generate Yao-Lee interaction, and what are other interactions generated during the exchange the processes?

How do we generate such Yao-Lee-like interaction in honeycomb lattice?

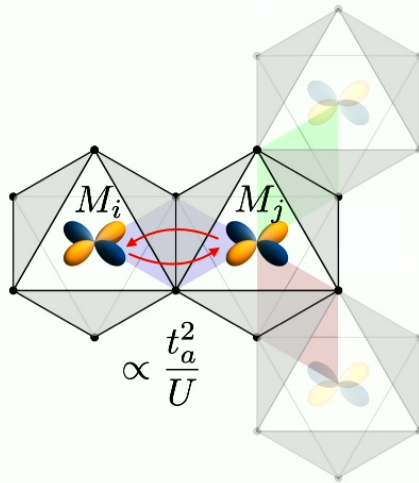
- Let us consider two orbitals which are degenerate such as d^7 or d^9



where \mathbf{T}_i and \mathbf{S}_i are orbital and spin degrees of freedom

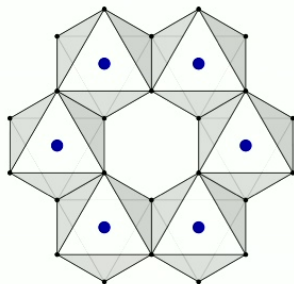
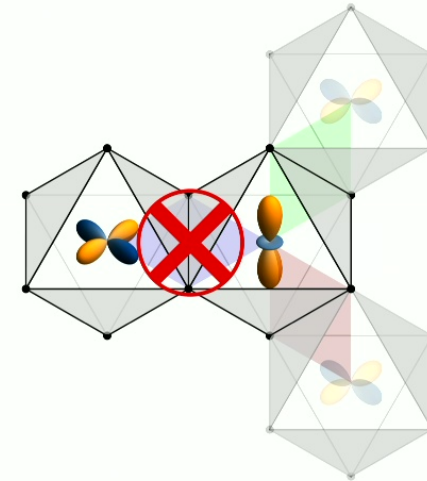
Consider direct exchange between nearest neighbour M sites

