

Title: Measurement incompatibility implies irreversible disturbance

Speakers: Davide Rolino

Collection/Series: Quantum Foundations

Subject: Quantum Foundations

Date: November 04, 2024 - 11:00 AM

URL: <https://pirsa.org/24110060>

Abstract:

To justify the existence of measurements that can not be performed jointly on quantum systems, Heisenberg put forward a heuristic argument, involving the famous gamma-ray microscope Gedankenexperiment, based on the existence of measurements that irreversibly alter the physical system on which they act. Today, the impossibility of jointly measuring some physical quantities, termed measurement incompatibility, and irreversible disturbance, namely the existence of operations that irreversibly alter the system on which they act, are understood to be distinct but related features of quantum mechanics. In our work, we formally characterized the relationship between these two properties, showing that measurement incompatibility implies irreversible disturbance, though the converse is false. The counterexamples are two toy theories: Minimal Classical Theory and Minimal Strongly Causal Bilocal Classical Theory. These two are distinct as counterexamples because the latter allows for classical conditioning. Our research followed an operational approach exploiting the framework of Operational Probabilistic Theories. In particular, it required the development of two new classes of operational theories: Minimal Operational Probabilistic Theories and Minimal Strongly Causal Operational Probabilistic Theories. These theories are characterized by a restricted set of dynamics, limited to the minimal set consistent with the set of states. In Minimal Strongly Causal Operational Probabilistic Theories, classical conditioning is also allowed.

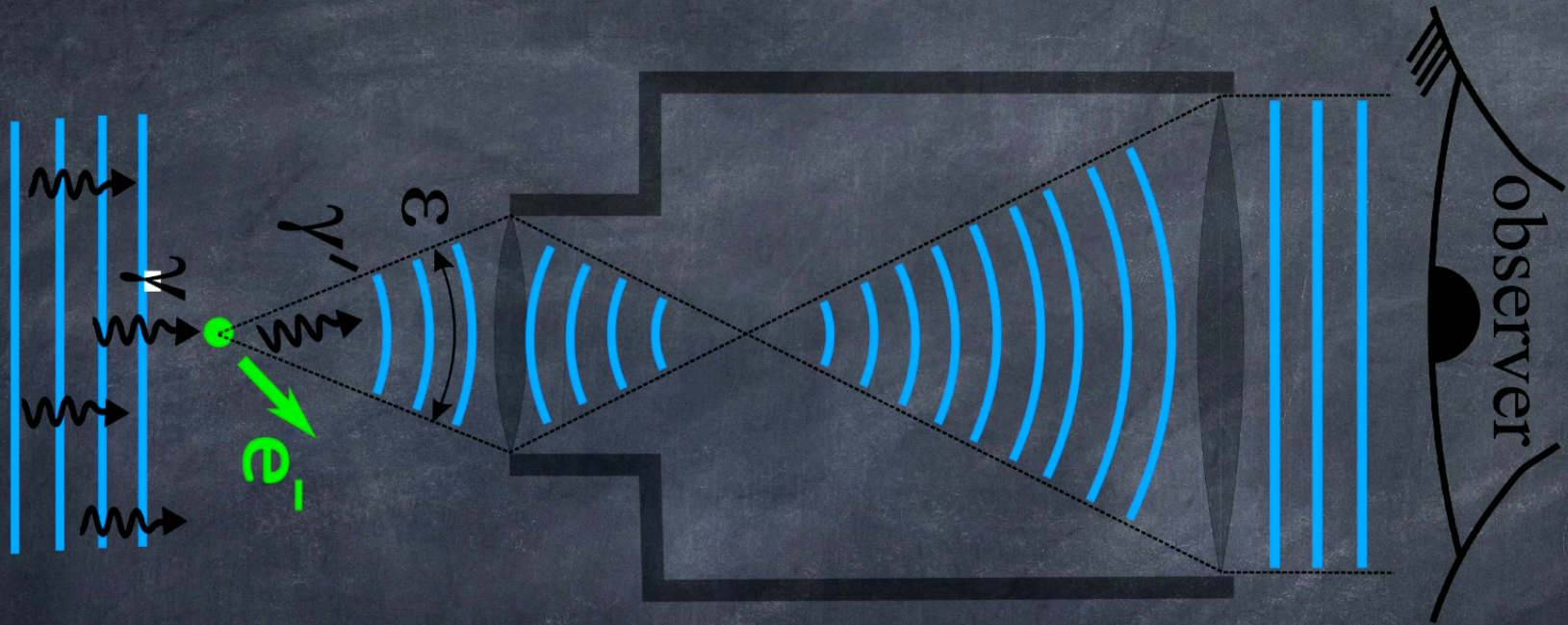
Measurement
incompatibility
implies irreversible
disturbance

Davide Rolino

Perimeter Institute for Theoretical Physics

Waterloo, November 4, 2024





$$\Delta x \Delta p_x \approx h$$



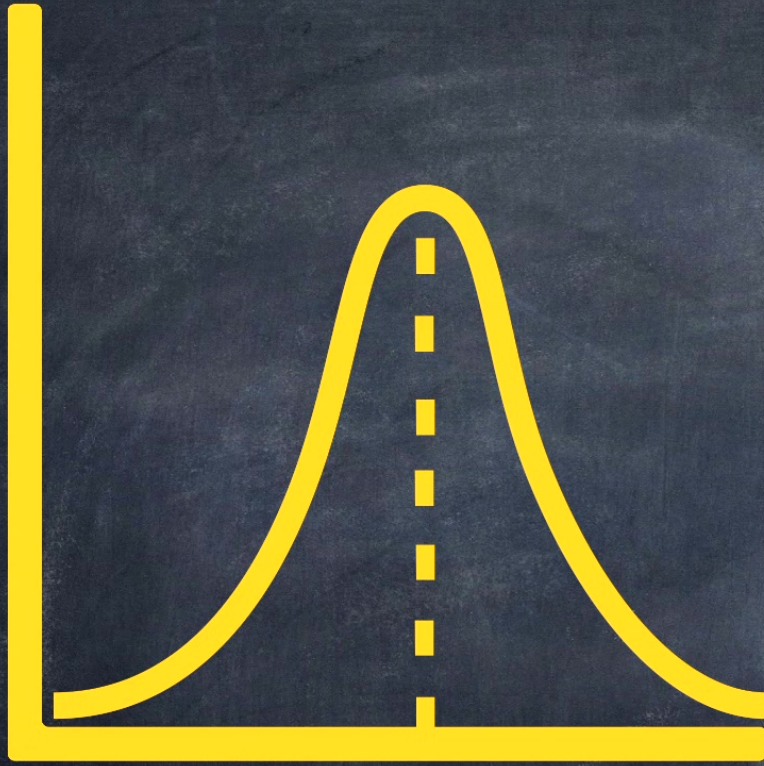
My work

$\Delta \times \text{JP} \approx \frac{B}{2}$

MEASUREMENT INCOMPATIBILITY



Measurement
incompatibility
implies irreversible
disturbance



X



\mathcal{P}

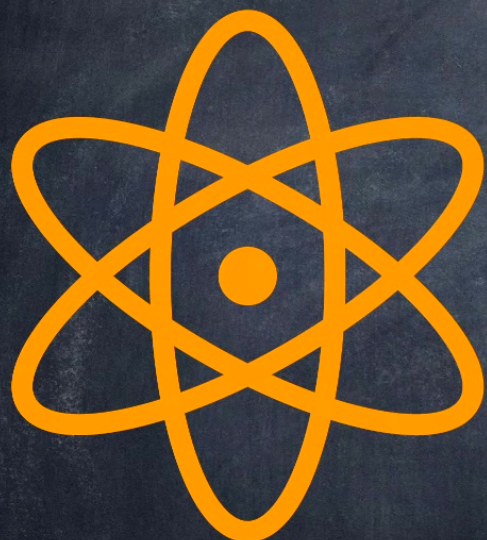
Measurement
incompatibility

Irreversible
disturbance

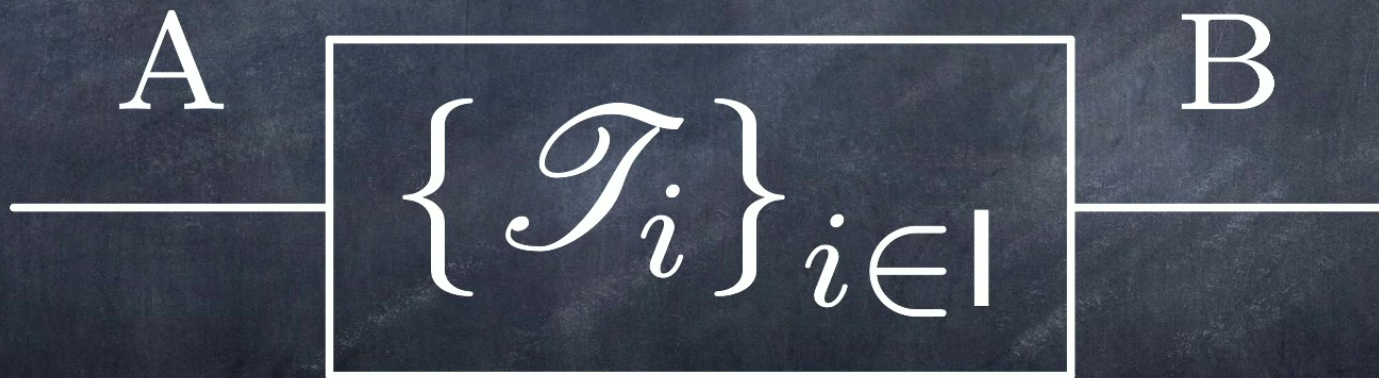


OPERATIONAL PROBABILISTIC THEORIES (OPTS)

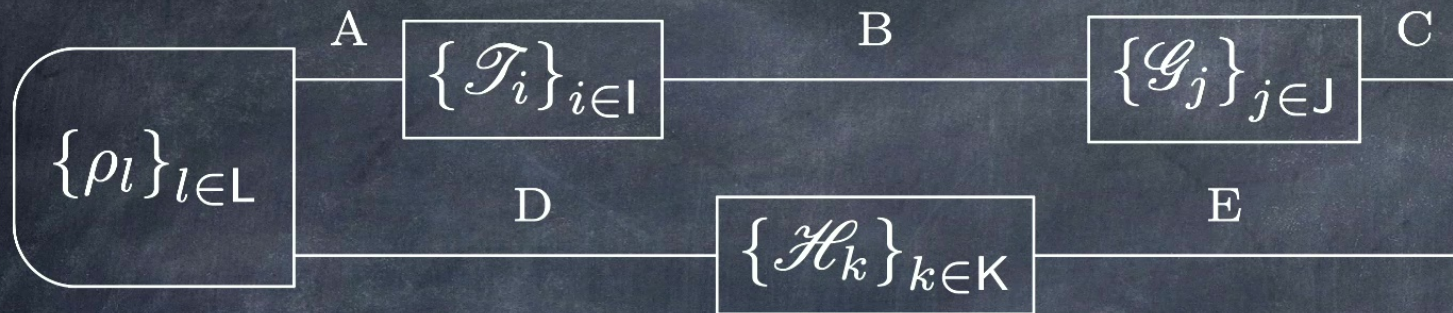
G. Chiribella, G. M. D'Ariano, and P. Perinotti, *Phys. Rev. A* 81, 062348 (2010),
G. M. D'Ariano, G. Chiribella, and P. Perinotti, "Quantum Theory from First Principles", CUP (2017)

