Title: Testing General Relativity with Ensembles of Compact Binary Mergers: the Importance of Astrophysics and Statistical Assumptions

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Collection/Series: Strong Gravity

Subject: Strong Gravity

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Abstract:

Observations of gravitational waves from binary black-hole mergers provide a unique testbed for General Relativity in the strong-field regime. To extract the most information, many gravitational-wave signals can be used in concert to place constraints on theories beyond General Relativity. Although these hierarchical inference methods have allowed for more informative tests, careful consideration is needed when working with astrophysical observations. Assumptions about the underlying astrophysical population and the detectability of possible deviations can influence hierarchical analyses, potentially biasing the results. In this talk, I will address these key assumptions and discuss their mitigation. Finally, I will demonstrate how we can leverage the astrophysical nature of gravitational-wave observations to our advantage to empirically bound the curvature dependence of extensions to General Relativity.

Testing General Relativity with Ensembles of Compact Binary Mergers:
The Importance of Astrophysics and Statistical Assumptions

Ethan Payne

Strong Gravity Seminar - Nov 14
Perimeter Institute

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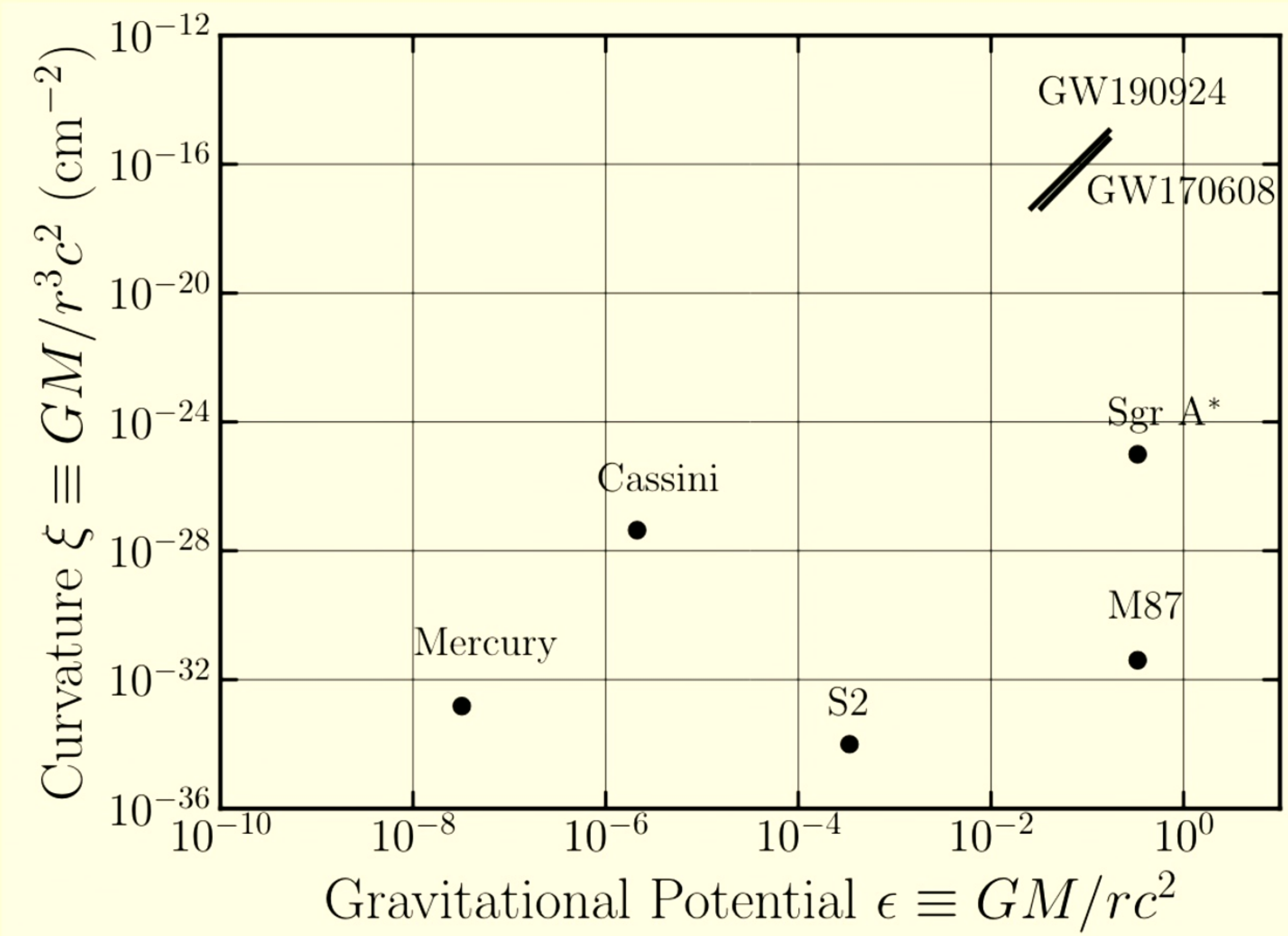
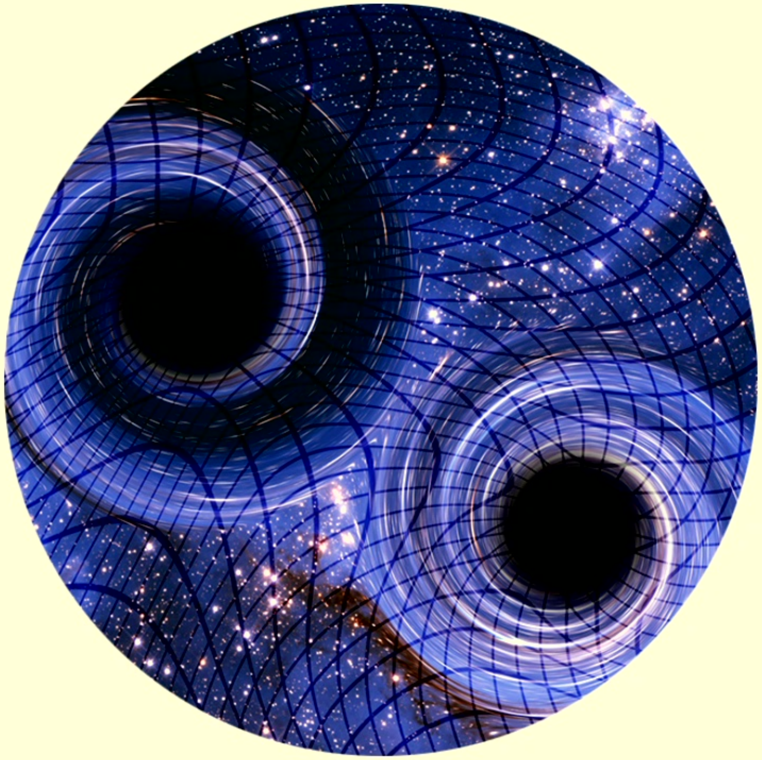


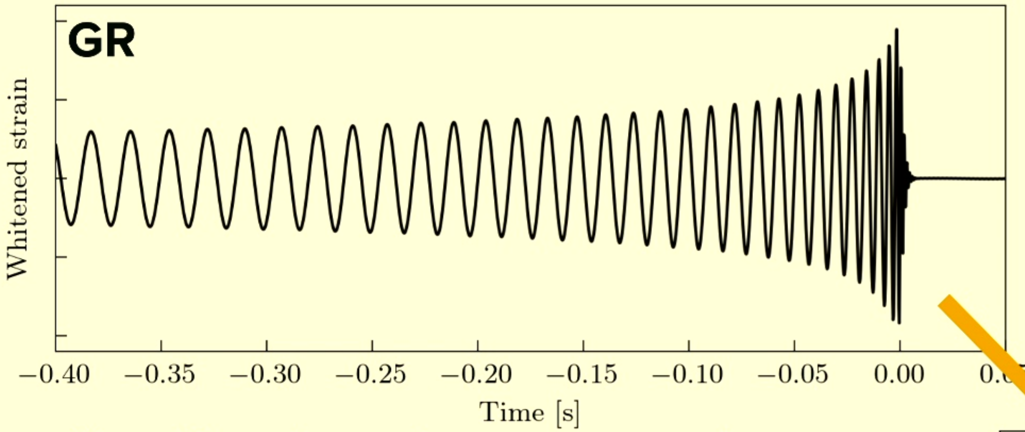
Summary plot from Psaltis, Talbot, **EP**, Mandel (2021)

Overview of the talk

1. How do we analyse gravitational-wave signals for potential violations of general relativity (GR)?
2. The roles of astrophysical and statistical assumptions in testing GR
3. How can we leverage the astrophysical nature of gravitational-wave (GW) sources to improve our tests of GR?

