Title: Why there is (almost) nothing rather than something? On the cosmological constant problem.

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Abstract:

The failure to calculate the vacuum energy remains a central problem in theoretical physics. In my talk I present a new understanding of the cosmological constant problem, grounded in the insight that vacuum energy density can be expressed in terms of phase space volume. Introduction of a UV-IR regularization implies a relationship between the vacuum energy and entropy. Combining this insight with the holographic bound on entropy then yields a bound on the cosmological constant consistent with observations. It follows that the universe is large, and the cosmological constant is naturally small, because the universe is filled with a large number of degrees of freedom. The talk is based on our papers Phys.Rev.D 107 (2023) 12, 126016; e-Print: 2212.00901 [hep-th] and Int.J.Mod.Phys.D 32 (2023) 14, 2342004; e-Print: 2303.17495 [hep-th].

Why there is (almost) nothing rather than something?

On the cosmological constant problem

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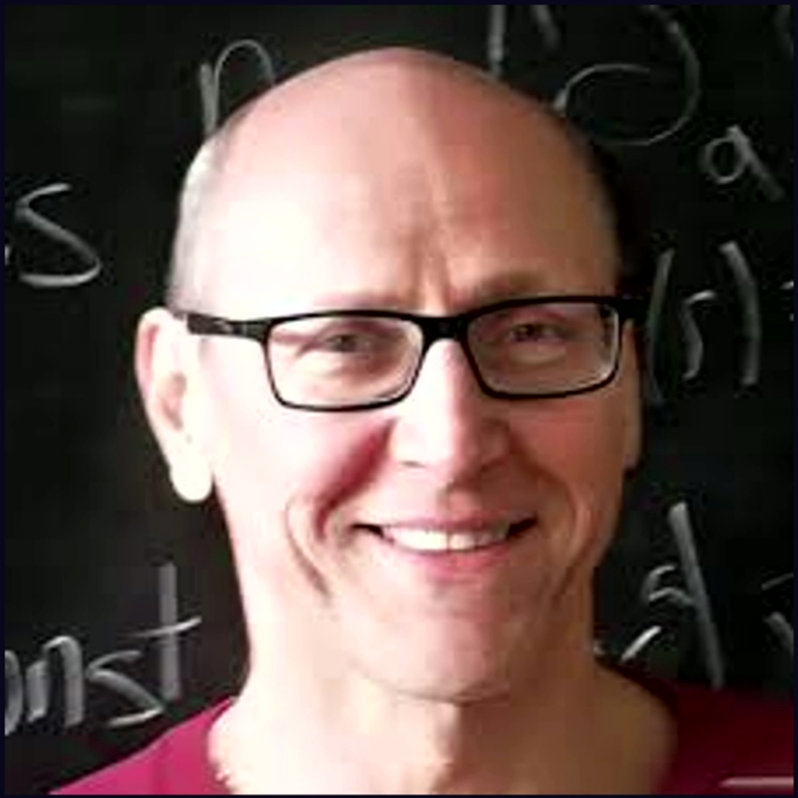
PI Quantum Gravity Seminar
14 November 2024

Dedicated to the memory of
Jurek Lewandowski
(1959-2024)



Why there is something rather than nothing? (1714)

Why there is nothing rather than something? (1988)

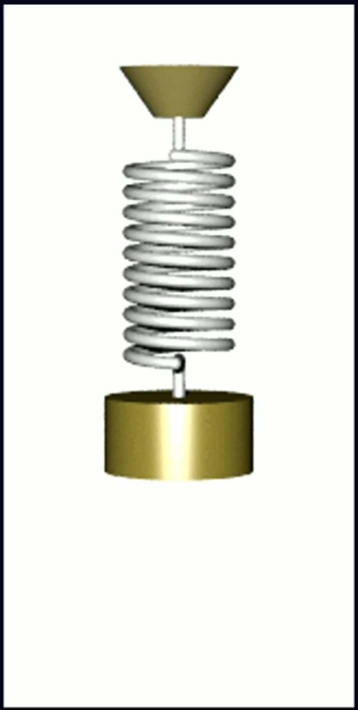
Leibniz vs Coleman

metaphysics vs physics

Vacuum energy

- In quantum mechanics, the ground (vacuum) state of an oscillator of frequency ω has energy $E_0 = 1/2\ \hbar\omega$.
- Field theory describes an infinite number of oscillators (one per momentum), and the total vacuum energy density is infinite.

