Title: Stringy Gregory-Laflamme

Speakers: Marija Tomašević

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Abstract:

Thin enough black strings are unstable to rippling along their length, and the instability threshold indicates that static inhomogeneous black strings exist. These have indeed been constructed with increasing inhomogeneity until a high-curvature singular pinch appears. We study the string-scale version of this phenomenon: "string-ball strings", which are linearly extended, self-gravitating configurations of string balls obtained within the Horowitz- Polchinski (HP) approach to near-Hagedorn string states. We construct inhomogeneous HP strings in spatial dimension $d \leq 6$, and show that, as the inhomogeneity increases, they approach localized HP balls when $d \leq 5$ or cease to exist when $d = 6$. We then discuss how string theory can smooth out the naked singularities that appear in the Kaluza-Klein black hole/black string transition, and we propose scenarios for the final stage of the evolution of the black string instability after string theory takes over.

Outline

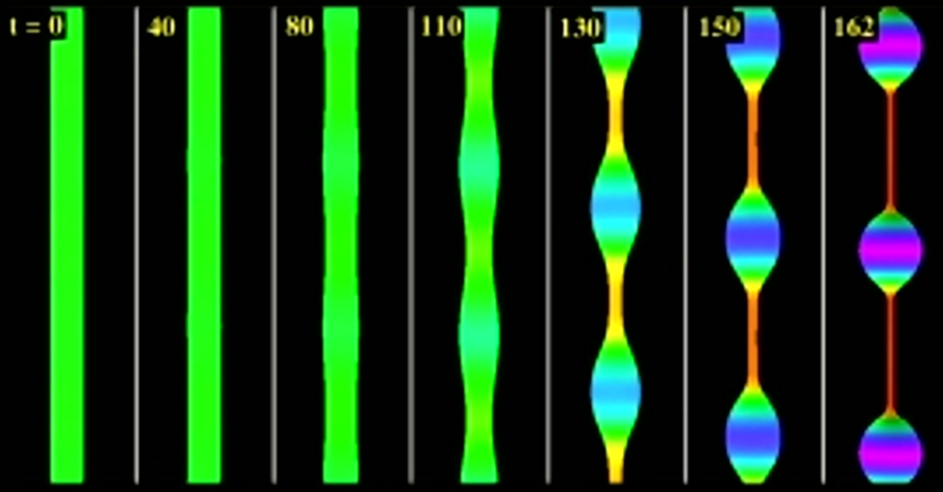
- Gregory-Laflamme instability of black strings
- The black hole---fundamental string correspondence
- Gregory-Laflamme instability of stringy strings
- A proposal

See also: Chu 24

The basic story of GL

A long enough black string is susceptible to breaking apart

One can see this thermodynamically and with linearized perturbations (GL 93) and with the full non-linear evolution of black strings (Lehner, Pretorius 11)



We can give a a simple argument for why black strings might be expected to evolve into localized black holes.

The basic story of GL

- The entropy of a black hole in D=d+1 dimensions takes the form

$$S_{BH} = c_d M^{\frac{d-1}{d-2}}$$

- Now we imagine that this black hole is localized in a circle of length L, and neglect the finite-size distortions that would modify the entropy formula above.
- For a black string in the same number of dimensions, the formula above applies after replacing d → d-1and scaling S → S/L and M → M/L, so the entropy is

$$S_{BS} = c_{d-1} M^{\frac{d-2}{d-3}} L^{-\frac{1}{d-3}}$$

The basic story of GL

$$S_{BH} = c_d M^{\frac{d-1}{d-2}} \qquad S_{BS} = c_{d-1} M^{\frac{d-2}{d-3}} L^{-\frac{1}{d-3}}$$

- It is now clear that if we compare a black hole and a black string of the same mass, then for L sufficiently larger than M, it will be entropically favorable for a *black string to transition into a localized black hole*.

This topology change implies a naked singularity!

- If the black string is unstable, it may happen that the classical evolution takes it to a *stable non-uniform black string*, and not all the way down to a fully localized black hole (for finite L).

