Title: Measurement incompatibility implies irreversible disturbance

Speakers: Davide Rolino

Collection/Series: Quantum Foundations

Subject: Quantum Foundations

Date: November 04, 2024 - 11:00 AM

URL: https://pirsa.org/24110060

Abstract:

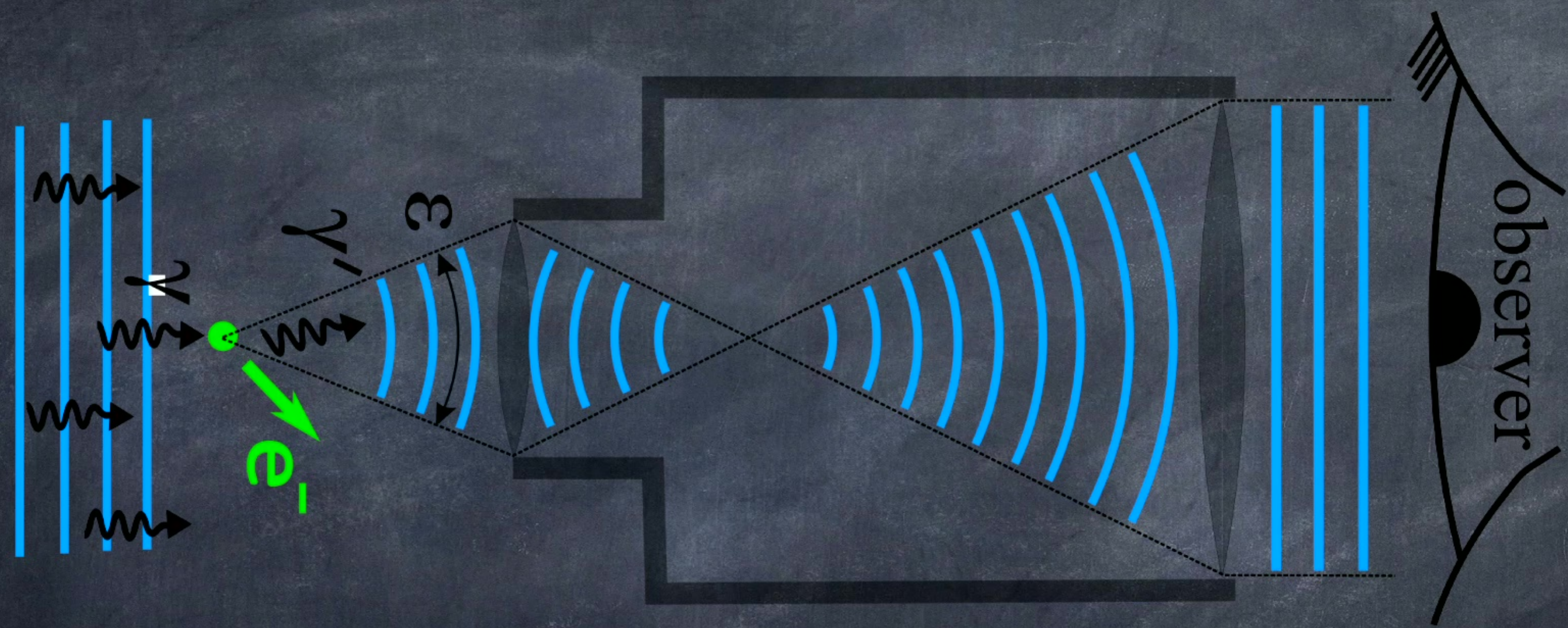
To justify the existence of measurements that can not be performed jointly on quantum systems, Heisenberg put forward a heuristic argument, involving the famous gamma-ray microscope Gedankenexperiment, based on the existence of measurements that irreversibly alter the physical system on which they act. Today, the impossibility of jointly measuring some physical quantities, termed measurement incompatibility, and irreversible disturbance, namely the existence of operations that irreversibly alter the system on which they act, are understood to be distinct but related features of quantum mechanics. In our work, we formally characterized the relationship between these two properties, showing that measurement incompatibility implies irreversible disturbance, though the converse is false. The counterexamples are two toy theories: Minimal Classical Theory and Minimal Strongly Causal Bilocal Classical Theory. These two are distinct as counterexamples because the latter allows for classical conditioning. Our research followed an operational approach exploiting the framework of Operational Probabilistic Theories. In particular, it required the development of two new classes of operational theories: Minimal Operational Probabilistic Theories and Minimal Strongly Causal Operational Probabilistic Theories. These theories are characterized by a restricted set of dynamics, limited to the minimal set consistent with the set of states. In Minimal Strongly Causal Operational Probabilistic Theories, classical conditioning is also allowed.

Measurement incompatibility implies irreversible disturbance

Davide Rolino

Perimeter Institute for Theoretical Physics

Waterloo, November 4, 2024



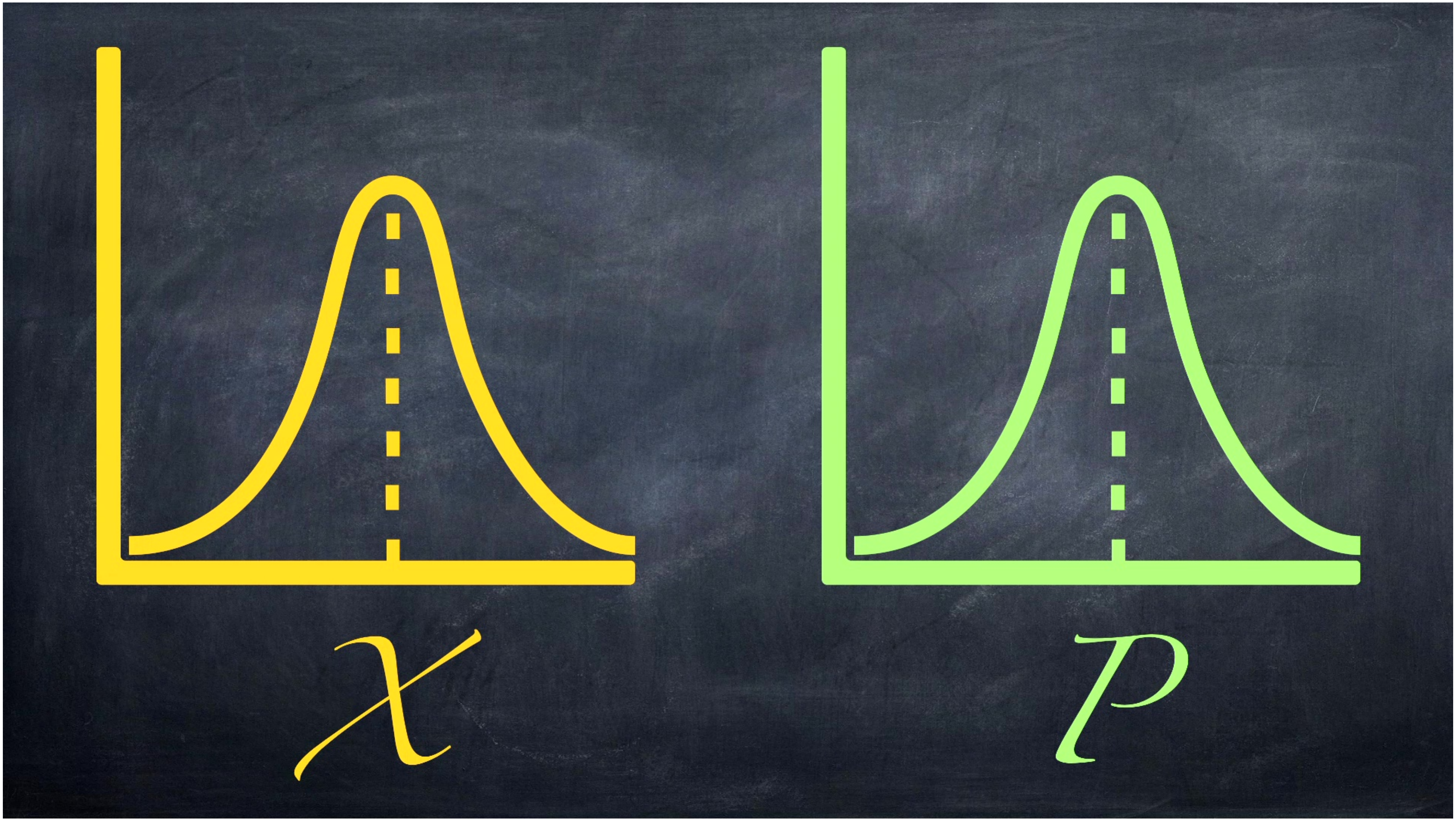
$$\Delta x \Delta p_x \approx h$$

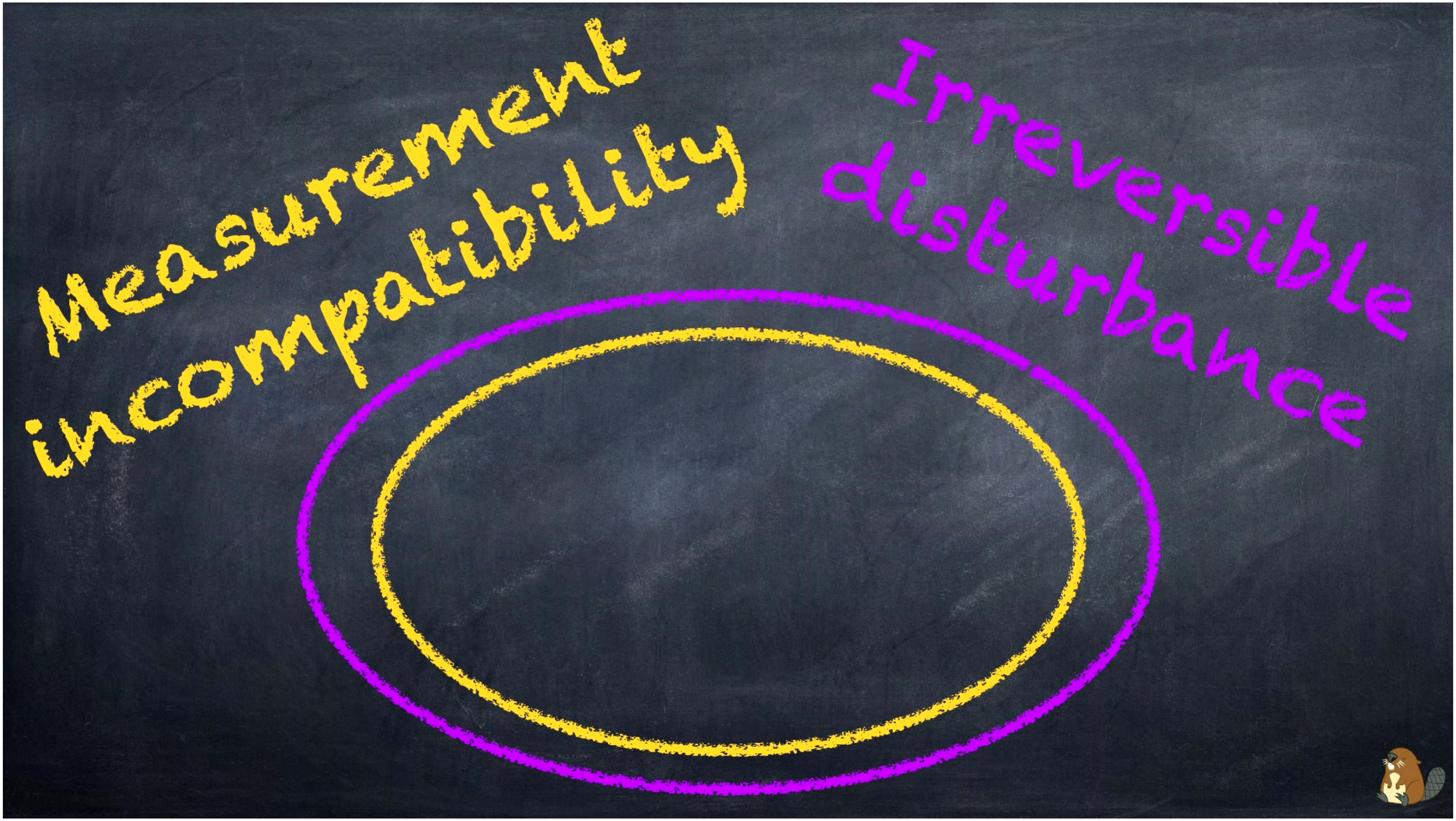

My Work


$\Delta x \Delta p \geq \hbar/2$

MEASUREMENT INCOMPATIBILITY

Measurement incompatibility implies irreversible disturbance

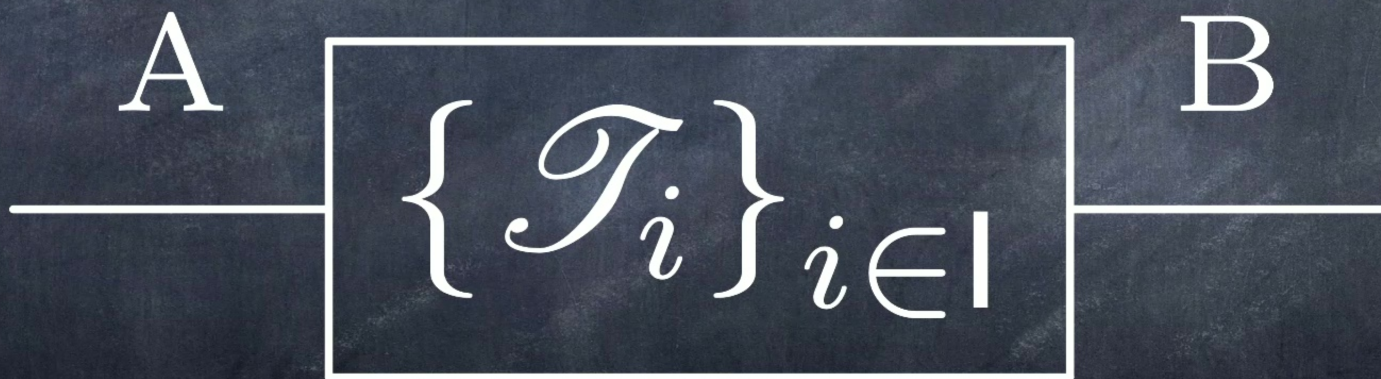

x
p

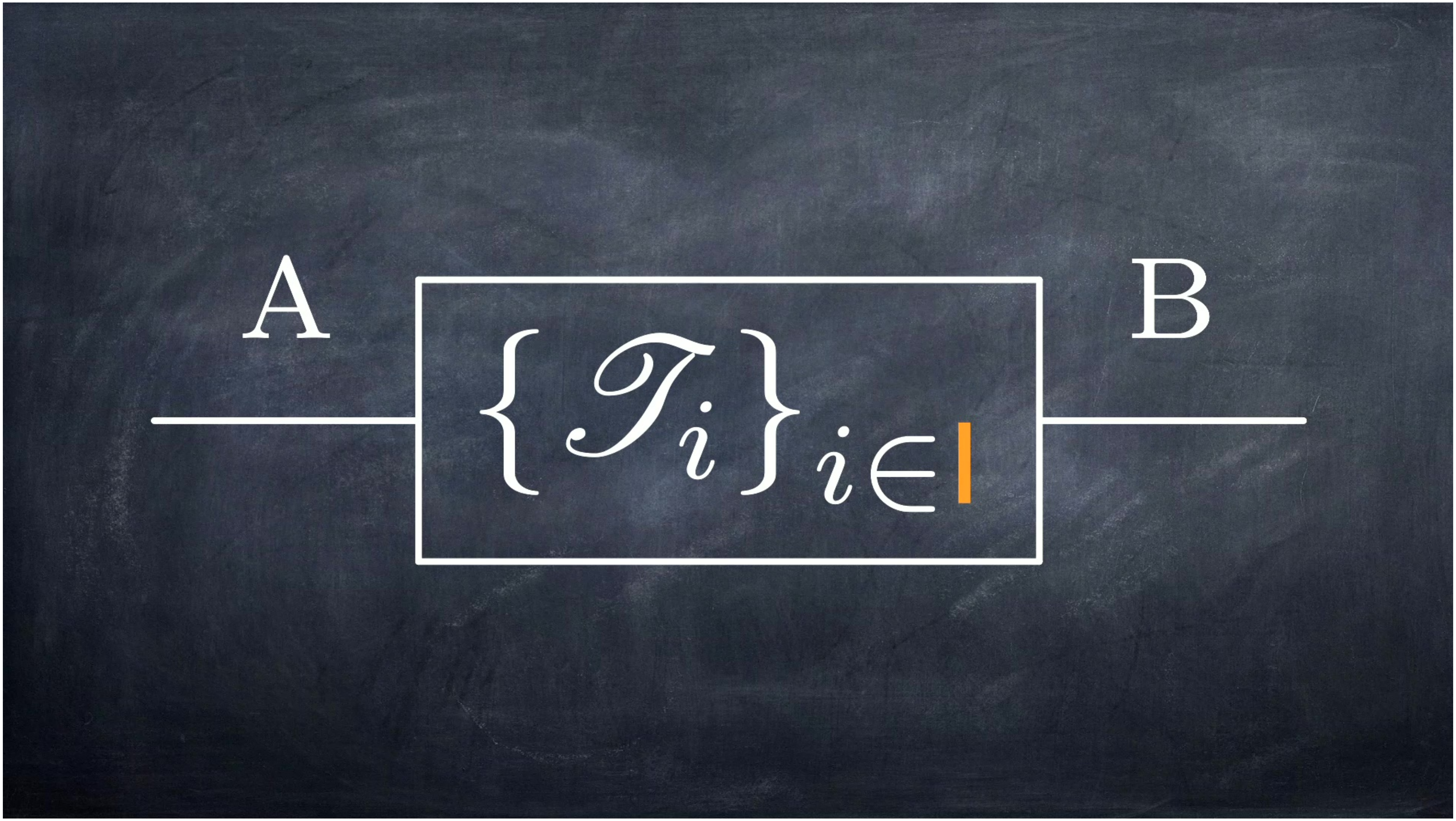

Measurement incompatibility
Irreversible disturbance

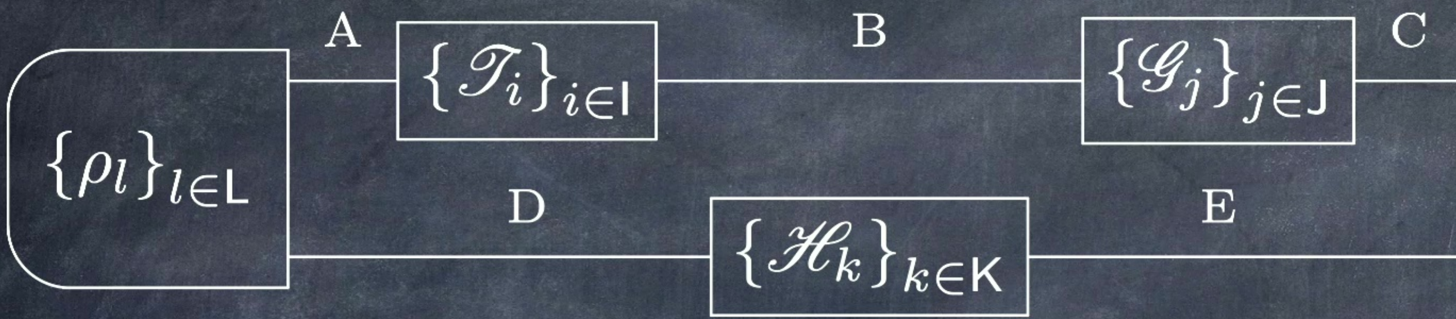
OPERATIONAL PROBABILISTIC THEORIES (OPTs)