$$E_0 = \int \frac{d^3p}{(2\pi)^3}\frac{1}{2}\omega_p \sim \lim_{\Lambda\to\infty}\frac{1}{16\pi^2}\Lambda^4$$



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The cosmological constant problem

- This is not a problem if gravity is not there, because only the energy differences matter, and one can always shift the excited states energies by infinite amount (using normal ordering).
- If, however, gravity is present*, due to its universal nature the infinite vacuum energy produces an infinite gravitational field.
- But do quantum fluctuations (vacuum energy) really gravitate? (Assume they do.)

*With the notable exception being the unimodular gravity, which is specifically constructed NOT to couple to vacuum energy.

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The cosmological constant problem

- The gravity action contains the cosmological constant

$$S \sim \int \sqrt{-g}R + \sqrt{-g}\Lambda_{cc} + \text{matter}$$

- You may argue that the value of this parameter is just a constant defining the action that must be fixed observationally, but this misses the point.
- The point is that there are non-controllable contributions to the cosmological constant from matter loop diagrams.

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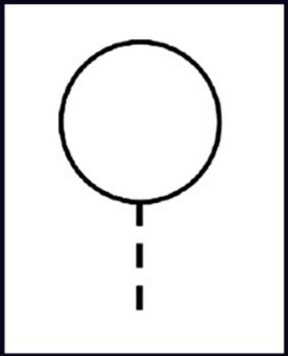
Rough calculation*

- In the leading order the matter-linearized gravity (graviton) coupling is

$$S_{int} \sim \int h_{\mu\nu} T^{\mu\nu}$$



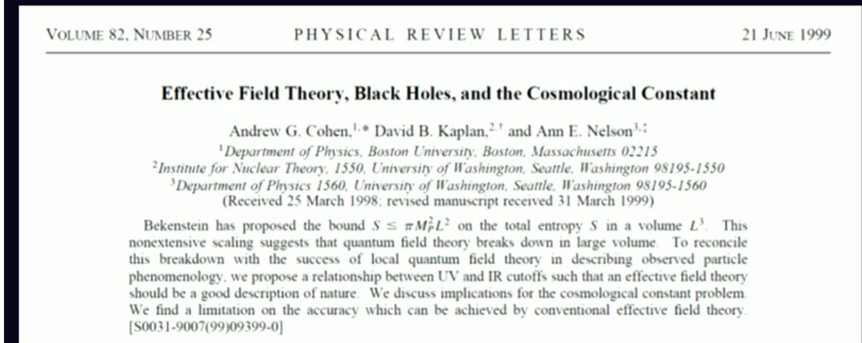
- Computing the tadpole diagram, we get

$$\Delta\mathcal{L} \sim h_{\mu\nu} \times \int \frac{d^4p}{(2\pi)^4} \frac{2p^\mu p^\nu - \eta^{\mu\nu}\left(p^2 - m^2\right)}{p^2 - m^2 + i\epsilon}$$
$$\sim h_{\mu\nu} \times \eta^{\mu\nu} \frac{1}{64\pi^2} \Lambda^4$$

- Notice that that loop contribution to cosmological constant is proportional to the regularized volume of momentum space.