intraorbital hopping



$\propto \frac{t_b^2}{U}$

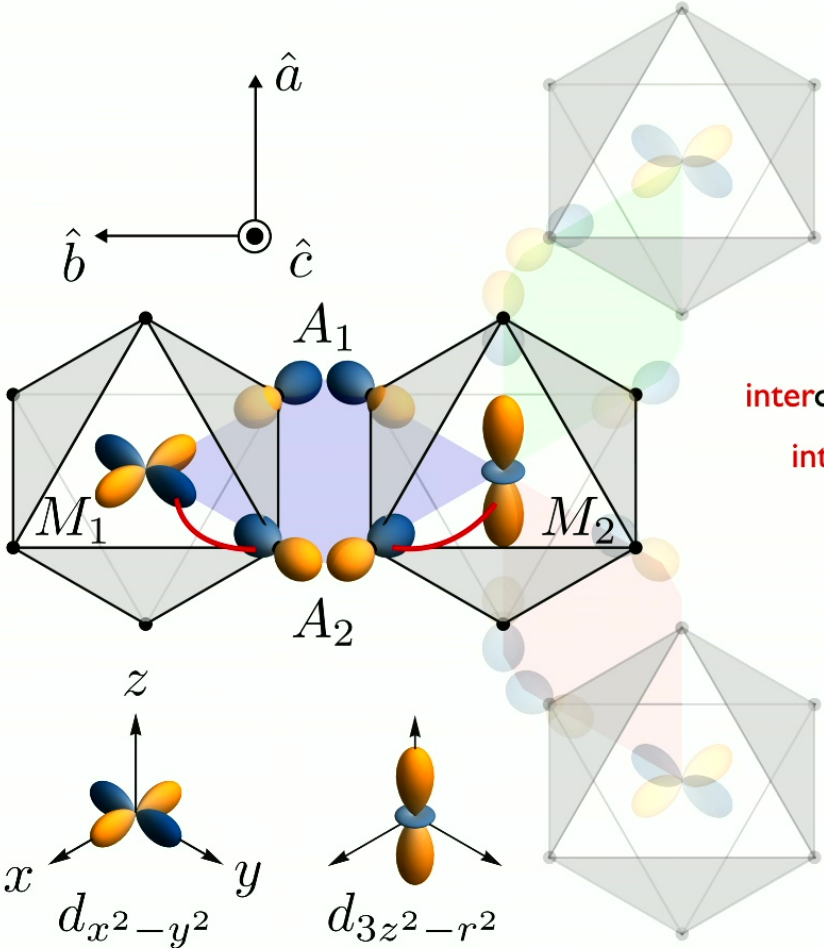
interorbital



$$t_a \approx t_b \implies H_{\text{eff}} = \frac{t^2}{U} \sum_{\langle ij \rangle} \left(S_i \cdot S_j + \frac{1}{4} \right) \left(T_i \cdot T_j + \frac{1}{4} \right)$$

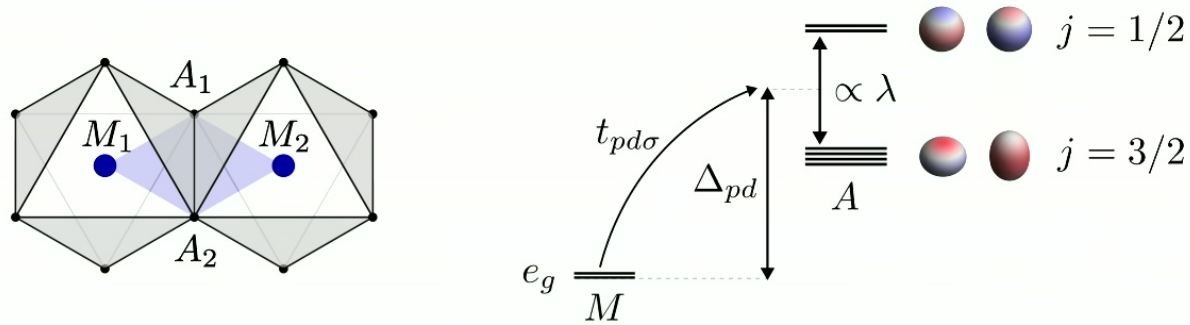
Kugel-Khomskii model: no bond-dependence - due to missing spin-orbit coupling!

Consider **interorbital** hopping that changes the angular momentum



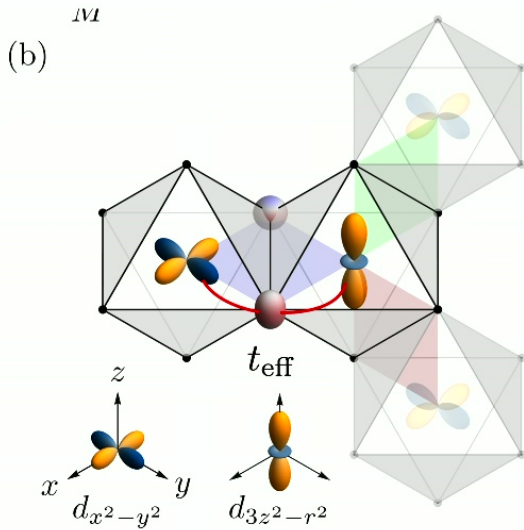
interorbital hopping is enabled via p-orbital and when p-orbital is mixed
interorbital hopping via p-orbital - dominant hopping in eg orbital

Consider an intermediate ligand (A site, p^6 configuration) with strong SOC



Hopping between M sites through ligands becomes spin-dependent!

$$t_{\text{eff}} = \frac{t_{pd\sigma}^2}{4\sqrt{3}} \left(\frac{1}{\Delta_{pd} - \frac{\lambda}{2}} - \frac{1}{\Delta_{pd} + \lambda} \right)$$

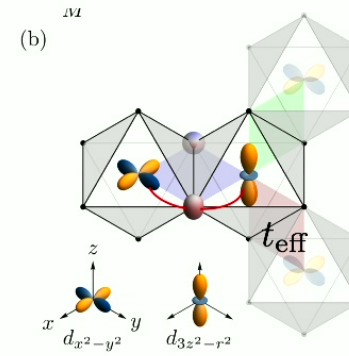
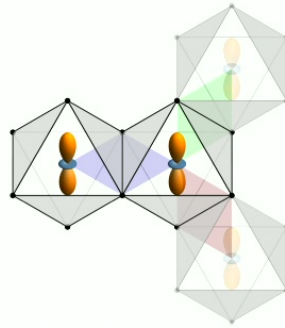
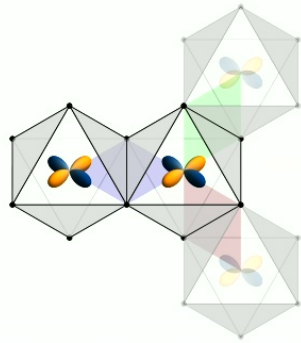