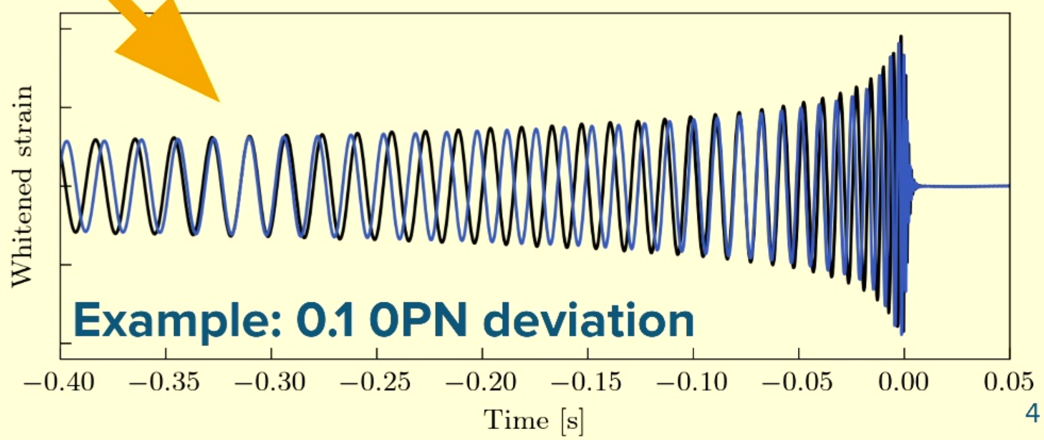


Testing general relativity with gravitational waves



Primarily focus on modelled waveforms to capture one of:

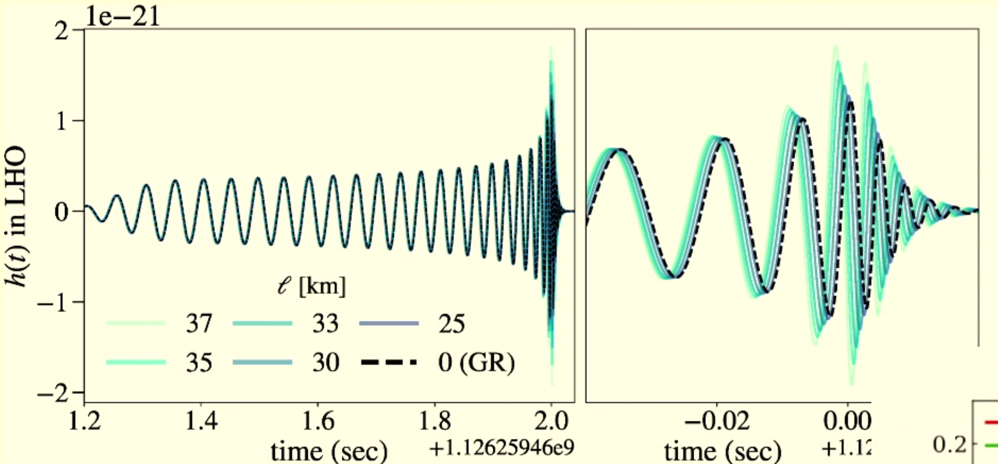
1. **Post-Newtonian (PN) phase deviations**
2. **Dispersion due to propagation effects (e.g. a massive graviton)**
3. Ringdown frequency and damping time differences
4. Theory-specific behaviour and coupling coefficients
5. Gravitational-wave birefringence



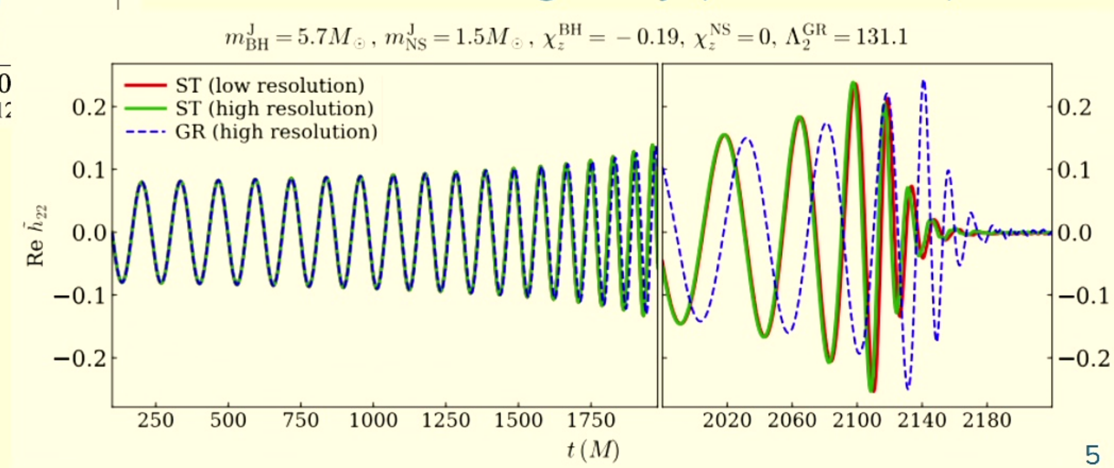
and more!

Are these “simple” GW dephasing models appropriate?

Dynamical Chern-Simons (Okounkova+, 2023)



Scalar-tensor gravity (Ma+, 2023)

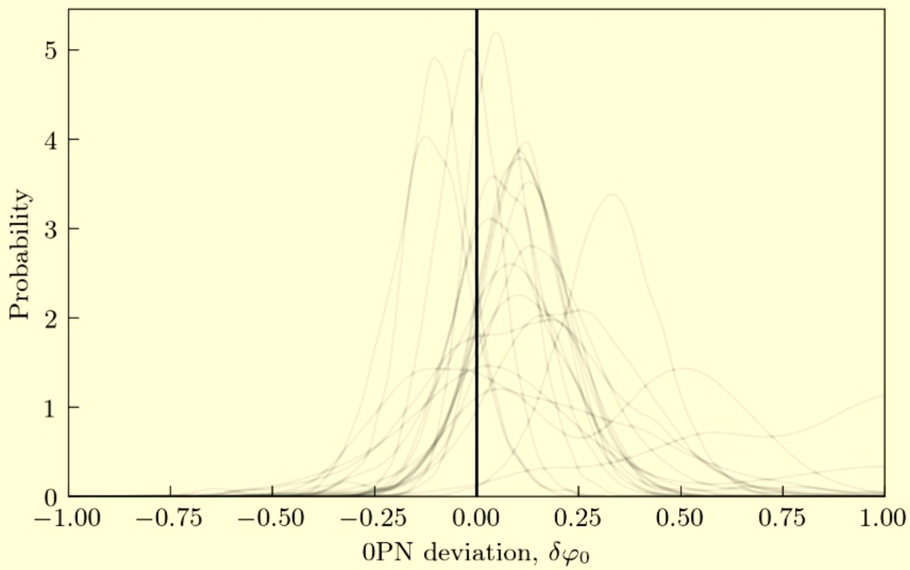


- Seems so
- Use PN tests as main example for the remainder of the talk
  - Discussions here are applicable to all other tests as well

5

Post-Newtonian deviation constraints

- Model as a fractional deviation to GW phase evolution at a specific PN order
  - Null test of general relativity
- Motivated by specific theories (previous slide)
- Can infer probability distribution on deviation and astrophysical parameters
- Any individual observational constraints are not particularly strong...



6

Post-Newtonian deviation constraints

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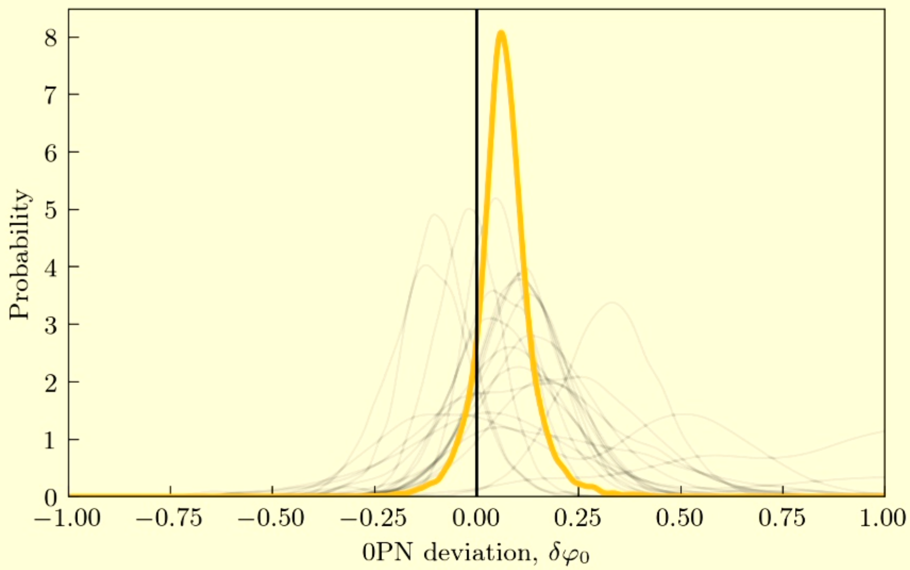
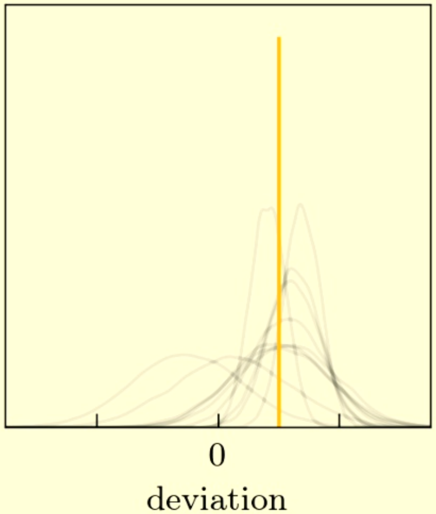