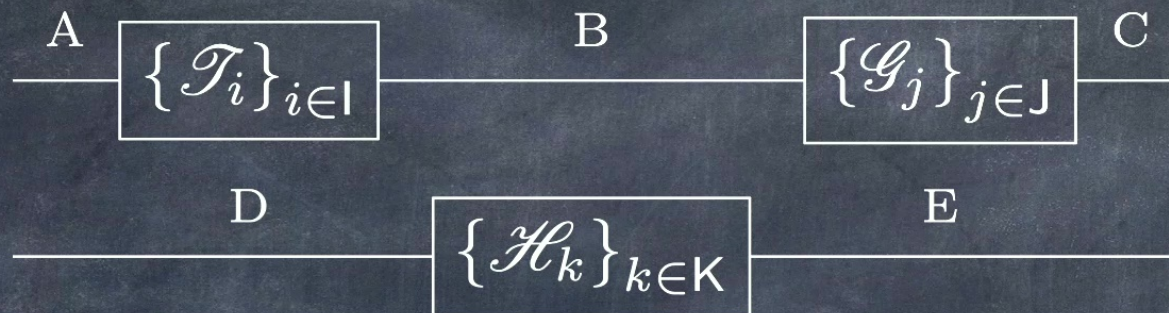


INSTRUMENTS

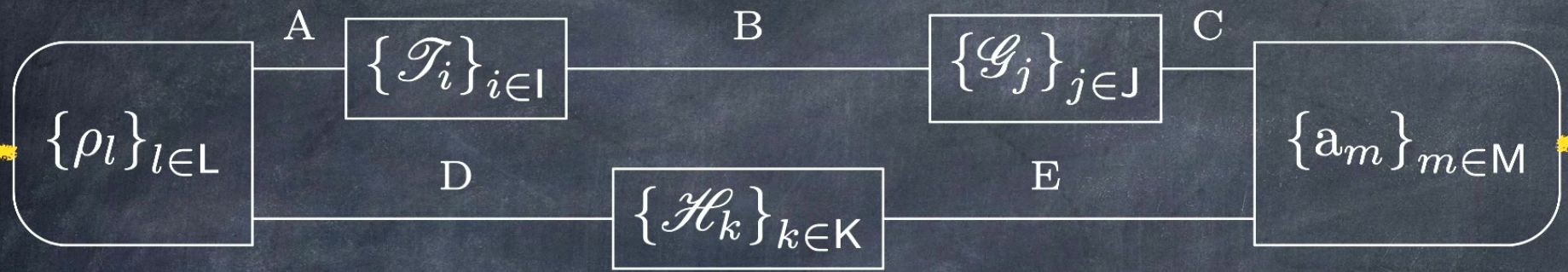




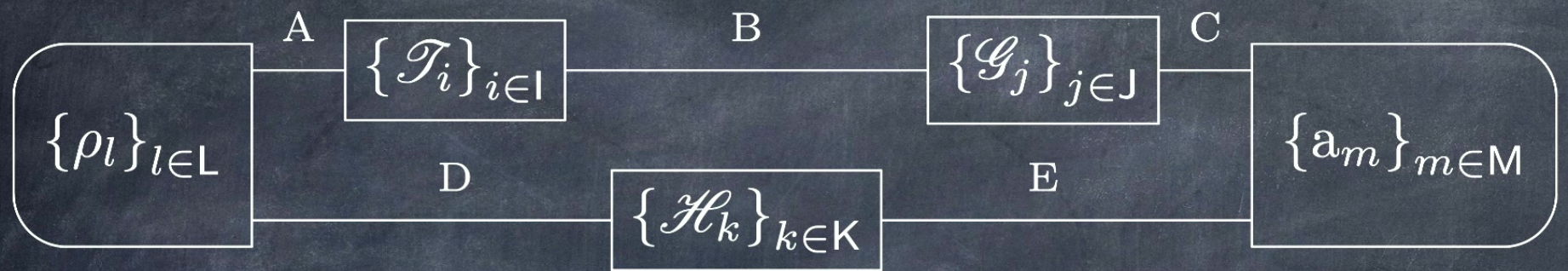




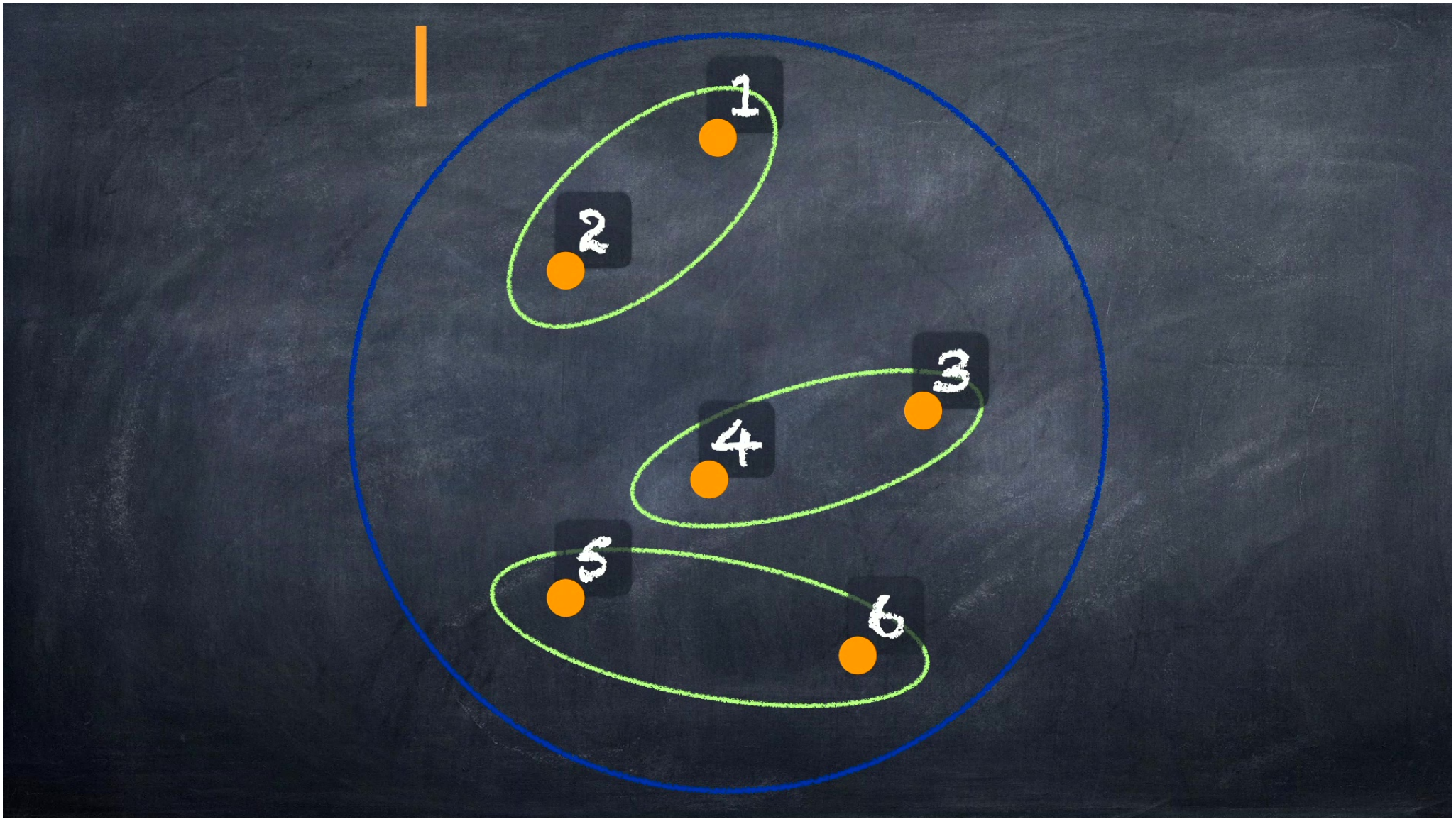
CLOSED UNDER
COMPOSITION

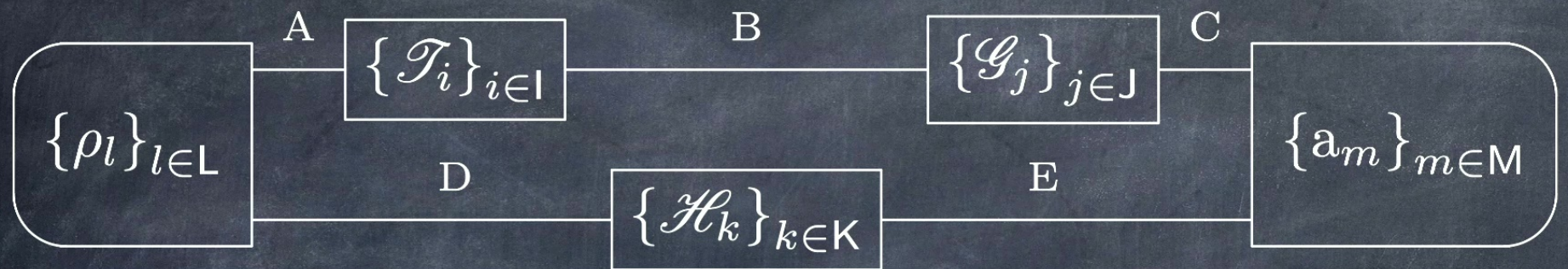


I

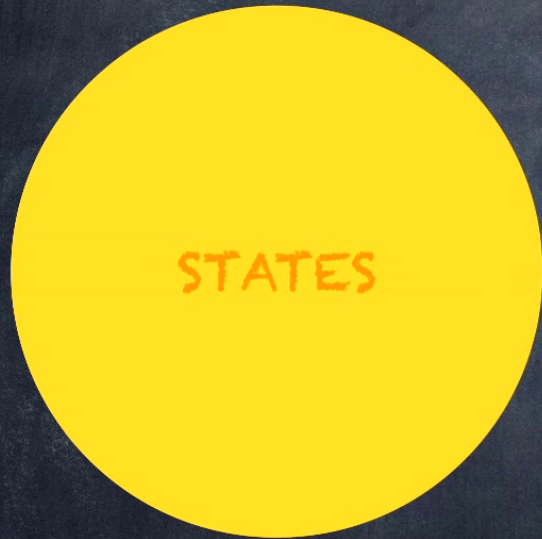


BRAIDED
(STRICT)
MONOIDAL
CATEGORY

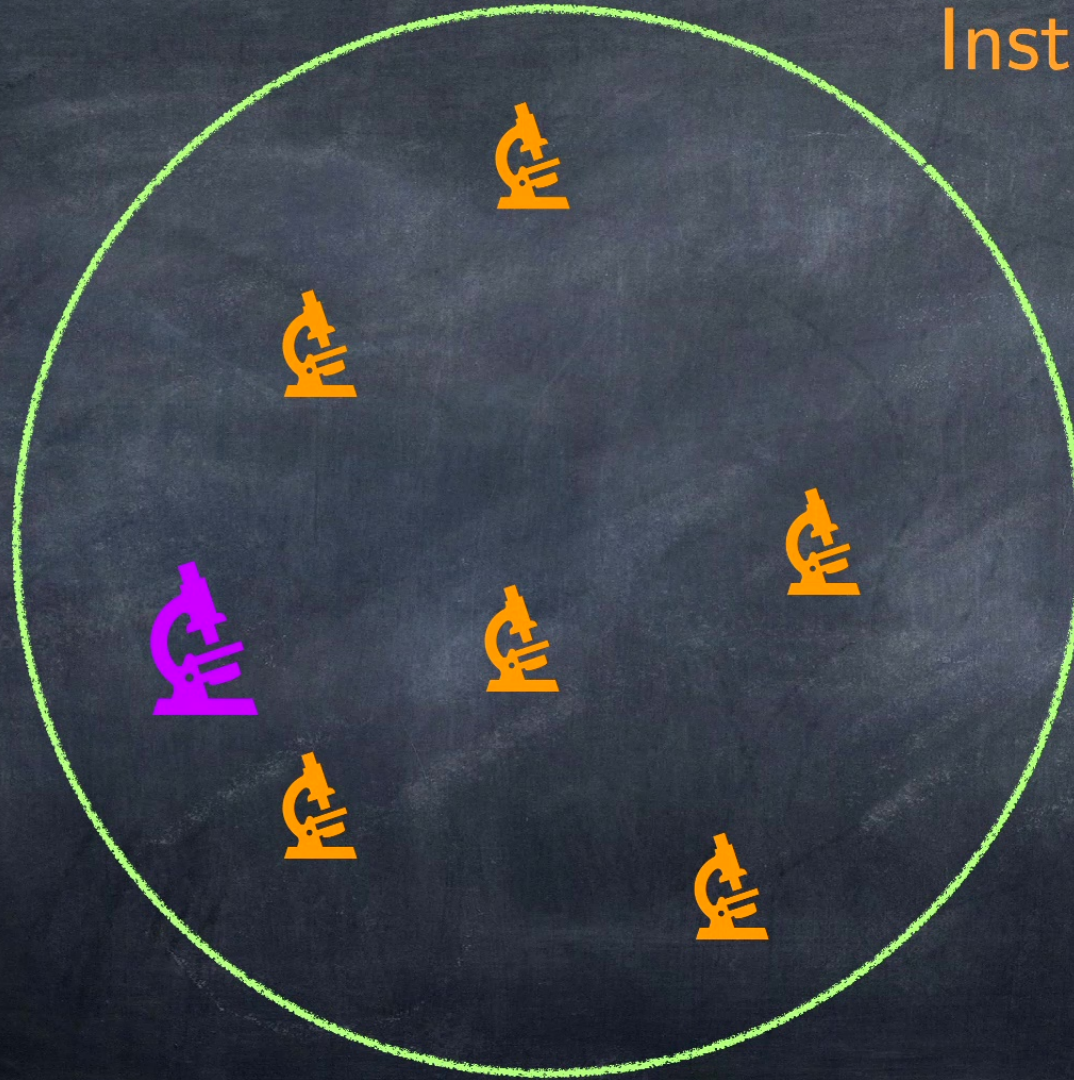




$$p(l, i, j, k, m | \rho_L, \mathcal{T}_I, \mathcal{G}_J, \mathcal{H}_K, a_M)$$



Instr (A \rightarrow B)



DISCRIMINATION PROBABILITY

$$d \left(\text{microscope}, \text{microscope} \right)$$

CAUSAL THEORIES

COMPATIBILITY

G. M. D'Ariano, P. Perinotti, and A. Tosini J. Phys. A: Math. Theor. 55 394006 (2022)

$$P_i = \sum_{j \in J} R_{(i,j)} \quad \forall i \in I$$

$$Q_j = \sum_{i \in I} R_{(i,j)} \quad \forall j \in J$$

$$\overset{A}{\text{---}} \boxed{a_i} = \sum_{j \in J} \overset{A}{\text{---}} \boxed{c(i,j)} \quad \forall i \in I$$

$$\overset{A}{\text{---}} \boxed{b_j} = \sum_{i \in I} \overset{A}{\text{---}} \boxed{c(i,j)} \quad \forall j \in J$$

$$\begin{array}{c}
 \text{A} \quad \boxed{\mathcal{T}_i} \quad \text{B} \\
 \text{---} \quad \quad \quad \text{---}
 \end{array}
 =
 \sum_{j \in J}
 \begin{array}{c}
 \text{A} \quad \quad \quad \text{B} \\
 \text{---} \quad \quad \quad \text{---} \\
 \boxed{\mathcal{C}_{(i,j)}} \\
 \text{C} \quad \quad \quad \boxed{e}
 \end{array}
 \quad \forall i \in I$$

$$\begin{array}{c}