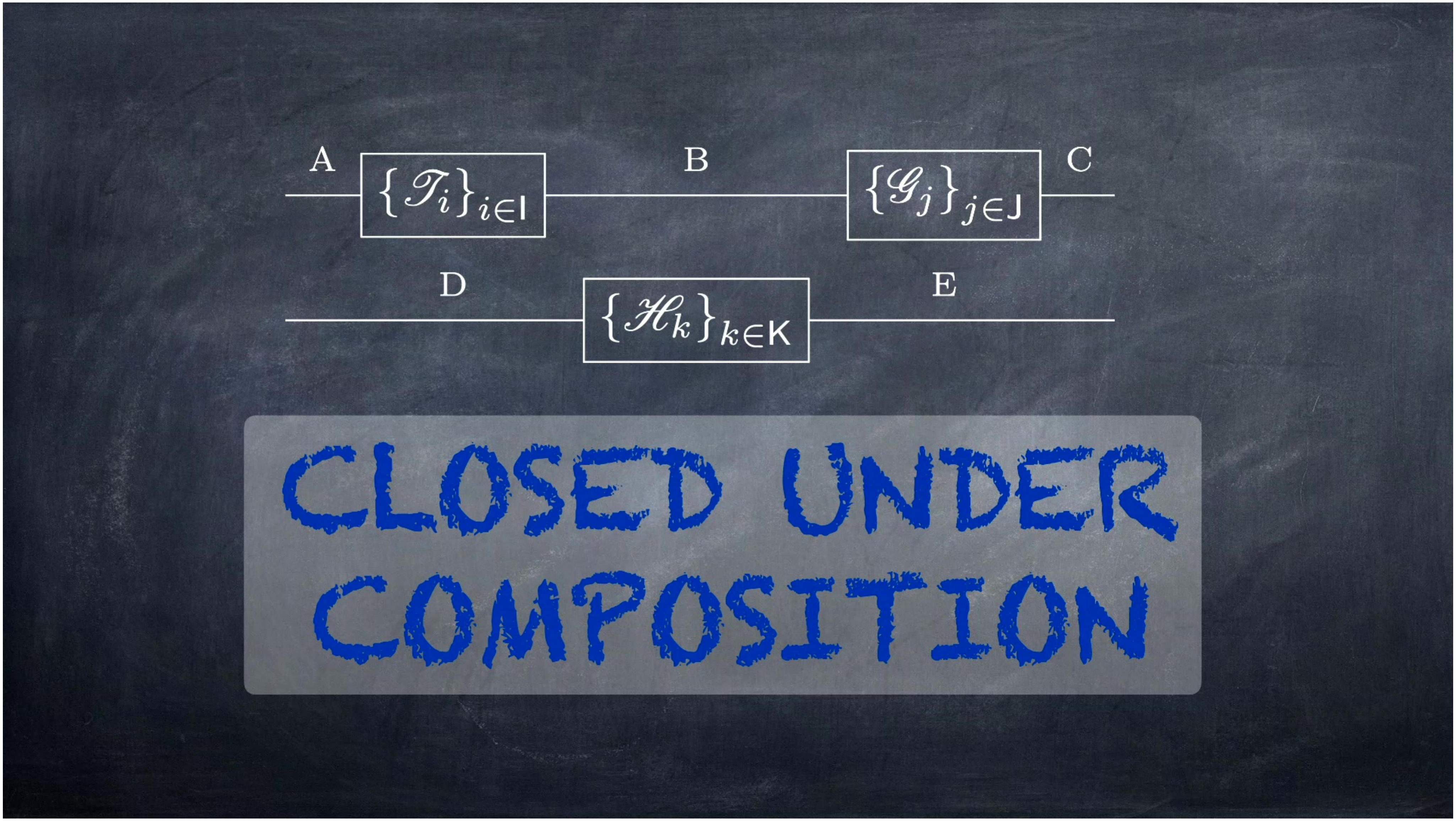
G. Chiribella, G. M. D'Ariano, and P. Perinotti, Phys. Rev. A 81, 062348 (2010),
G. M. D'Ariano, G. Chiribella, and P. Perinotti, "Quantum Theory from First Principles", CUP (2017)

INSTRUMENTS

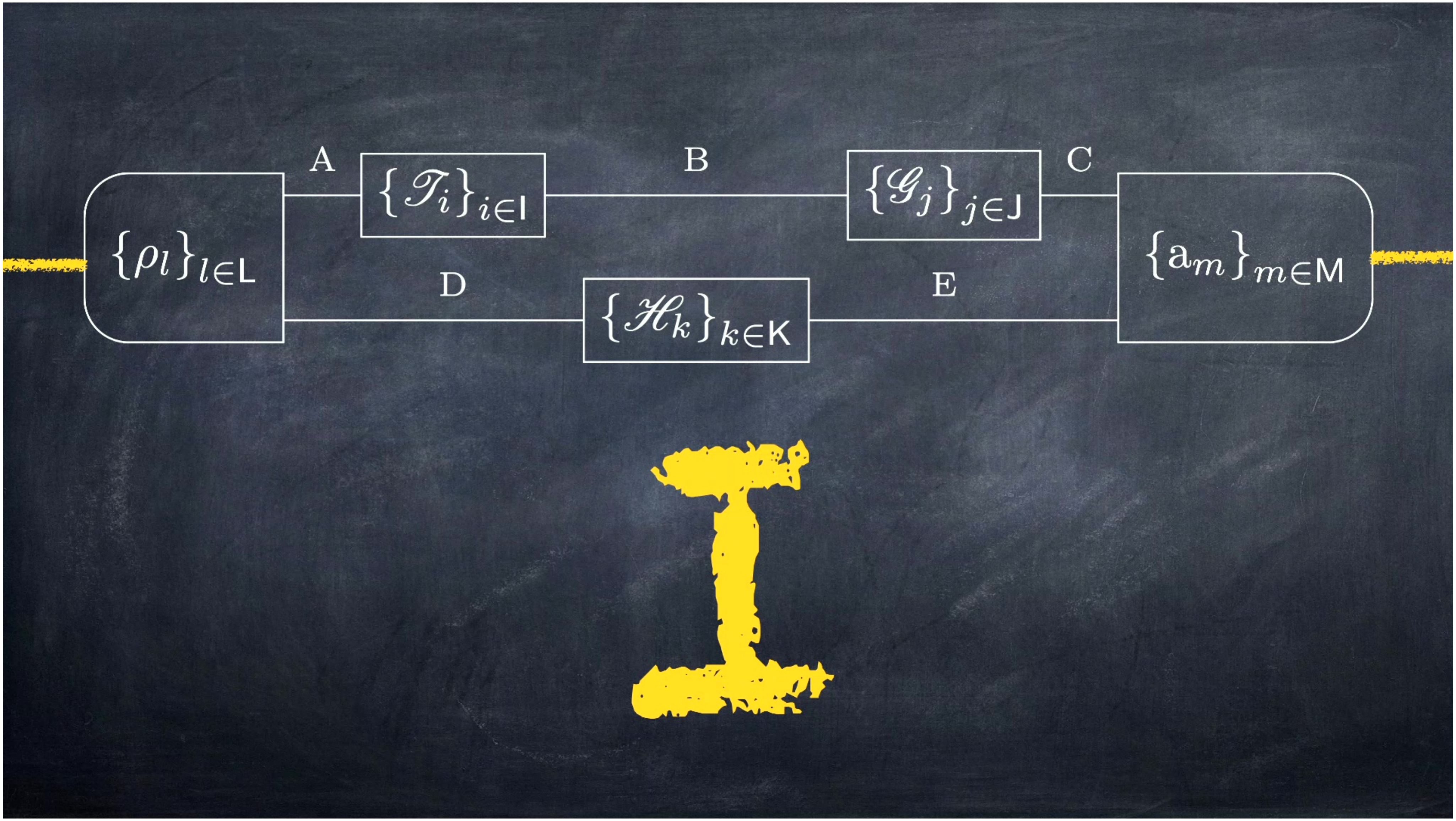



A
$\{\mathscr{T}_i\}_{i \in I}$
B

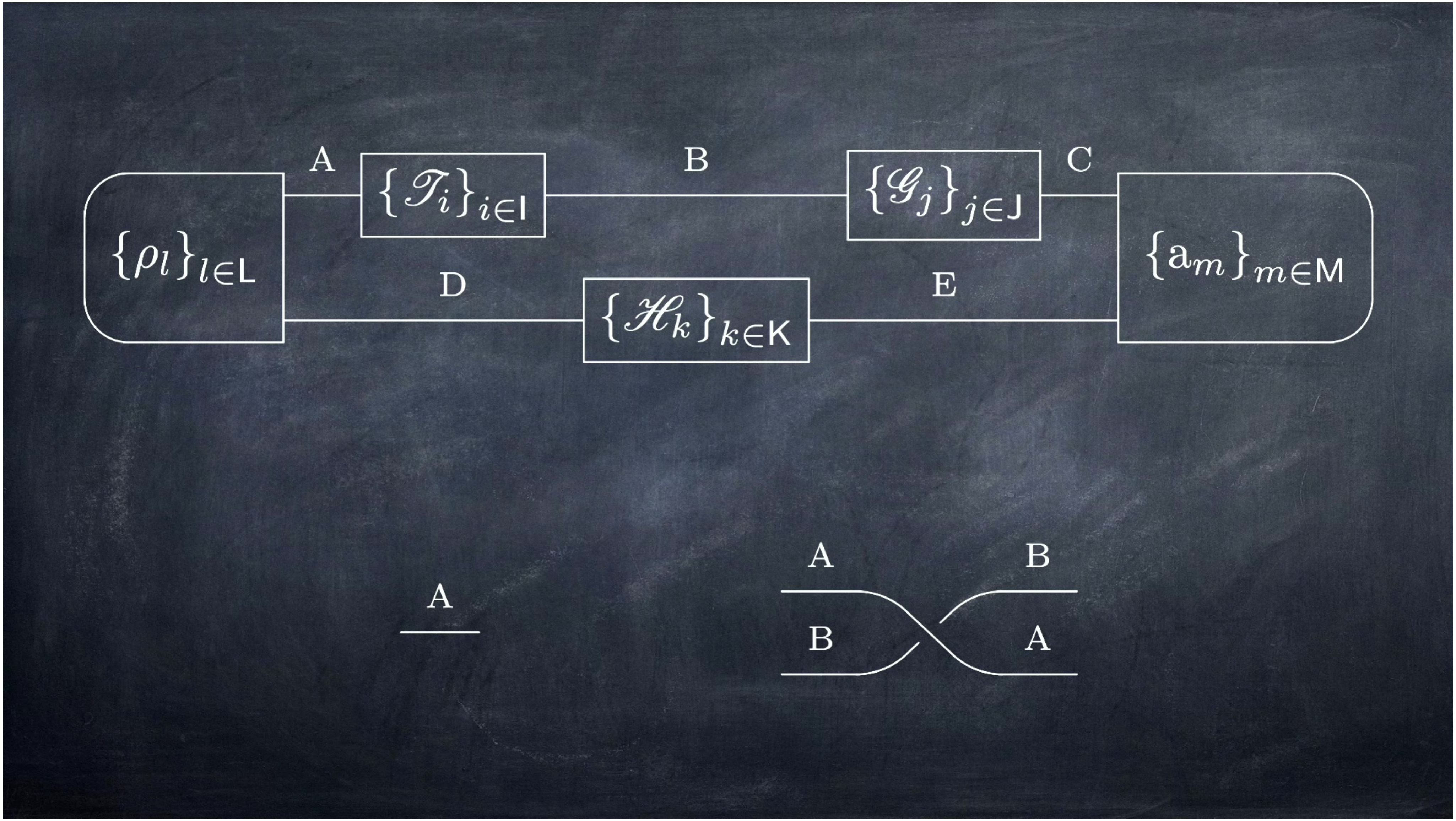

$\{\rho_l\}_{l\in\mathsf{L}}$
A
$\{\mathscr{T}_i\}_{i\in\mathsf{I}}$
B
$\{\mathscr{G}_j\}_{j\in\mathsf{J}}$
C
D
$\{\mathscr{H}_k\}_{k\in\mathsf{K}}$
E



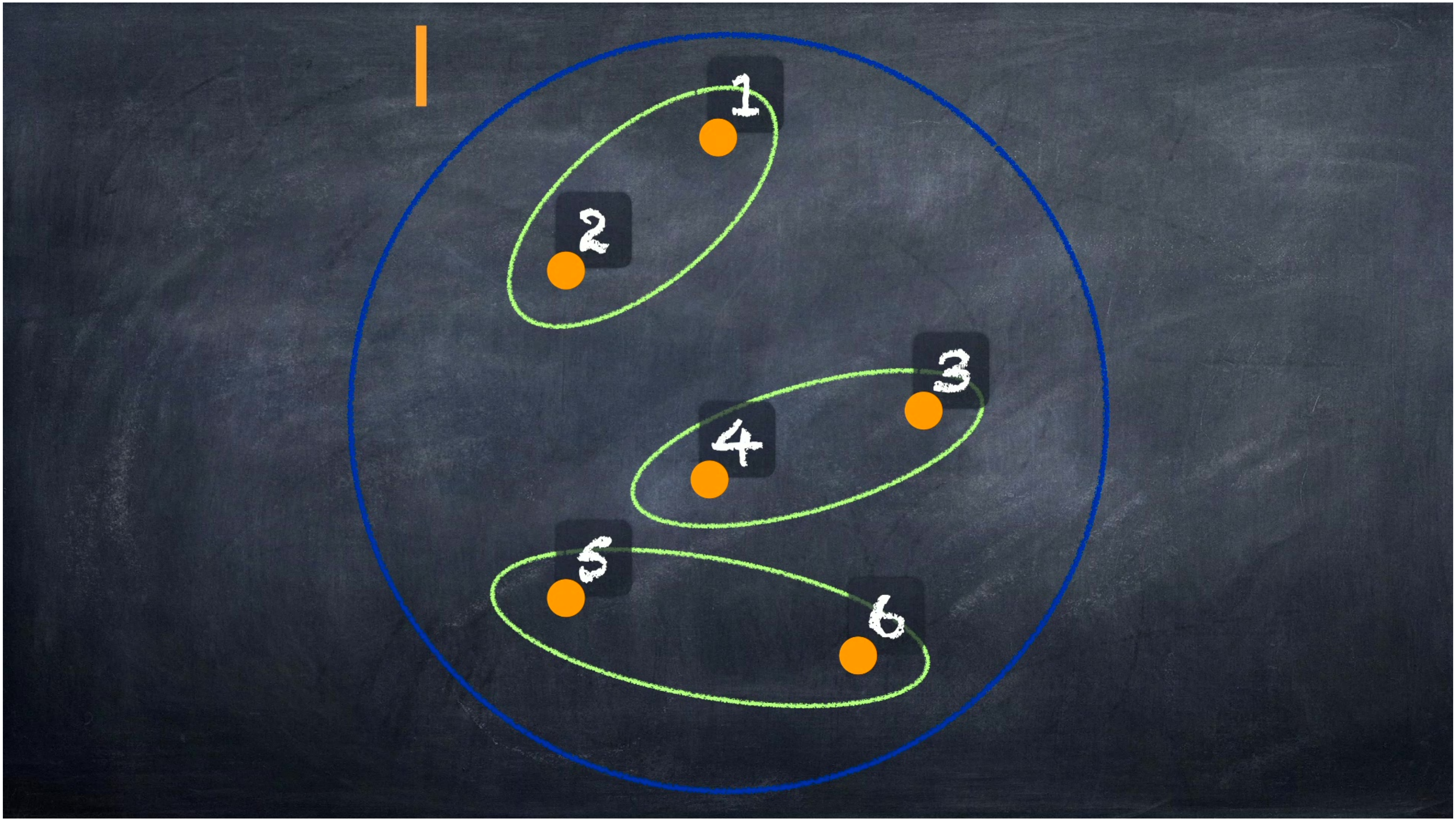
A
$\{\mathscr{T}_i\}_{i\in\mathsf{I}}$
B
$\{\mathscr{G}_j\}_{j\in\mathsf{J}}$
C
D
$\{\mathscr{H}_k\}_{k\in\mathsf{K}}$
E
CLOSED UNDER COMPOSITION

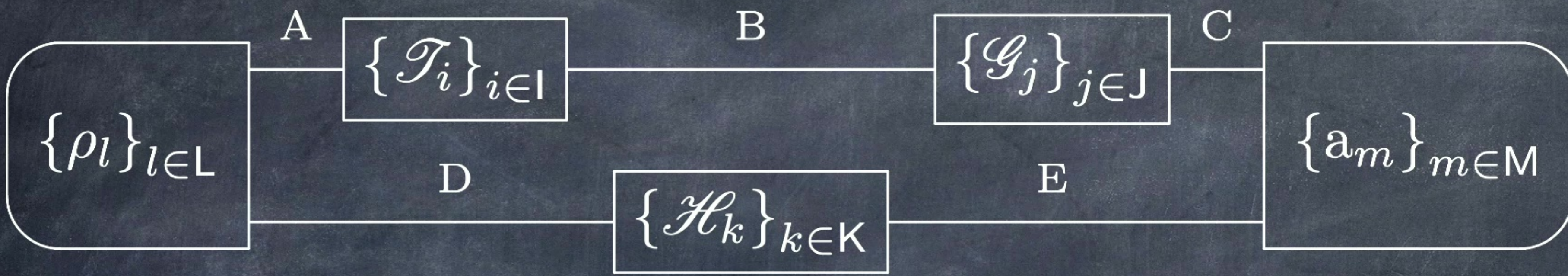


A
B
C
D
E
$\{\rho_l\}_{l\in L}$
$\{\mathscr{T}_i\}_{i\in I}$
$\{\mathscr{G}_j\}_{j\in J}$
$\{\mathscr{H}_k\}_{k\in K}$
$\{a_m\}_{m\in M}$


A
$\{\mathscr{T}_i\}_{i\in\mathsf{I}}$
B
$\{\mathscr{G}_j\}_{j\in\mathsf{J}}$
C
$\{\rho_l\}_{l\in\mathsf{L}}$
D
$\{\mathscr{H}_k\}_{k\in\mathsf{K}}$
E
$\{\mathrm{a}_m\}_{m\in\mathsf{M}}$
A
A
B
B
A