*J. F. Donoghue, Phys. Rev. D **104**, 045005

The cosmological constant problem


VOLUME 82, NUMBER 25 PHYSICAL REVIEW LETTERS 21 JUNE 1999

Effective Field Theory, Black Holes, and the Cosmological Constant

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Bekenstein has proposed the bound $S \leq \pi M_P^2 L^2$ on the total entropy S in a volume L^3. This nonextensive scaling suggests that quantum field theory breaks down in large volume. To reconcile this breakdown with the success of local quantum field theory in describing observed particle phenomenology, we propose a relationship between UV and IR cutoffs such that an effective field theory should be a good description of nature. We discuss implications for the cosmological constant problem. We find a limitation on the accuracy which can be achieved by conventional effective field theory. [S0031-9007(99)09399-0]

- 25 years ago, in a renowned paper Cohen, Kaplan, and Nelson postulated that to solve the cosmological constant problem one should append EFT with a constraint of the QG origin relating IR cutoff (the size of the spacetime region L) and UV cutoff (the size of momentum space Λ)

$$L^3\Lambda^4 \lesssim L M_P^2$$

- If L is identified with the Hubble size H^{-1}, the UV cutoff is bounded by 10^{-3} eV, which agrees with the current value of the cosmological constant.

- Not surprisingly the CKN bound can be understood as a dramatic depletion of the number of states in UV.

- Could we obtain a bound like that from the first principles? Which "degrees of freedom" should be employed?

Theory vs observation/comments

- The measured value of cosmological constant is 10^{-120} in Planck units.
- Depending on what is your favorite cut-off scale* the parameter Λ is between 10^{-17} (Standard Model scale) and 1 (Quantum Gravity scale). Whichever you choose, the discrepancy is huge.
- The cosmological constant problem, or the vacuum energy problem, is also associated with the enormous hierarchy of scales between the observed vacuum energy scale and the naive quantum gravity scale set by the Planck energy.

* String theory is UV finite, but this does not solve the problem, because in the string loop calculation the cutoff scale Λ is just replaced by the string scale (and the mass of the lightest string state), again many orders of magnitudes off the desired result. Besides, superstrings are incompatible with positive cosmological constant.

An idea

- The crucial observation is that the problem is not only about UV, but also about IR.

- To see this let us revisit the computation from slightly different perspective. Instead of computing the loop diagram with no external legs, following Polchinski we start with a particle moving on a circle S^1. The circle amplitude is

$$Z_{S^1} = \int_0^\infty \frac{d\tau}{2\tau} \mathrm{Tr}\, e^{i\hat{\mathscr{H}}\tau} \sim \rho V_4$$

Polchinski, String theory, ch. 7

Equivalence of loops and circles

- Consider the vacuum partition function

$$Z_{vac} = \int D\phi e^{-\int \frac{1}{2}\phi(-\partial^2+m^2)\phi}$$

$$\sim \left(\det\left(-\partial^2 + m^2\right)\right)^{-1/2} = e^{-\frac{1}{2}\mathrm{Tr}\log\left(-\partial^2+m^2\right)}$$

- In momentum space trace is an integral, while

$$-\frac{1}{2}\log\left(k^2 + m^2\right) = \int \frac{\mathrm{d}\tau}{2\tau} e^{-\left(k^2+m^2\right)\tau/2}$$

- so that $Z_{vac} = \exp(Z_{S^1})$

$$Z_{S^1} = V_4 \int \frac{d^4k}{(2\pi)^4}\frac{d\tau}{2\tau} e^{-(k^2+m^2)\tau/2} \sim V_4 \int \frac{d^3k}{(2\pi)^3}\omega_p$$

UV & IR

- A bit of massaging

$$\delta^4(p) = \int d^4q e^{-ipq}$$

$$\delta^4(0) = \int d^4q e^{-i0q} = V_q$$

$$Z_{S^1} = \int_0^\infty \frac{d\tau}{2\tau} \int \frac{d^4p}{(2\pi)^4} \langle p_\mu | e^{i\hat{\mathscr{H}}(\hat{p})\tau} | p_\mu \rangle$$

$$= \delta^{(4)}(0) \int_0^\infty \frac{d\tau}{2\tau} \int \frac{d^4p}{(2\pi)^4} e^{i\mathscr{H}(p)\tau}$$

$$= V_q \int_0^\infty \frac{d\tau}{2\tau} \int \frac{d^4p}{(2\pi)^4} e^{i\mathscr{H}(p)\tau}$$

$\Leftarrow$ divergent IR volume contribution

$$= \int_0^\infty \frac{d\tau}{2\tau} \int \frac{d^4q d^4p}{(2\pi)^4} e^{i\mathscr{H}(p)\tau}$$

$\Leftarrow$ phase space integration

In the standard treatment it is assumed that V_q is finite and fixed.