 \text{A} \quad \boxed{\mathcal{G}_j} \quad \text{B} \\
 \text{---} \quad \quad \quad \text{---}
 \end{array}
 =
 \sum_{i \in I}
 \begin{array}{c}
 \text{A} \quad \quad \quad \text{B} \\
 \text{---} \quad \quad \quad \text{---} \\
 \boxed{\mathcal{C}_{(i,j)}} \\
 \text{C} \quad \quad \quad \boxed{e}
 \end{array}
 \quad \forall j \in J$$

G. M. D'Ariano, P. Perinotti, and A. Tosini J. Phys. A: Math. Theor. 55 394006 (2022)

STRONG
COMPATIBILITY

$$\begin{array}{c}
 \text{A} \quad \boxed{\mathcal{T}_i} \quad \text{B} \\
 \text{---} \quad \quad \quad \text{---}
 \end{array}
 = \sum_{z \in Z^{(i)}} \begin{array}{c}
 \text{A} \quad \quad \quad \text{B} \\
 \text{---} \quad \quad \quad \text{---} \\
 \quad \quad \quad \boxed{\mathcal{C}_z} \\
 \quad \quad \quad \text{E} \quad \text{---} \quad \boxed{e}
 \end{array} \quad \forall i \in I$$

$$\begin{array}{c}
 \text{A} \quad \boxed{\mathcal{G}_j} \quad \text{C} \\
 \text{---} \quad \quad \quad \text{---}
 \end{array}
 = \sum_{z \in Z} \begin{array}{c}
 \text{A} \quad \quad \quad \text{B} \quad \quad \quad \text{C} \\
 \text{---} \quad \quad \quad \text{---} \quad \quad \quad \text{---} \\
 \quad \quad \quad \boxed{\mathcal{C}_z} \quad \quad \quad \boxed{\mathcal{P}_i^{(z)}} \\
 \quad \quad \quad \text{E} \quad \quad \quad \text{---}
 \end{array} \quad \forall j \in J$$

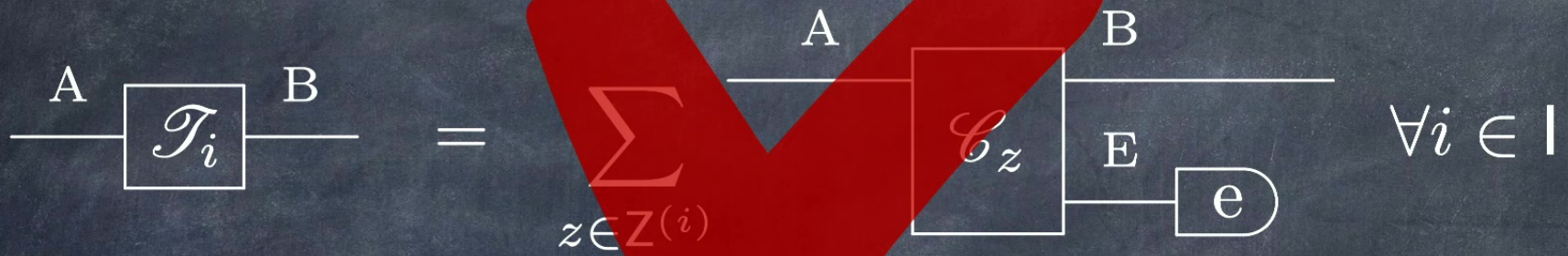
G. M. D'Ariano, P. Perinotti, and A. Tosini J. Phys. A: Math. Theor. 55 394006 (2022)