The basic story of GL

We can find naked curvature singularities in two different circumstances:

1. Dynamical evolution of generic initial perturbations of the unstable black string
2. Evolution along the *space of static solutions* of increasingly inhomogeneous black strings.

- This singularity is small, of the order of the cutoff scale of the theory

The big picture

- A black hole whose horizon curvature is near the string scale is expected to morph into a highly excited string ball.

- When the curvature along the horizon reaches the string scale, a transition of roughly this kind should prevent the appearance of naked singularities.

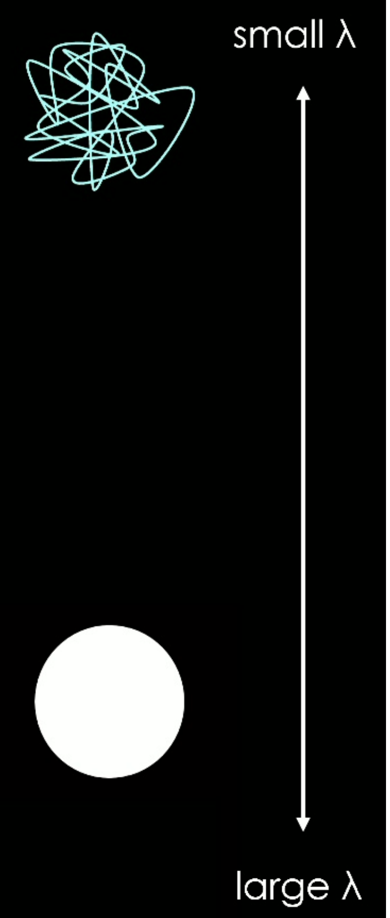
- In the time-evolving situation, string theory should control the further evolution of the string ball and provide a plausible mechanism by which the black string is severed into separate horizons.

The big picture

Let's first see how black holes are understood in string theory.

$$\delta\lambda = S\delta g^2$$

What comprises a (neutral) black hole



Observation:

- Black holes are highly degenerate objects with a large entropy
- However, strings, when highly excited, are also highly degenerate

How can we relate them without any SUSY-like protection?

simply **fix the entropy*** while changing the string coupling!

- Of course, without SUSY-like protection, the mass will get renormalized as we change the coupling---need to check that the mass changes *adiabatically* from one side to another

*recall, we are trying to give a description of bh microstates in terms of known states

Black hole side

$R_{\rm sch} = MG = M g_s^2 \ell_s^2$ $\qquad \frac{R_{\rm sch}}{\ell_s} = M g_s^2 \ell_s$

$S_{\rm BH} = \frac{Area}{4G} = M^2 G = M^2 g_s^2 \ell_s^2$

String side

$M = \frac{L}{\ell_s^2}$ $\qquad S_s = \frac{L}{\ell_s}$

$S_s = M \ell_s$

$g_s^2 \sim \frac{1}{S}$

Matching:

$\frac{R_{\rm sch}}{\ell_s} \sim 1 \qquad M \sim \frac{1}{g_s^2 \ell_s}$

$S_{\rm BH} \sim \frac{1}{g_s^4 \ell_s^2}\, g_s^2 \ell_s^2 \longrightarrow S_{\rm BH} \sim \frac{1}{g_s^2}$

$S_s \sim \frac{1}{g_s^2 \ell_s}\, \ell_s \longrightarrow S_s \sim \frac{1}{g_s^2}$