$$H_{\text{eff}} = -J \sum_{\langle ij \rangle_\gamma} \left[\left(\mathbf{s}_i \cdot \mathbf{s}_j - 2S_i^\gamma S_j^\gamma - \frac{1}{4} \right) \otimes \left(\mathbf{T}_i \cdot \mathbf{T}_j - 2T_i^y T_j^y - \frac{1}{4} \right) \right], \quad J \propto \frac{t_{\text{eff}}^2}{U}$$

$$T_i^x \rightarrow \tilde{T}_i^x, \quad T_i^y \rightarrow (-1)^i \tilde{T}_i^y, \quad T_i^z \rightarrow \tilde{T}_i^z$$

$$H_{\text{eff}} = -J \sum_{\langle ij \rangle_\gamma} \left[\left(\mathbf{s}_i \cdot \mathbf{s}_j - 2S_i^\gamma S_j^\gamma - \frac{1}{4} \right) \otimes \left(\tilde{\mathbf{T}}_i \cdot \tilde{\mathbf{T}}_j - \frac{1}{4} \right) \right]$$

Yao-Lee interaction



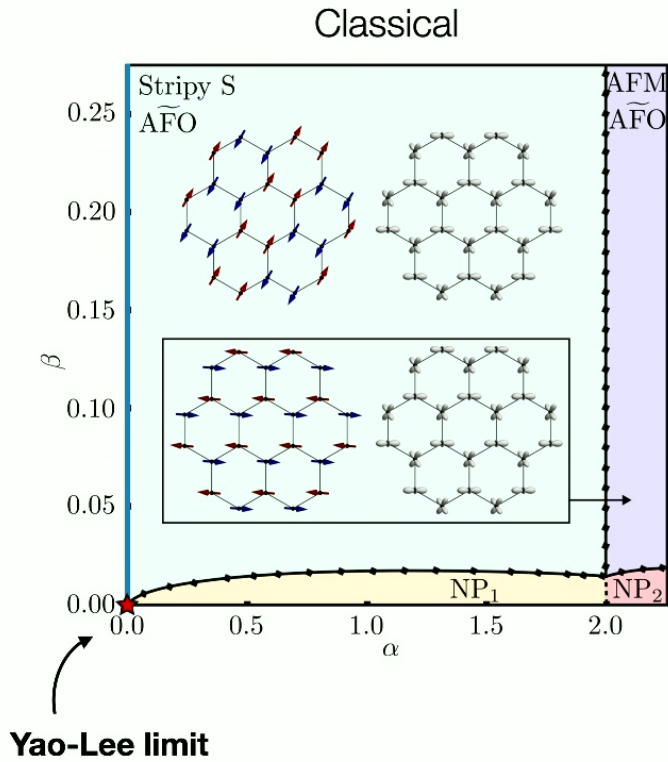
$$H_{\text{KK}} = \frac{t^2}{U} \sum_{\langle ij \rangle} \left(S_i \cdot S_j + \frac{1}{4} \right) \left(T_i \cdot T_j + \frac{1}{4} \right)$$

$$H_{\text{eff}} = -J \sum_{\langle ij \rangle_\gamma} \left[\left(\mathbf{S}_i \cdot \mathbf{S}_j - 2S_i^\gamma S_j^\gamma - \frac{1}{4} \right) \otimes \left(\tilde{\mathbf{T}}_i \cdot \tilde{\mathbf{T}}_j - \frac{1}{4} \right) \right]$$

Introduce α and β and investigate the phase diagram

$$H_{\text{model}} = - \sum_{\langle ij \rangle_\gamma} \left[\left(\alpha \mathbf{S}_i \cdot \mathbf{S}_j - 2S_i^\gamma S_j^\gamma - \beta \right) \otimes \left(\tilde{\mathbf{T}}_i \cdot \tilde{\mathbf{T}}_j - \beta \right) \right].$$

