Title: TBA - Quantum Fields & Strings Seminar

Speakers: Laura Donnay

Collection/Series: Quantum Fields and Strings

Subject: Quantum Fields and Strings

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$w_{1+\infty}$ symmetries in Carrollian holography

w/ Laurent Freidel, Yannick Herfray

Laura DONNAY

Quantum Fields & Strings Seminar
Perimeter Institute
Nov 5 2024

SISSA

Motivation

The holographic principle → **How general is it?**

Quantum gravity is *encoded* in a different theory that lives in a *lower-dimensional* spacetime.

['t Hooft '93; Susskind '94; Maldacena '97]

Flat space holography: a *structure X* ?

(13)

THE REAL OBSTACLE TO AN ANALOGOUS SUCCESS WHEN $\Lambda=0$ SEEMS TO BE THAT THE NATURAL BOUNDARY OF MINKOWSKI SPACE IS NOT AT SPATIAL INFINITY BUT AT PAST AND FUTURE NULL INFINITY

(24)

A HOLOGRAPHIC DESCRIPTION FOR $\Lambda=0$, IF THERE REALLY IS SUCH A THING, MUST INVOLVE NOT C.F.T. BUT SOMETHING ELSE – CALL IT "STRUCTURE X" AS WE DON'T KNOW WHAT IT IS.

E. Witten's talk - *Strings* 1998

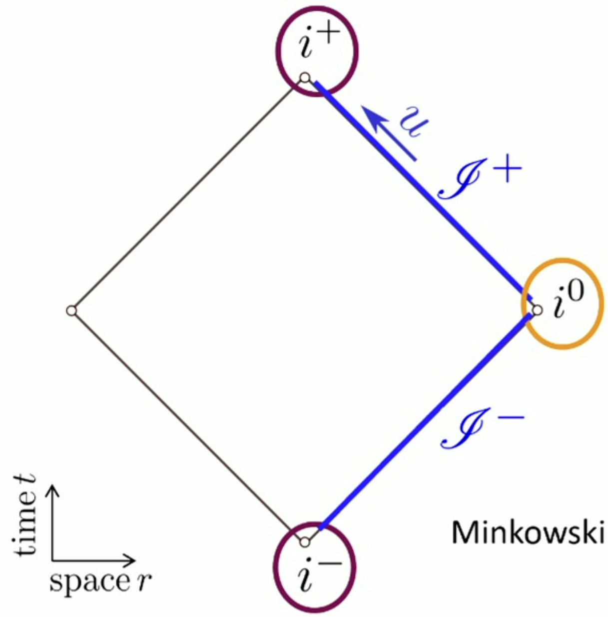
Flat space holography

Holographic description of quantum gravity in 4d asymptotically flat spacetimes?

Main obstructions/difficulties:

① The conformal **boundary** includes

future/past timelike infinity
future/past null infinity
spatial infinity



Flat space holography

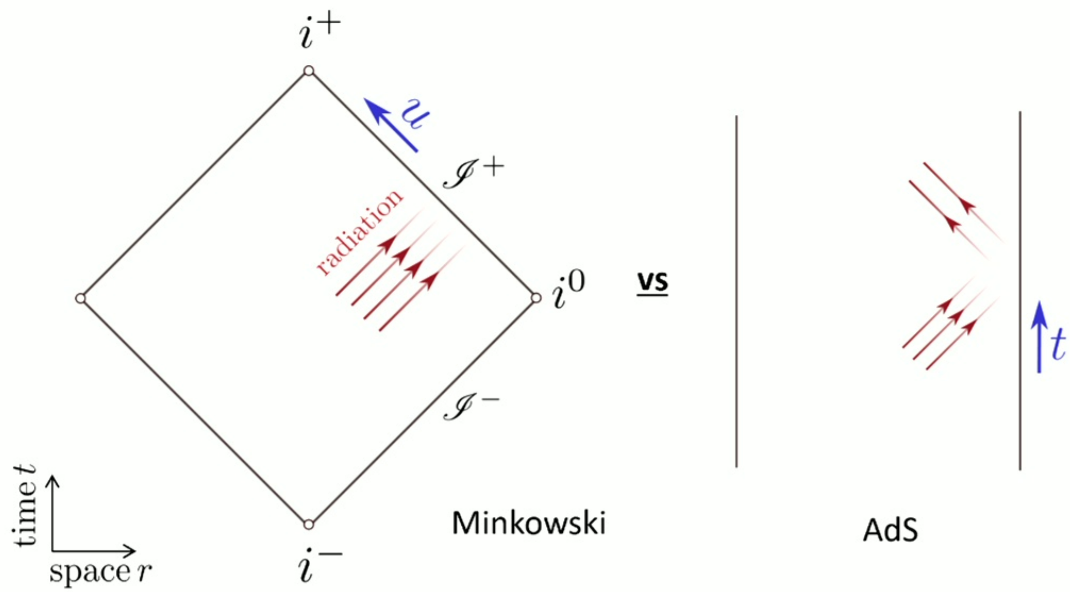
Holographic description of quantum gravity in 4d asymptotically flat spacetimes?

Main obstructions/difficulties:

1. The conformal **boundary** includes

   future/past timelike infinity
   future/past null infinity
   spatial infinity

2. There are **fluxes** leaking out the boundary



Flat space holography

Holographic description of quantum gravity in 4d asymptotically flat spacetimes?