STRONG
COMPATIBILITY



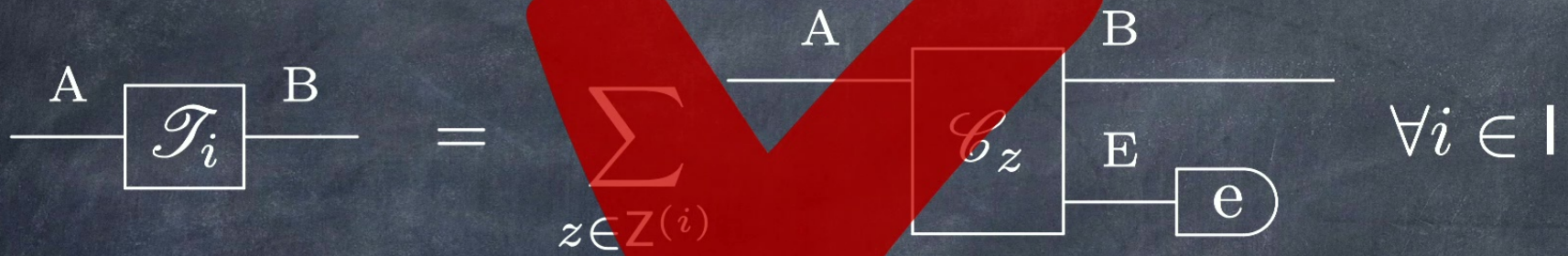
WEAK
COMPATIBILITY

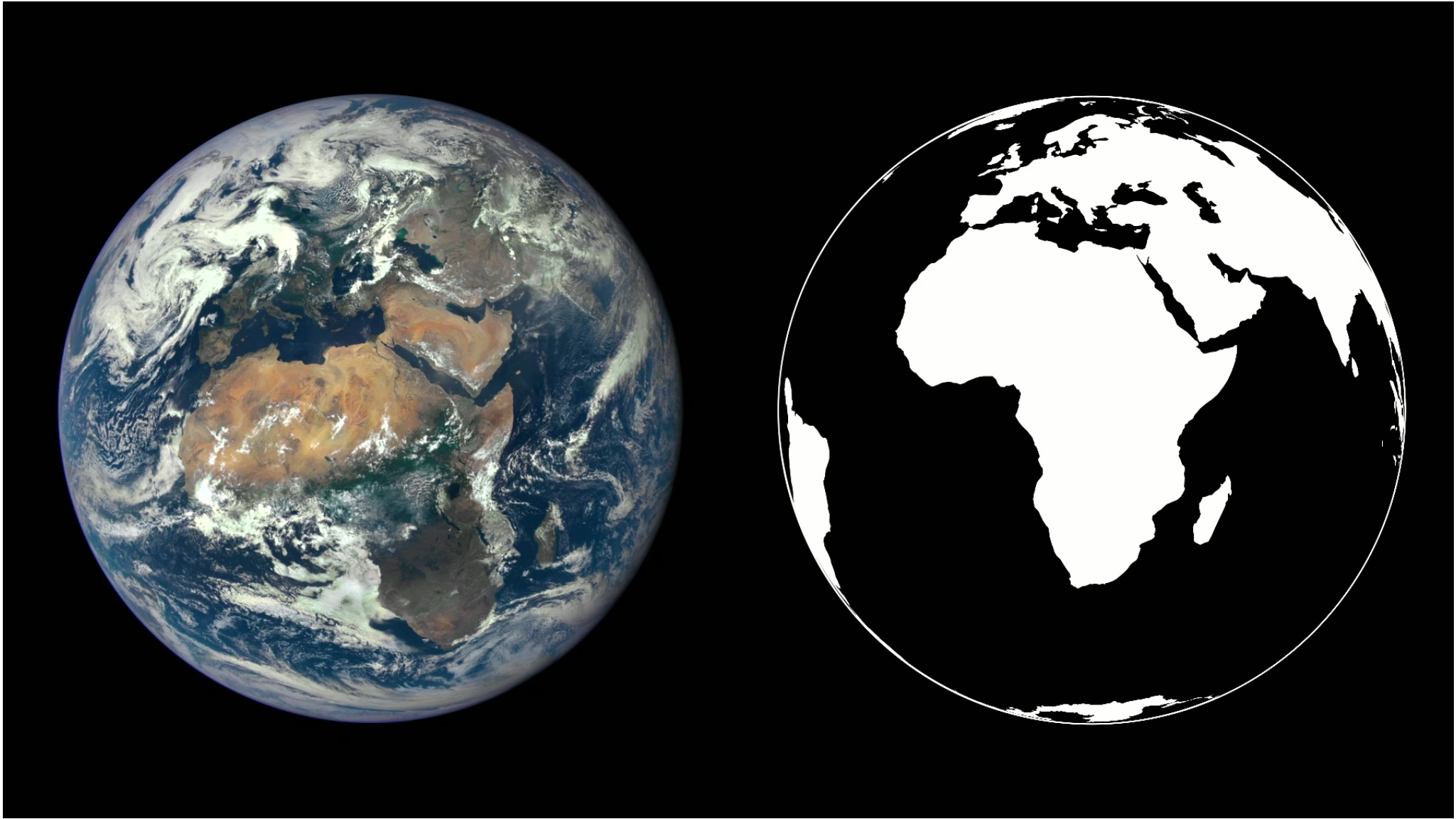




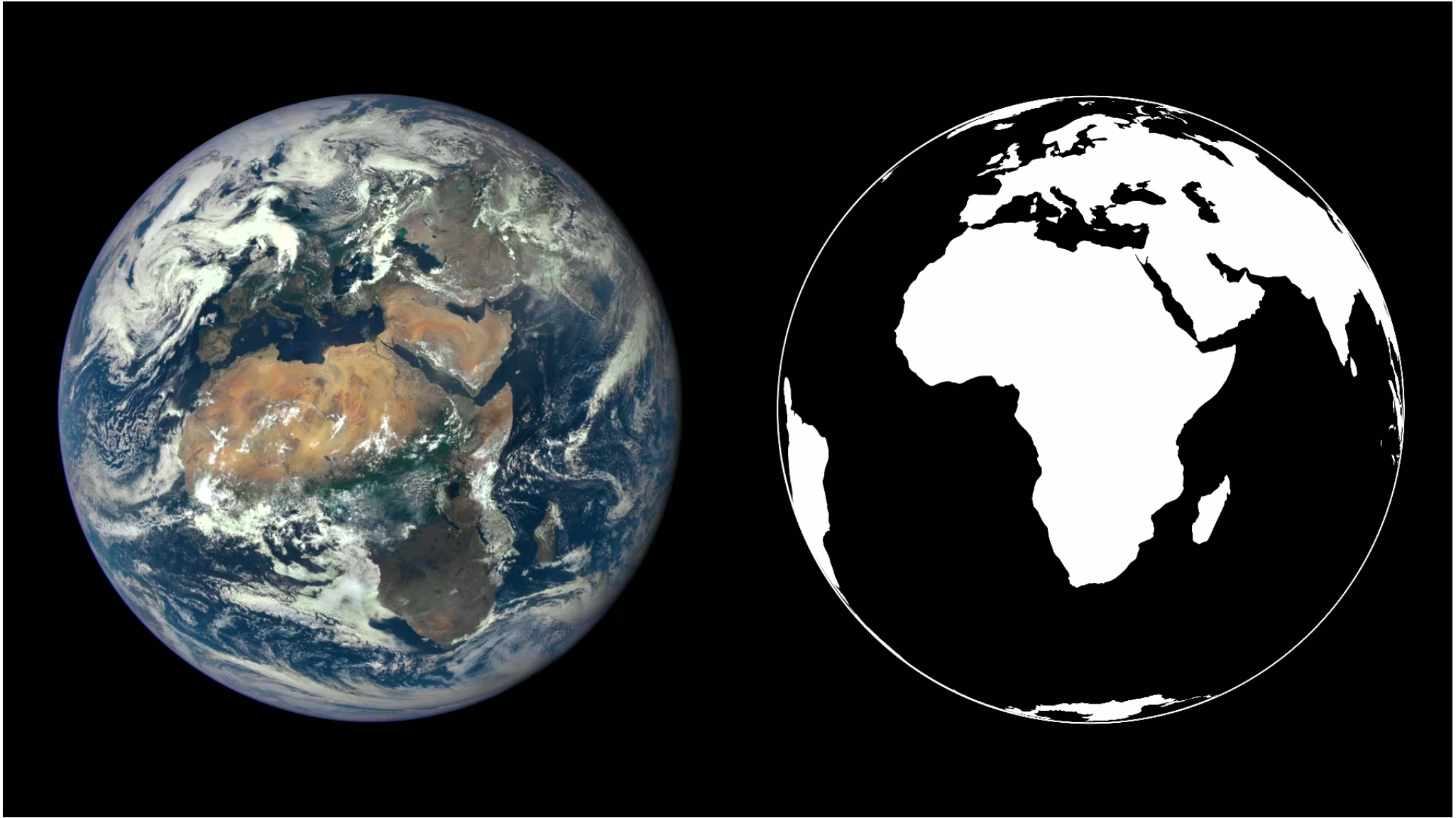
$$\begin{array}{c} \text{---} \text{A} \text{---} \boxed{a_i} \\ \text{---} \text{A} \text{---} \boxed{b_j} \end{array} = \sum_{j \in J} \text{---} \text{A} \text{---} \boxed{c(i,j)} \quad \forall i \in I$$

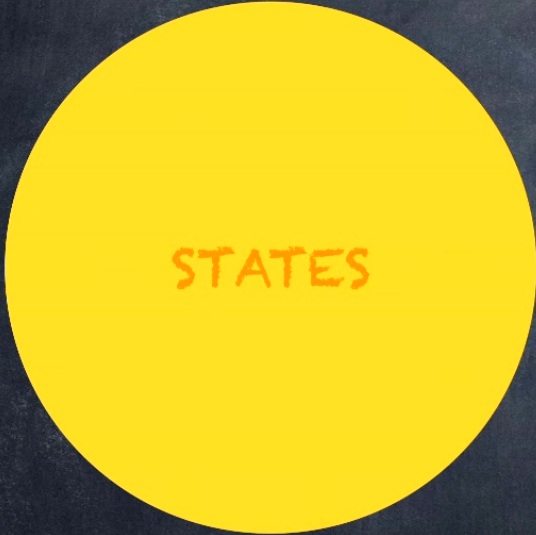
$$\text{---} \text{A} \text{---} \boxed{b_j} = \sum_{i \in I} \text{---} \text{A} \text{---} \boxed{c(i,j)} \quad \forall j \in J$$