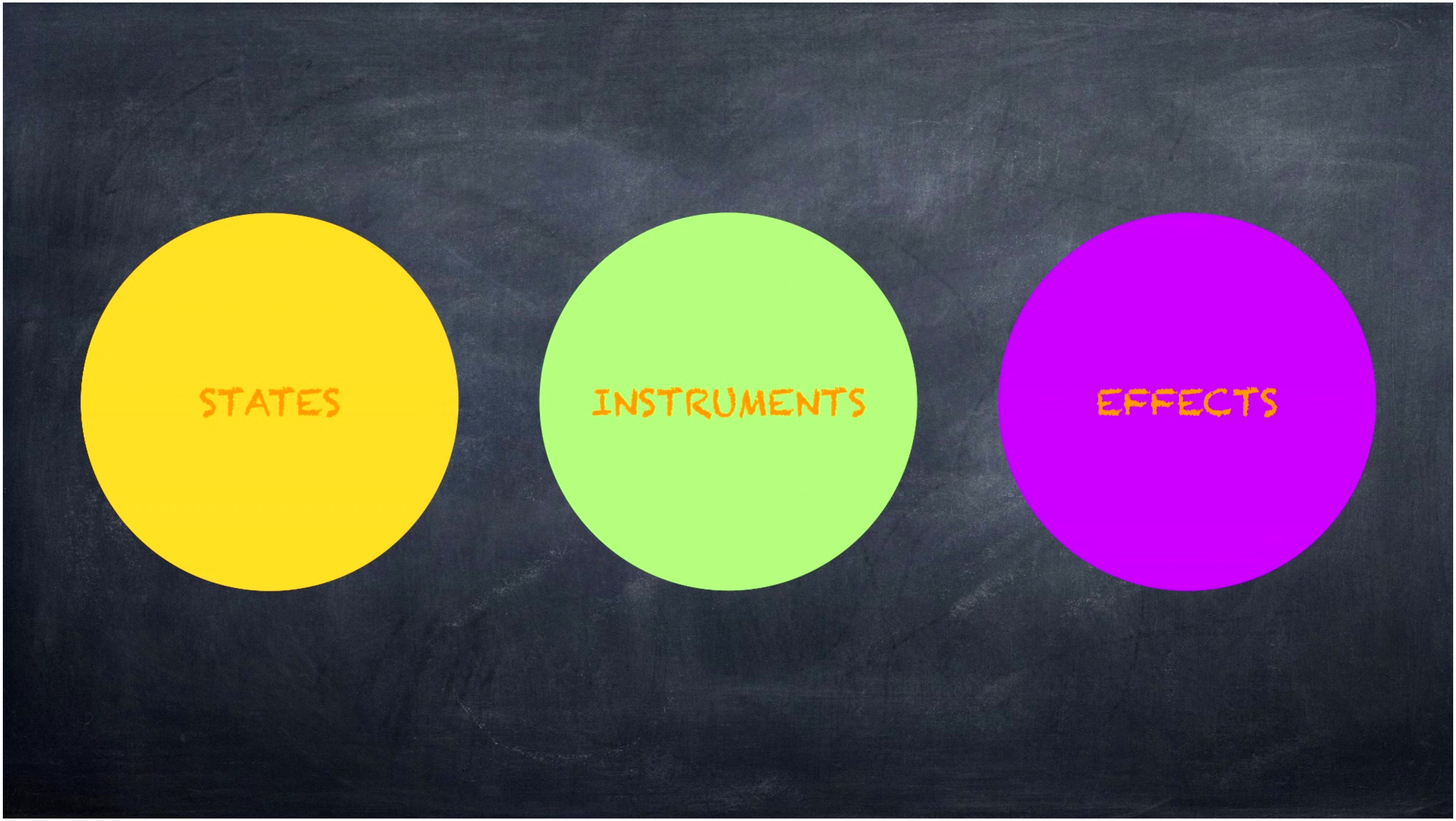
BRAIDED (STRICT) MONOIDAL CATEGORY

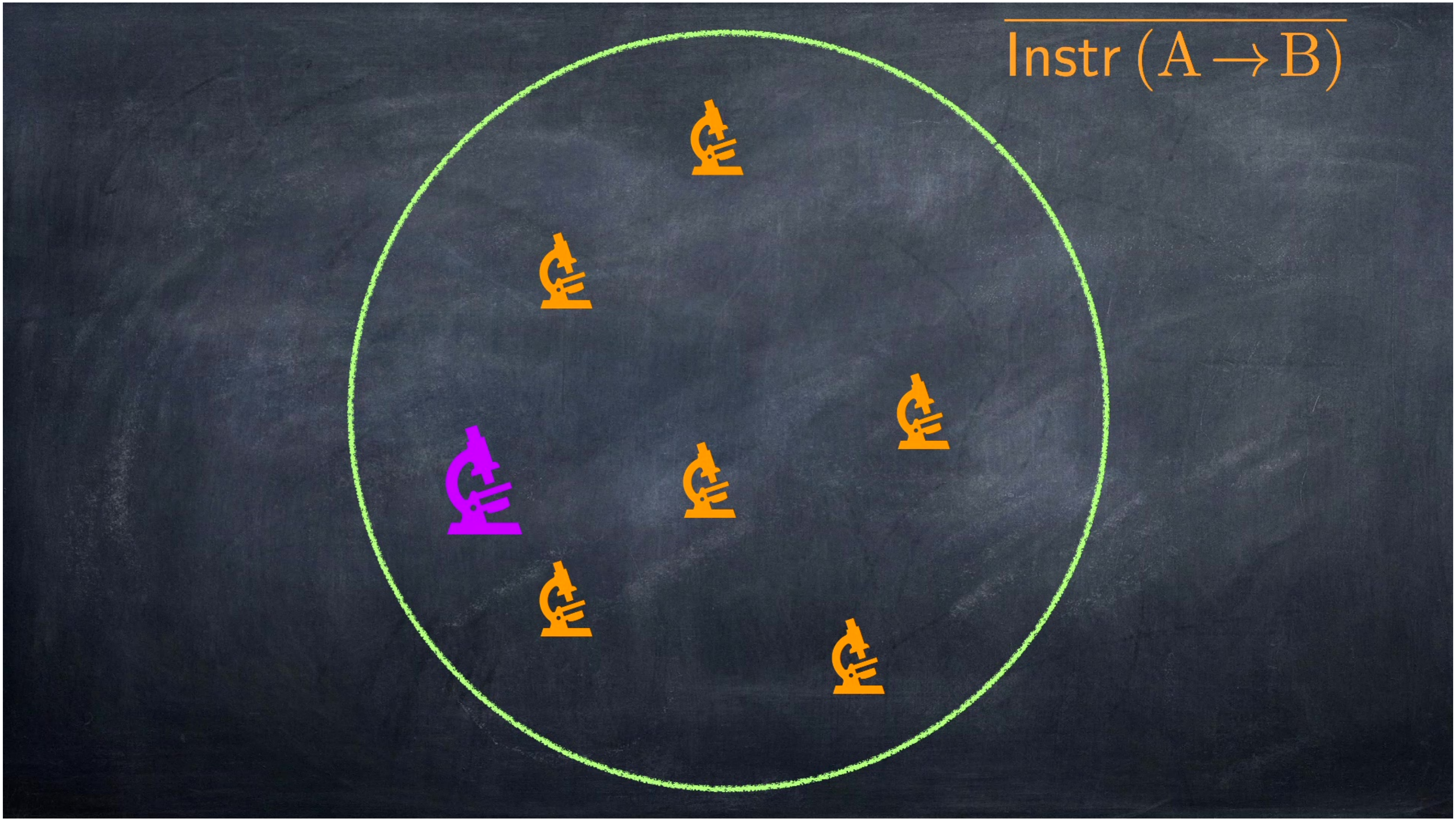

1
2
3
4
5
6



$$:=$$

$$p\,(l, i, j, k, m | \rho_{\mathsf{L}}, \mathsf{T}_{\mathsf{I}}, \mathsf{G}_{\mathsf{J}}, \mathsf{H}_{\mathsf{K}}, \mathrm{a}_{\mathsf{M}})$$


STATES
INSTRUMENTS
EFFECTS


$\overline{\mathsf{Instr}\,(A \to B)}$

DISCRIMINATION PROBABILITY

d ( , )

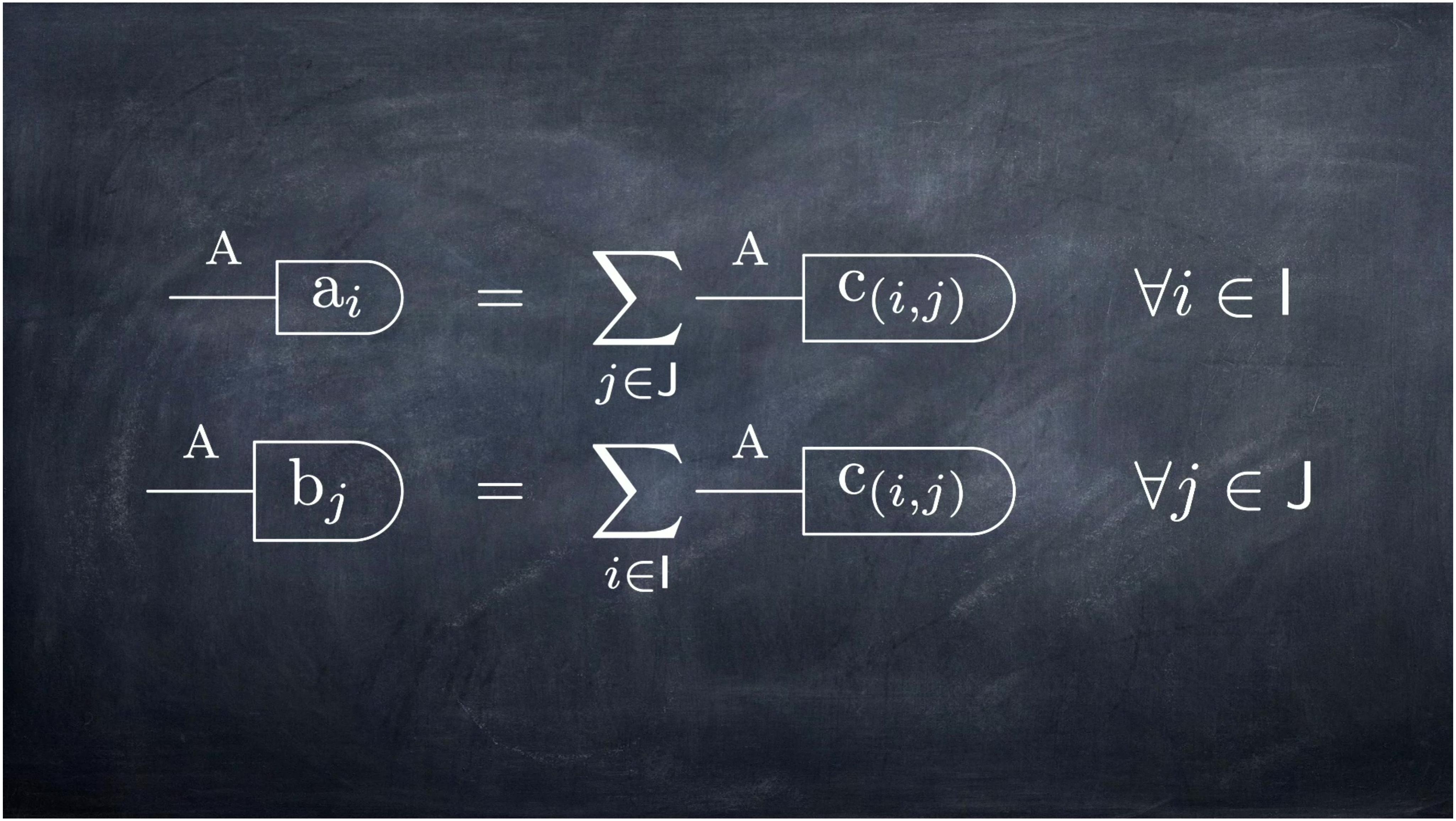
CAUSAL THEORIES

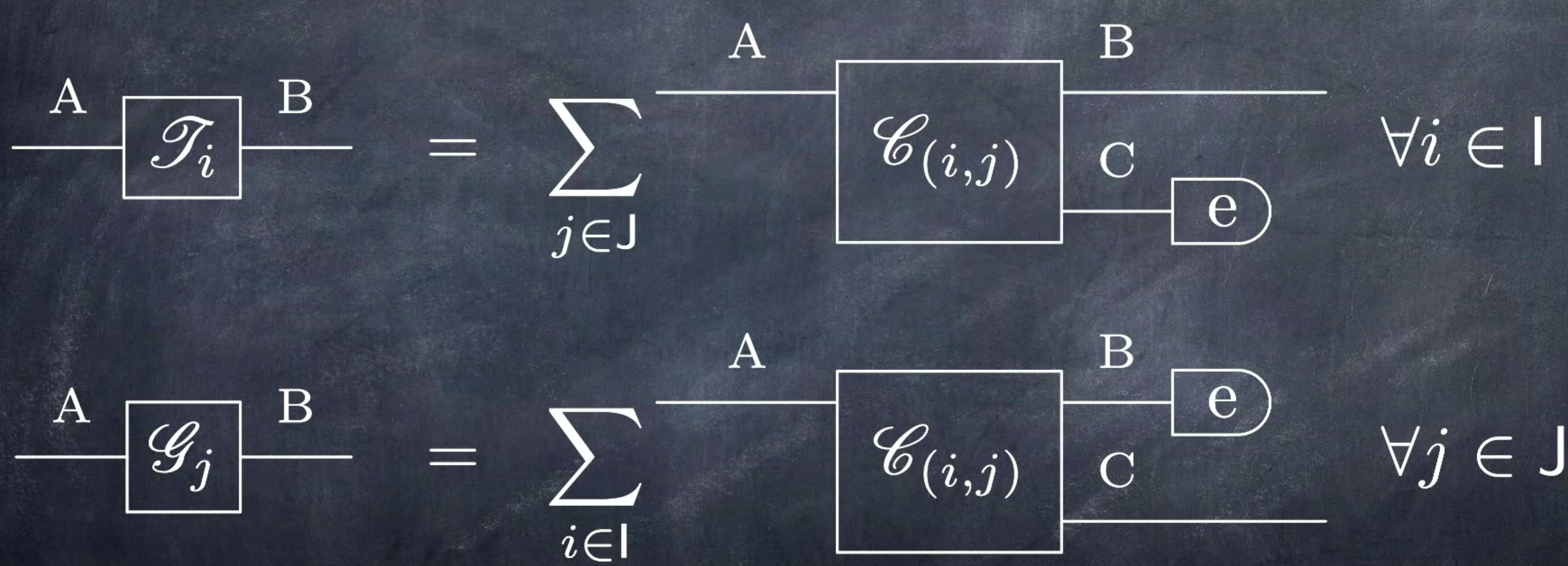
COMPATIBILITY

G. M. D'Ariano, P. Perinotti, and A. Tosini J. Phys. A: Math. Theor. 55 394006 (2022)

$$\mathrm{P}_i = \sum_{j \in \mathsf{J}} \mathrm{R}_{(i,j)} \quad \forall i \in \mathsf{I}$$

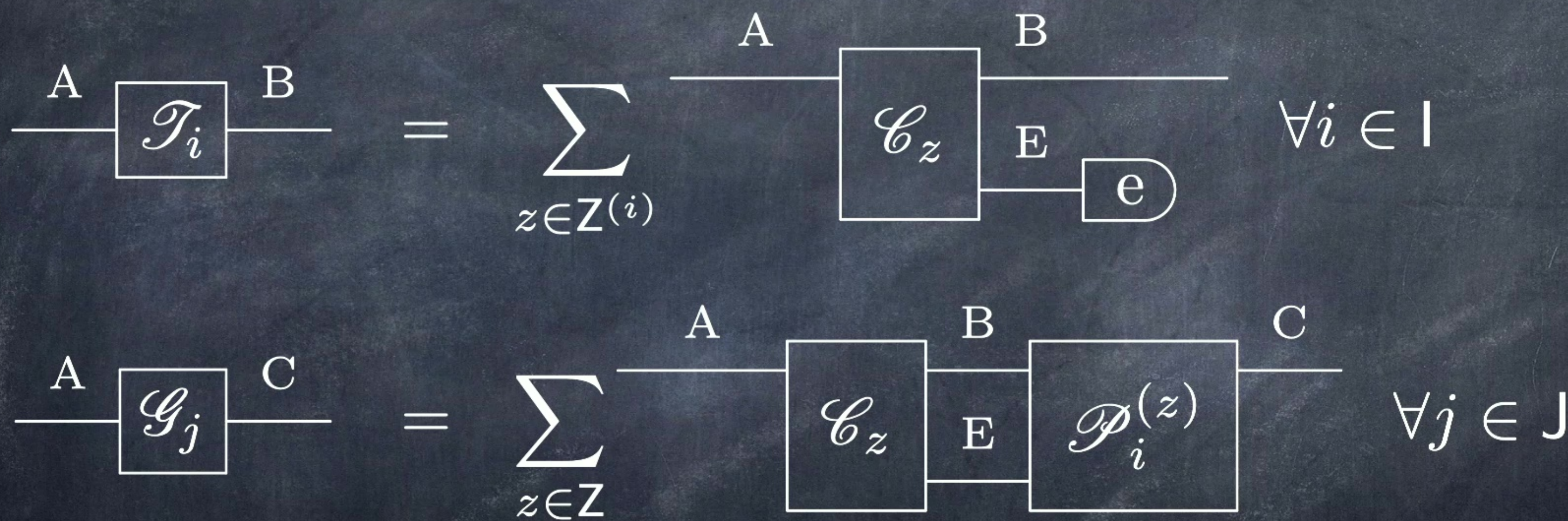
$$\mathrm{Q}_j = \sum_{i \in \mathsf{I}} \mathrm{R}_{(i,j)} \quad \forall j \in \mathsf{J}$$


A
a_i
$=$
$\sum_{j \in J}$
A
$c_{(i,j)}$
$\forall i \in I$
A
b_j
$=$
$\sum_{i \in I}$
A
$c_{(i,j)}$
$\forall j \in J$



G. M. D'Ariano, P. Perinotti, and A. Tosini J. Phys. A: Math. Theor. 55 394006 (2022)

STRONG COMPATIBILITY

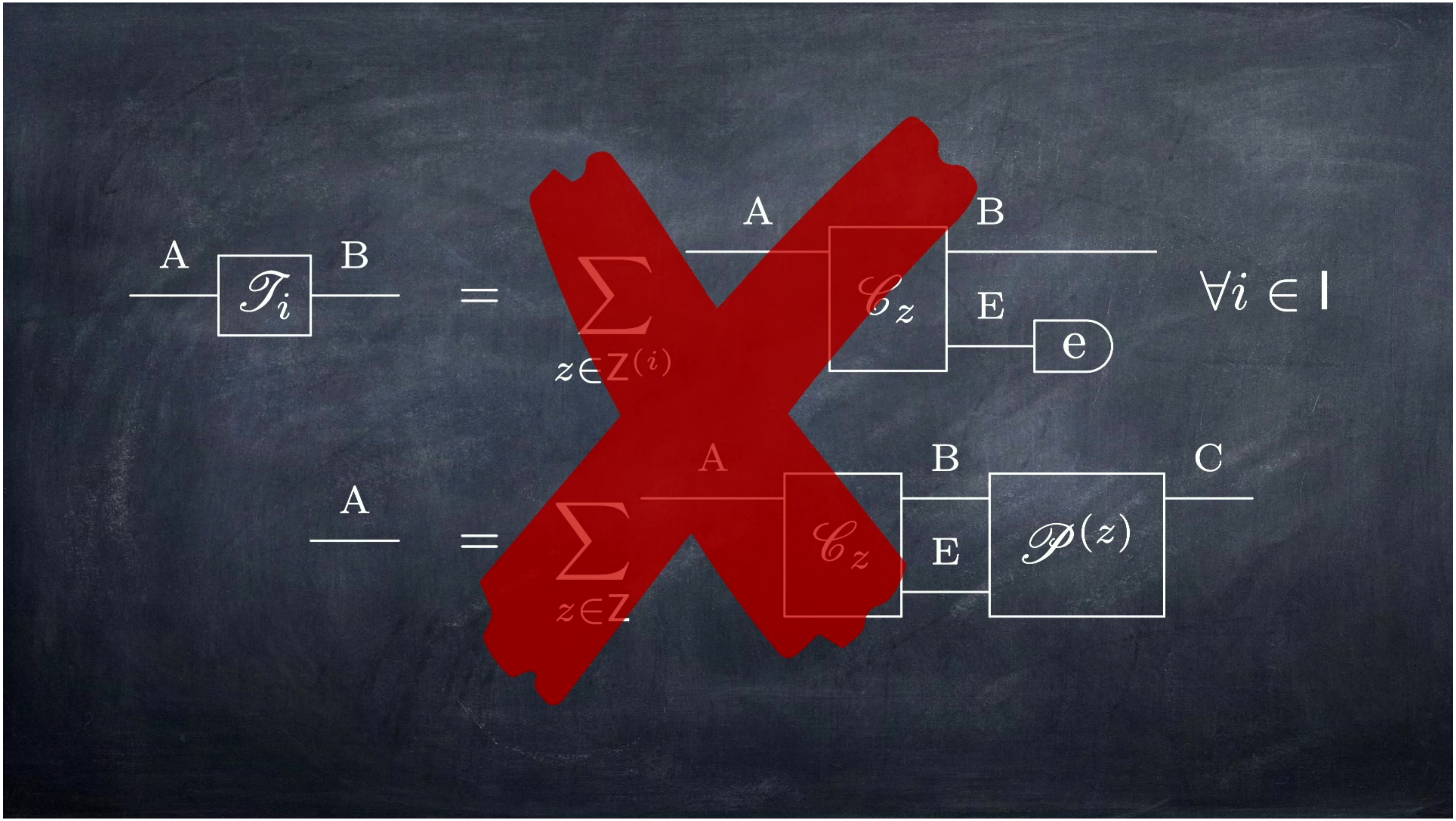


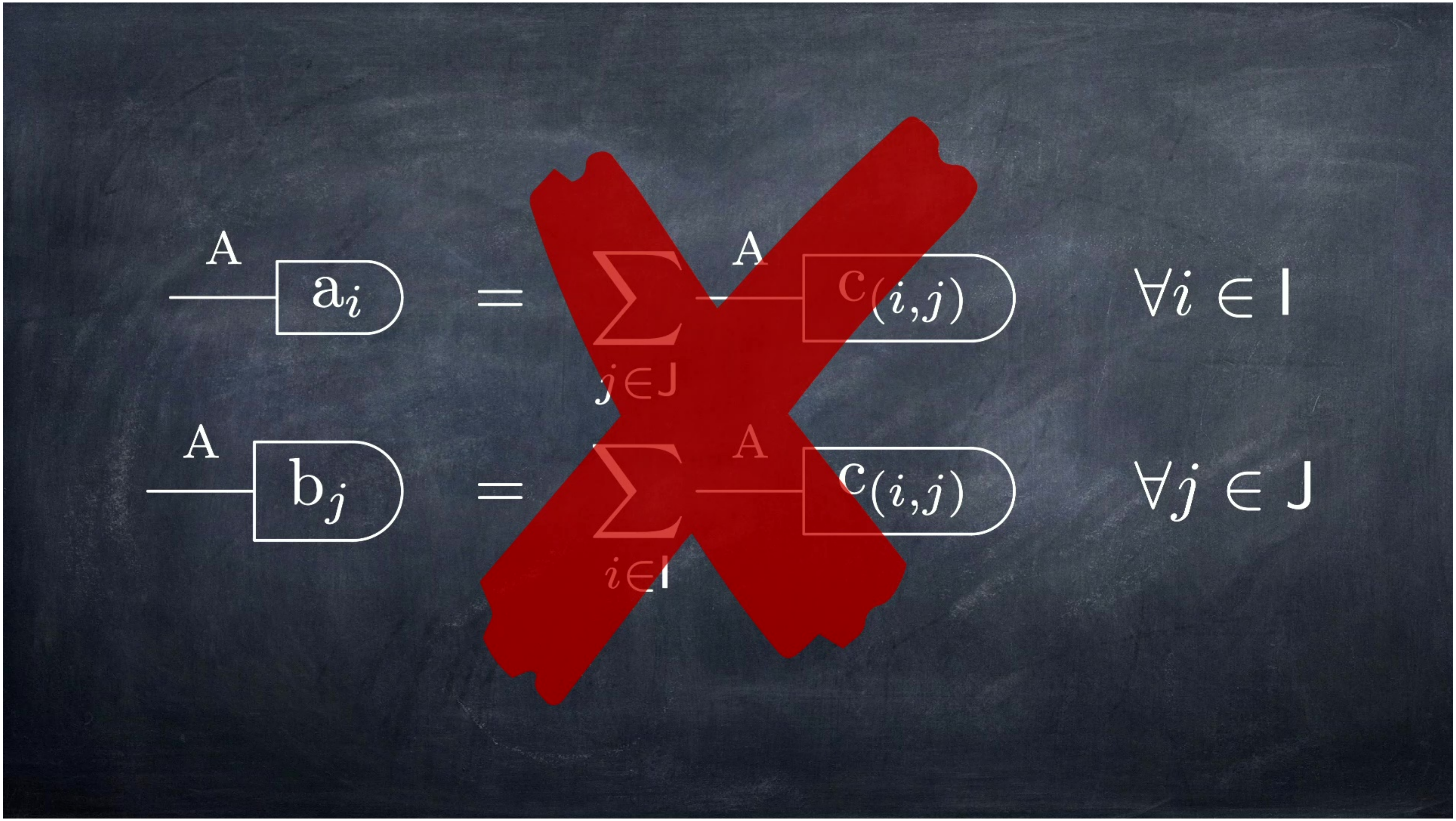
G. M. D'Ariano, P. Perinotti, and A. Tosini J. Phys. A: Math. Theor. 55 394006 (2022)