Phase diagrams



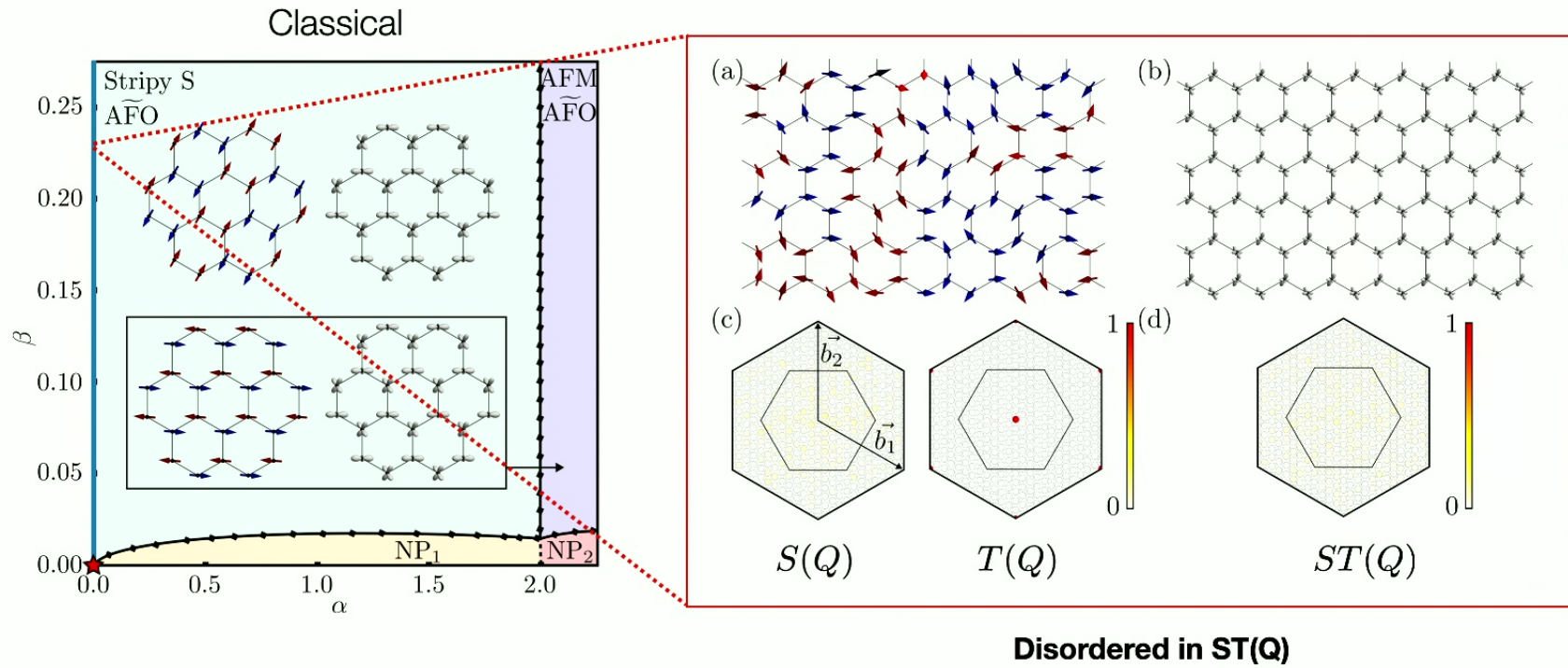
$$S(Q) = \frac{1}{N^2} \sum_{ij} \langle (S_i \cdot S_j) \rangle e^{-iQ \cdot (r_i - r_j)}$$

$$T(Q) = \frac{1}{N^2} \sum_{ij} \langle (T_i \cdot T_j) \rangle e^{-iQ \cdot (r_i - r_j)}$$

$$ST(Q) = \frac{1}{N^2} \sum_{ij} \langle (S_i \cdot S_j) (T_i \cdot T_j) \rangle e^{-iQ \cdot (r_i - r_j)}$$