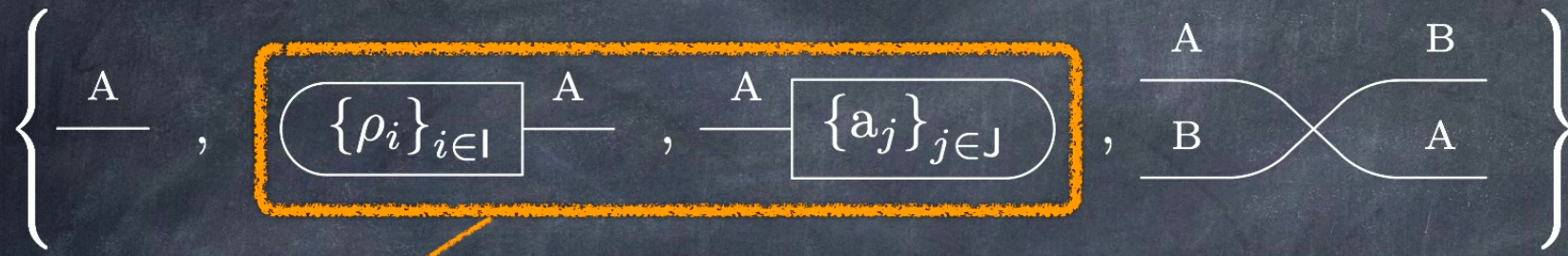
INSTRUMENTS



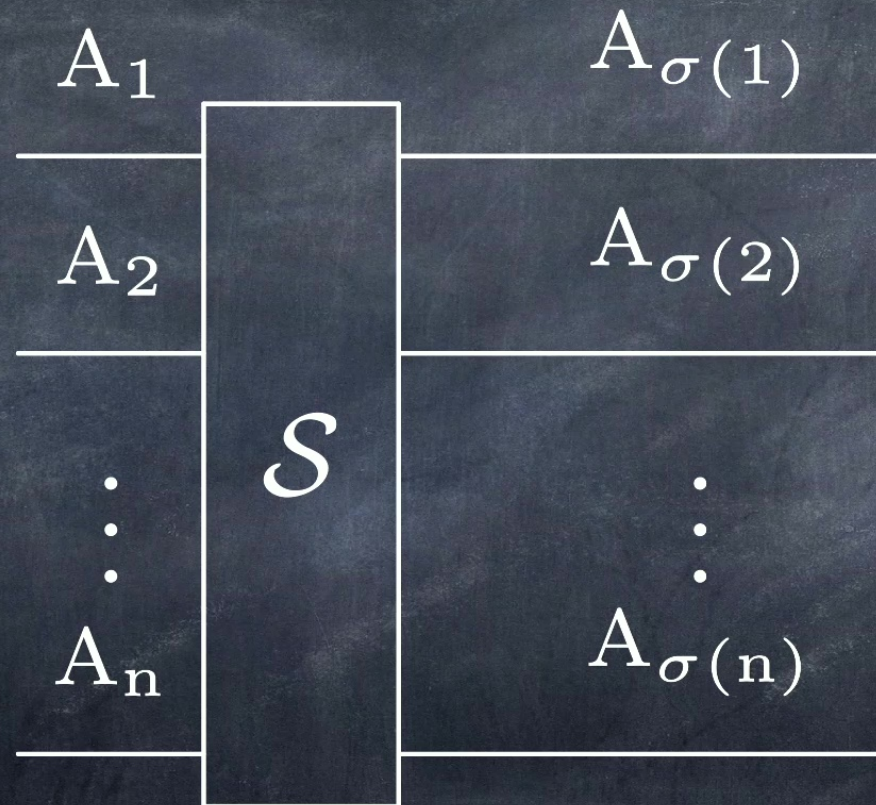
$$\left\{ \frac{A}{\quad} \right\}$$

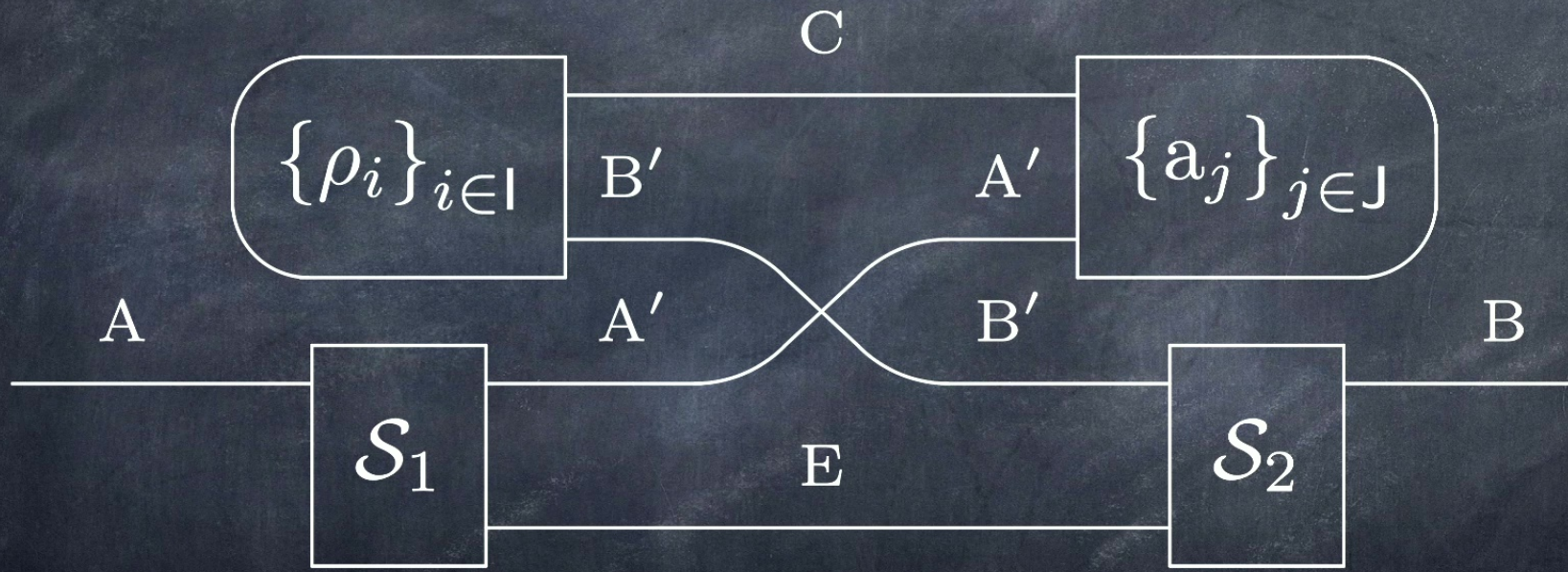
$$\left\{ \overline{A}, \quad \boxed{\{\rho_i\}_{i \in I}} \overline{A}, \quad \overline{A} \boxed{\{a_j\}_{j \in J}}, \quad \begin{array}{c} \overline{A} \quad \overline{B} \\ \overline{B} \quad \overline{A} \end{array} \right\}$$

IRREVERSIBLE
DISTURBANCE

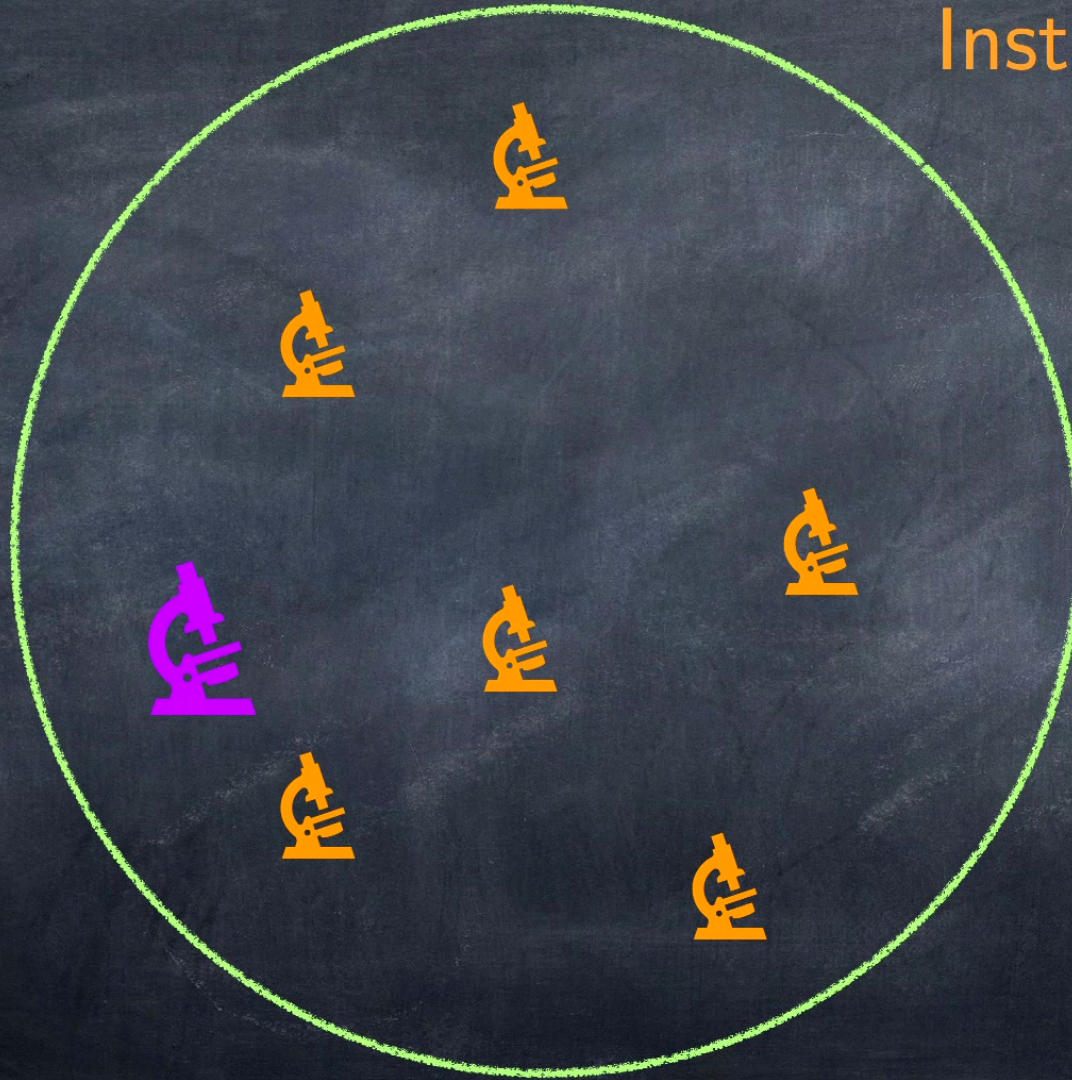


States and
effects of
Classical Theory



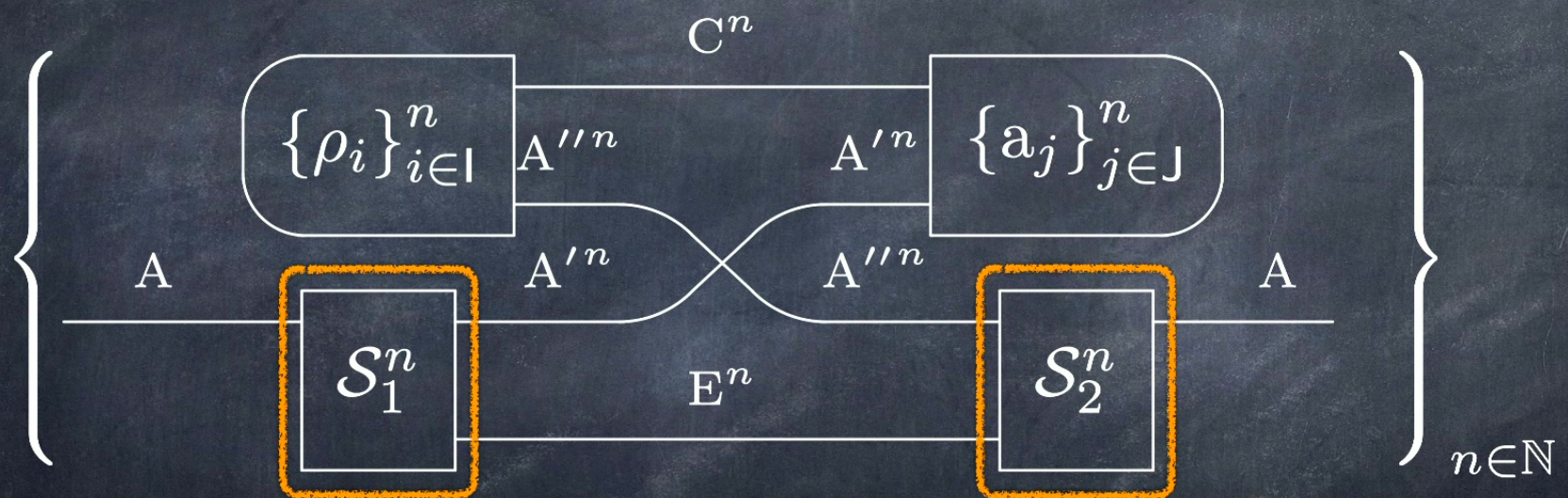


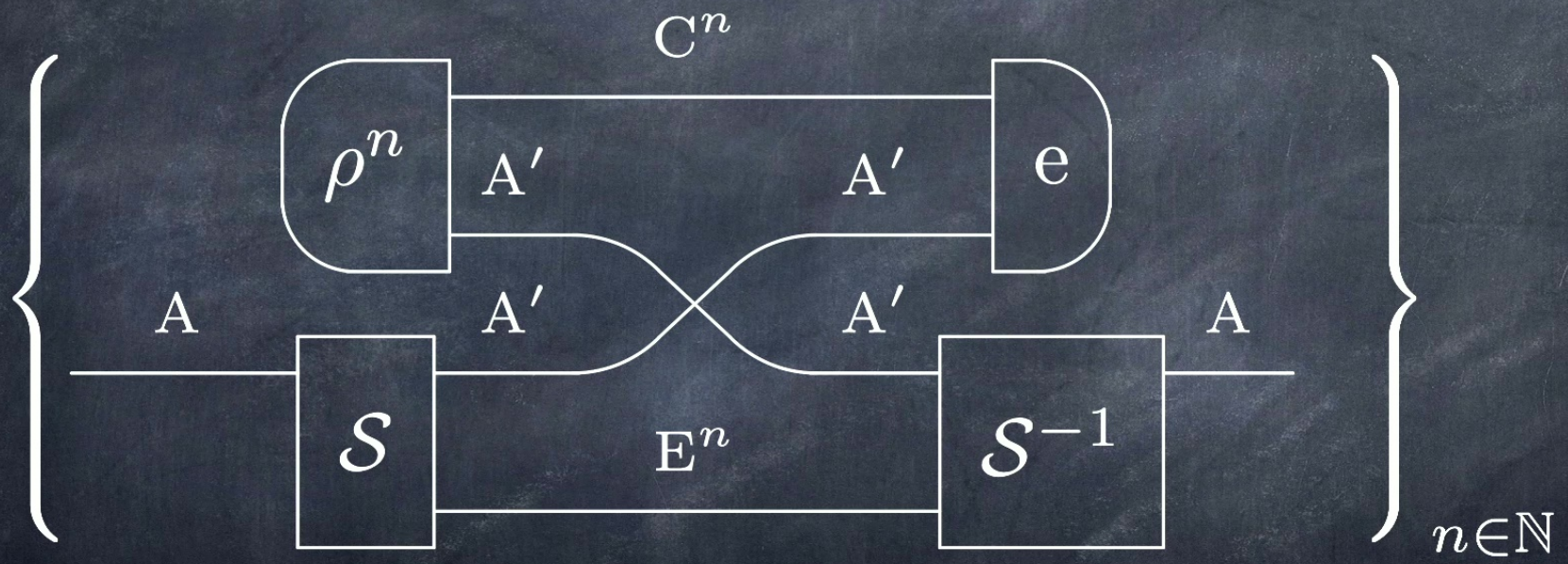
Instr (A \rightarrow B)