4D results

The black hole/string correspondence

Susskind 93
Horowitz, Polchinski 96/7

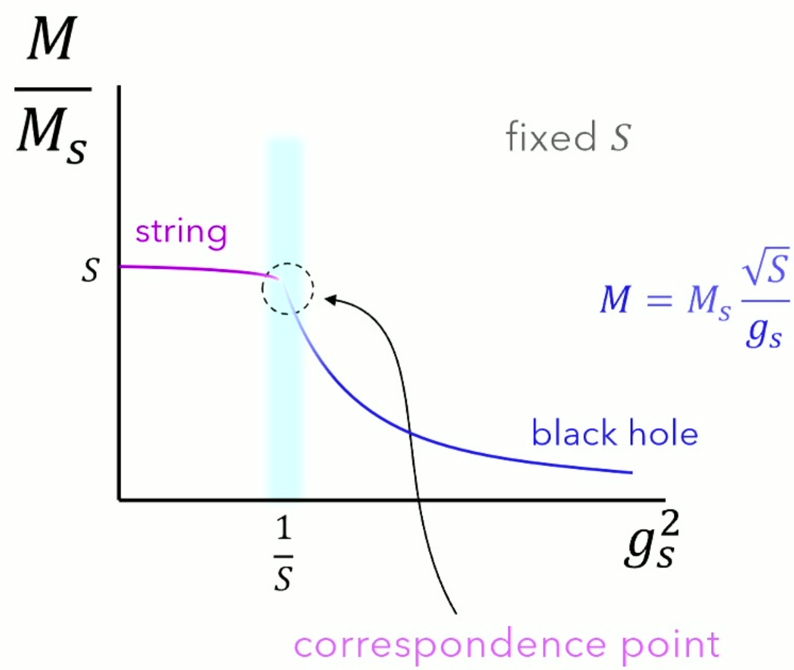
The black hole-string transition

The blue line is the line of fixed black hole mass in Mp

The pink line is the line of fixed string mass in Ms

They match up to O(1) factors for the coupling constant ~ 1/S

We can adiabatically* switch between the black hole and the fundamental string



The black hole/string correspondence: FAQ

- **What does it mean to change the string coupling?**

--- The string coupling is not a constant, but can vary in space and time, and is given by an expectation value of the dilaton field

- **Does the correspondence work for charged and/or rotating configurations?**

--- Yes, the charged case was discussed in the original HP paper, and the rotating version was constructed recently (more involved)

- **Does the correspondence include quantum effects?**

--- Yes, one can match the rate of evaporation of a black hole and a string, giving us a Goldilocks window of opportunity for the correspondence

- **Do the sizes match?**

--- No, until we include self-gravitation (but the J-dependent corrections agree)

Čeplak, Emparan, Puhm, MT 23 + to appear

Damour, Veneziano 99

What do we need?

- We want to study stringy GL
- This means that we want to study string configurations that include gravitational backreaction
- In order to compare different phases of the string, we need to perform either a dynamical evolution (hard) or a thermodynamic analysis (less hard)

- Both of these can be addressed within the **thermal scalar formalism**, which captures the essential mean-field features of a highly-excited string

Self-gravitating strings

The highly excited states of string theory near the Hagedorn temperature $T = \beta - \beta_H$ can be collectively described, in the Euclidean time formalism, by an effective mean-field χ.

This is the **winding mode** of the string around the Euclidean time circle, which becomes almost massless when its length is $\beta \simeq \beta_H$.

Being light, *this field must be added to the effective action* of string theory at low energies, which also contains the graviton and dilaton.

The coupling between the latter and χ allows to describe self-gravitating configurations of highly-excited strings, often called *string balls or string stars*.

Polchinski 86
Atick, Witten 88

Self-gravitating strings

After integrating the Euclidean time circle, the effective action in the d non-compact spatial directions becomes

$$I_d = \frac{1}{16\pi G_N}\int d^dx\sqrt{g}e^{-2\phi_d}\left(-\mathcal{R} - 4(\nabla\phi_d)^2 + (\nabla\varphi)^2 + |\nabla\chi|^2 + m(\varphi)^2|\chi|^2\right)$$

Here ϕ_d and g_{ab} are the d-dimensional dilaton and spatial metric. The field φ measures the length βexp(φ) of the Euclidean time circle, so φ is the gravitational potential in d dimensions.

The mass of the thermal scalar χ depends on φ and takes the value m_∞^2 at large distances where φ → 0.

$$m(\varphi)^2 = m_\infty^2 + \frac{\kappa}{\alpha'}\varphi + \mathcal{O}(\varphi^2), \qquad m_\infty^2 = \frac{\kappa}{\alpha'}\frac{\beta-\beta_H}{\beta_H}$$

Self-gravitating strings

Very close to the Hagedorn temperature, when the winding scalar is very light, m_∞^2 << κ/α', the dominant interaction is the one between φ and χ.