Gain more information via combining observations

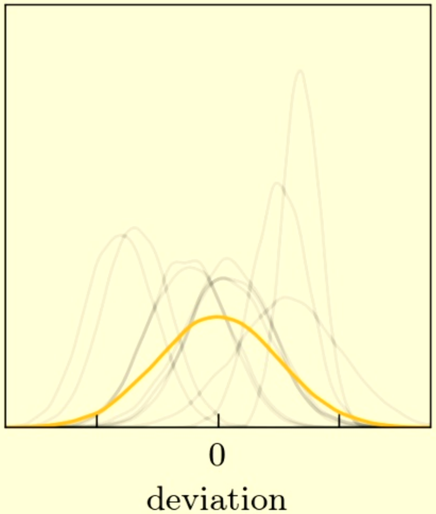
Classes of GR deviation populations

Shared parameter



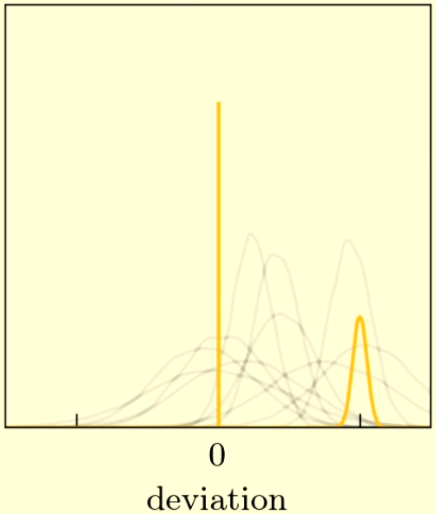
Examples: graviton mass, birefringence, specific theories

Distribution of deviations



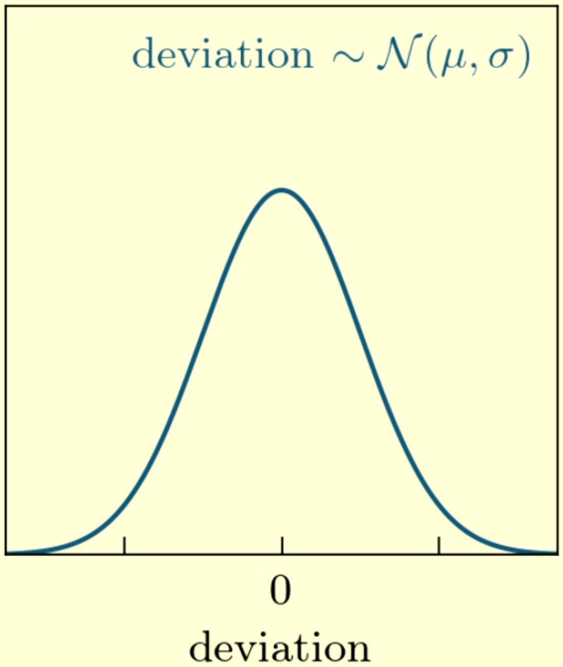
Examples: null tests, stochastic quantum effects

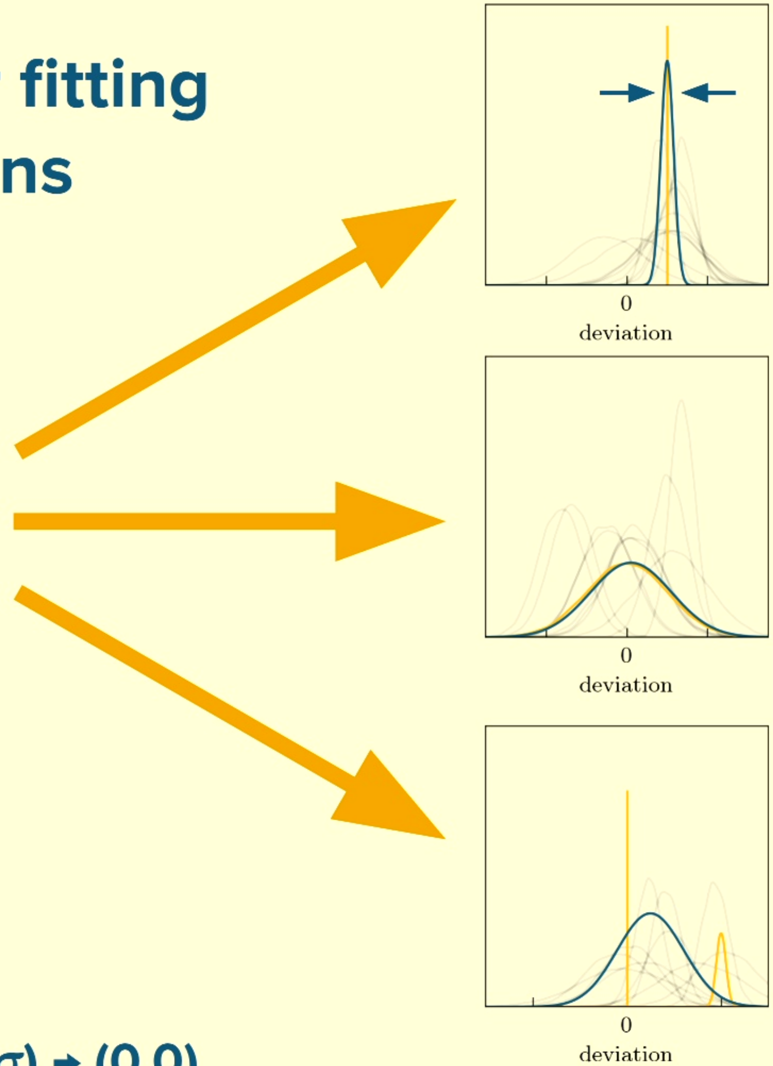
Distinct exotic objects



Examples: exotic compact objects

Gaussian model for fitting deviation populations





Shared parameter

Distribution of deviations

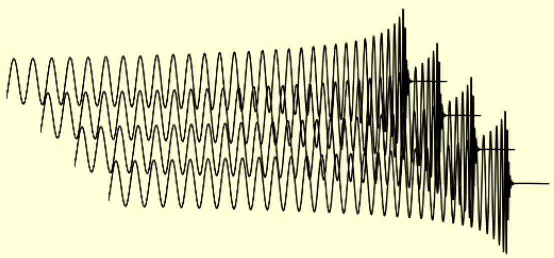
Distinct exotic objects

Following from Isi+ (2019)

Infer (μ,σ); **recovers GR as (μ,σ) → (0,0)**

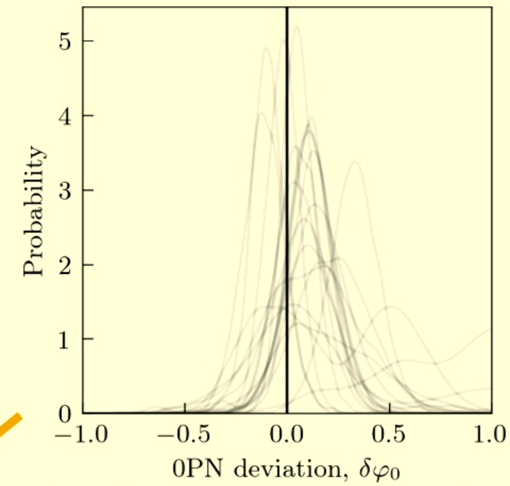
Testing of GR hierarchically

Ensemble of observations

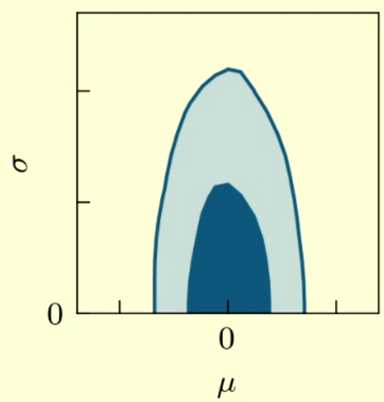


Bayesian inference

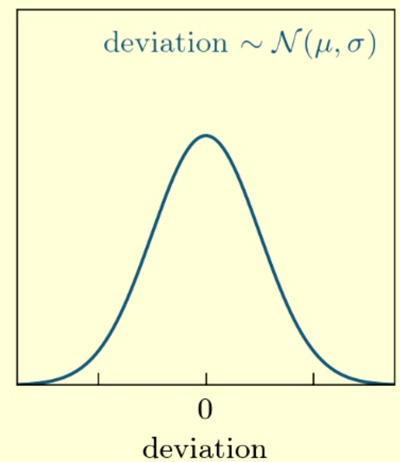
$$p(\theta|d) \propto p(d|\theta)\pi(\theta)$$



+ astrophysical parameters (masses, spins, redshifts)