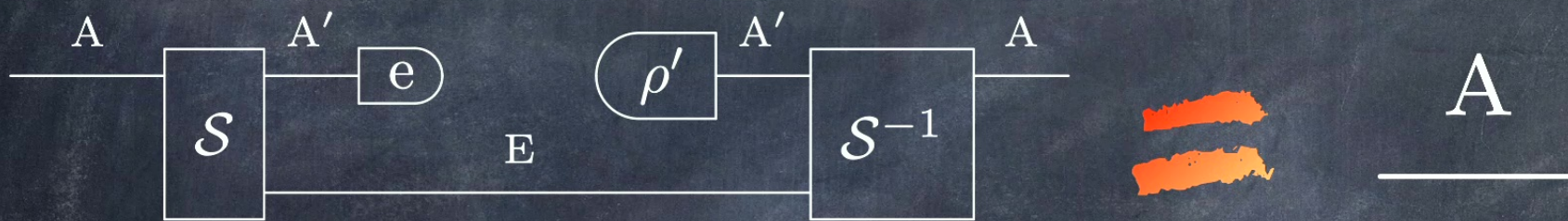


ATOMIC

$$G_j \propto \mathcal{I} \quad \forall j \in J'$$



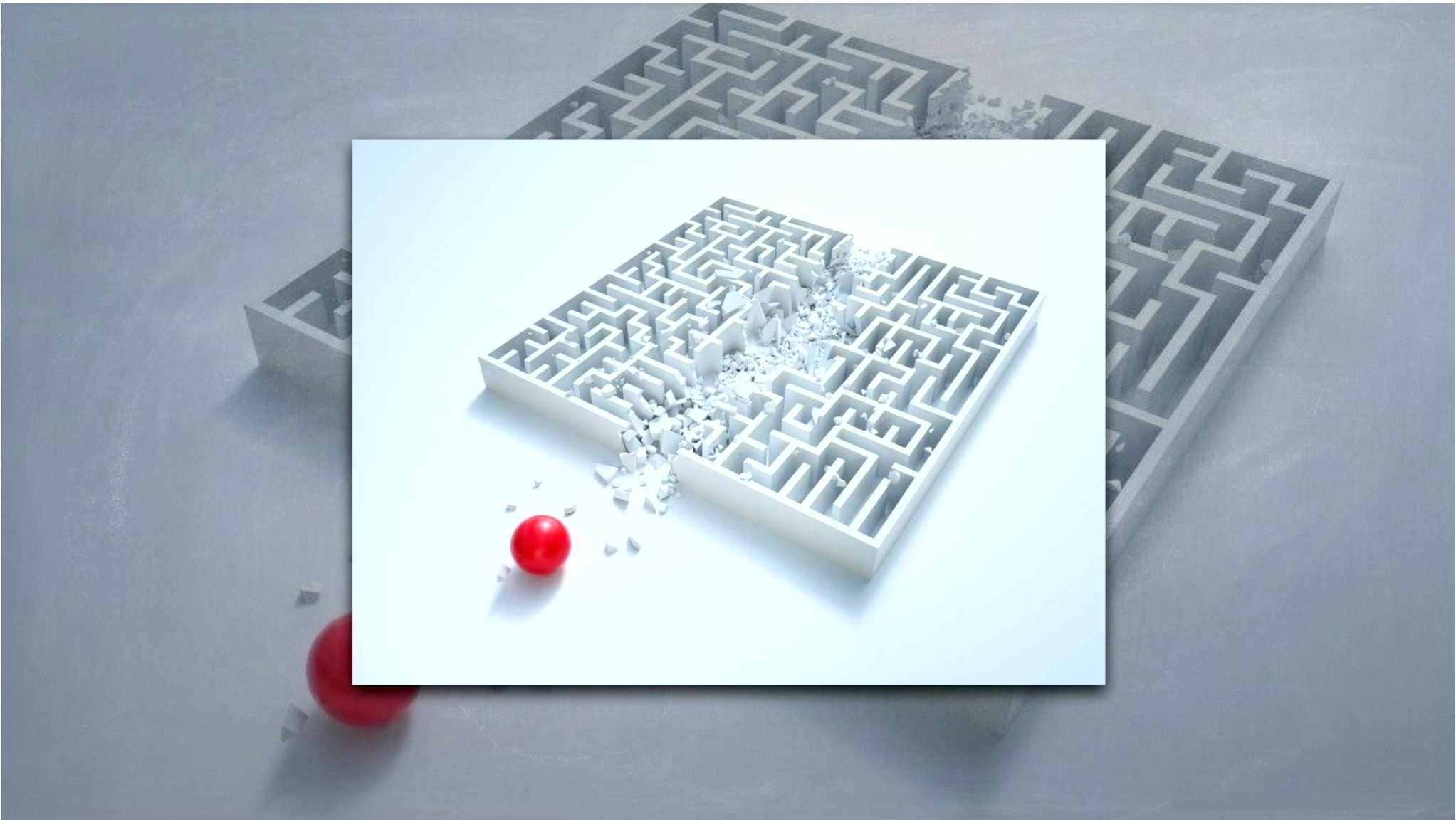




IRREVERSIBLE
DISTURBANCE



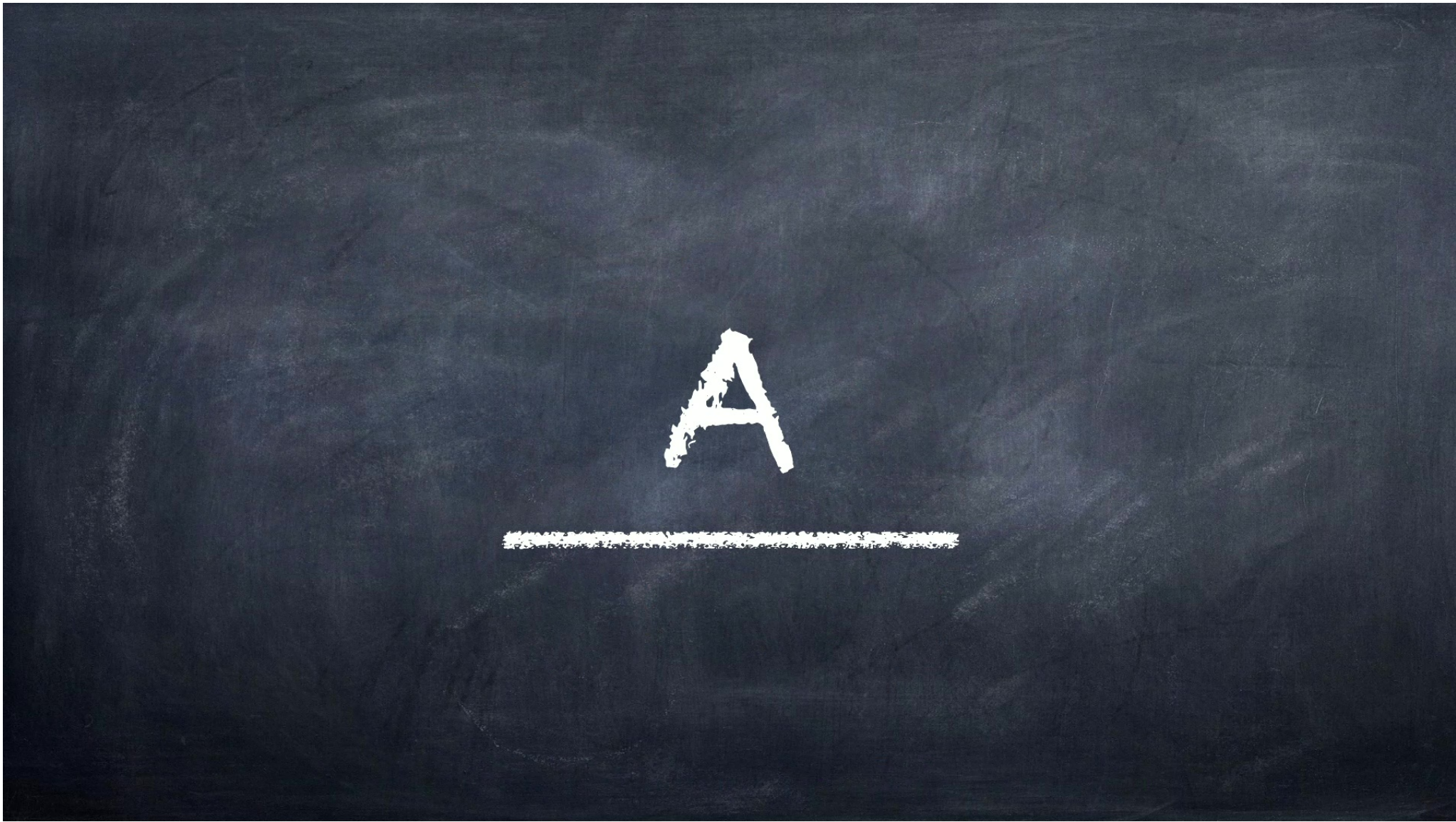
MEASUREMENT
INCOMPATIBILITY



MINIMAL CLASSICAL THEORY



~~MINIMAL~~
CLASSICAL
THEORY



$$\left\{ \begin{array}{c} \text{A} \\ \hline \end{array} , \quad \boxed{\begin{array}{c} \text{A} \\ \hline \{ \rho_i \}_{i \in I} \\ \hline \end{array}} , \quad \begin{array}{c} \text{A} \\ \hline \{ a_j \}_{j \in J} \\ \hline \end{array} , \quad \begin{array}{cc} \text{A} & \text{B} \\ \hline & \hline \end{array} \begin{array}{c} \text{B} \\ \hline \text{A} \end{array} \right\}$$

States and effects
of Bilocal
Classical Theory

\$S_{\mathcal{L}}\$ (PT) tho. of entangled for every sys. admit entangled states that are a different from the null state is equivalent to equipment in Theor. sampled states and that any MSOFT a sequence of instr. coarse grain to the in. \$\{G_{\mathcal{L}}^{(i)}\}_{i=1}^n\$ be a Cauchy sequence of instr. crossed above where for simplicity in the here highlight only the last conditioning step. \$\forall (A \rightarrow A_{k-1}^n)\$ with \$\mathcal{X} = (x_1, \dots, x_{k-1}) \in \text{near}(A_{k-1}^n \rightarrow A)\$ we it that considering the w. transformations obtained by the outcome space \$\mathcal{X}\$ i.e., are of the under which

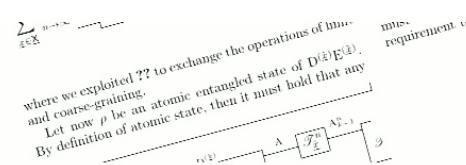
To be precise, \$\mathcal{I}^n\$ is defined up to this is not important for the or highlight the fact that \$\mathcal{I}^n \dots\$

But, wait a moment! This is one less conditioning step procedure discussed till now. Unfortunately, however are the consequences of straightforward, there are which attention must be One is related to the a like \$e(\mathcal{L})\$, that appear what we mean by this, exploiting the result we