The dilaton ϕ_d and the spatial metric g_{ab} can consistently remain fixed and the field equations for ϕ and χ are

$$\nabla^2\chi - (\Delta_\beta + \varphi)\,\chi = 0,$$
$$\nabla^2\varphi - \frac{1}{2}\chi^2 = 0.$$

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$$\Delta_\beta \equiv \frac{\beta - \beta_H}{\beta_H} = \frac{T_H}{T} - 1$$

These equations are the *same as for a non-relativistic boson star* where a boson condensate χ is coupled to the Newtonian potential φ.

One important difference is that our formalism is purely Euclidean and we cannot study time-dependent fluctuations (e.g., quasi-normal modes) of the string ball.

Self-gravitating HP balls

- Let us first construct HP string balls. These correspond to spherically symmetric configurations where φ and χ vanish asymptotically and the condensate is regular at the origin,

$$\varphi(r)\,,\chi(r)\to 0 \quad \text{as } r\to\infty\,, \qquad \partial_r\chi(0)=0$$

- Our EoMs also allow a rescaling of variables: this will help us later on

$$(x^i,\chi,\varphi,\Delta_\beta)\to(\lambda^{-1/2}x^i,\lambda\chi,\lambda\varphi,\lambda\Delta_\beta)$$

- This implies that we can arbitrarily fix the overall amplitude of the condensate, e.g., by selecting a value for χ(0), and if we find a solution, then a simple rescaling gives a solution for any other amplitude.

Self-gravitating HP balls

- From the solution we can extract its mass from the asymptotic fall-off of φ,

$$\varphi(r) = -\frac{8\pi}{(d-2)\omega_{d-1}} \frac{G_N M}{r^{d-2}} + O(r^{1-d})$$

- Thus we obtain the temperature and mass of a solution of a given amplitude. *By rescaling it, we can find the relation β(M) for the string ball states in d dimensions.* Observe that the combination

$$\frac{G_N M}{\Delta_\beta^{(4-d)/2}} \equiv \mathrm{g}_d^{-(4-d)/2}$$

is a pure number (in string units) that is invariant under the rescaling of EoMs

Self-gravitating HP balls

- If we compute it in some (arbitrary) reference solution ($\varphi_0(r)$, $\chi_0(r)$) with mass M_0 and temperature $\Delta_{\beta 0}$, then the mass and temperature of any other solution are related by

$$G_N M = \left(\frac{\Delta_\beta}{\Delta_{\beta 0}}\right)^{\frac{4-d}{2}} G_N M_0 = \left(\frac{\Delta_\beta}{\mathrm{g}_d}\right)^{\frac{4-d}{2}} \longrightarrow \frac{\beta}{\beta_H} = 1 + \mathrm{g}_d (G_N M)^{\frac{2}{4-d}}$$

- From here, we can integrate the first law to obtain the entropy

Note that HP balls exist only for d = 3, 4, 5!

$$S(M) = \int \beta(M) dM \longrightarrow S_b = \beta_H M + \mathrm{g}_d \frac{d-4}{d-6} M^{\frac{d-6}{d-4}}$$

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- Let us also restore units for a second

$$\frac{S}{\beta_H M} - 1 \propto \frac{d-4}{d-6}\left(g^2 \frac{M}{M_s}\right)^{\frac{2}{4-d}}$$

The size of the corrections is measured by the ‘t Hooft-like coupling $g^2 S$

Self-gravitating HP balls

- Notice a peculiar feature: we obtained this entropy from a purely classical analysis
- Another way to obtain is directly from the effective action $S = (1 - \beta\partial_\beta)$ (-I)
- This is emphasizing that this entropy is a *classical* entropy---just like the entropy of a black hole!
- This represents some compelling evidence that the thermal scalar formalism and the string star have something in common with black hole physics
- Of course, we don't know all the details of black hole microscopics, but in the thermal scalar formalism, it is clear that the entropy is a result of a mean-field theory approach---perhaps gravity is doing something similar?