Hierarchical inference with Gaussian GR deviation population model



Understanding the hierarchical inference likelihood

Calculation heavy-lifting contained within the hierarchical likelihood

$$p(\{d\}|\Lambda) = \frac{1}{\xi(\Lambda)^N} \prod_{i=1}^{N} \int \mathrm{d}\theta_i \, p(d_i|\theta_i) \, \pi(\theta_i|\Lambda)$$

$\Lambda = \text{population parameters}$

$\theta = \text{individual event parameters}$

$$\xi(\Lambda) = \int \mathrm{d}\theta \, p_{\mathrm{det}}(\theta)\pi(\theta|\Lambda)$$

Individual GW event likelihood:

Many studies on the impact of:

- Waveform systematics (Moore+, 2021)
- Detector noise (glitches; Kwok+, 2022)
- Missing physics (e.g. eccentricity; Saini+, 2022)

Assumptions in hierarchical tests of GR

However, little consideration for the surrounding assumptions

$$p(\{d\}|\Lambda) = \frac{1}{\underline{\xi(\Lambda)^N}} \prod_{i=1}^{N} \int \mathrm{d}\theta_i \, p(d_i|\theta_i) \, \underline{\pi(\theta_i|\Lambda)}$$

$\Lambda = \text{population parameters}$

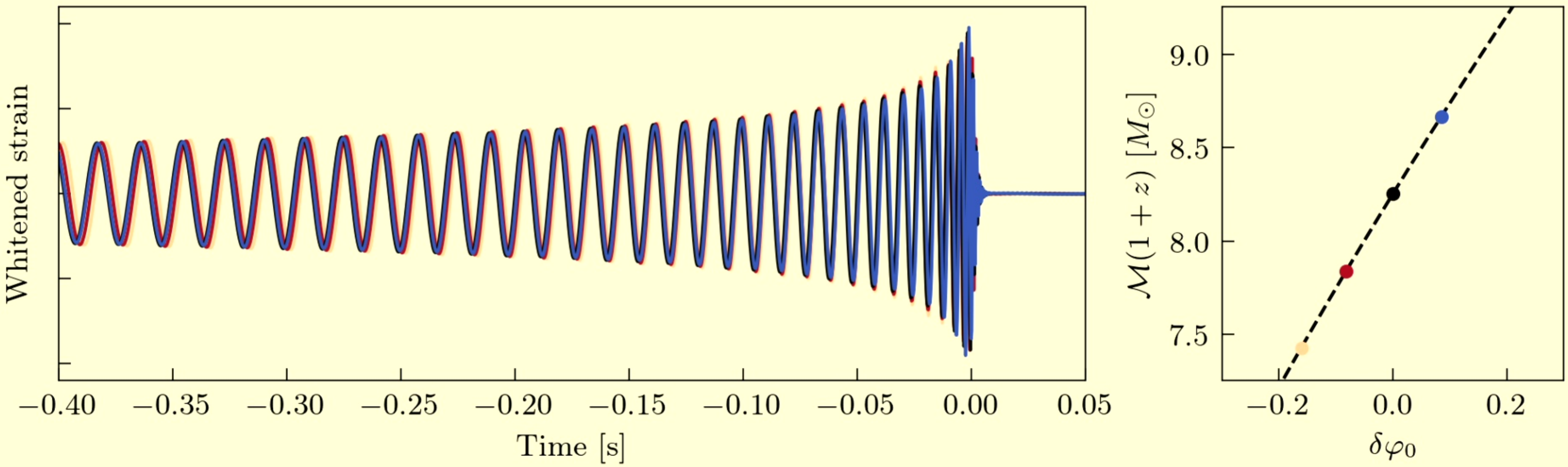
$\theta = \text{individual event parameters}$

Choice of astrophysical distributions:

Assume astrophysical distribution chosen during initial analysis good enough while testing GR

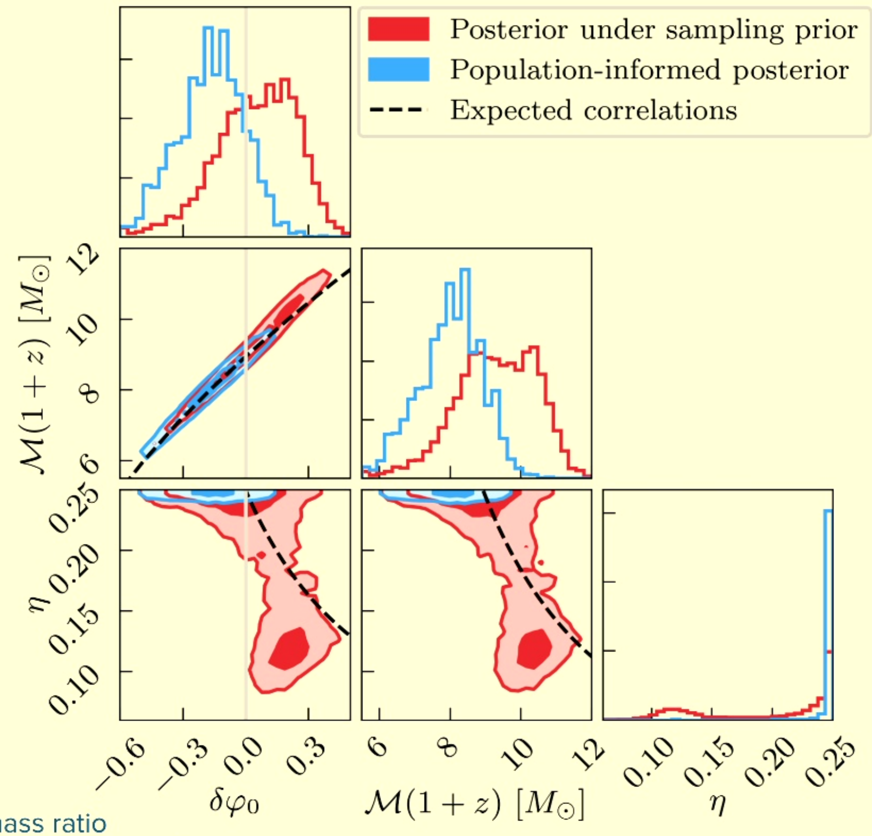
Why does the astrophysical population matter?

$$\Phi_{0\mathrm{PN}}(f) = \frac{3(1+\delta\varphi_0)}{128}\Big(\pi\mathcal{M}(1+z)f\Big)^{-5/3}$$ → **Correlation!**



12

Implications for gravitational-wave observations



The inferred astrophysical population:

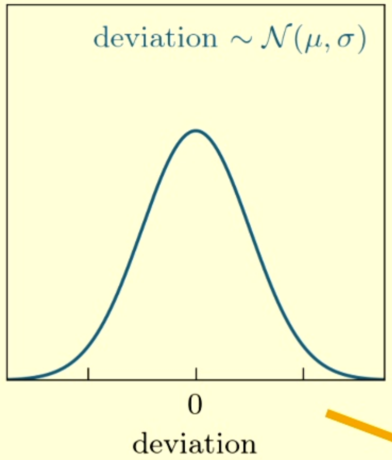
1. Prefers lighter BBHs
2. Prefers more equal-mass BBHs

Moves support to negative deviations

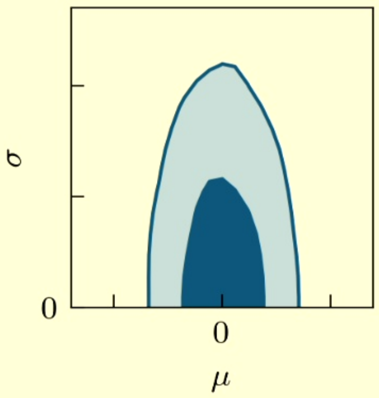
Astrophysical assumptions matter!

η = symmetric mass ratio
50% and 90% credible intervals

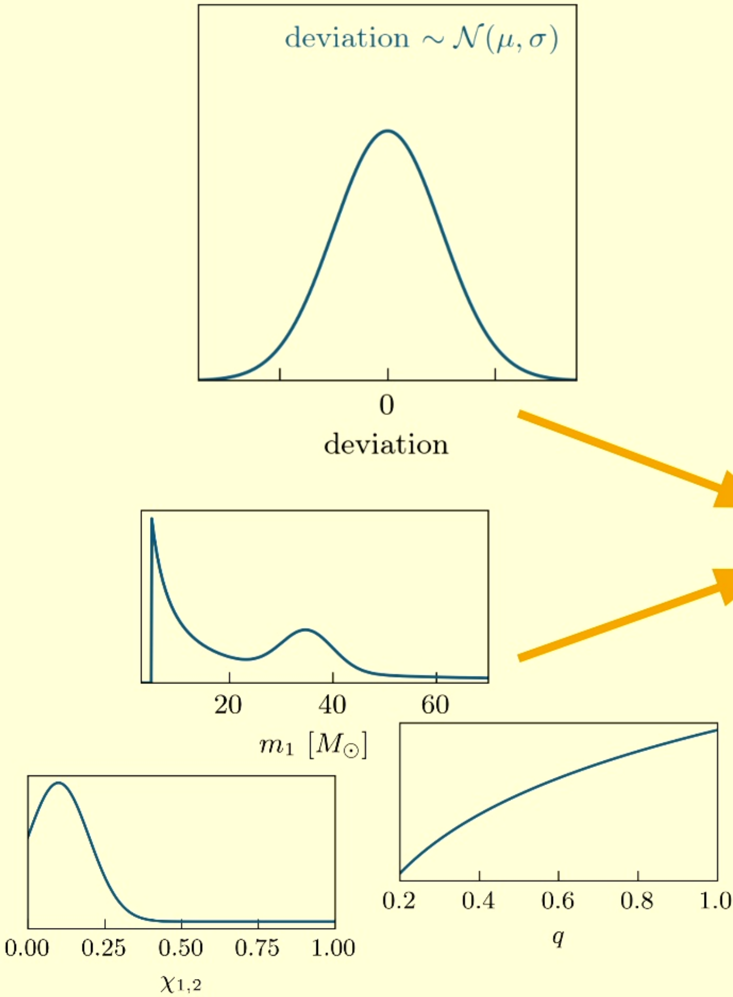
What is the solution?