Phase space integral

- To proceed, we leave the τ integration to the end and consider the Wick rotated integral

$$Z(\tau) = \int \frac{d^4q\, d^4p}{(2\pi\hbar)^4} e^{-p_\mu^2 \tau/2} = \mathrm{Tr}\, e^{-\hat{p}_\mu^2 \tau/2}$$

- Then we split the integral over phase space into a sum of integrals over a finite cell of unit (phase space) volume. The cell is defined by

$$p \to \varepsilon\, \tilde{x}\,, \quad \tilde{x} \in [0,1]$$

$$q \to \lambda\, x\,, \quad x \in [0,1]$$

$$Z(\tau) = \left[\frac{\lambda \varepsilon}{2\pi\hbar} \sum_{k,\tilde{k} \in \mathbb{Z}} \int_0^1 dx \int_0^1 d\tilde{x}\, e^{-(\tilde{x}+k)^2 \varepsilon^2 \tau/2} \right]^4$$

Modular polarization

- These manipulations are just rearrangements of the integral, but the resulting expression has an interpretation of the trace done in another basis, a so-called modular polarization. It is unitarily equivalent (via so called Zak transform) to the momentum basis.
- The phase space decomposes into modular cells of size

$$\varepsilon\lambda = 2\pi\hbar$$

- We call this quantum area constraint. It should be stressed that apart the area constraints the scales ε and λ are arbitrary; nothing forces us to identify them with the Planck scales.
- The sums can then be interpreted as counting of modular cells.

*Y. Aharonov, D. Rohrlich, "Quantum Paradoxes", Wiley 2005

Phase space integral

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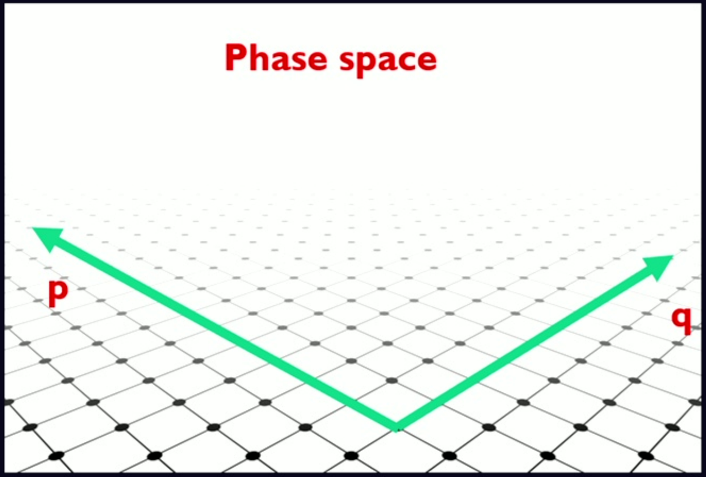
The phase space

- Instead of the minimal length, area, volume … we have here the minimal phase space cell of size $2\pi\hbar$, the notion that lies at heart of quantum mechanics. The exact size of the cell is fixed contextually, relative to the actual physical situation. For example, in the double slit experiment λ is the distance between the slits



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Regularization

- The expression $Z(\tau)$ is divergent and must be regularized introducing UV and IR cutoffs

$$Z(\tau)_{m.r.} := \left[\sum_{k=0}^{N_q-1} \sum_{\tilde{k}=0}^{N_p-1} \int_0^1 dx\, d\tilde{x}\, e^{-(\tilde{x}+k)^2 \varepsilon^2 \tau/2} \right]^4$$

- Here N_q and N_p are finite integers. Knowing them and λ, ε we can determine the size of spacetime and momentum space

$$L = N_q \lambda \,, \quad M = N_p \varepsilon$$

- Also, we know that the total number of cells

$$N = (N_q N_p)^4$$

Regularization

- Further,

$$\left(\frac{LM}{2\pi\hbar}\right)^4 = N$$

- If we regard N as fixed, then the cutoffs on space and momentum are not separately arbitrary but are inversely related.
- Also, if the effective cut-off is to be understood as a bound on N, it is manifestly Lorentz-invariant (contrary to bounds on energy and/or length).