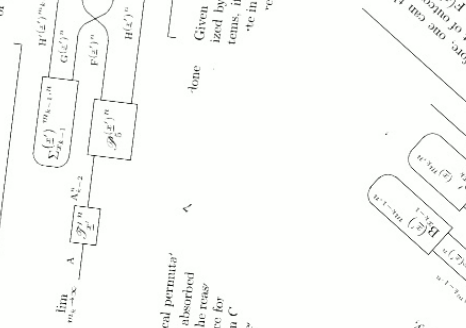
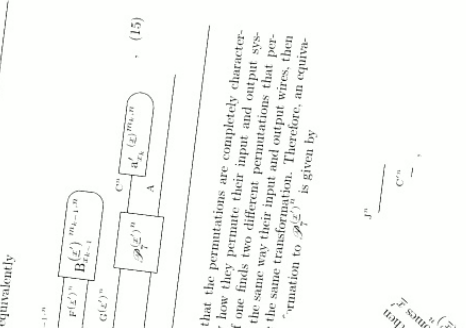
To understand what kind of instruments can decompose the identity, we have to transcribe all the operations in the following discussion are still as

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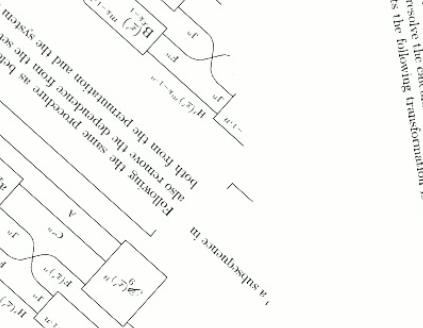
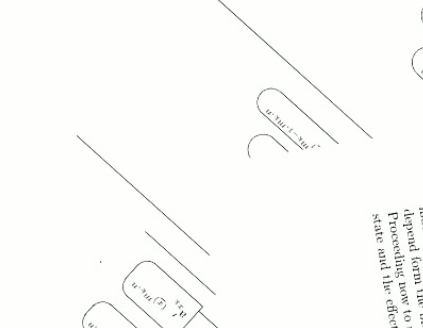
$$\lim_{m \rightarrow \infty} \sum_{\mathcal{X} \in \mathcal{X}} \dots$$



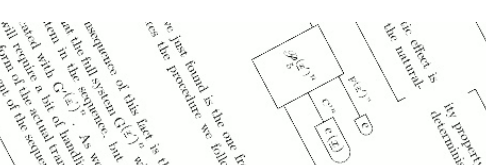
where the local parameter \$n\$ (10), was absorbed now explicit the new dependence for \$(\mathcal{L})\$, acting on the intrinsic edge instrument. have, and be used by

where the limits with respect to the initial state are due to the fact that the transformation \$\mathcal{I}^n\$ are the instruments of (3) are those of MSOFT, they may also be limits of sequences of \$n\$

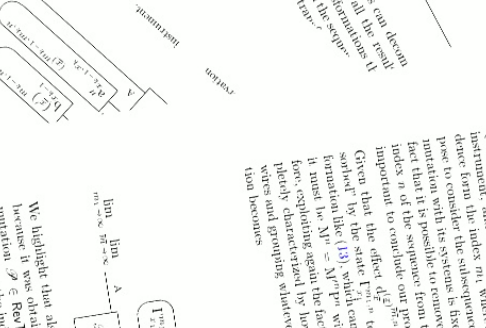
$$\lim_{m \rightarrow \infty} \sum_{\mathcal{X} \in \mathcal{X}} \dots$$



This proves our result: MSOFT whose full-course formation are randomized then itself.

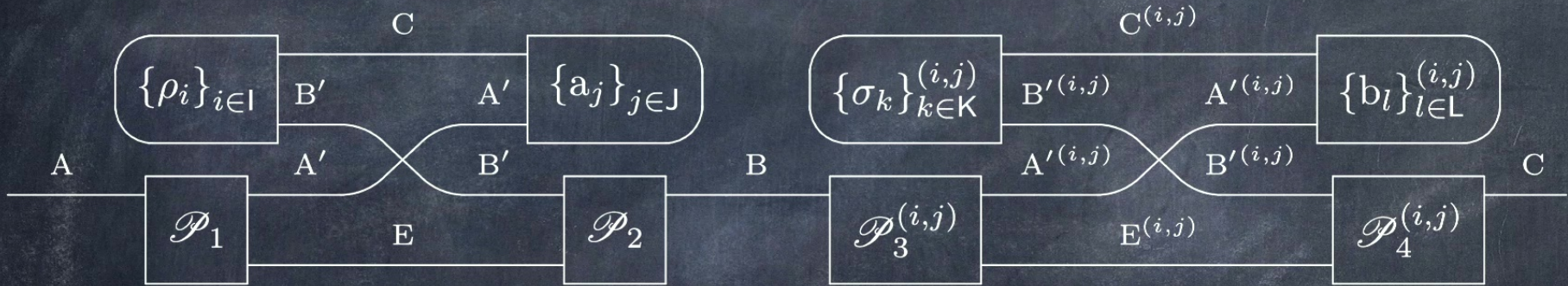


To understand what kind of instruments can decompose the identity, we have to transcribe all the operations in the following discussion are still as

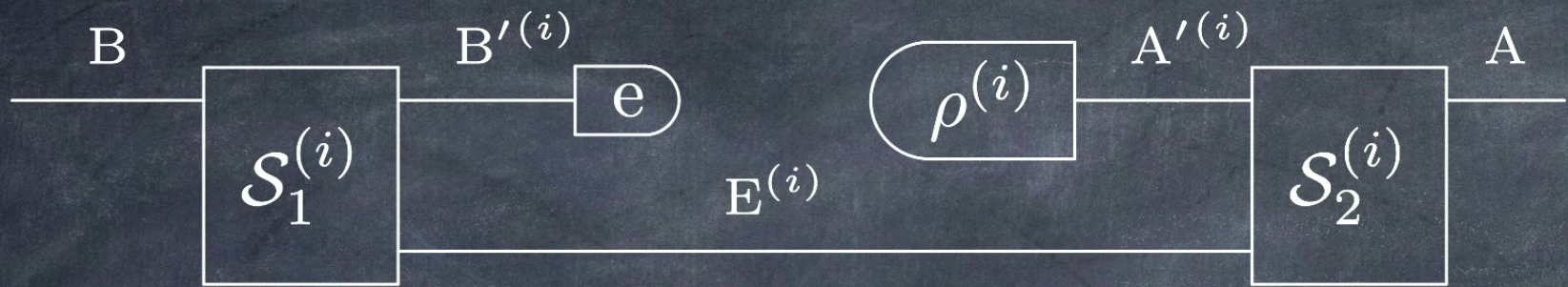


If one wants the full-course grain to be equal to the identity, i.e., for \$\mathcal{I}^n\$ to be the identity, we have to transcribe all the operations in the following discussion are still as

If one wants the full-course grain to be equal to the identity, i.e., for \$\mathcal{I}^n\$ to be the identity, we have to transcribe all the operations in the following discussion are still as



$$\sum_{i \in I} \sum_{j \in J} \xrightarrow{A} \boxed{\{\mathcal{T}_i\}_{i \in I}} \xrightarrow{B} \boxed{\{\mathcal{G}_j^{(i)}\}_{j \in J}} \xrightarrow{A} = \frac{A}{\quad}$$



ENTANGLEMENT
BREAKING !

Measurement
incompatibility

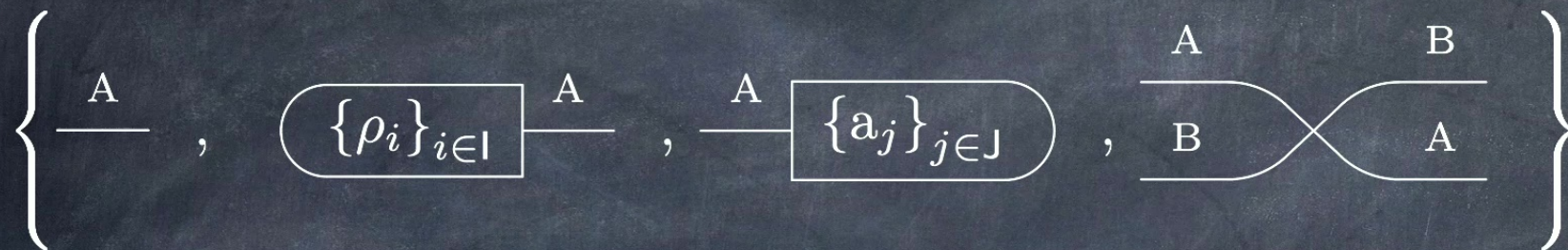
Irreversible
disturbance

MSBCT

MCT

GENERALISE





+

Classical
conditioning

The background of the slide is a photograph of water ripples on a dark surface. The ripples are concentric circles that spread outwards from a point of impact. The lighting is dramatic, with a bright light source from the top left, creating a strong highlight on the water's surface and casting deep shadows. The overall color palette is dark, with shades of blue, black, and white, accented by the bright orange text.

NO-INFORMATION WITHOUT DISTURBANCE

G. M. D'Ariano, P. Perinotti, and A. Tosini *Quantum* 4, 363 (2020)



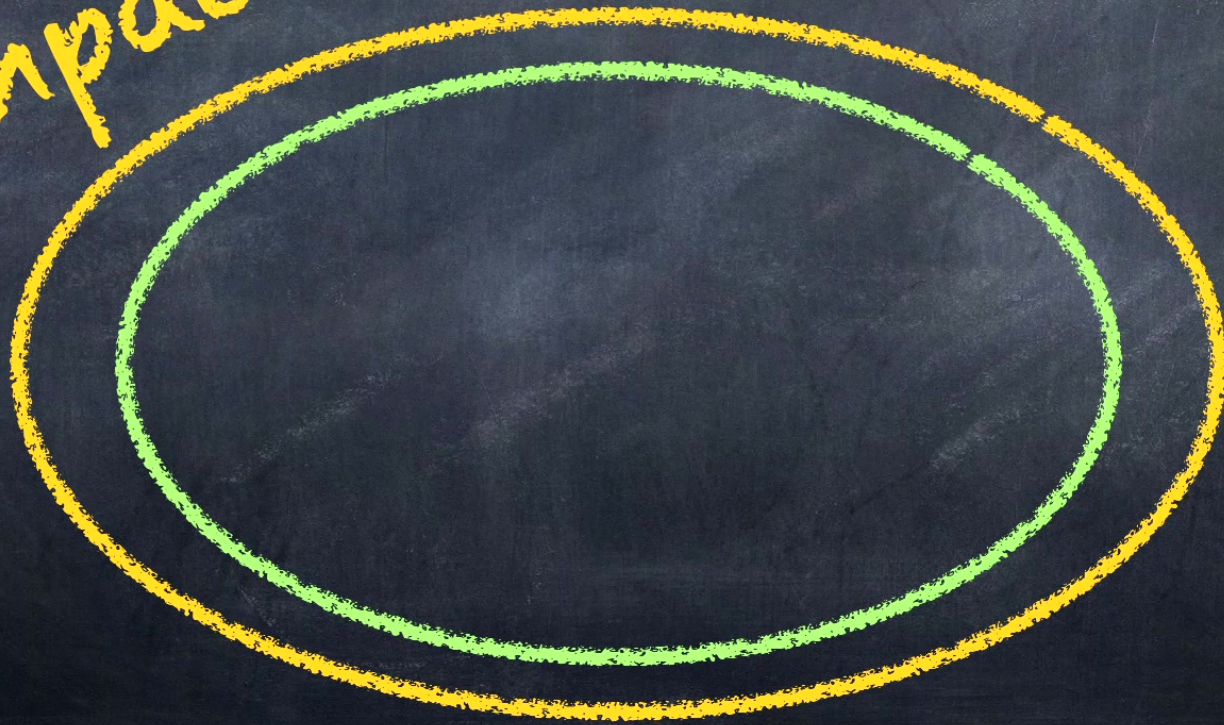
MINIMAL STRONGLY CAUSAL OPTs (MSOPTs)