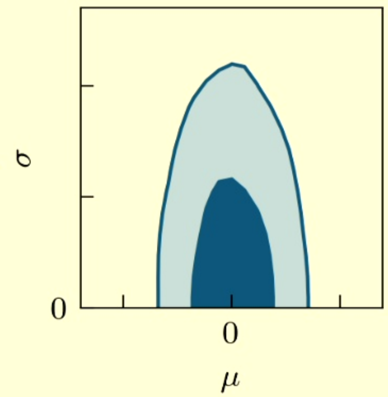
Hierarchical inference with Gaussian GR deviation population model



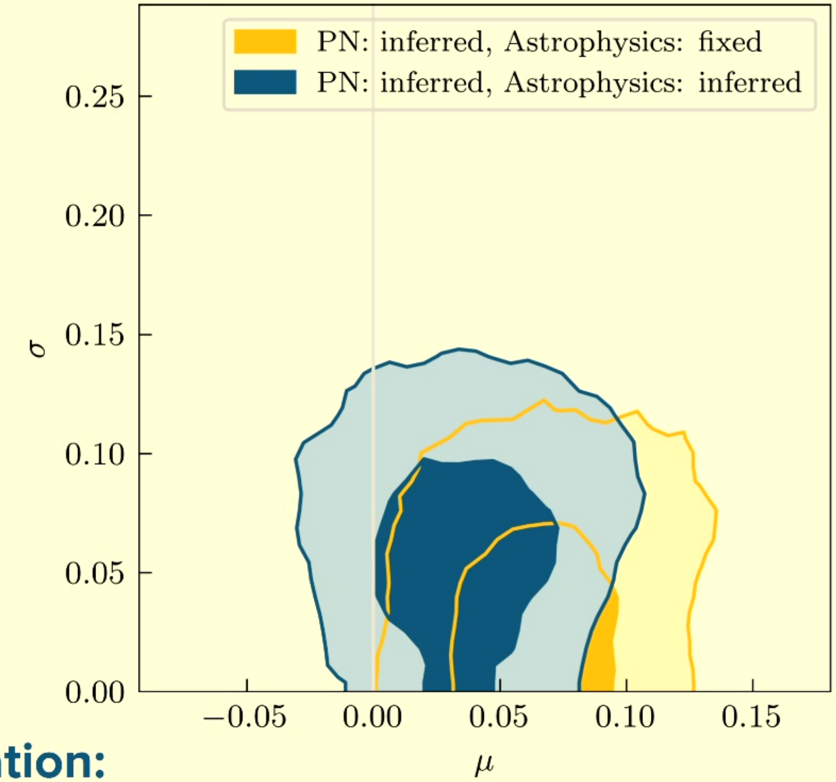
What is the solution?



Hierarchical inference with Gaussian GR deviation **and astrophysical** population models

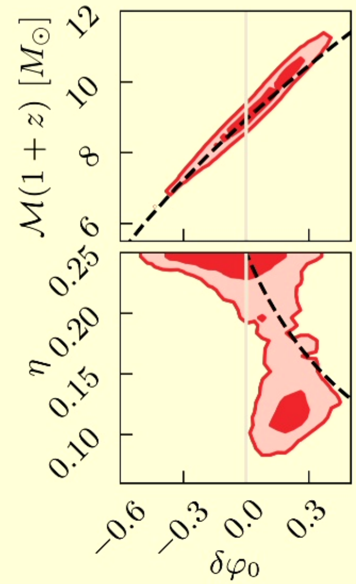


Constraints on 0PN deviations from gravitational waves



Incorporating the astrophysical population **pulls distribution to be more consistent with GR**

Related directly to these features...

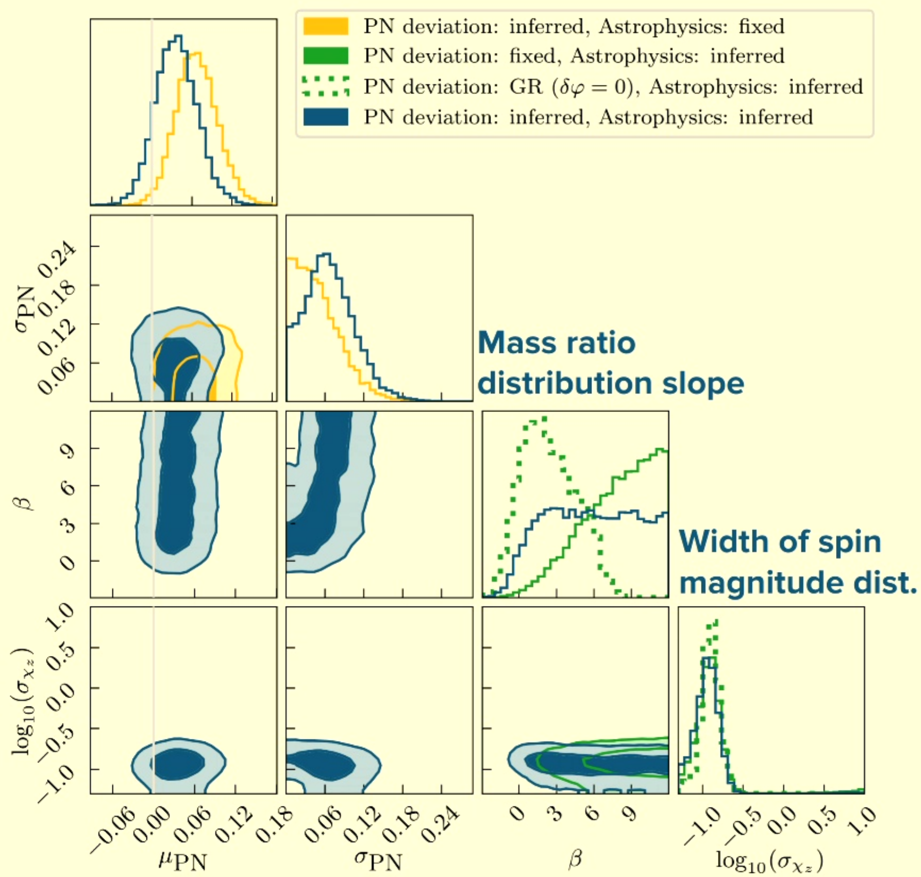


Notation:

Fixed: Use default sampling priors

Inferred: Model the population distribution

Example: 0PN deviation coefficient



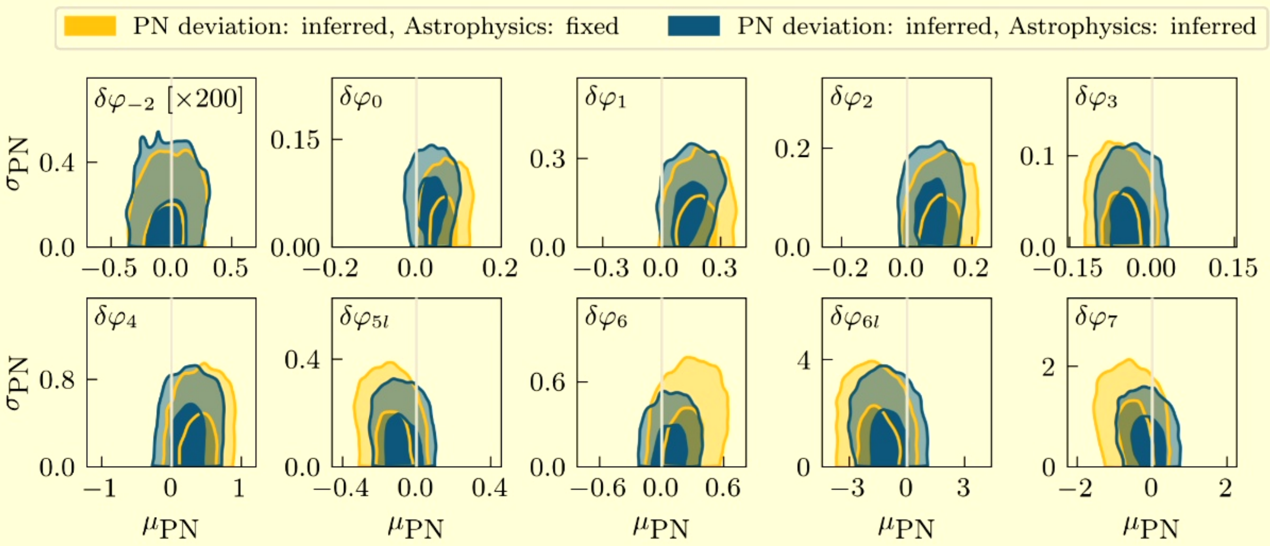
- Inference for more support for similar black-hole masses in a binary (q=1)
- Interplay between the inferred mass-ratio distribution slope and width of the 0PN deviation distribution
- Also highlights why the astrophysical prior for tests of GR cannot be set to one specific choice
  - All astrophysical parameter values are equally “valid”

17

Impact on post-Newtonian gravitational-wave tests

$\delta\varphi_k$ corresponding to deviations in the (k/2)PN term

Constrain mean and standard deviation of PN deviations in O3



More consistent with GR by **0.4σ, on average**, when modelling the astrophysics

Assumptions in hierarchical tests of GR

Addressed one assumption, what about the selection function?