Vacuum energy

- From regularized $Z(\tau)_{\mathrm{m.r.}}$ we get the vacuum energy density

$$\rho(\tau)_{m.r.} = \hbar \left[\frac{\varepsilon N_p}{2\pi\hbar} \frac{1}{N_q} \sum_{k=0}^{N_q-1} \int_0^1 d\tilde{x} e^{-(\tilde{x}+k)^2 \varepsilon^2 \tau/2} \right]^4$$

- Assuming that the τ integration does not change things substantially, we get a bound

$$\rho_{m.r.} \leq \hbar \left[\frac{\varepsilon N_p}{2\pi\hbar} \right]^4 = \hbar \left[\frac{M}{2\pi\hbar} \right]^4$$

- and if M is identified with a large mass scale such as Planck mass, then the usual conundrum pertains. ***But there is nothing here that makes it necessary/natural.***

Vacuum energy

- The bound can be also written as

$$\rho_{m.r.} \leq \hbar \frac{N}{V_q} , \quad V_q = L^4$$

- It relates the product of vacuum energy density and space-time volume, ρV_q, to N.
- Moreover, *N* can be identified with the entropy of the system.

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Entropy and N

- The gravitational entropy scales as an area

$$S_{grav} \sim \ell_{Pl}^{-2}\,\text{Area} \sim \left(\frac{\ell}{\ell_{Pl}}\right)^2$$

- The holographic principle states that matter entropy N cannot exceed de Sitter gravitational entropy which gives the vacuum energy bound

$$\rho_{m.r.} \leq \hbar\,\frac{N}{V_q} \lesssim \frac{\hbar}{\ell^2 \ell_{Pl}^2}\,, \quad V_q = \ell^4$$

- which gives the value of the vacuum energy contribution to cosmological constant

$$\Lambda_{cc} \sim \rho G \sim \frac{1}{\ell^2}$$

The cosmological constant

$$\Lambda_{cc} \leq \rho G \sim \frac{1}{\ell^2}$$

- The scale ℓ is the size of the system. In our universe it is the cosmic horizon size, and since our universe is essentially de Sitter, it equals de Sitter horizon. Now everything fits together perfectly, because N equals de Sitter entropy.

Why the cosmological constant is small?

- The cosmological constant is small because the universe is large. This is almost tautological: a nearly empty universe, corresponding to a small *N*, would have small entropy and thus be of Planckian size possessing an extremely large cosmological constant.
- Why the universe is large? It is large, because it is stable against fluctuations. If we have N degrees of freedom, the statistical fluctuations are of order of $N^{1/2}$ and are (relatively) small if N is large.

The Leibniz's revenge

Why there is nothing rather than something? (1988)

Because there is something rather than nothing (1714)

Conclusions: what has just happened?

- **The steps**:

1. The trace involved in computation of vacuum energy can be understood in terms of phase space geometry.
2. Then the computation of (regularized) vacuum energy density reduces to counting of elementary cells in phase space.
3. This calculation involves an undetermined length scale; this scale is fixed by bounding the microscopic count of vacua with the macroscopic value of gravitational entropy.
4. Finally, we get the result that the vacuum energy density contribution to cosmological constant is consistent with observations.
5. The number of degrees of freedom is huge: $N \approx 10^{124}$

Conclusions

- In a vast universe, the vacuum energy contribution to the cosmological constant must be small.
- The universe is vast because it contains a huge number of degrees of freedom.
- The universe must contain a huge number of degrees of freedom to exist.