Chen, Maldacena, Witten 21

Self-gravitating HP strings

- To construct the stringy string solutions, we proceed in a similar manner as before, with the assumption that our solutions are cylindrical, not spherical, with $z \sim z + L$
- One writes down the scaling again and obtains the solutions for uniform strings as a translationally invariant ball solution in one dimension less.
- We can already make some heuristic predictions just based on the *uniform* string solution, similar to the original GL argument for black strings.

GL-like argument

- Namely, the entropies of an HP ball and HP string are given by

$$S_b = \beta_H M + g_d \frac{d-4}{d-6} M^{\frac{d-6}{d-4}} \qquad S_s = \beta_H M + g_{d-1} \frac{d-5}{d-7} M^{\frac{d-7}{d-5}} L^{\frac{2}{d-5}}$$

Now we have a dimension-dependent situation:

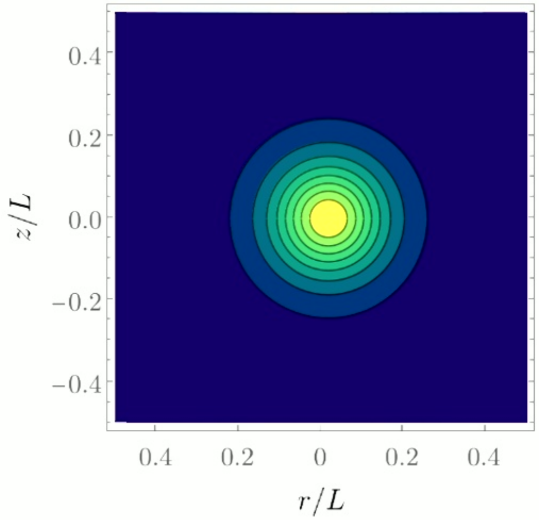
- **d = 4:** the correction for a string is $\sim +1/L^2$ while the ball receives no correction. So for any nonzero length L the *string* is always more entropic than the ball and therefore *will be thermodynamically preferred*
- **d = 5:** the ball entropy is now corrected by a negative term, while the string receives no correction. So, again, *the string is thermodynamically favored*.
- **d = 6:** the string entropy is now reduced by $\sim -L^2$. The approximations should break down for large enough L, but, in any case, in this dimension, the string cannot evolve into a localized string ball since the latter does not exist.

Self-gravitating HP strings

- So, from the thermodynamic argument we see that the strings dominate over ball solutions!
- Bear in mind that this is valid for very large L's, and also, it cannot tell us if there are any stable non-uniform solutions
- So, we must (numerically) construct the non-uniform solutions
- We start by finding the *zero mode* of the uniform string.
- This is a small, linearized perturbation, which signals the appearance of a family of non-uniform string configurations, namely the non-linear extensions of the zero mode.

Self-gravitating HP strings

- Using relaxation methods, we obtain a family of non-uniform stringy strings, which tend to a higher-dimensional ball solution for large enough L



Contour plots of χ of the non-uniform string configurations in d = 5 for increasing L

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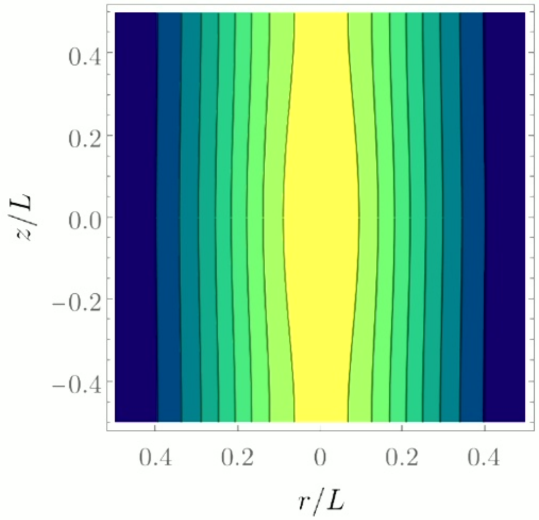