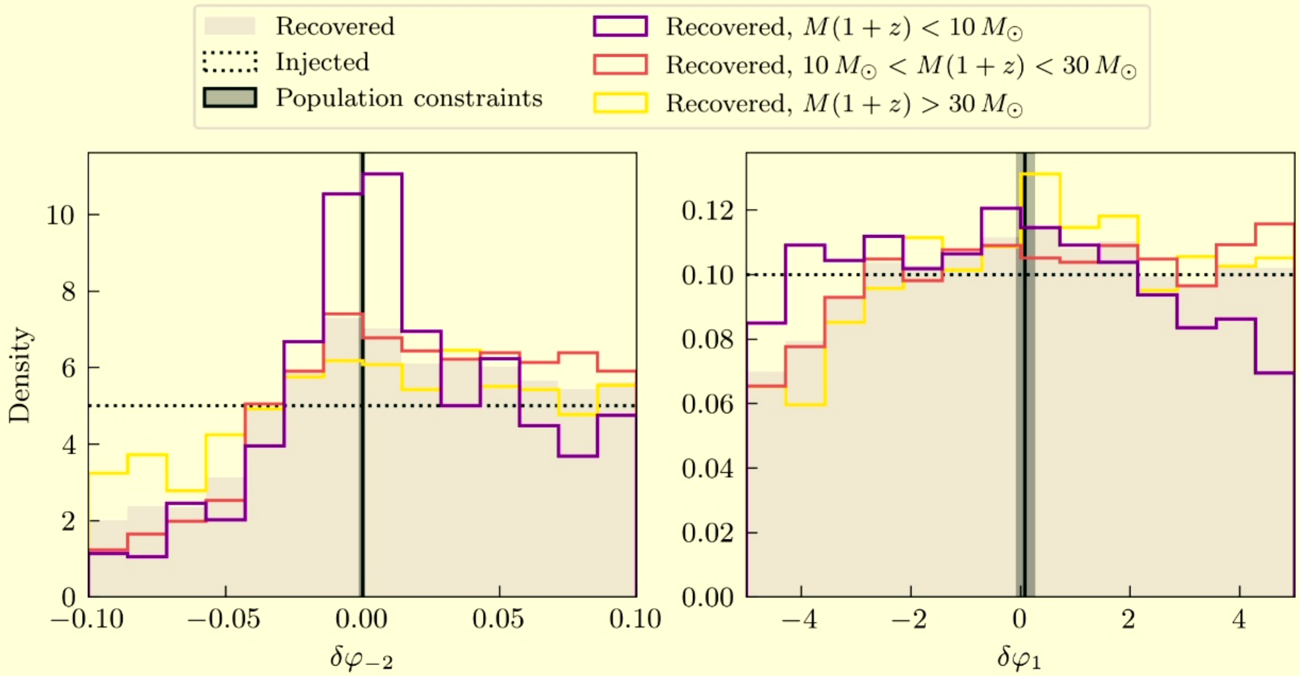
$$p(\{d\}|\Lambda) = \frac{1}{\xi(\Lambda)^N} \prod_{i=1}^{N} \int \mathrm{d}\theta_i \, p(d_i|\theta_i) \, \pi(\theta_i|\Lambda)$$

Detectability of the GR deviation population:

Assume all GR deviation populations are equally detectable, i.e. $\xi(\Lambda) = \mathrm{constant}$

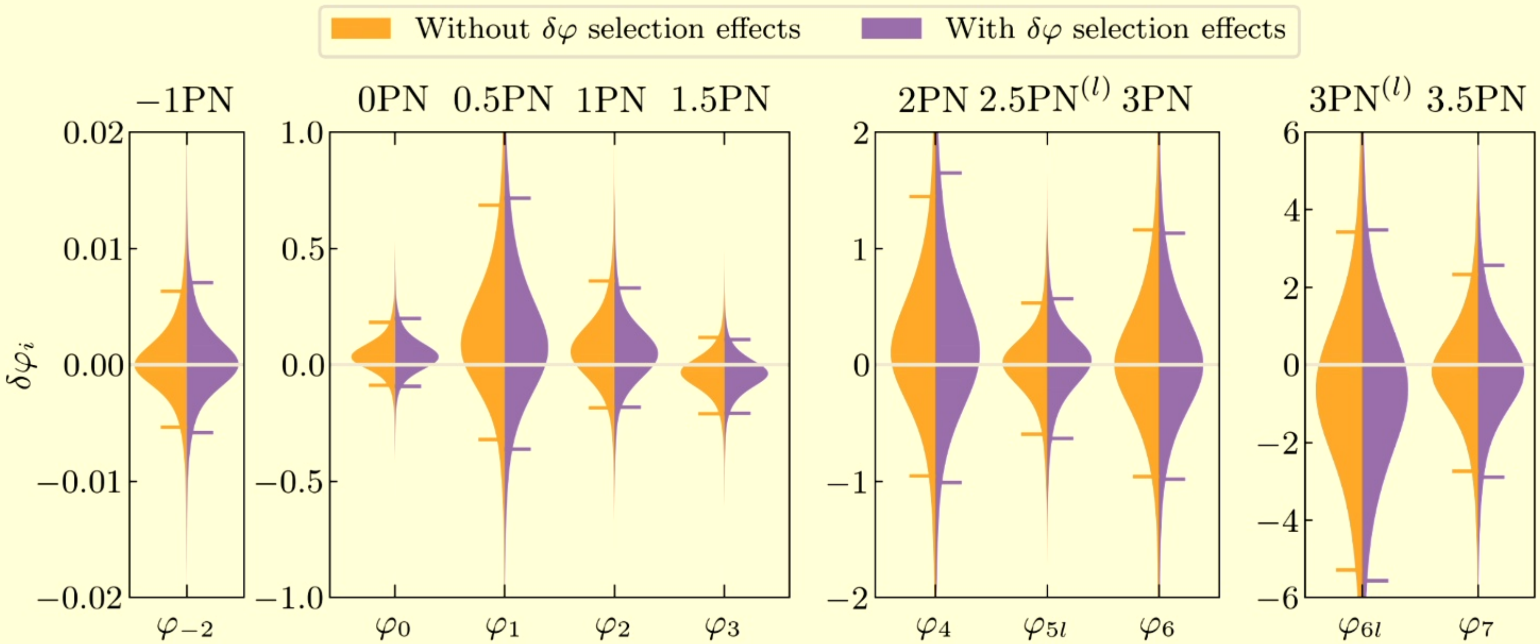
Detection probability of GR deviations

Measure the detectability of a particular deviation through simulated signals:



20

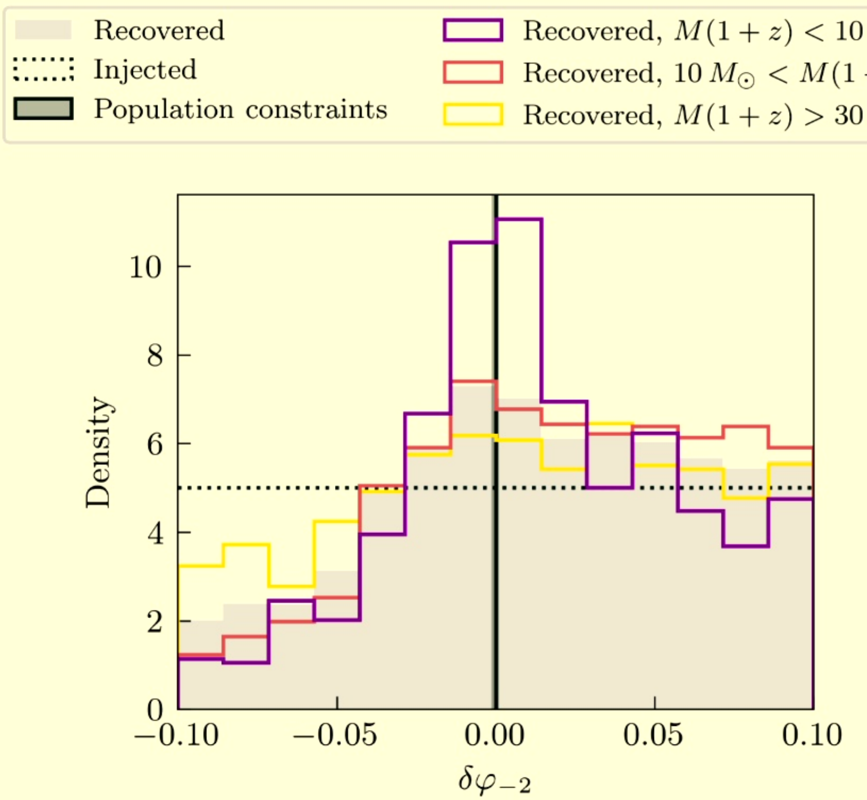
Impact on hierarchical PN deviations tests