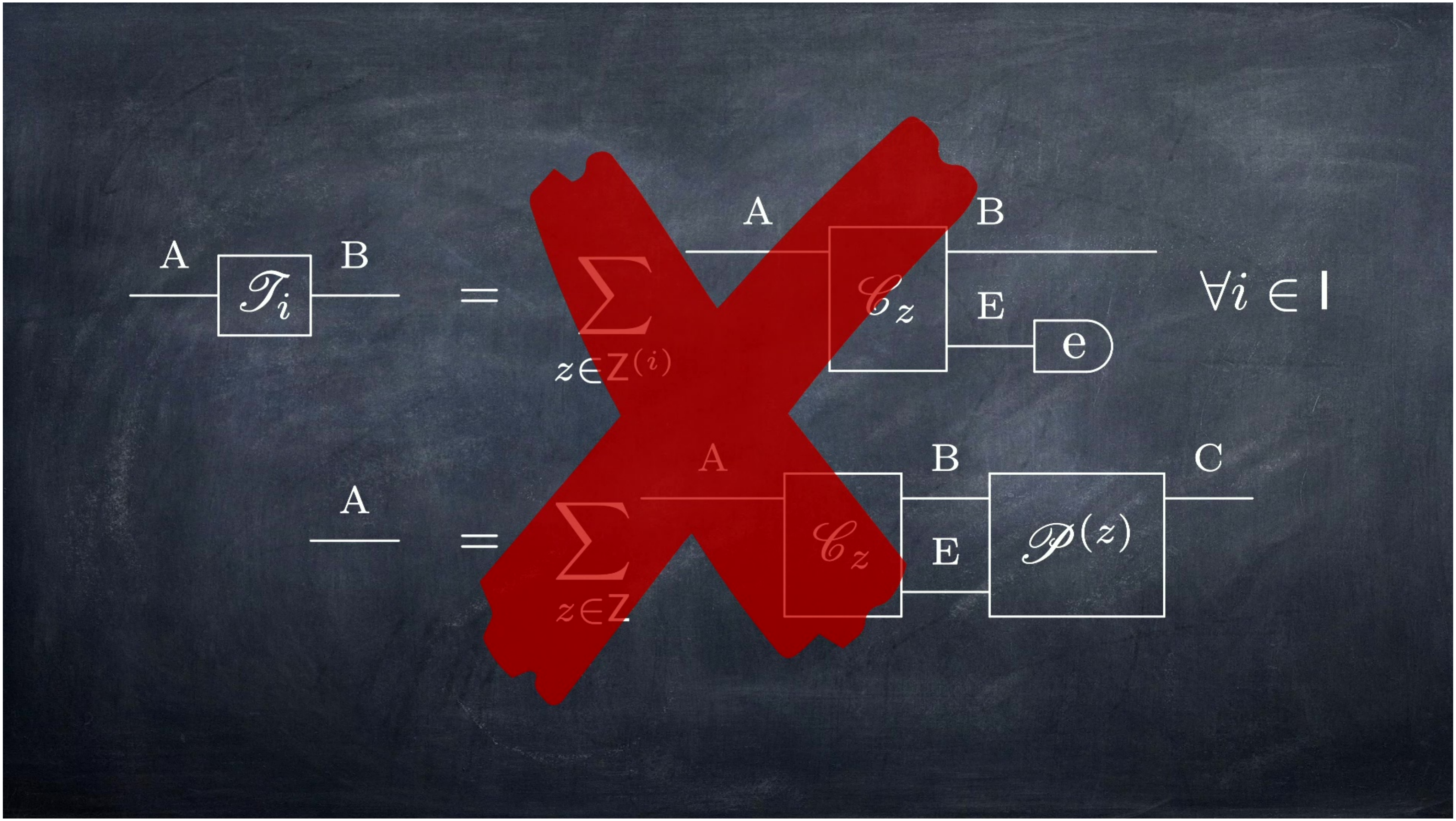
STRONG COMPATIBILITY

$\Downarrow$ $\quad$ ~~$\Uparrow$~~

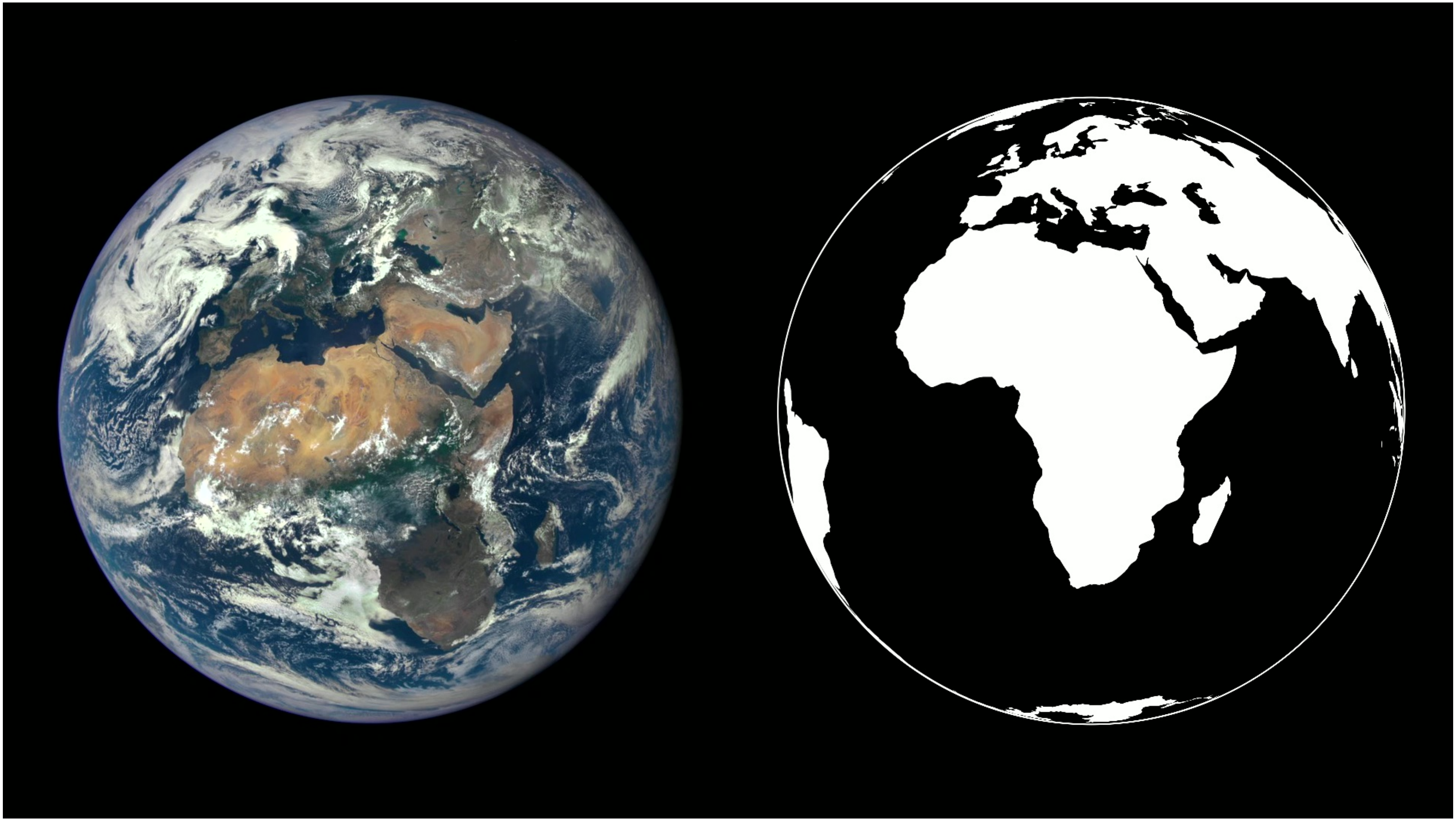
WEAK COMPATIBILITY

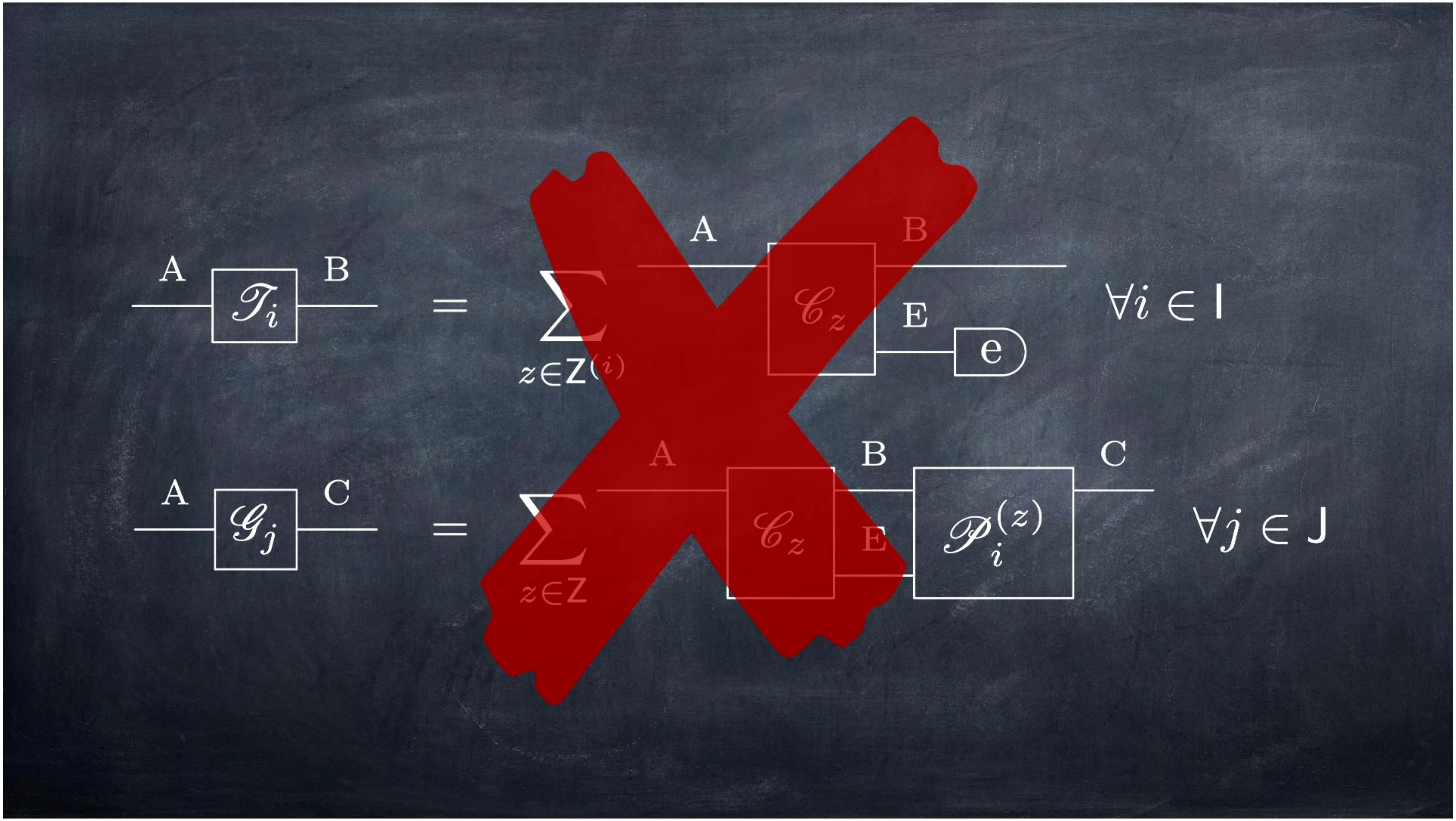

A
B
$\mathscr{T}_i$
=
$\sum_{z \in Z^{(i)}}$
A
B
$\mathscr{C}_z$
E
e
$\forall i \in I$
A
=
$\sum_{z \in Z}$
A
B
C
$\mathscr{C}_z$
E
$\mathscr{P}^{(z)}$


A
a_i
$=$
$\sum_{j \in J}$
A
$c_{(i,j)}$
$\forall i \in I$
A
b_j
$=$
$\sum_{i \in I}$
A
$c_{(i,j)}$
$\forall j \in J$

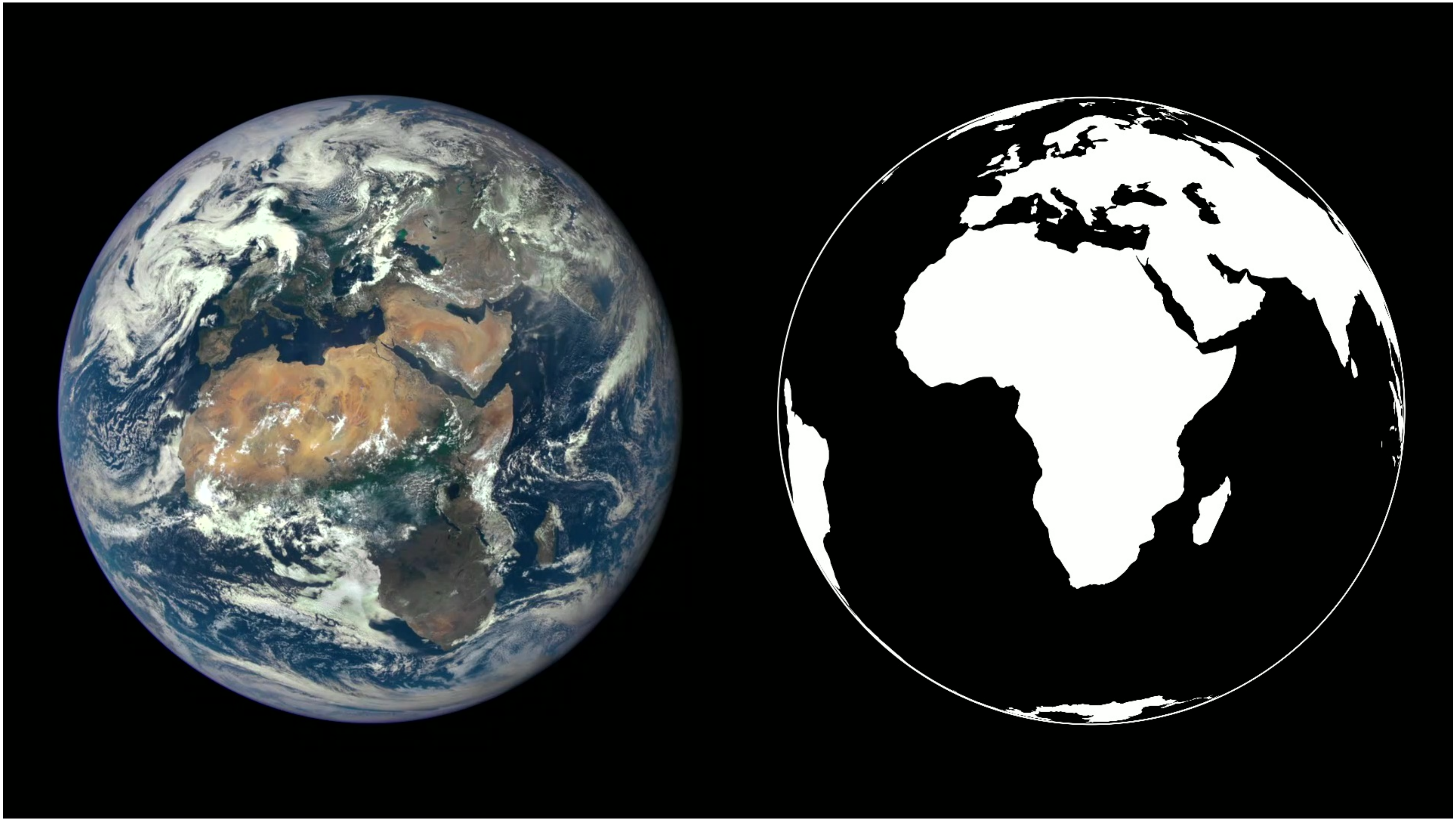


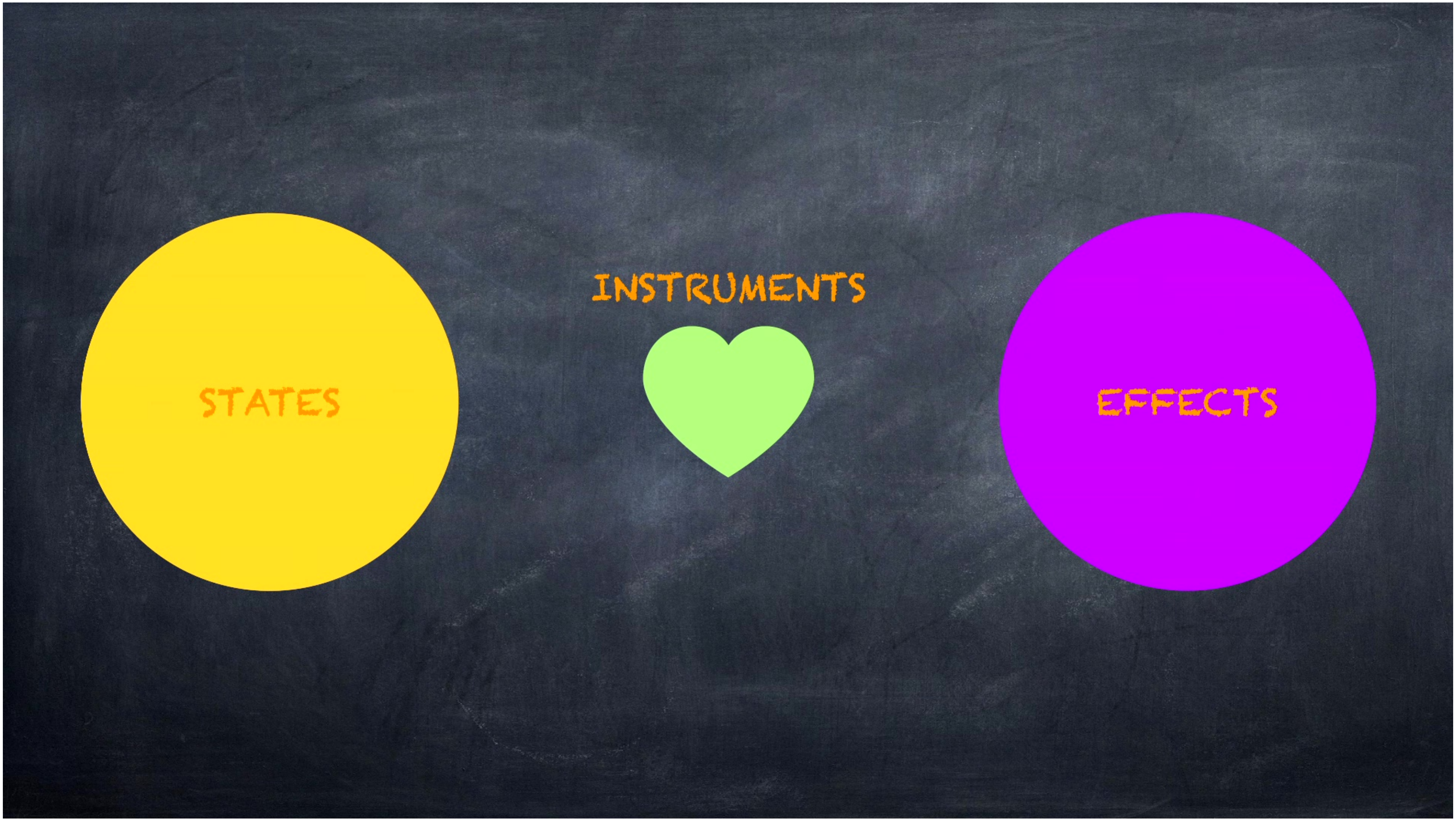
A
B
$\mathscr{T}_i$
=
$\sum_{z \in Z^{(i)}}$
A
B
$\mathscr{C}_z$
E
e
$\forall i \in I$
A
=
$\sum_{z \in Z}$
A
$\mathscr{C}_z$
B
E
$\mathscr{P}^{(z)}$
C



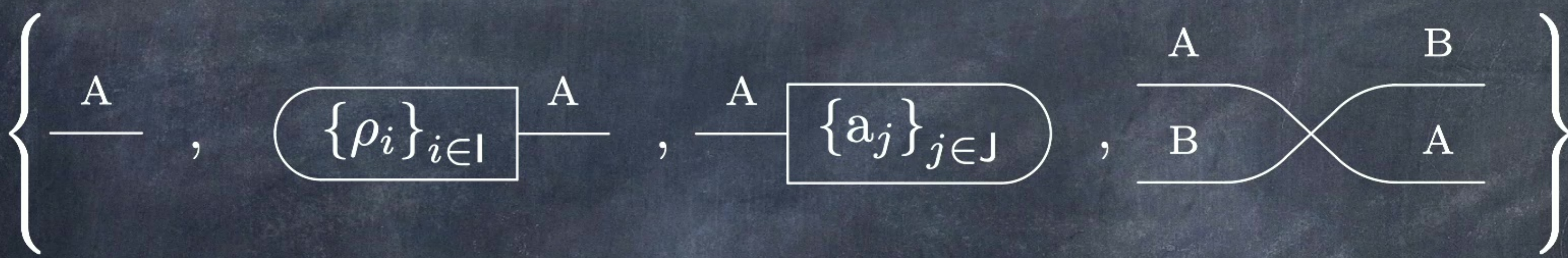


A
B
$\mathscr{T}_i$
$=$
$\sum_{z\in Z^{(i)}}$
A
B
$\mathscr{C}_z$
E
e
$\forall i \in I$
A
C
$\mathscr{G}_j$
$=$
$\sum_{z\in Z}$
A
B
C
$\mathscr{C}_z$
E
$\mathscr{P}_i^{(z)}$
$\forall j \in J$