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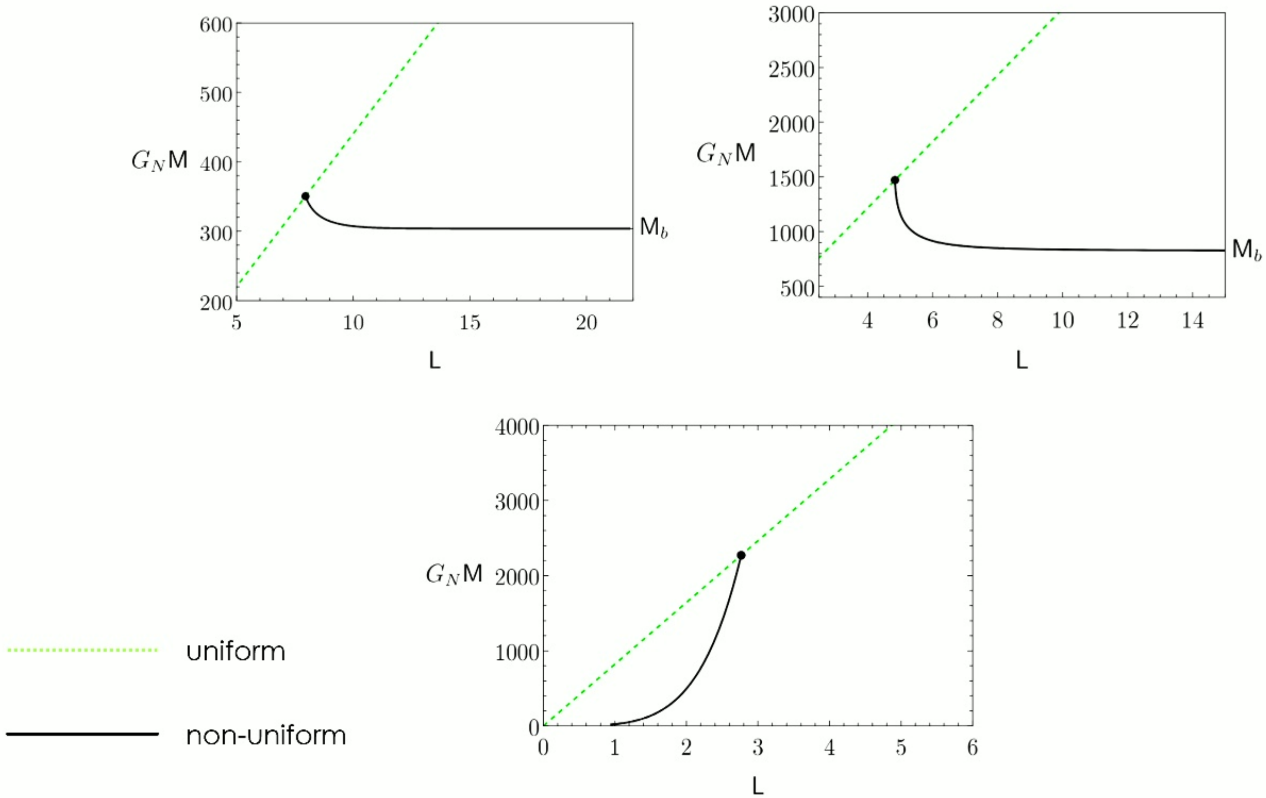
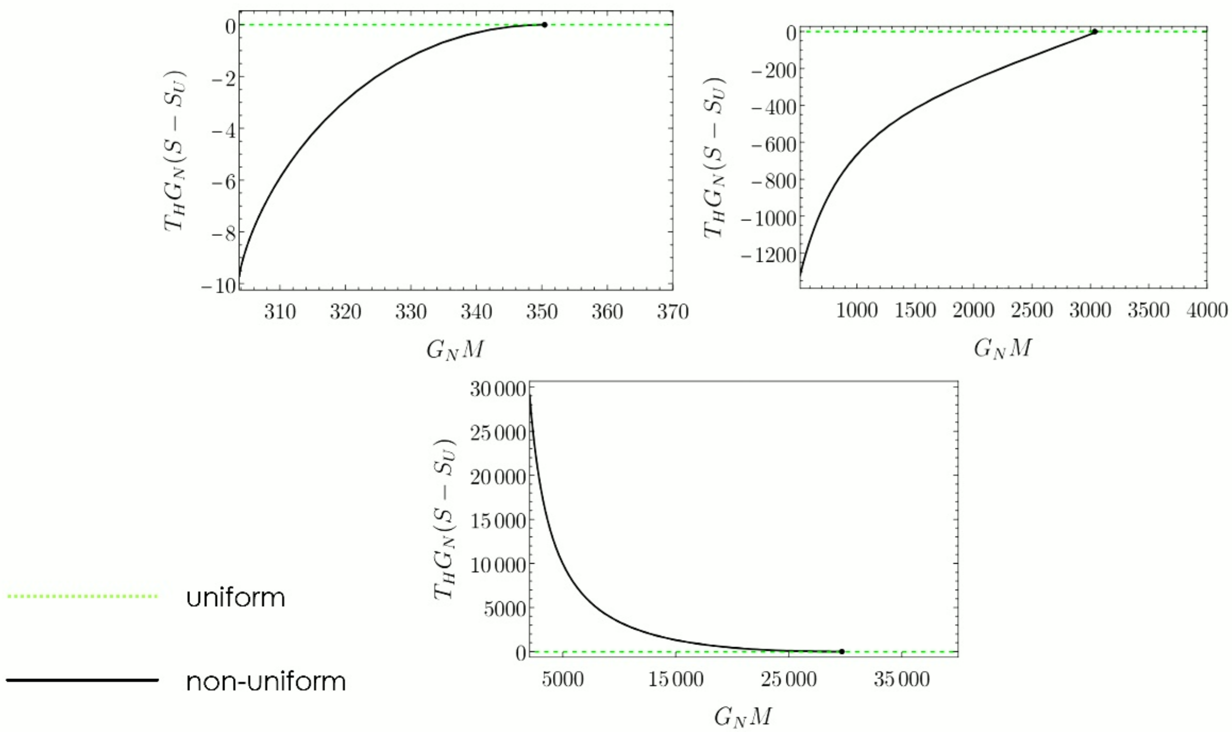