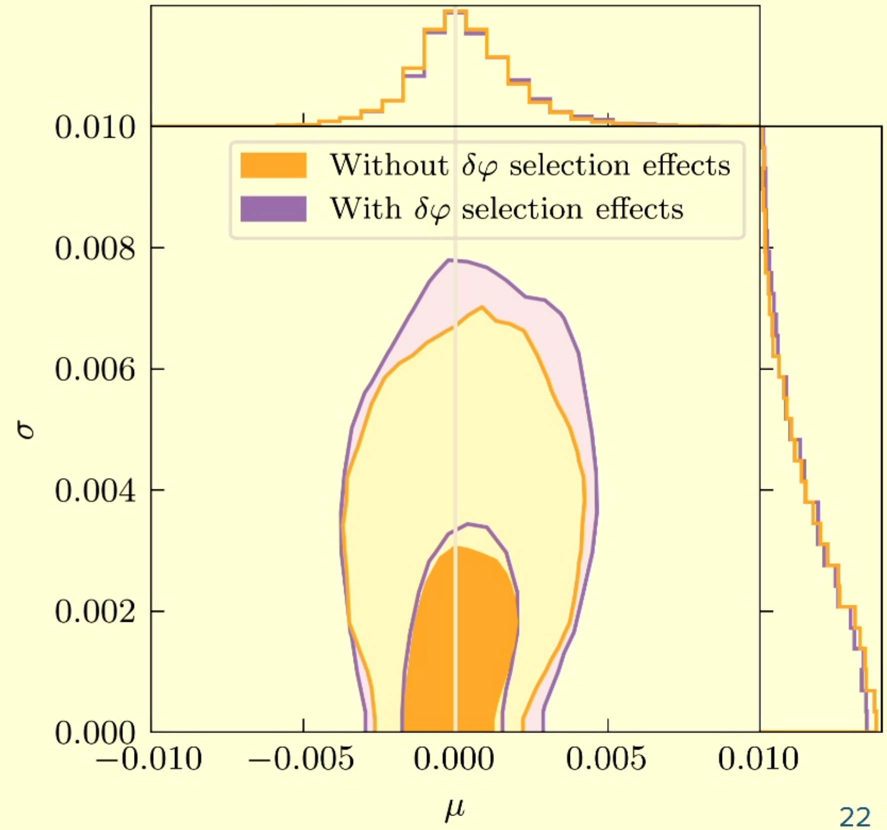


Show possible range of deviations with and without selection effects

Only a small change in the inferred bounds on PN deviations coefficients

Why these selection effects are not very impactful





22

Summary of the solutions presented

Addressed how we can deal with other assumptions surrounding hierarchical tests of general relativity:

$$p(\{d\}|\Lambda) = \frac{1}{\xi(\Lambda)^N} \prod_{i=1}^{N} \int \mathrm{d}\theta_i \, p(d_i|\theta_i) \, \pi(\theta_i|\Lambda)$$

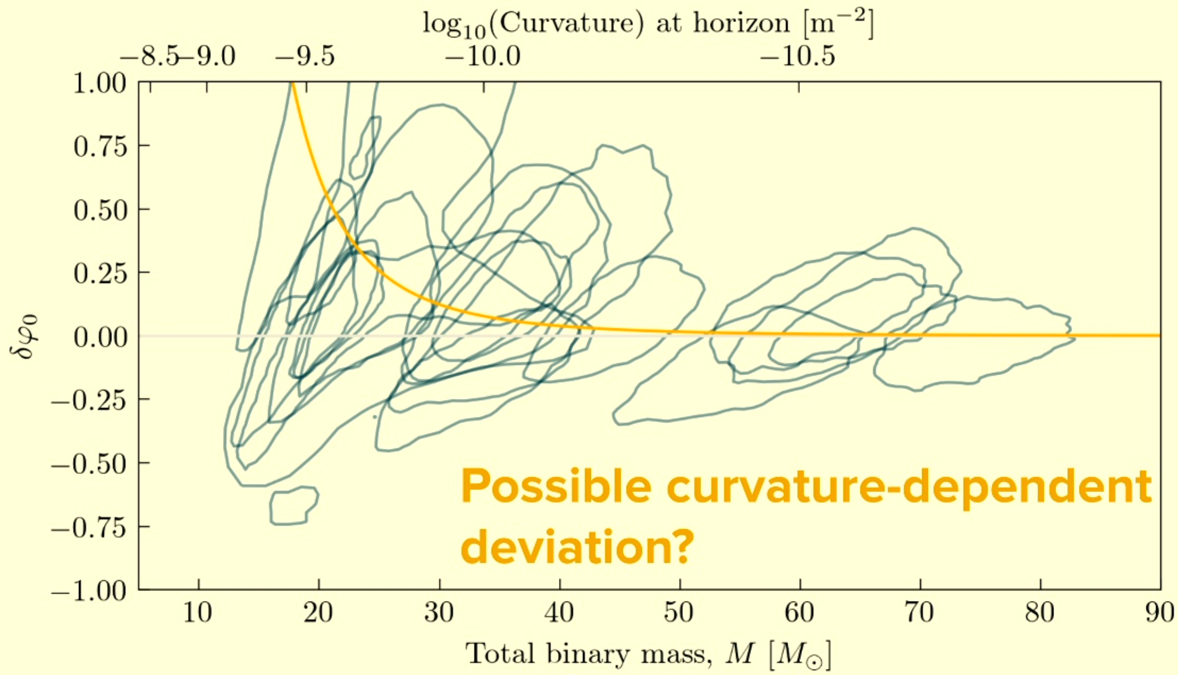
Detectability of the GR deviation population:

Solution: estimate the selection function within the GW detectors as a function of the deviation (Magee, Isi & **EP**+, 2024)

Choice of population distributions:

Solution: jointly model the astrophysical population when testing GR (**EP**+, 2023)

Leveraging the mass - deviation distribution



Possibility for extracting more information regarding the curvature dependence of any possible GR deviations present (**EP**+, 2024)

Expectations from extensions to GR

- Generically write extensions to GR as a modification of the Lagrangian:

$$\mathcal{L} = \mathcal{L}_{\rm GR} + \lambda\, F_\gamma(\mathcal{R}, \phi)$$

$$F_\gamma(\mathcal{R}, \phi) \sim \text{curvature}^\gamma \quad [\lambda] = \text{length}^{2(\gamma-1)}$$

- Specific theories possess specific predictions on the gravitational-wave
- There are **near-universal predictions** regarding the curvature dependence

$$\text{deviation} \sim \frac{\lambda}{\ell_{\rm obs}^{2(\gamma-1)}}$$

Expectation for gravitational-wave observations

Therefore, dimensionless deviations GW observations measure must scale as:

$$\lambda/M^{2(\gamma-1)}$$

no additional fields

$$(\lambda/M^{2(\gamma-1)})^2$$

additional fields

$$M^{-p}$$

Example from classes of theories:

p = 0: propagation-based effects

p = 4: scalar fields (dCS, EdGB), cubic EFT extensions

p = 6: quartic EFT extensions

From the masses of our GW observations, we can infer p and make statements about the underlying theory - without any explicit modelling