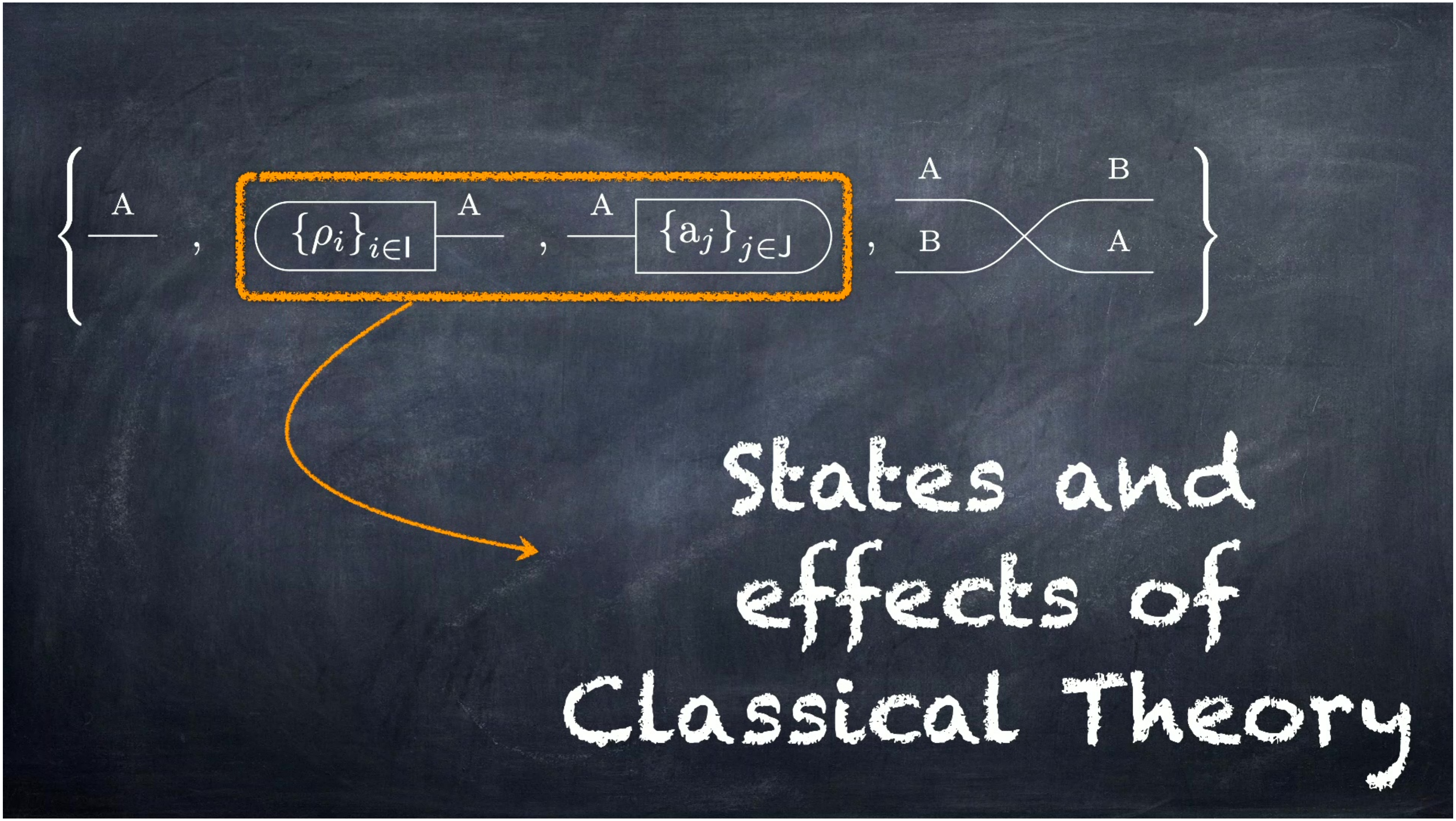


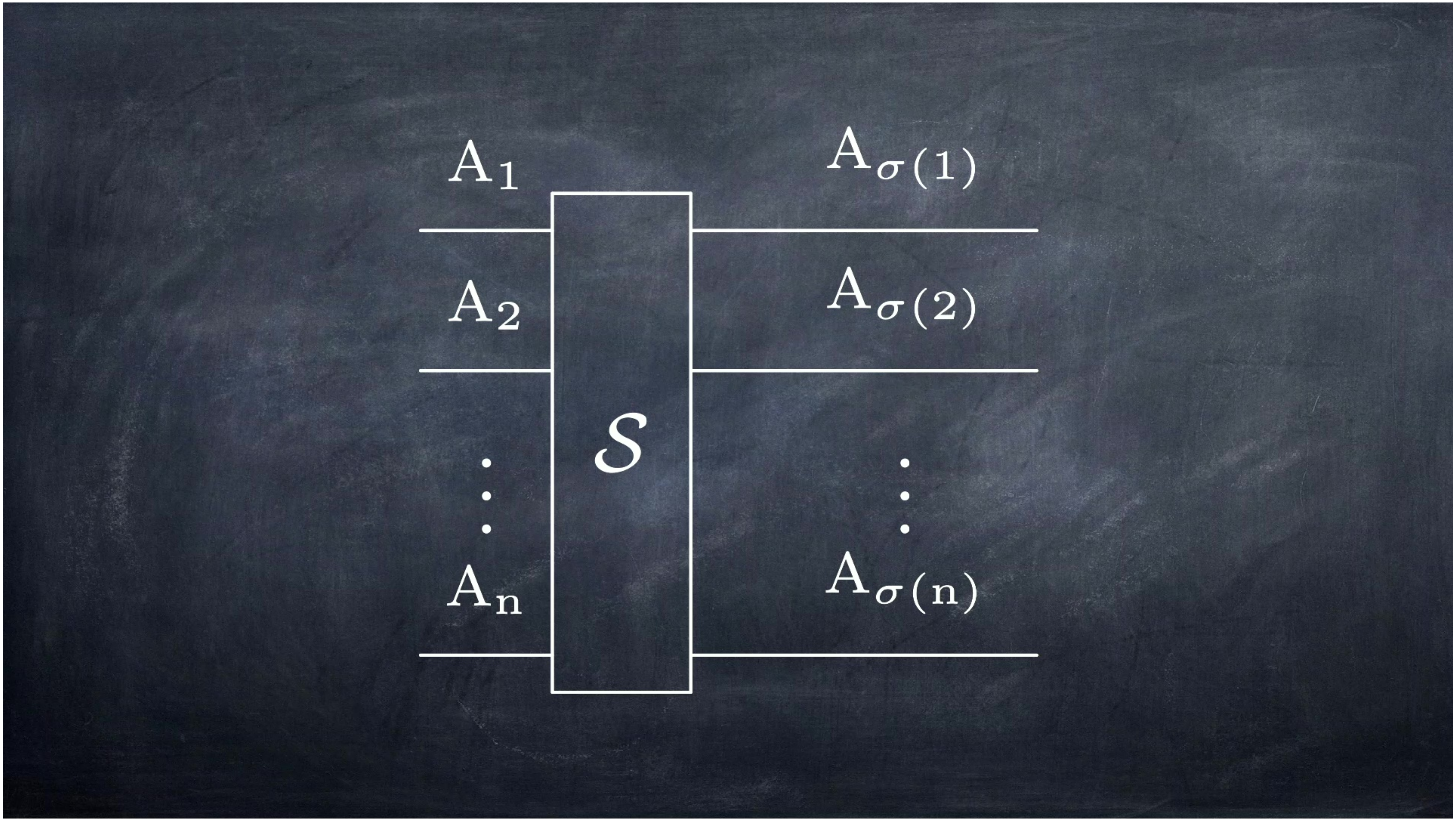

INSTRUMENTS
STATES
EFFECTS

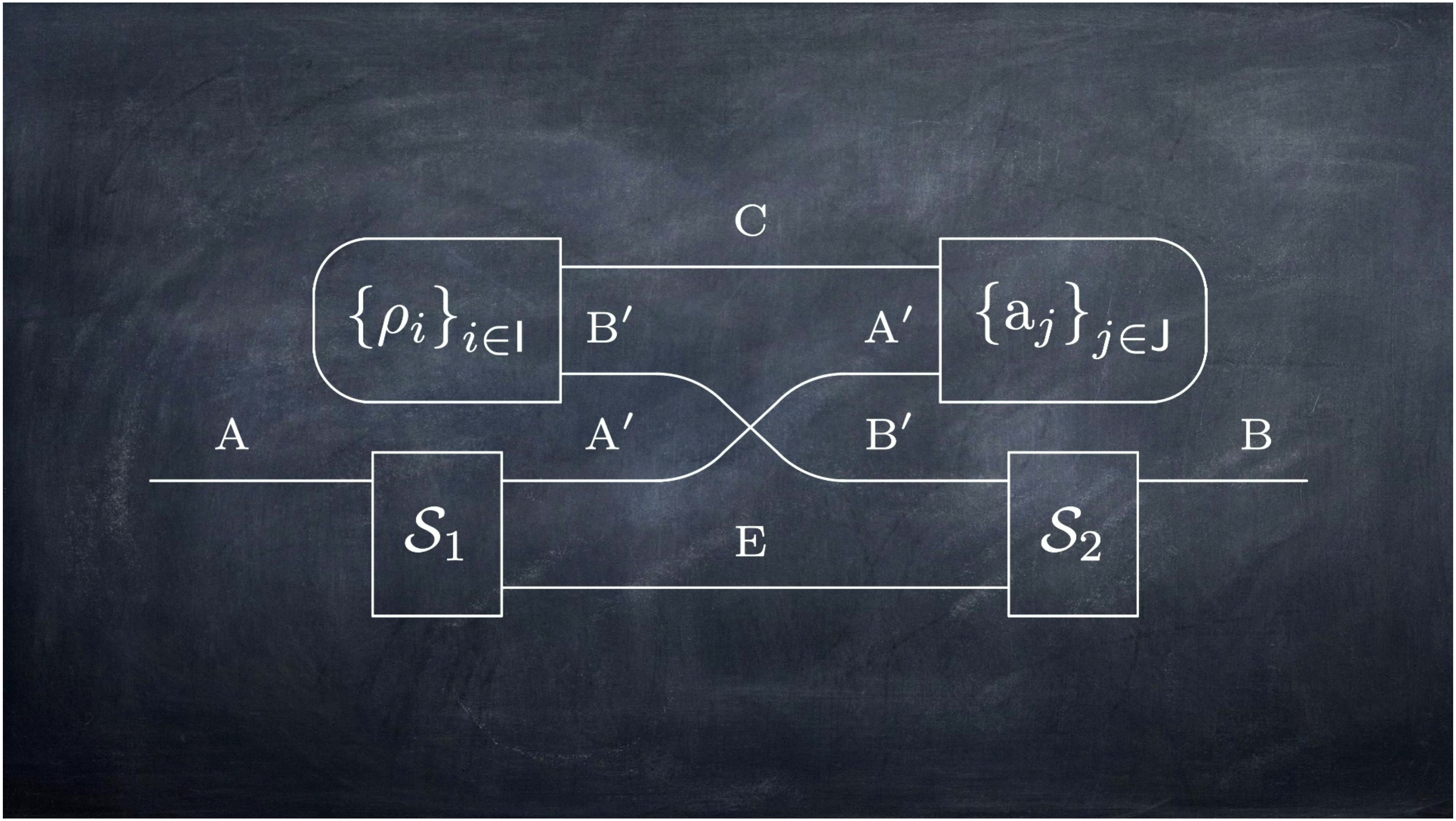
$$\left\{ \frac{A}{\quad} \right\}$$

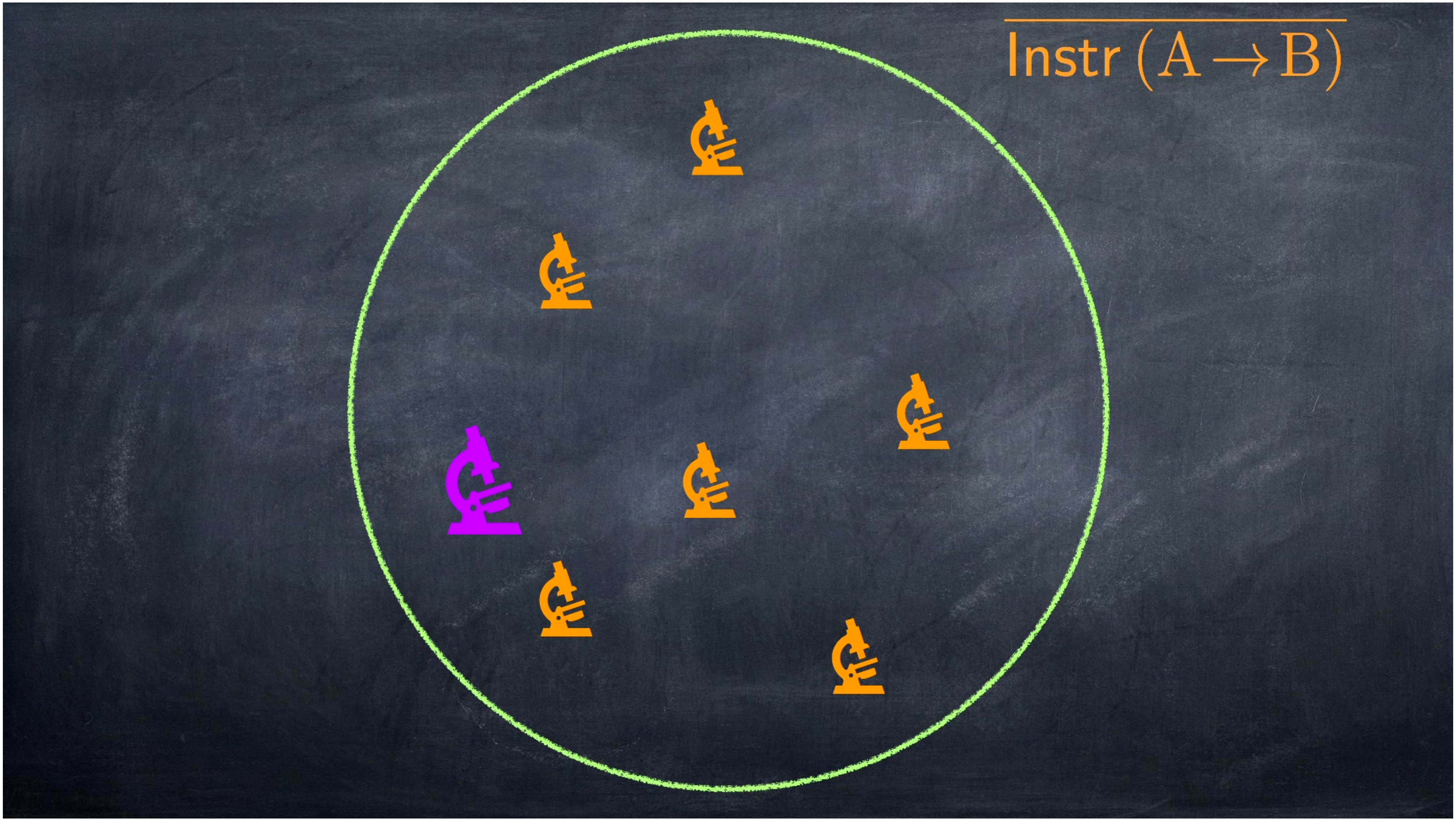

A
$\{\rho_i\}_{i\in \mathsf{I}}$
A
A
$\{\mathrm{a}_j\}_{j\in \mathsf{J}}$
A
B
B
A

IRREVERSIBLE DISTURBANCE


A
{ρi}i∈I
A
A
{aj}j∈J
A
B
B
A
States and effects of Classical Theory


A_1
A_2
A_n
$\mathcal{S}$
$A_{\sigma(1)}$
$A_{\sigma(2)}$
$A_{\sigma(n)}$

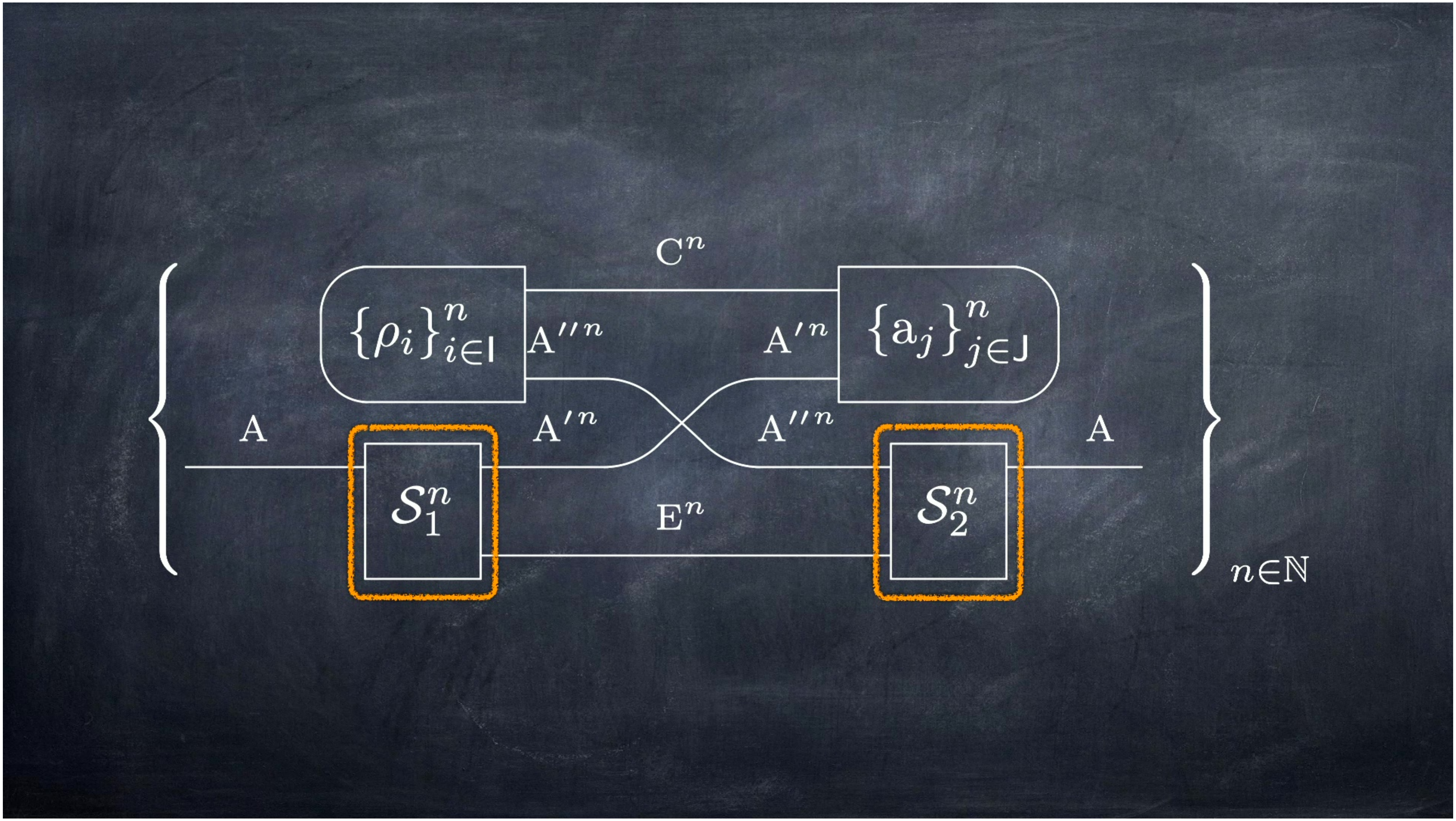

C
$\{\rho_i\}_{i\in I}$
B′
A′
$\{a_j\}_{j\in J}$
A
A′
B′
B
$\mathcal{S}_1$
E
$\mathcal{S}_2$