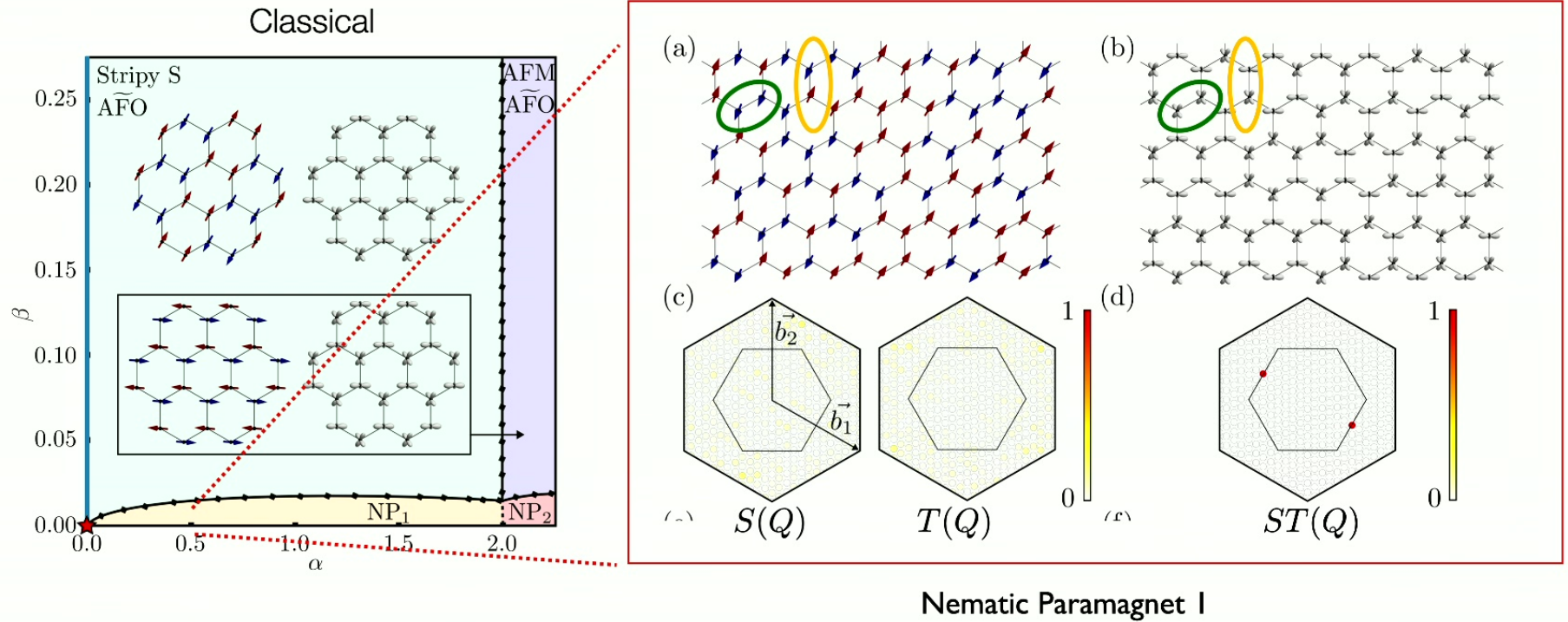
$$H_{\text{model}} = - \sum_{\langle ij \rangle_\gamma} \left[\left(\alpha \mathbf{S}_i \cdot \mathbf{S}_j - 2S_i^\gamma S_j^\gamma - \beta \right) \otimes \left(\tilde{\mathbf{T}}_i \cdot \tilde{\mathbf{T}}_j - \beta \right) \right].$$