- Let's see what does the thermodynamics tell us
- For the masses: in d = 4, 5 the non-uniform string tends to the ball solution, whereas for d = 6, the approximation breaks down at L = 1

Self-gravitating HP strings



- Let's see what does the thermodynamics tell us
- <u>For the masses</u>: in d = 4, 5 the non-uniform string tends to the ball solution, whereas for d = 6, the approximation breaks down at L = 1
- <u>For the entropies</u>: in d = 4, 5 the uniform string dominates over the non-uniform ones, while this behavior is reversed for d = 6

Figueras, Kovacs, Yao xx

Conclusion and a proposal

We see that the uniform solutions are preferred over the ball ones; so what's the endpoint of the GL instability of black strings then?

- Our proposal is that

Stringy physics slows down the GL instability, such that the uniform string simply evaporates at the Hagedorn temperature and fizzles out

- In fact, there is some preliminary evidence from Figueras et al. that this might be the case from the black hole side as well:

They do a full non-linear evolution of a black string with higher curvature corrections (EGB in 5d) and see that the GL instability switches off!

- **Key point:** the higher curvature corrections are of the type found in string theory

Summary and outlook

We studied a stringy version of the Gregory-Laflamme instability

We constructed stringy strings using the thermal scalar formalism and we obtain the relevant thermodynamic phases of this object

Unlike black strings, stringy strings do not lead to a pinch-off; instead, they settle on a uniform solution

We propose this uniform solution will evaporate away, providing a smoothening of the GL naked singularity

Outlook: given that the critical collapse singularity and the GL one share a host of similarities (crucially, the same symmetry) one should be able to see a slow-down of the critical collapse as well.

Thank you!