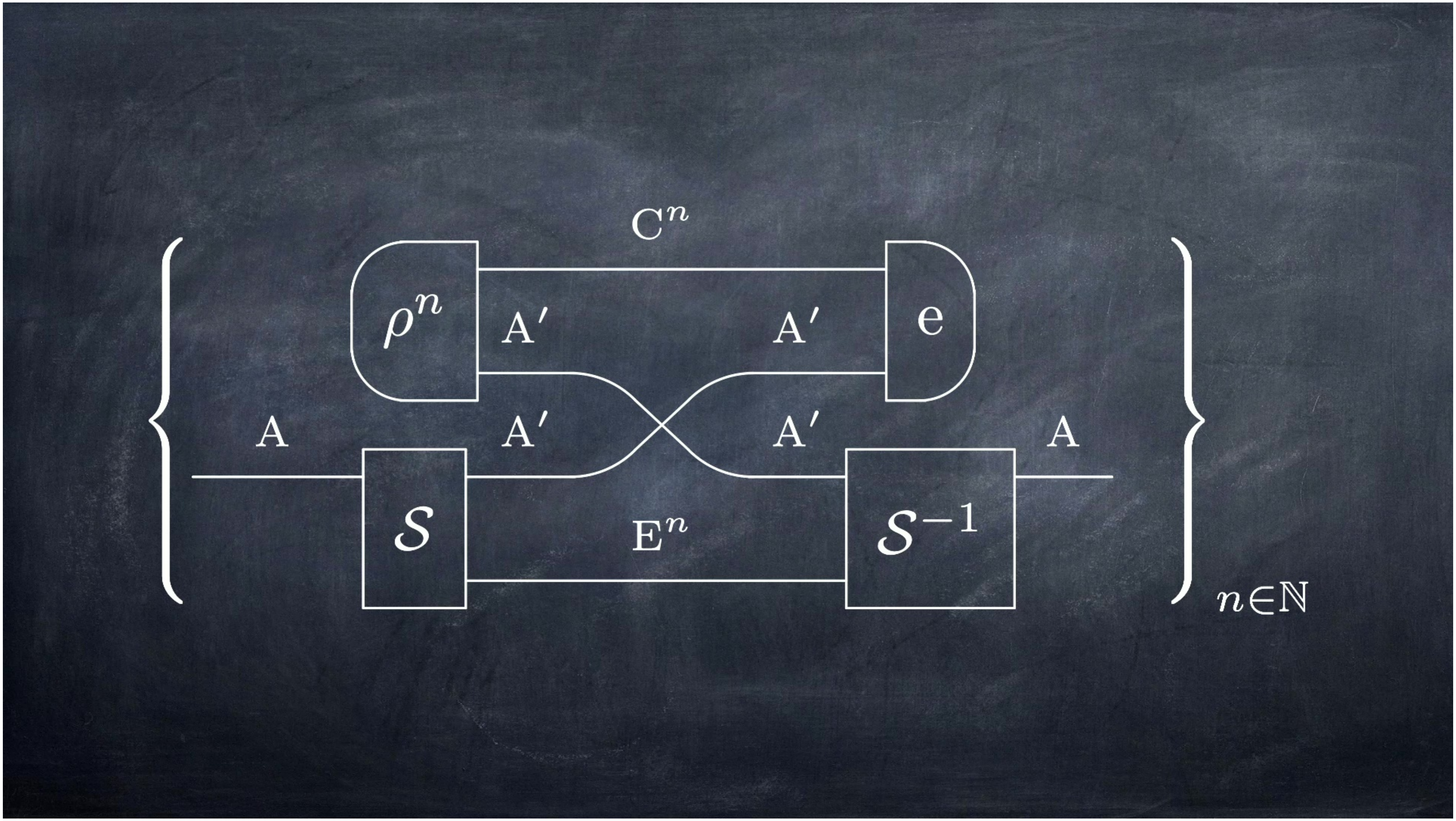


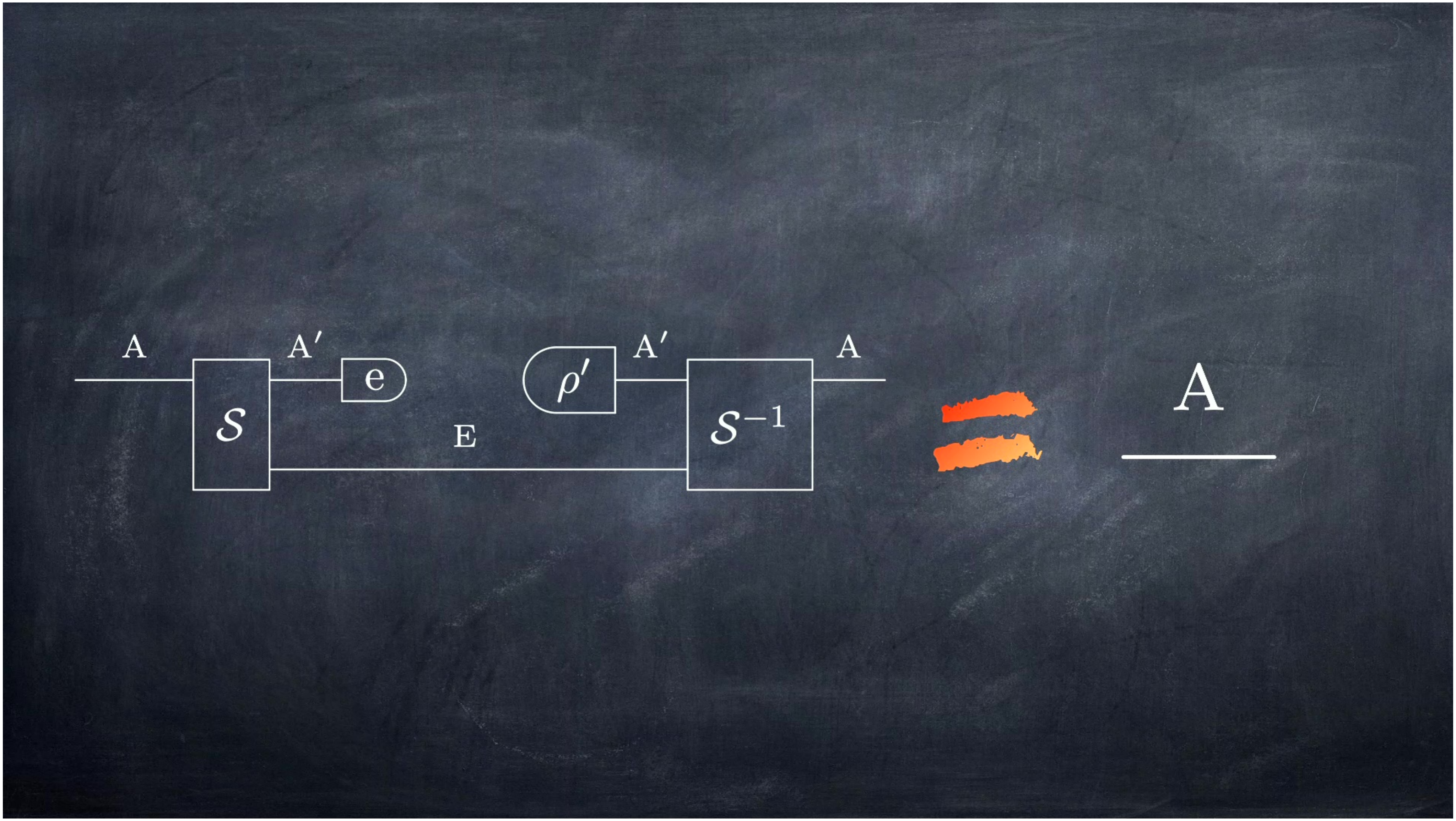
$\overline{\mathsf{Instr}\,(A \to B)}$

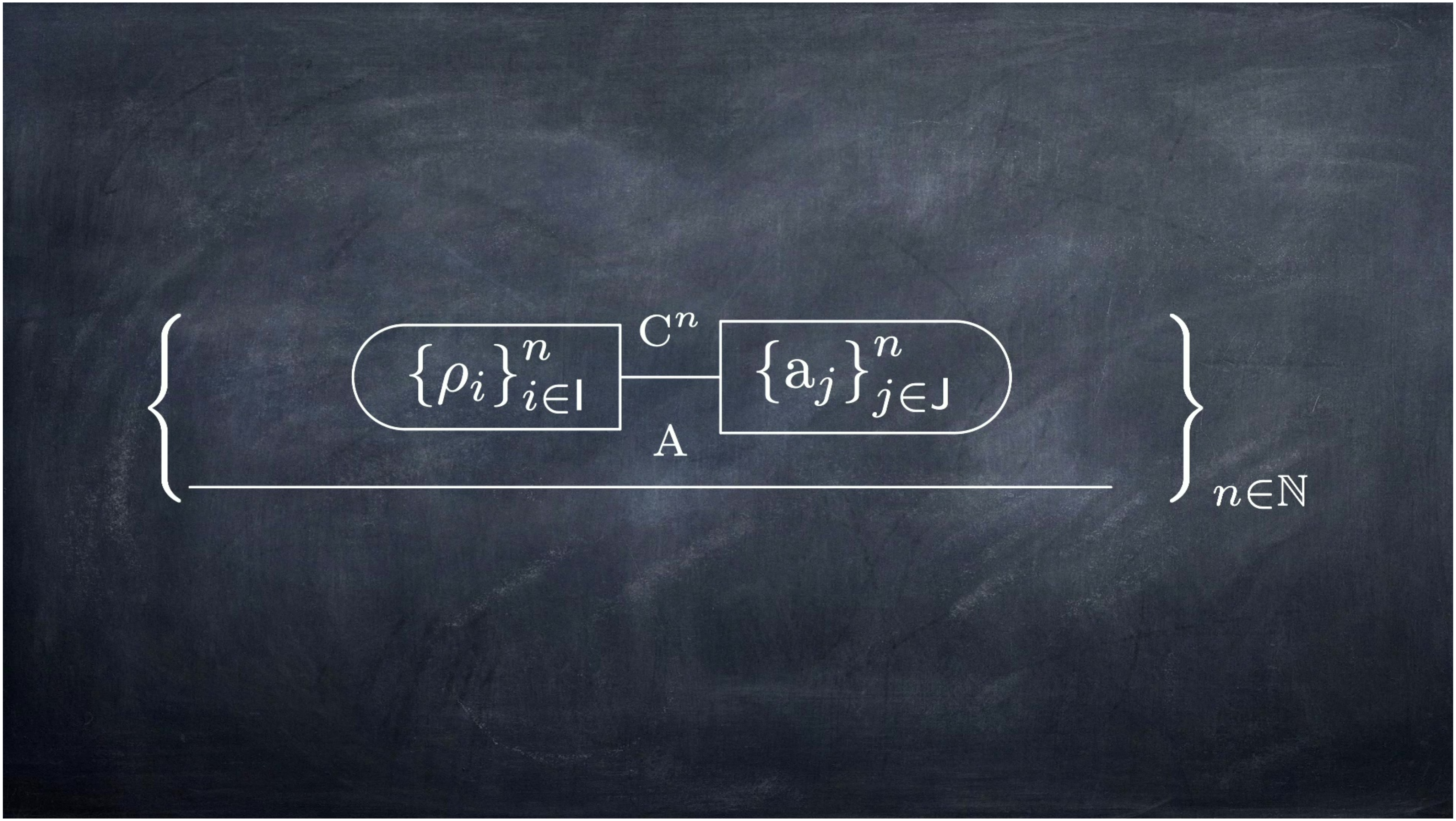
ATOMIC

$$\mathscr{G}_j \propto \mathscr{T} \quad \forall j \in \mathsf{J}'$$


C^n
$\{\rho_i\}^n_{i\in I}$
A''^n
A'^n
$\{a_j\}^n_{j\in J}$
A
A'^n
A''^n
A
$\mathcal{S}^n_1$
E^n
$\mathcal{S}^n_2$
$n\in\mathbb{N}$


C^n
ρ^n
A'
A'
e
A
A'
A'
A
$\mathcal{S}$
E^n
$\mathcal{S}^{-1}$
$n \in \mathbb{N}$

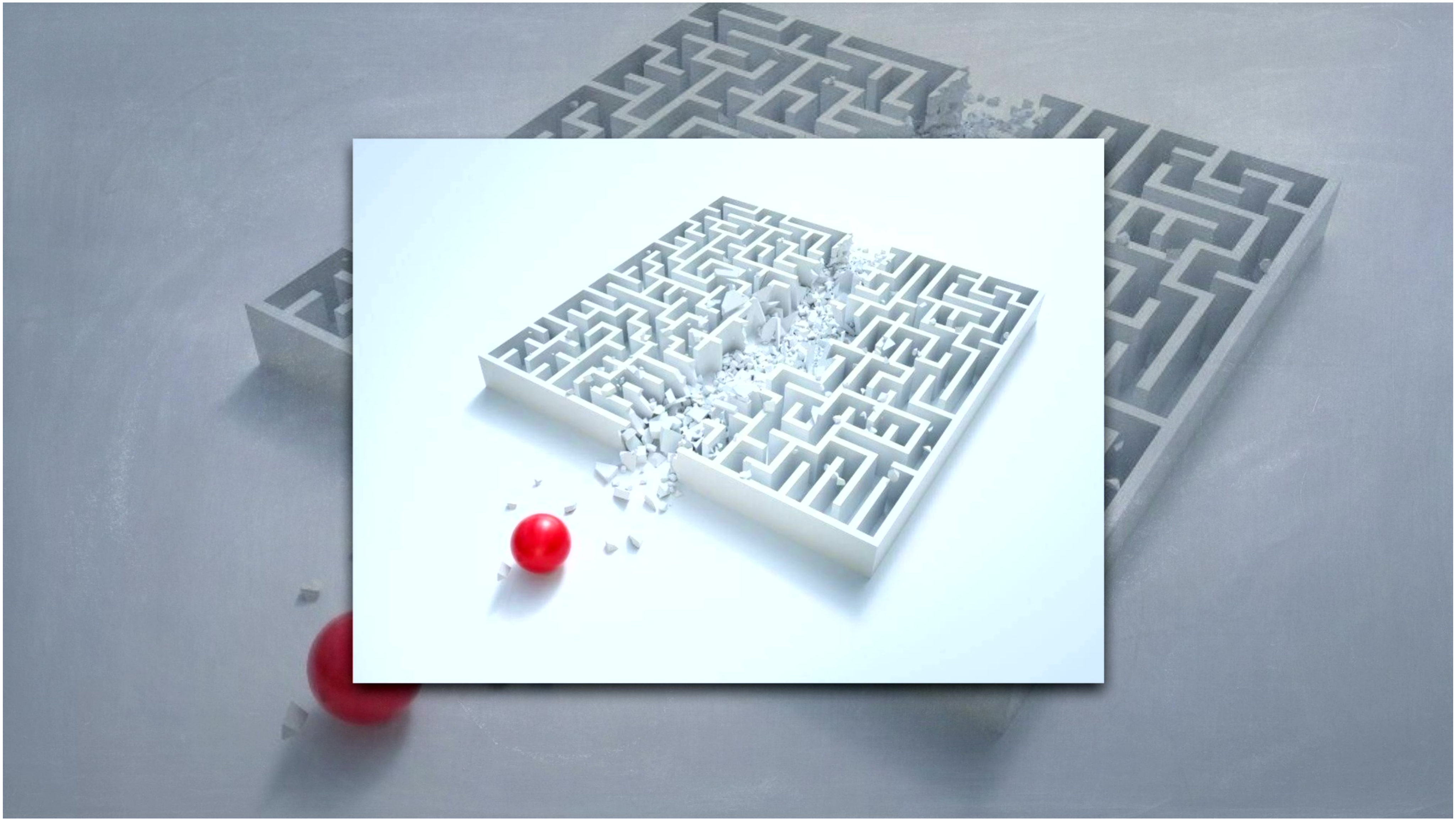

A
A′
e
E
ρ′
A′
A
$\mathcal{S}$
$\mathcal{S}^{-1}$
A


$\{\rho_i\}^n_{i\in \mathsf{I}}$
C^n
$\{\mathrm{a}_j\}^n_{j\in \mathsf{J}}$
A
$n\in\mathbb{N}$