Phase diagrams



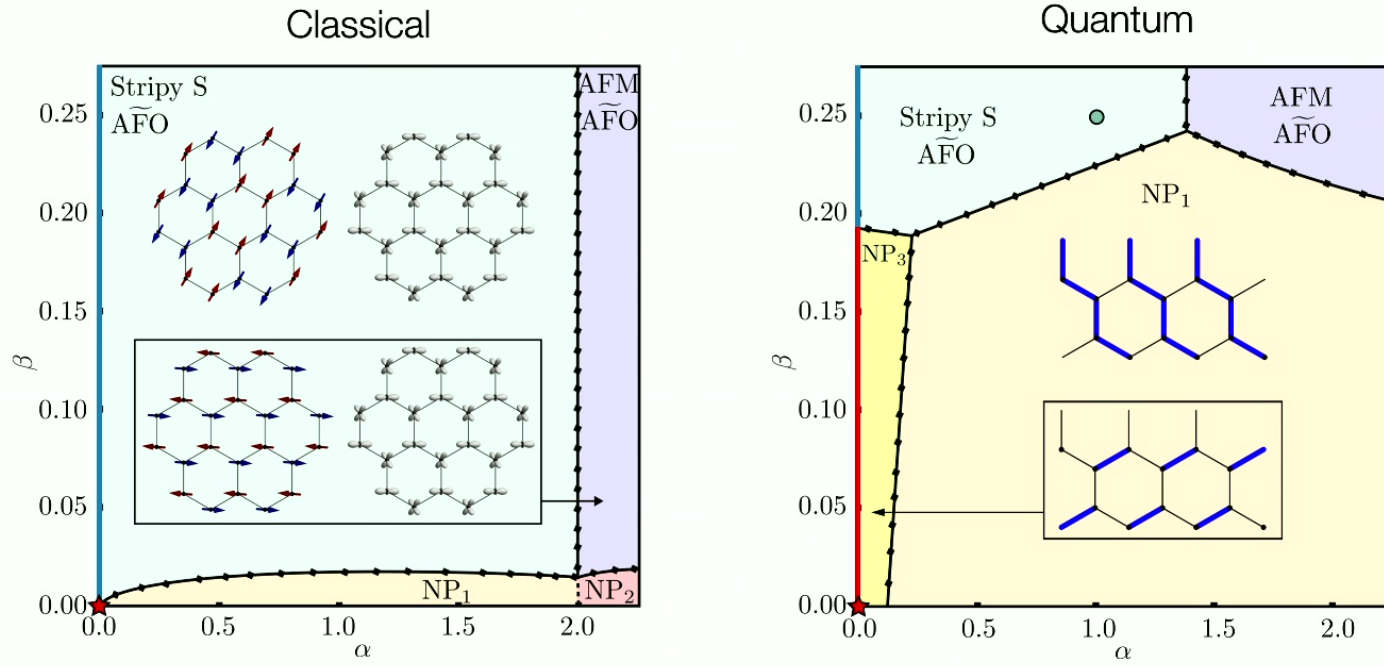
$$H_{\text{model}} = - \sum_{\langle ij \rangle_\gamma} \left[\left(\alpha \mathbf{S}_i \cdot \mathbf{S}_j - 2S_i^\gamma S_j^\gamma - \beta \right) \otimes \left(\tilde{\mathbf{T}}_i \cdot \tilde{\mathbf{T}}_j - \beta \right) \right].$$

Phase diagrams



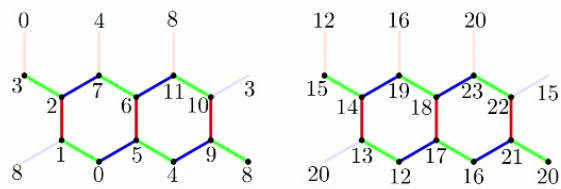
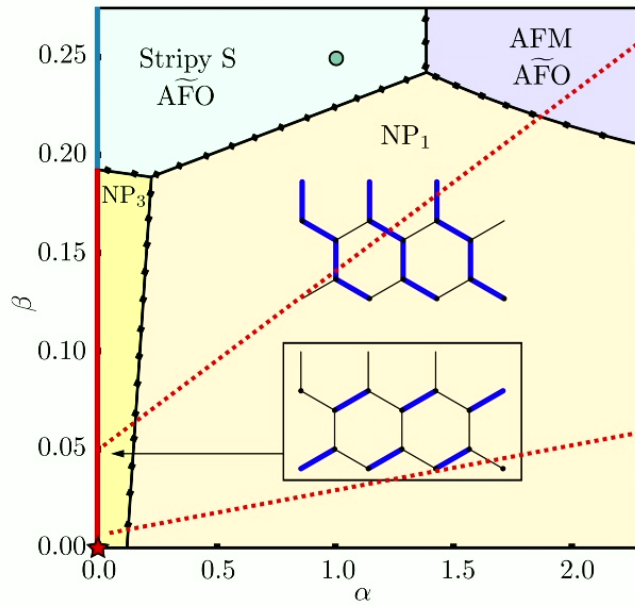
$$H_{\text{model}} = - \sum_{\langle ij \rangle_\gamma} \left[\left(\alpha \mathbf{S}_i \cdot \mathbf{S}_j - 2S_i^\gamma S_j^\gamma - \beta \right) \otimes \left(\tilde{\mathbf{T}}_i \cdot \tilde{\mathbf{T}}_j - \beta \right) \right].$$