M. Erba, P. Perinotti, DR, and A. Tosini, Phys. Rev. A 109, 022239 (2024)

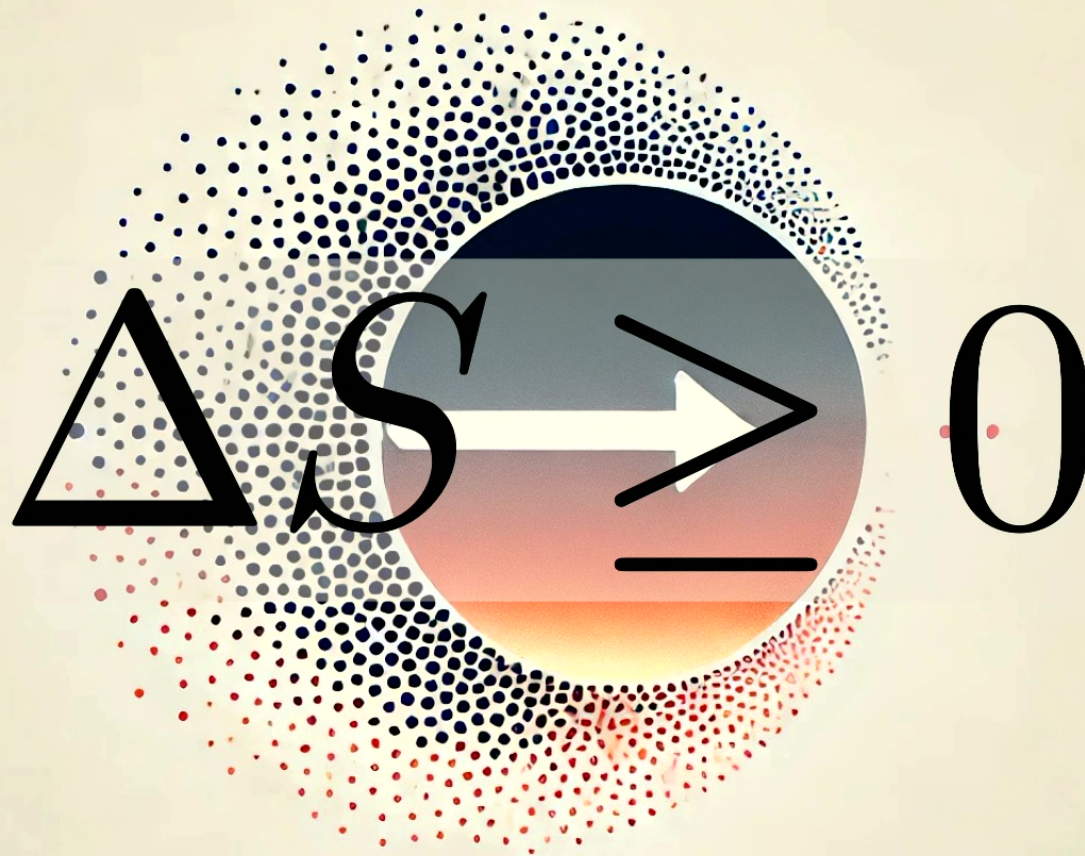


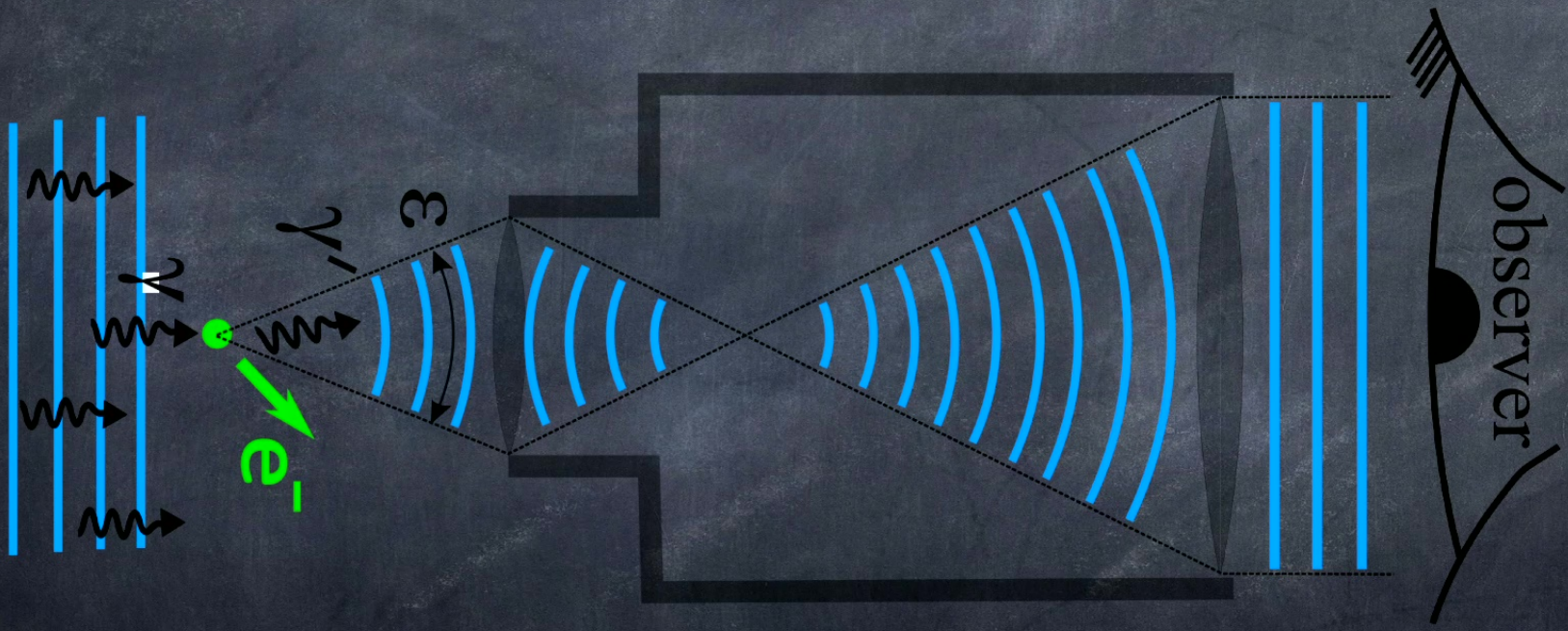
Measurement
incompatibility

Complementarity



UNCERTAINTY





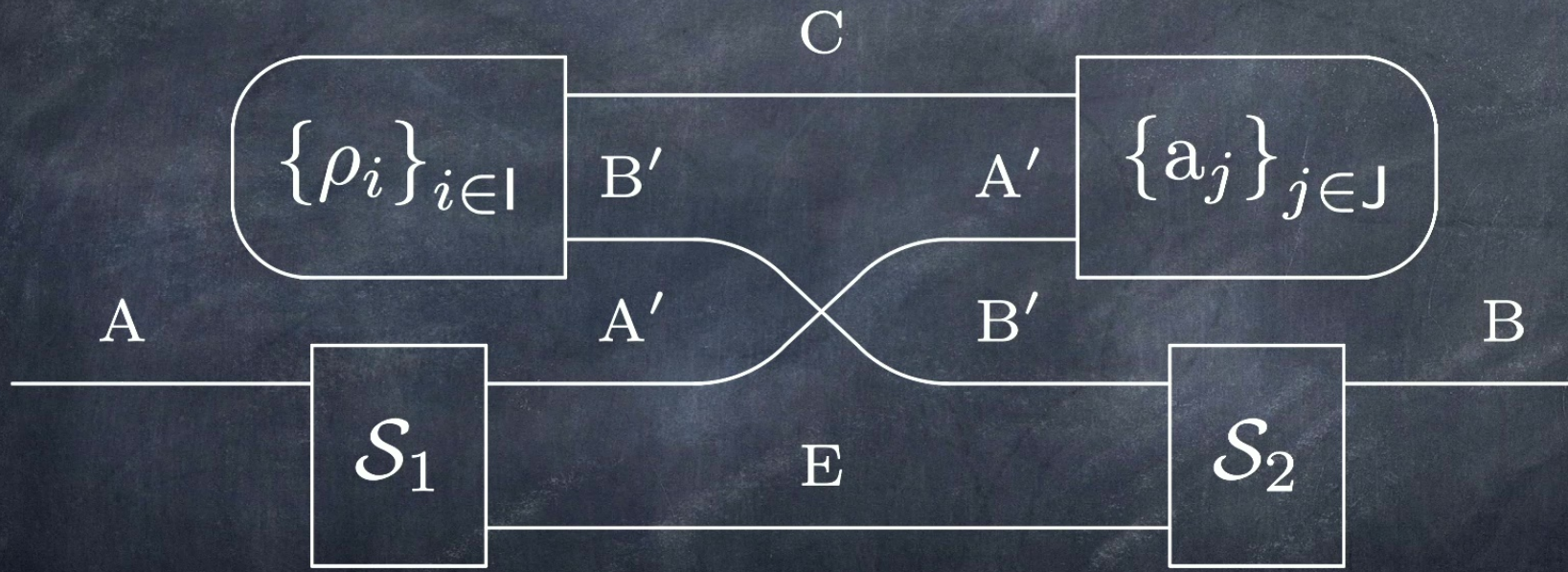


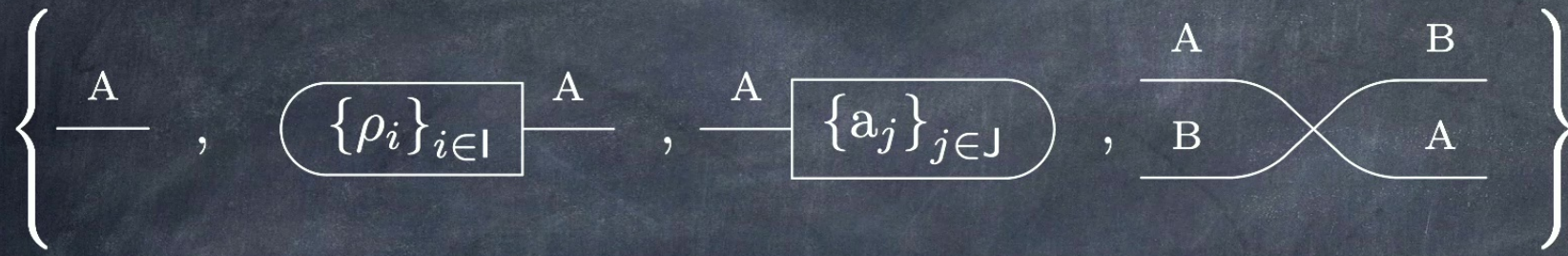
Measurement
incompatibility

Irreversible
disturbance

MSBCT

MCT





Classical
conditioning