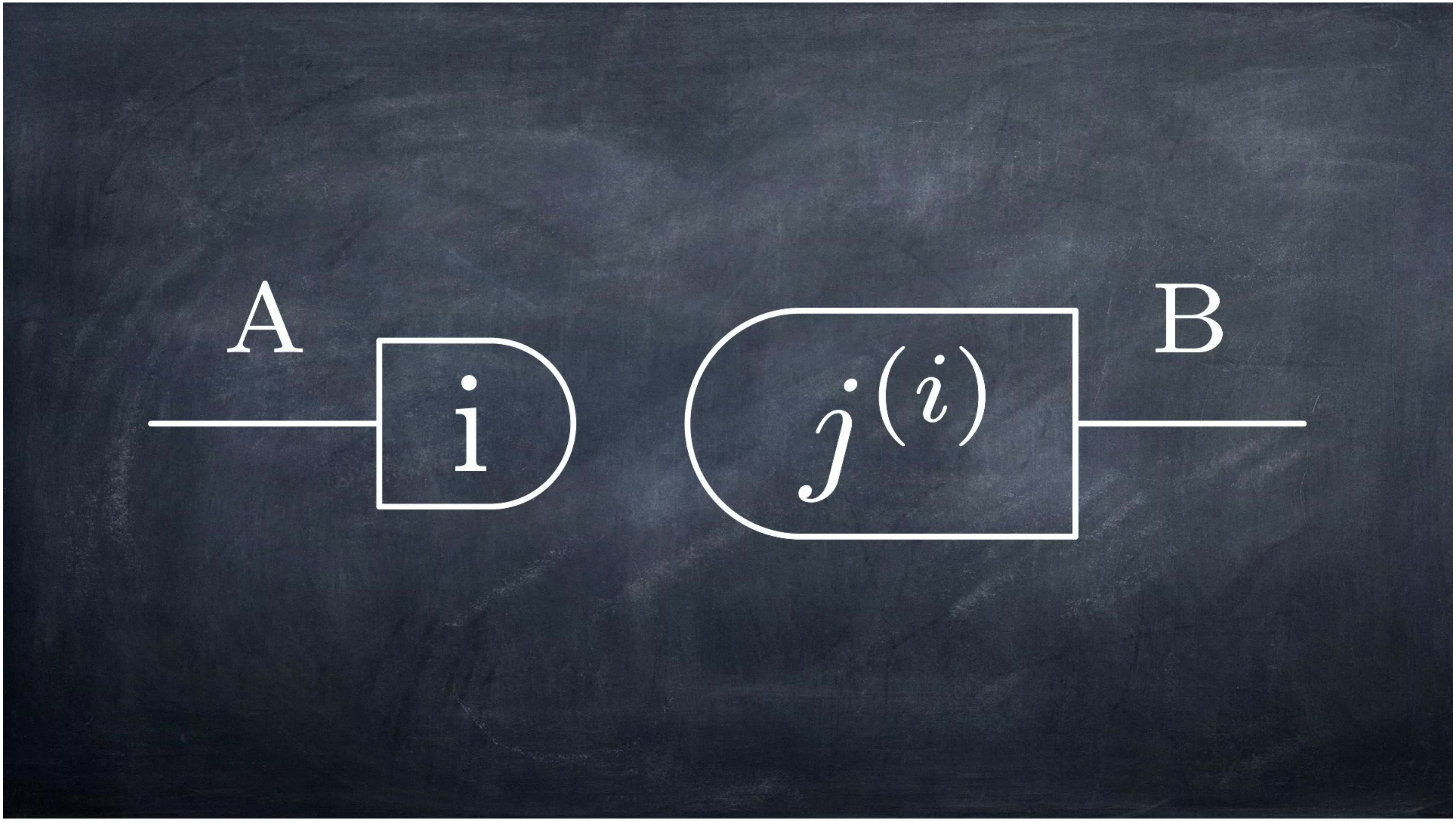
IRREVERSIBLE DISTURBANCE

$\not\Downarrow$

MEASUREMENT INCOMPATIBILITY



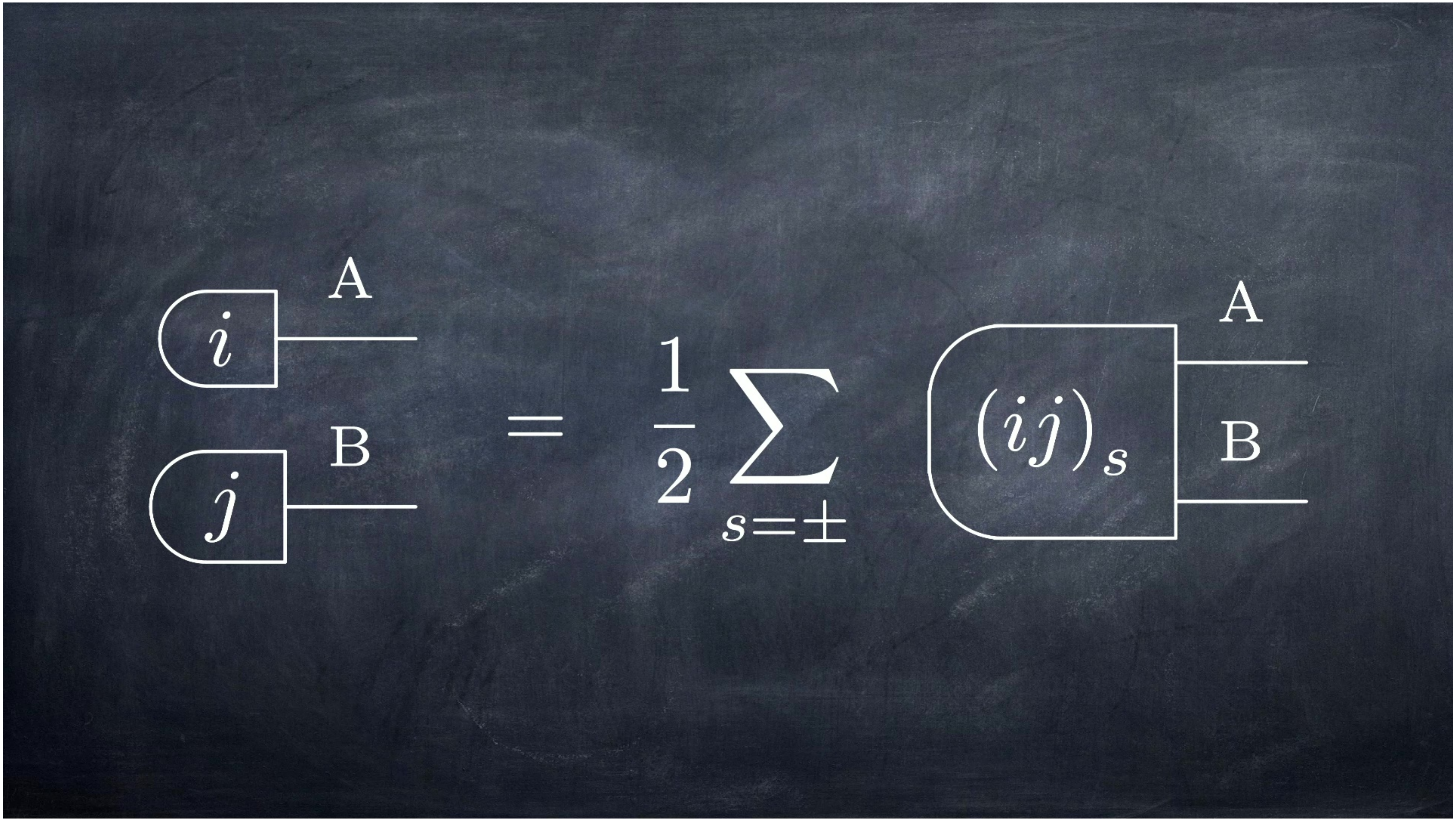
MINIMAL CLASSICAL THEORY

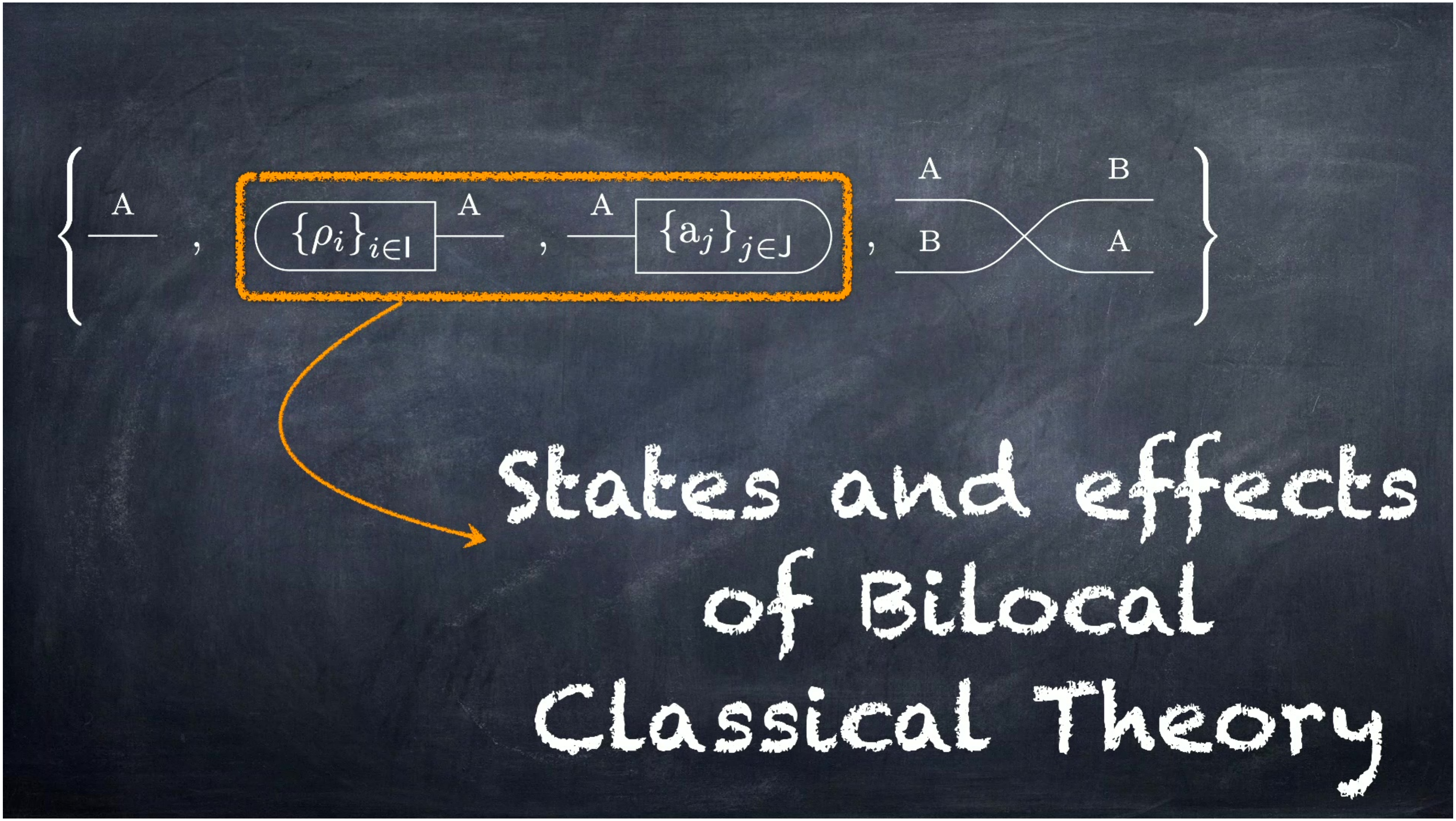

A
i
$j^{(i)}$
B

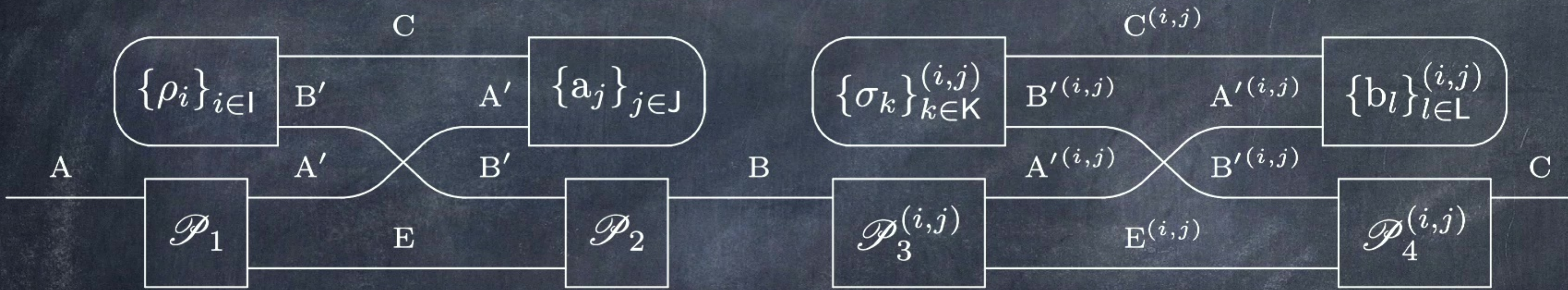
~~MINIMAL~~

CLASSICAL THEORY

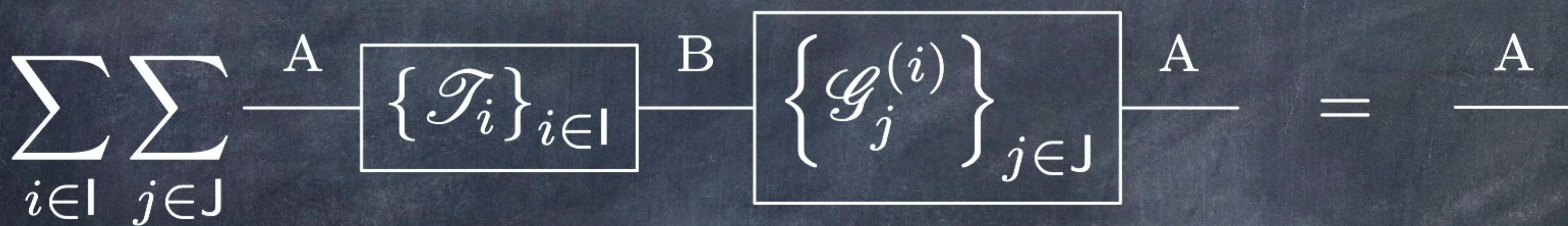
A

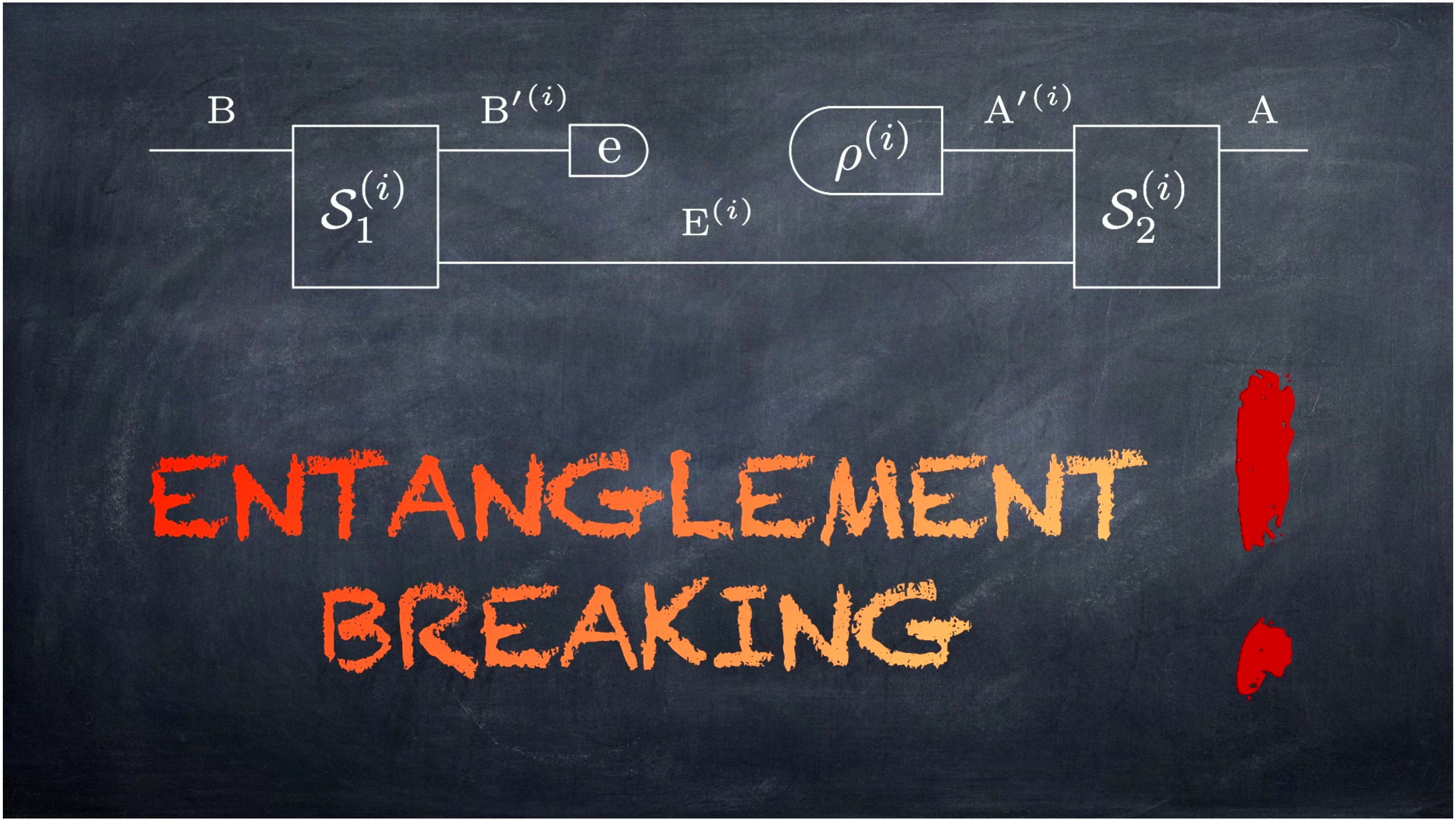

i
A
j
B
=
$\frac{1}{2}\sum_{s=\pm}$
$(ij)_s$
A
B

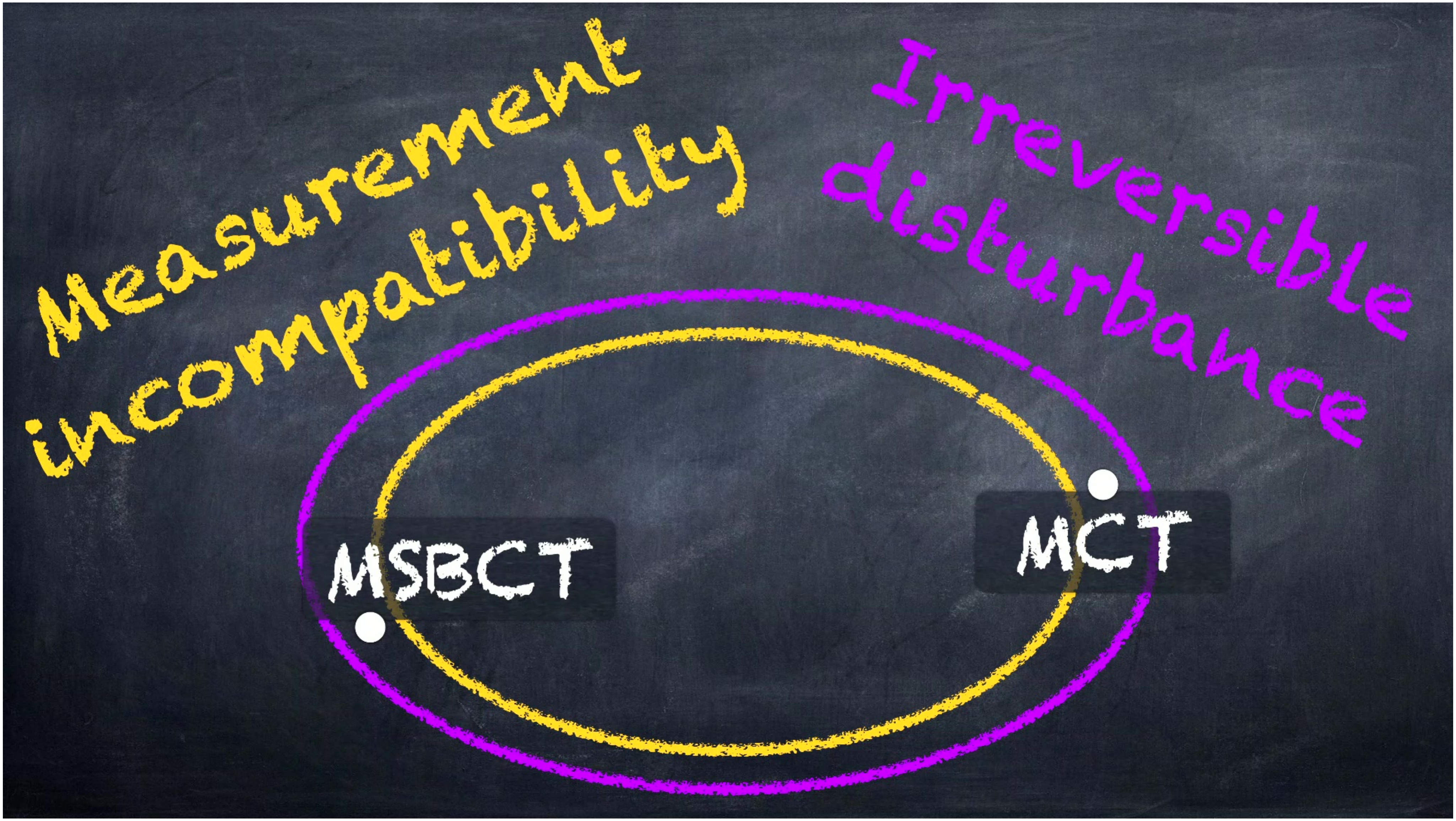

A
A
A
A
B
B
A
$\{\rho_i\}_{i\in I}$
$\{a_j\}_{j\in J}$
States and effects
of Bilocal
Classical Theory



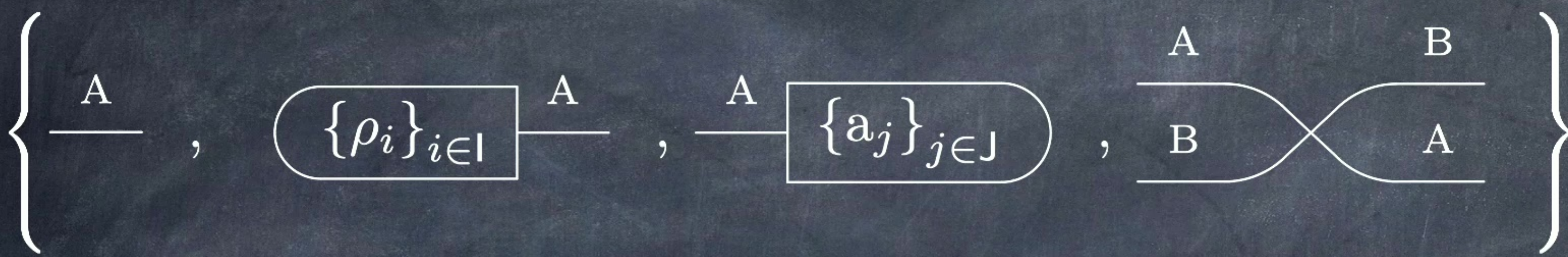
C
$\{\rho_i\}_{i\in I}$
B′
A′
$\{a_j\}_{j\in J}$
A
A′
B′
B
$\mathscr{P}_1$
E
$\mathscr{P}_2$
$C^{(i,j)}$
$\{\sigma_k\}_{k\in K}^{(i,j)}$
$B'^{(i,j)}$
$A'^{(i,j)}$
$\{b_l\}_{l\in L}^{(i,j)}$
$A'^{(i,j)}$
$B'^{(i,j)}$
C
$\mathscr{P}_3^{(i,j)}$
$E^{(i,j)}$
$\mathscr{P}_4^{(i,j)}$


$\sum_{i\in I}\sum_{j\in J}$
A
$\{\mathscr{T}_i\}_{i\in I}$
B
$\left\{\mathscr{G}_j^{(i)}\right\}_{j\in J}$
A
=
A


B
$B'^{(i)}$
e
$\rho^{(i)}$
$A'^{(i)}$
A
$\mathcal{S}_1^{(i)}$
$E^{(i)}$
$\mathcal{S}_2^{(i)}$
ENTANGLEMENT
BREAKING
!


Measurement incompatibility
Irreversible disturbance
MSBCT
MCT

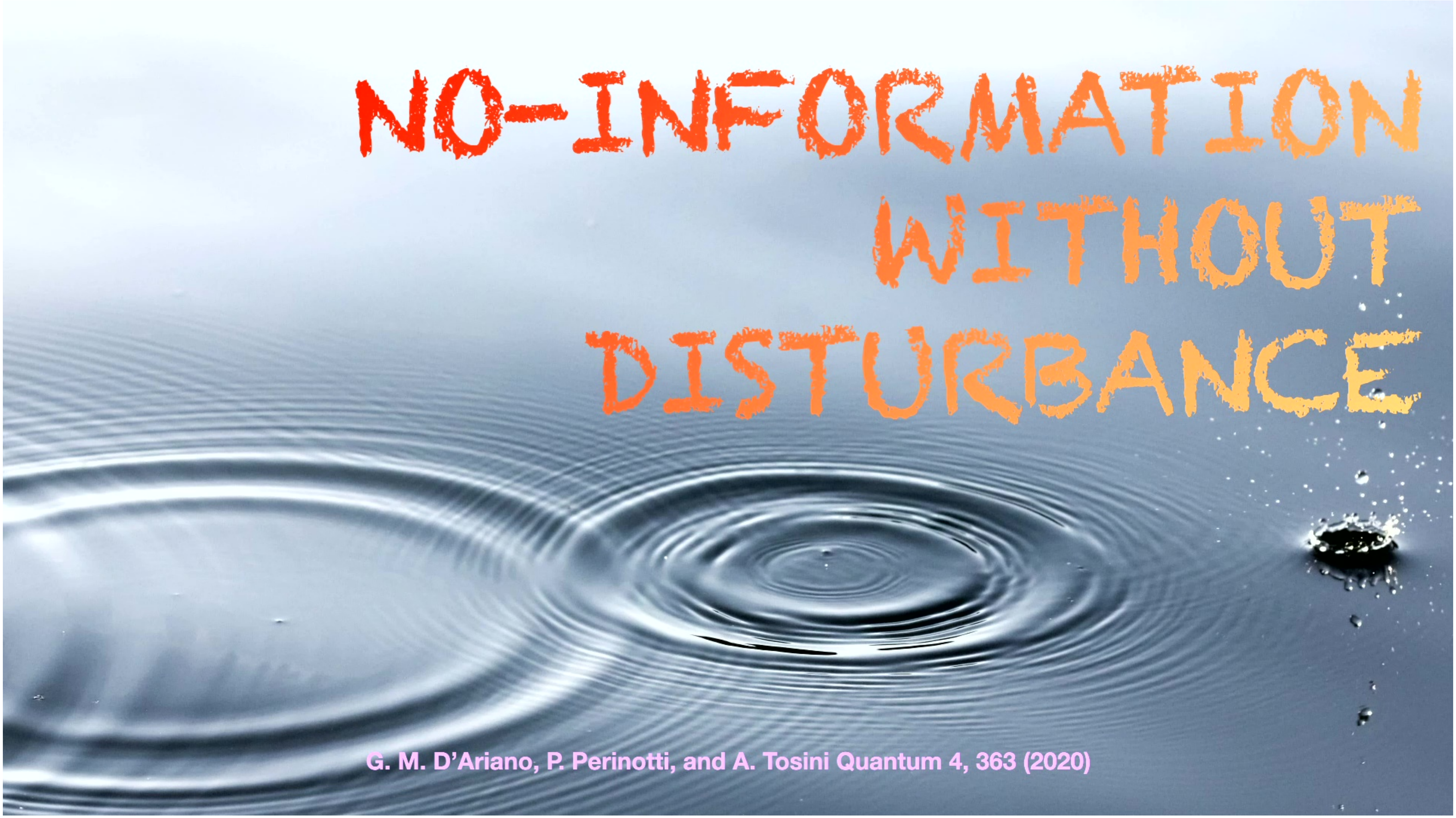
GENERALISE?