Phase diagrams

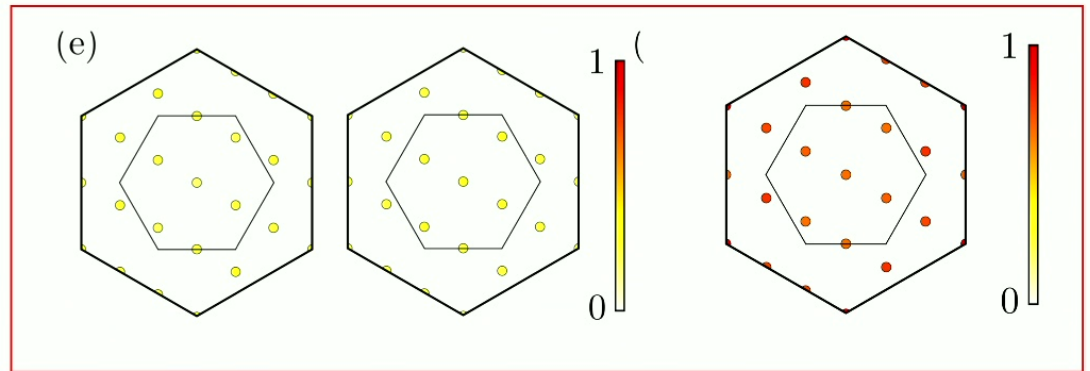
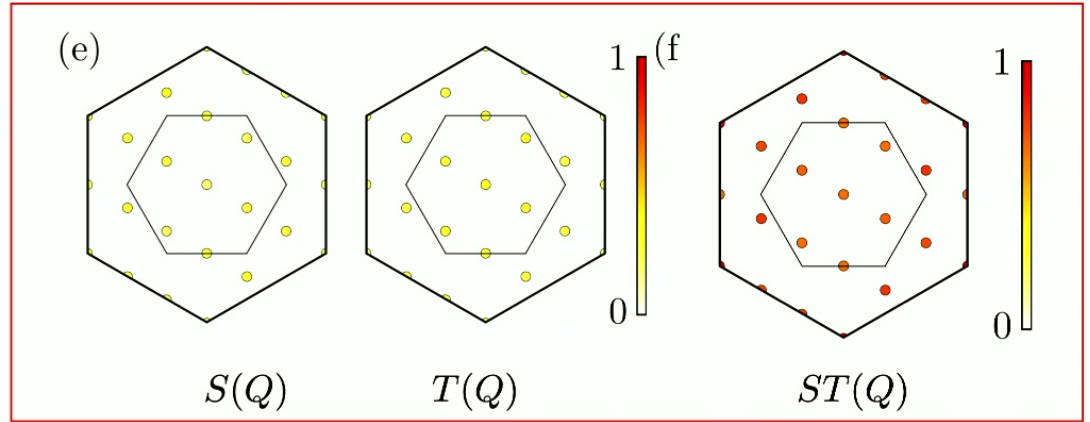


$$H_{\text{model}} = - \sum_{\langle ij \rangle_\gamma} \left[\left(\alpha \mathbf{S}_i \cdot \mathbf{S}_j - 2 S_i^\gamma S_j^\gamma - \beta \right) \otimes \left(\tilde{\mathbf{T}}_i \cdot \tilde{\mathbf{T}}_j - \beta \right) \right].$$

ED Phase diagrams



$(\alpha, \beta) = (0, 0.05)$



Yao-Lee limit $(\alpha, \beta) = (0, 0)$

Exactly solvable point

Define Majorana operators $S_i^\alpha = -\frac{i}{4}\epsilon^{\alpha\beta\gamma}c_i^\beta c_i^\gamma$ and $T_i^\alpha = -\frac{i}{4}\epsilon^{\alpha\beta\gamma}d_i^\beta d_i^\gamma$

Then when $\alpha = 0, \beta = 0$ defining the fermionic operator $f_i^y = \frac{1}{\sqrt{2}}(d_i^z - id_i^x)$

$$H = \frac{1}{8} \sum_{\langle ij \rangle} \hat{u}_{ij} \left(2 \left(if_{i,y}^\dagger f_{j,y} - if_{j,y}^\dagger f_{i,z} \right) - id_i^y d_j^y \right)$$