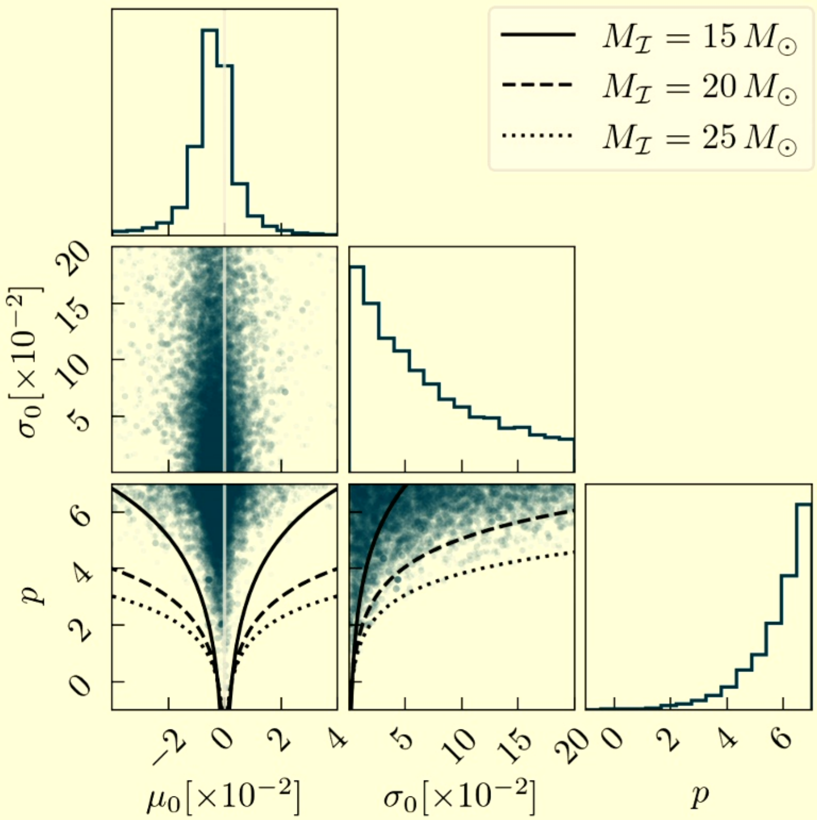
Implementation in hierarchical tests of GR

- Need to study this in the population to infer *p* over range of masses
- Use hierarchical inference methods laid out previously
  - Need mass distribution inference to accurately recover the scaling
- We extend the GR deviation distribution to a Gaussian conditioned on M

$$\mu = \mu_0 \left(\frac{M}{10\, M_\odot} \right)^{-p}, \quad \sigma = \sigma_0 \left(\frac{M}{10\, M_\odot} \right)^{-p}$$

We can infer μ_0, σ_0, and p to both determine if GR is violated — $(\mu_0,\sigma_0) \neq (0,0)$ — and, if it is, the curvature scaling at which it is violated

Constraints on -1PN deviation from GWTC-3



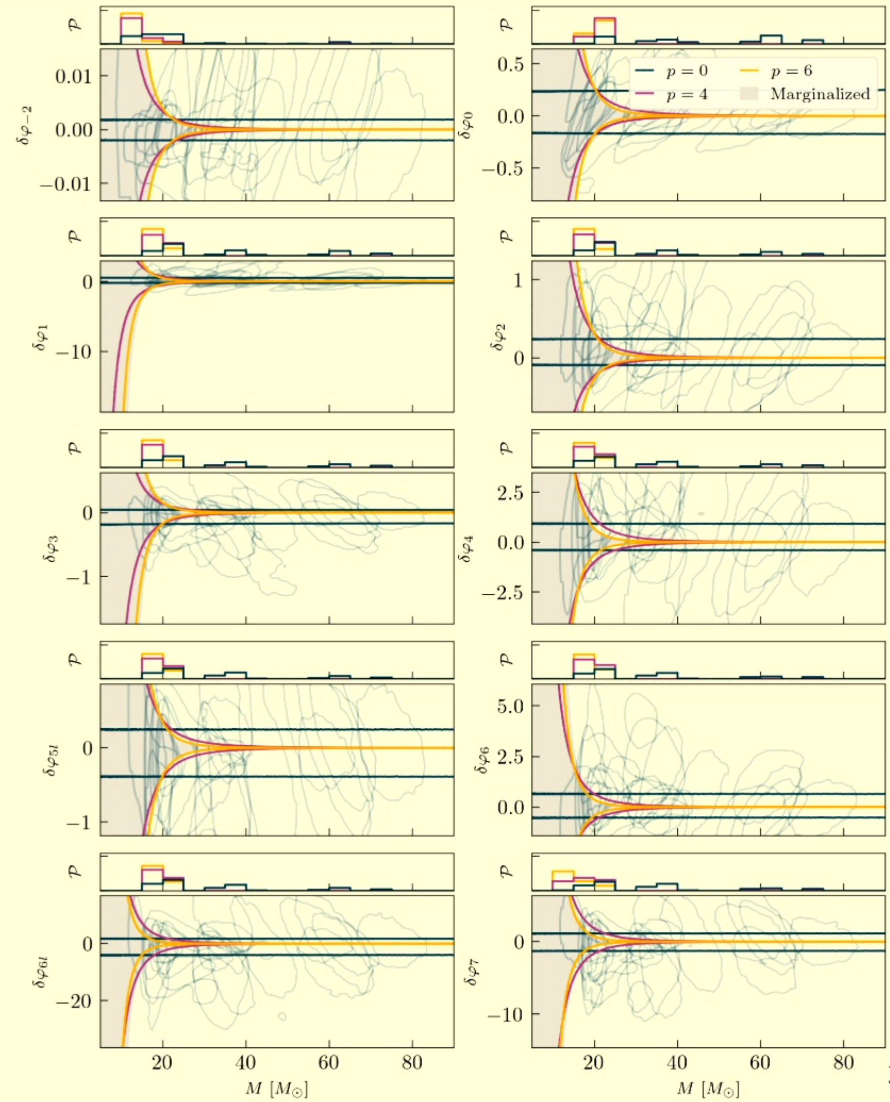
- Without measurable GR deviation, constraints follow:

$$\{\mu_0, \sigma_0\} \left(\frac{M_{\mathcal{I}}}{10\,M_\odot} \right)^{-p} \sim \text{const.}$$

- Features present for all analyses at all PN orders

Bounds in mass - deviation space

- Plot the inferred bounds:
  - Consistent with GR
  - Marginalized over distribution of p (grey)
  - For fixed values of p (colours)
- For p > 0, constraint dominated by the lighter binary BH systems
- Quantify with a curvature index weighted per-bin "precision"
  - Typically most informative around ~20 $M_{\odot}$



Conclusion

Gravitational-wave observations from **stellar-mass BBH mergers** provide a uniquely powerful **test-bed for general relativity**

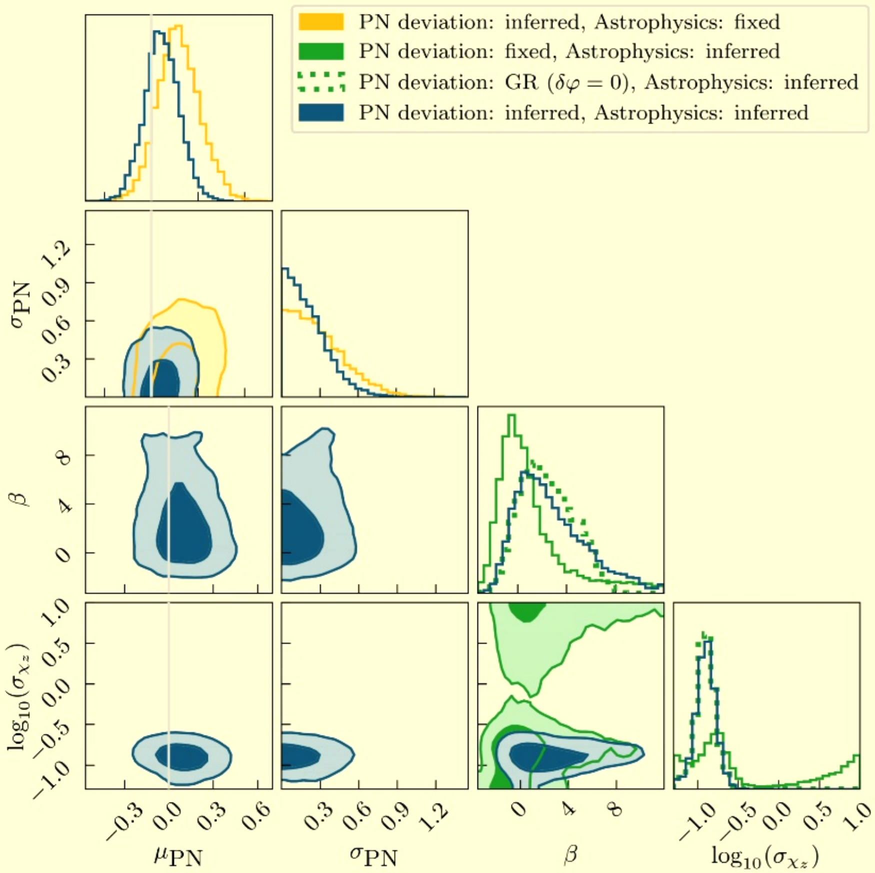
Though signals are quiet in GW detectors, **hierarchically combining observations can place stringent constraints** on deviations from general relativity

Care needs to be taken to not introduce biases from poor statistical assumptions

Demonstrated how we can mitigate the **impact of astrophysical prior choices and detection efficiency of GR deviations** on our hierarchical tests

Leveraged the astrophysical origin of gravitational-wave observations to show to **directly infer the curvature dependence** of modifications to GR without invoking any specific extension

Example: 3PN deviation coefficient



Deviation parameter with the **greatest increase in support for GR**

3PN deviation broadens the inferred mass ratio and spin distributions

Joint modelling infers both small-to-no GR deviations and reasonable mass ratio and spin distributions

37