+

Classical conditioning


where we exploited ?? to exchange the operations of
and coarse-graining.
Let now ρ be an atomic entangled state of
By definition of atomic state, then it must hold that any
or equivalently
to remove the dependence from the index n on the
transformation in our composition
This proves our result.


NO-INFORMATION WITHOUT DISTURBANCE
G. M. D'Ariano, P. Perinotti, and A. Tosini Quantum 4, 363 (2020)

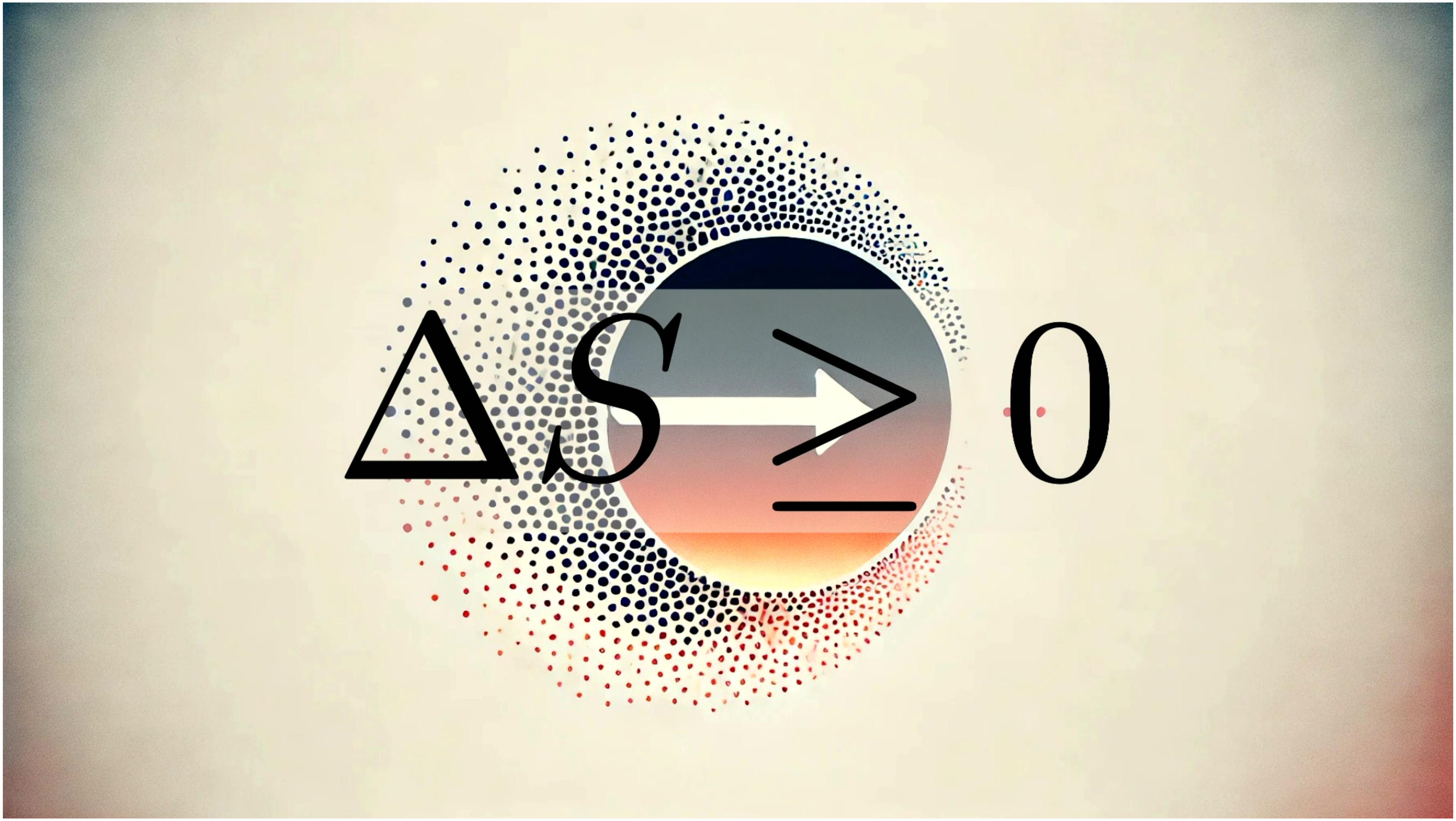

What's
Next?

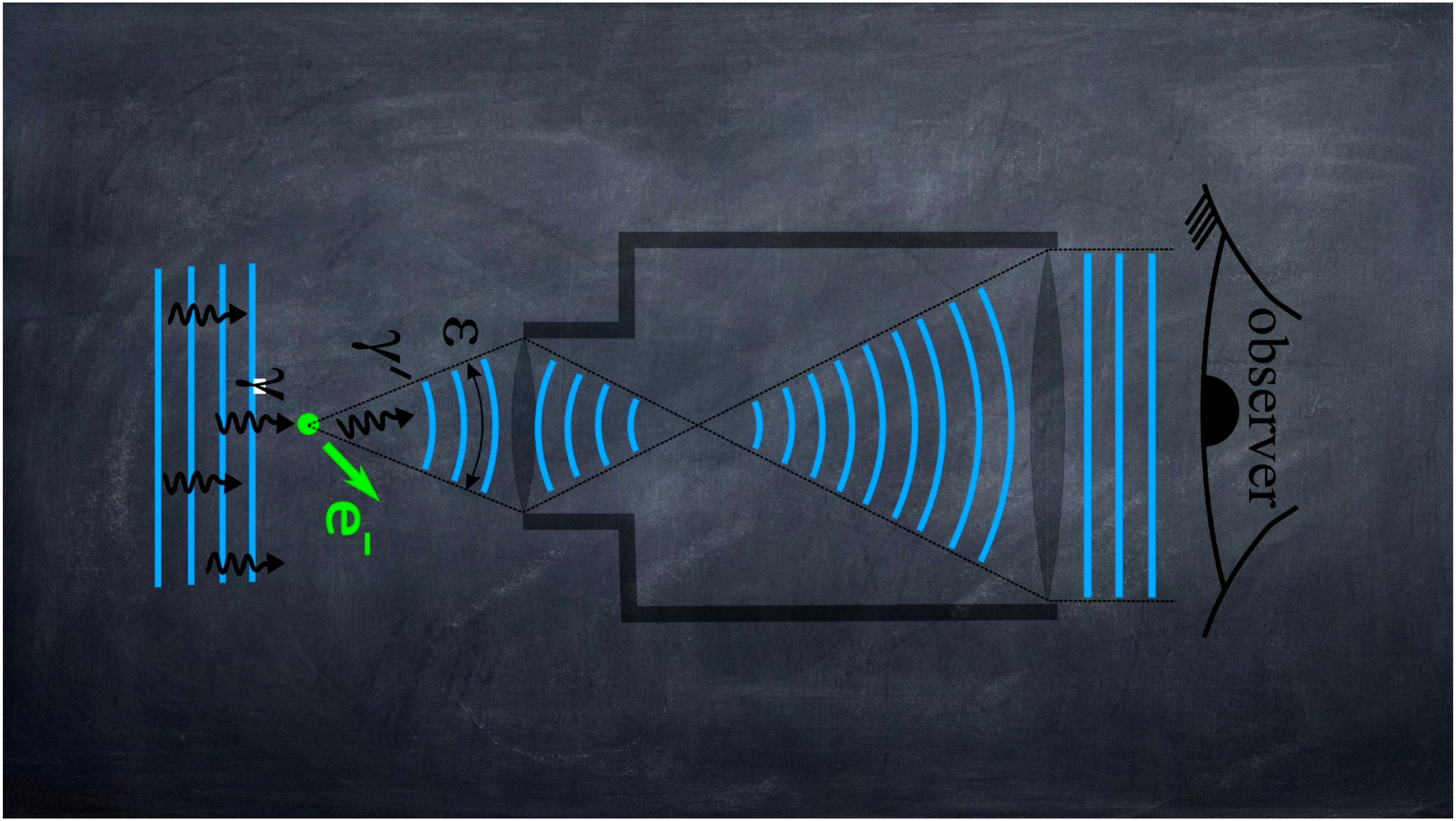
MINIMAL STRONGLY CAUSAL OPTs (MSOPTs)

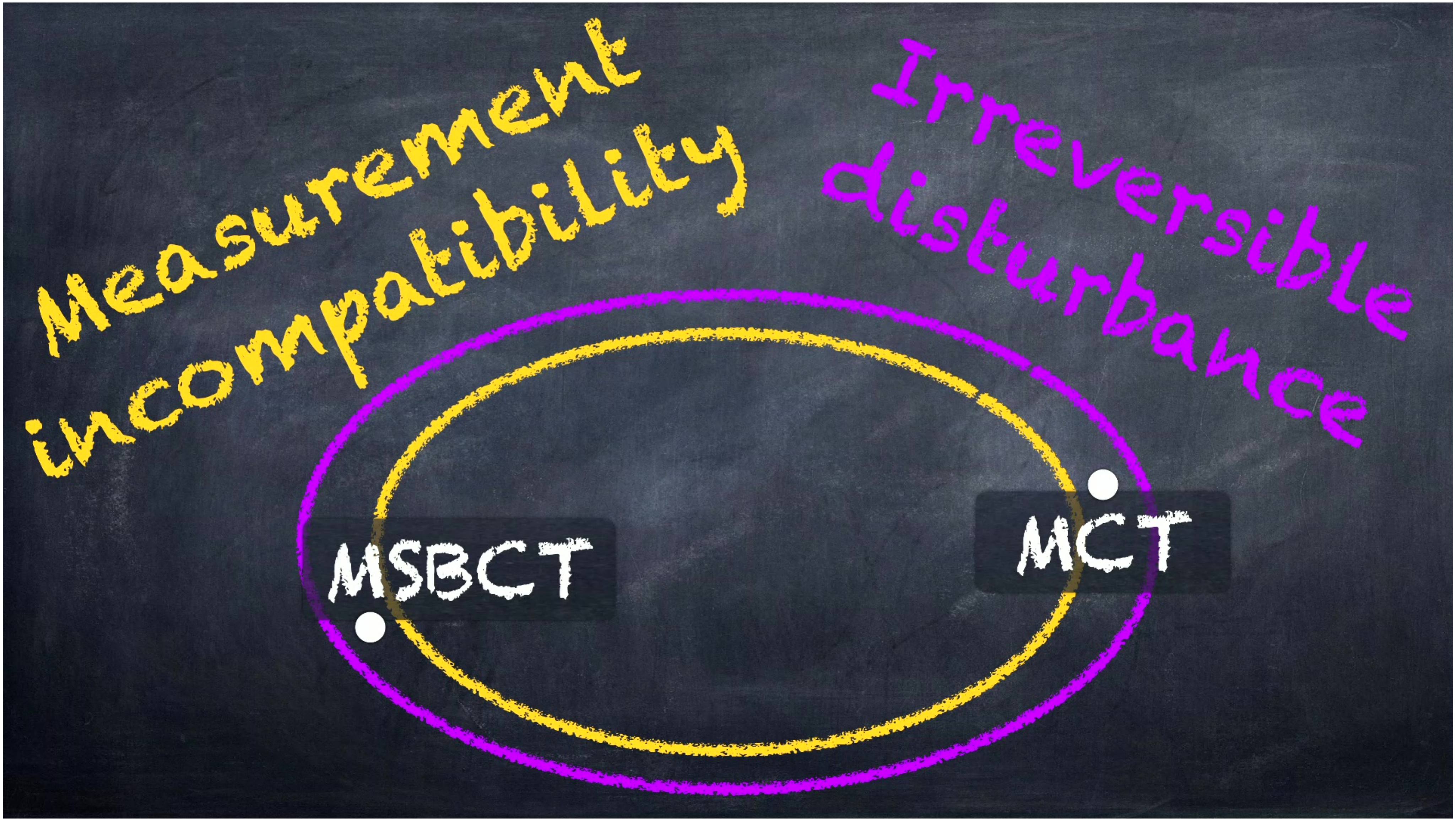
M. Erba, P. Perinotti, DR, and A. Tosini, Phys. Rev. A 109, 022239 (2024)

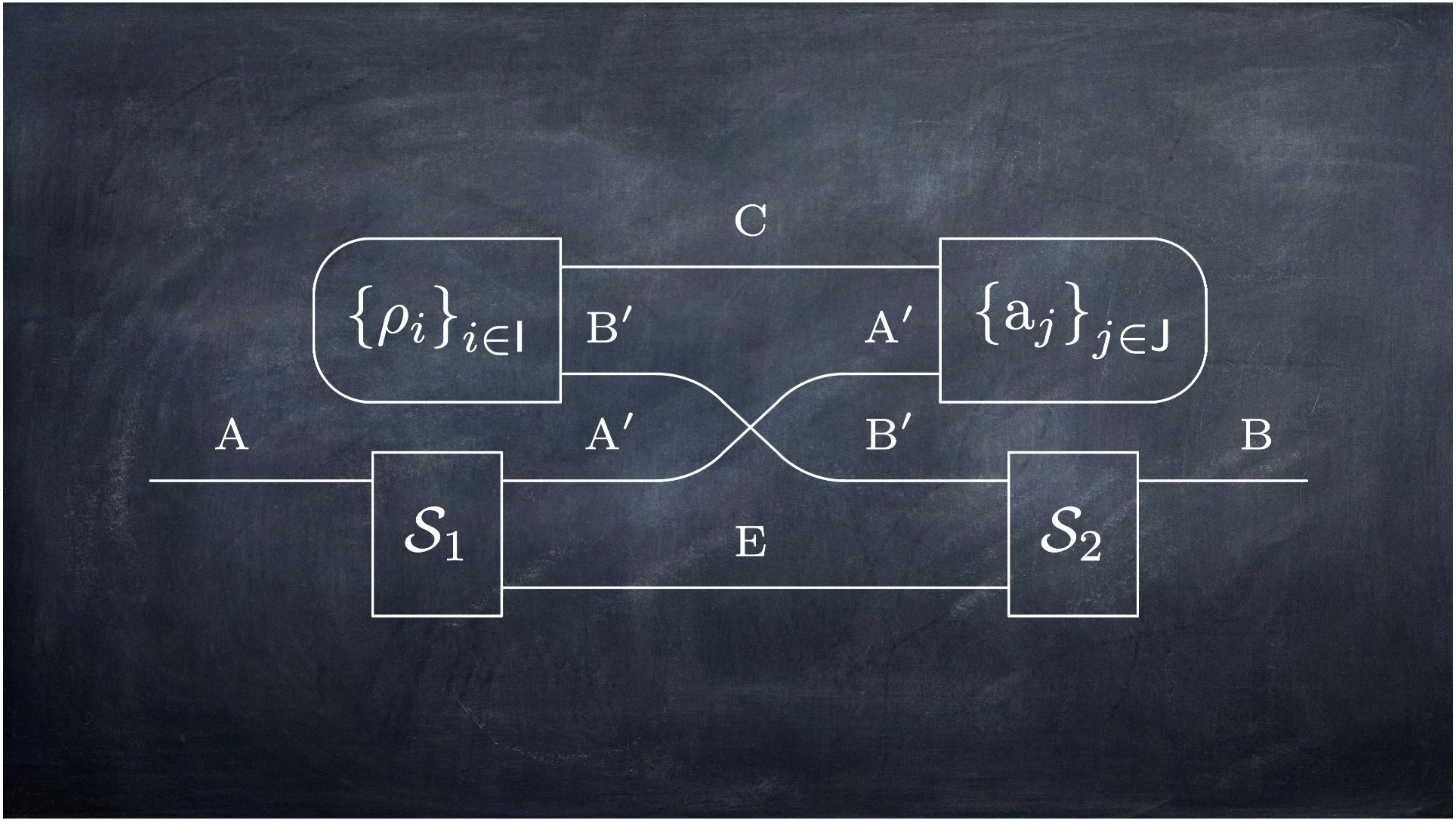

Measurement incompatibility
Complementarity

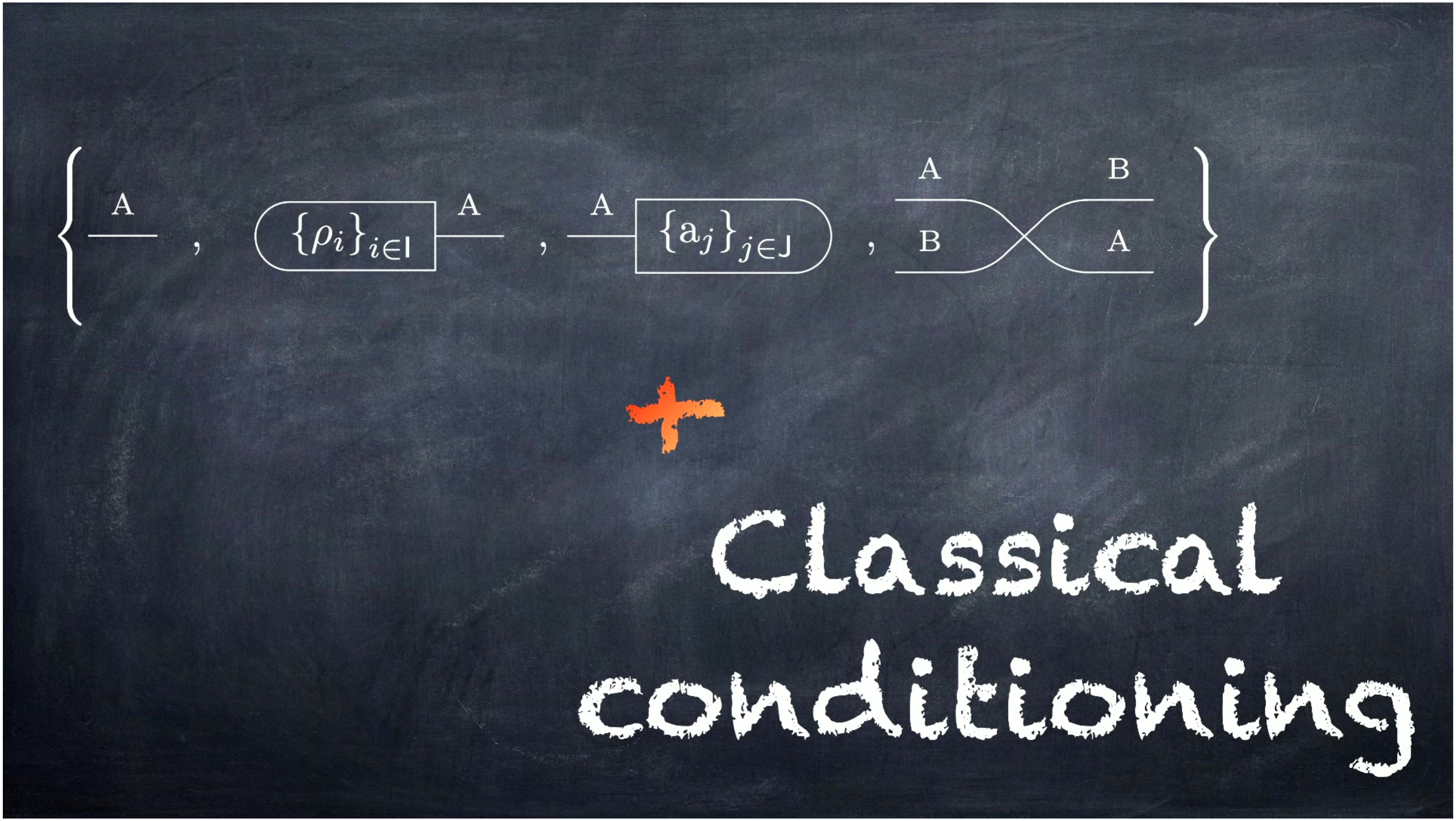
UNCERTAINTY


ΔS ≥ 0


γ
γ'
ω
e⁻
observer


Measurement incompatibility
Irreversible disturbance
MSBCT
MCT


C
$\{\rho_i\}_{i\in \mathsf{I}}$
B′
A′
$\{a_j\}_{j\in \mathsf{J}}$
A
A′
B′
B
$\mathcal{S}_1$
E
$\mathcal{S}_2$


A
$\{\rho_i\}_{i \in \mathsf{I}}$
A
A
$\{a_j\}_{j \in \mathsf{J}}$
A
B
B
A
+
Classical conditioning