The ground state lies in the zero-flux sector by Lieb's theorem

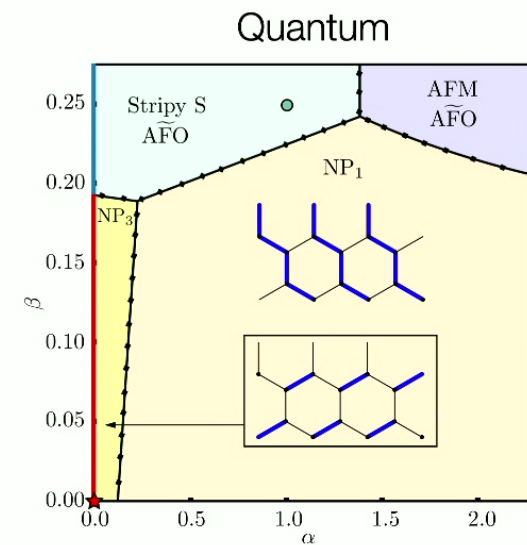
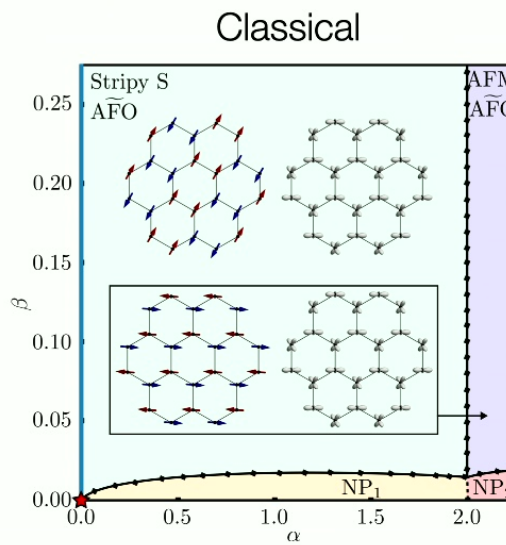
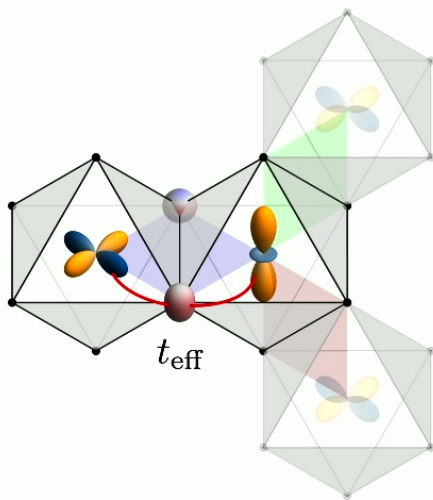
Fermions created by $f_{i,y}^\dagger$: fermionic octupolar excitation

Since $T^y = P^T \left(\frac{1}{3\sqrt{5}} O_{xyz} \right) P$, where $O_{xyz} = \frac{\sqrt{15}}{6} L_x L_y L_z$

$$\text{cf: } T_x = \frac{1}{2\sqrt{3}} Q_{x^2-y^2} \quad T_z = \frac{1}{2\sqrt{3}} Q_{3z^2-y^2}$$

Summary

- Provide a microscopic mechanism to obtain a flavoured (Yao-Lee-like) Kitaev interaction on a honeycomb lattice
- Show certain d^7 (d^9) compounds lie near swaths of nematic phases engulfing a Quantum Spin-Orbital Liquid (QSOL) point
- Revealed interesting features of the QSOL: fractionalized orbitals, octupolar fermionic excitation



Open questions

- Nature of transition between two SO liquids
- Candidate materials (Cu^{2+} , Co^{2+} , Ni^{3+} surrounded by heavy ions making a honeycomb)
- Finite size effects; different numerical techniques are needed
- Effects of other interactions; compass terms are generated in orbital part:
 - if small, they are not going to affect the final result

.....

Summary

- Provide a microscopic mechanism to obtain a flavoured (Yao-Lee-like) Kitaev interaction on a honeycomb lattice
- Show certain d^7 (d^9) compounds lie near swaths of nematic phases engulfing a Quantum Spin-Orbital Liquid (QSOL) point
- Revealed interesting features of the QSOL: fractionalized orbitals, octupolar fermionic excitation