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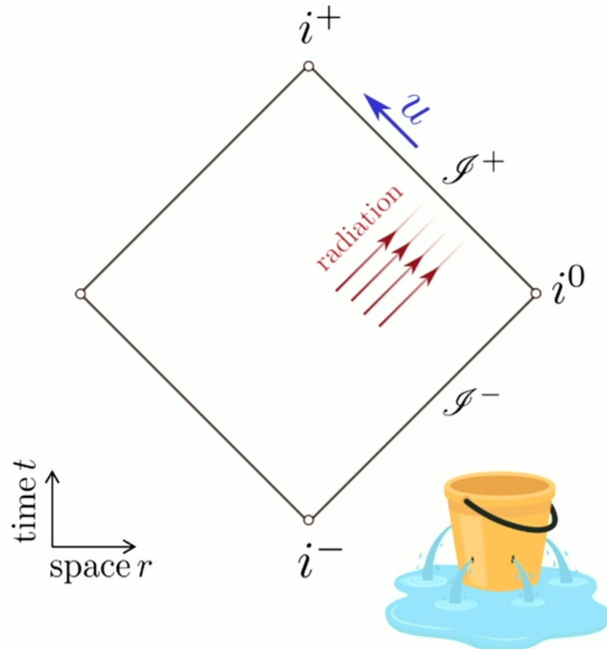
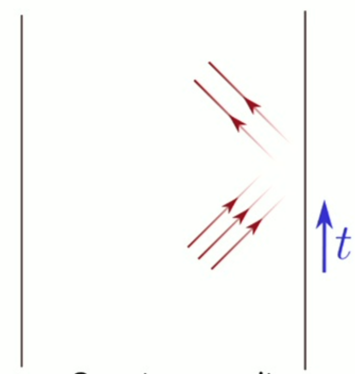


vs

AdS



Quantum gravity 'in a **box**'

Flat space holography

Road map: symmetries

- The phenomenon of **symmetry enhancement** is a key feature of asymptotically flat spacetimes, due to the presence of gravitational radiation

other example: AdS_3 / CFT_2

asymptotic symmetry group of AdS_3

↕

the conformal group in $d = 2$ gets enhanced to Virasoro

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Flat space holography

Road map: symmetries

- The phenomenon of **symmetry enhancement** is a key feature of asymptotically flat spacetimes, due to the presence of gravitational radiation

4 Poincaré translations

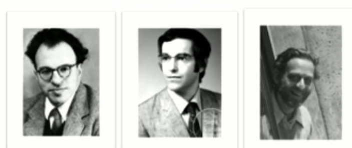
↓ *Symmetry enhancement*

∞ BMS supertranslations

what was expected

Poincaré

what was found



Bondi-Metzner-Sachs (BMS) ('62)

+ van der Burg

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Flat space holography

--> Road map: symmetries

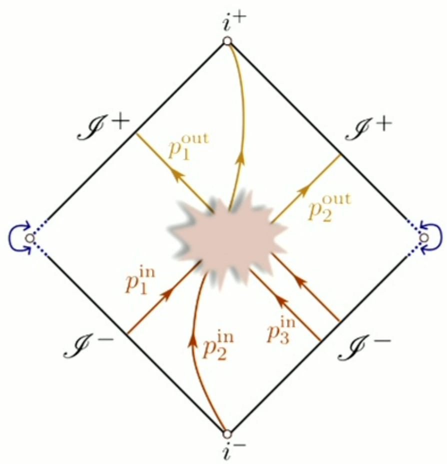
- The phenomenon of symmetry enhancement is a key feature of asymptotically flat spacetimes, due to the presence of gravitational radiation

While BMS symmetries were originally disregarded, it was realized (50 years later, [Strominger '13]) that they

- constrain the gravitational **S-matrix**
- have associated low-energy **observables** (memory effects)
- allow further extensions, including the local **conformal** group

↓

revival of **flat holography**



$\langle \text{out}|S|\text{in}\rangle$

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Which boundary?

null infinity

lighlike 3d hypersurface

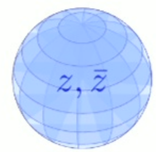
$$\mathscr{I} = \mathbb{R} \times S^2$$

Looking for a
3d 'BMS field theory'

Carroll
Holography

celestial sphere

Euclidean 2d-sphere



Looking for a
2d 'celestial CFT'

Celestial
Holography

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Salvador Dalí, illustrations for *Alice's Adventures in Wonderland*, 1969:

Outline

Carrollian holography

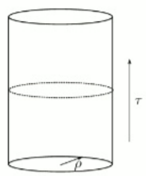
$\mathscr{L}w_{1+\infty}$ symmetries

Final remarks

Carroll symmetries

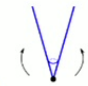
Holographic duality: step 0

- AdS$_{d+1}$ / CFT$_d$

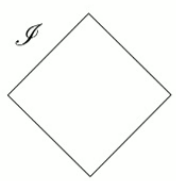


$SO(d,2)$ conformal group $=$ isometry group of $\boxed{\text{AdS}}$ $=$ conformal symmetries of $\boxed{\partial\text{AdS}}$

$\downarrow \Lambda \to 0$ $\quad\quad$ $c \to 0$ $\downarrow$ Carrollian limit



- Mink$_{d+1}$ / ?$_d$



$\text{CCarr}(d)$ conformal Carroll group $=$ isometry group of $\boxed{\text{Minkowski}}$ $\text{ISO}(d,1)$ $=$ conformal symmetries of $\boxed{\mathscr{I}}$

gets enhanced in presence of radiation

“Alice’s Adventures in Wonderland”
Lewis Carroll (1865)

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BMS = conformal Carroll

- Null infinity is endowed with a `Carrollian structure'

[Geroch][Penrose][Henneaux][Duval, Gibbons, Horvathy][Hartong][Ciambelli, Leigh, Marteau, Petropoulos][Bekaert, Morand][Herfray]...

Carrollian geometry

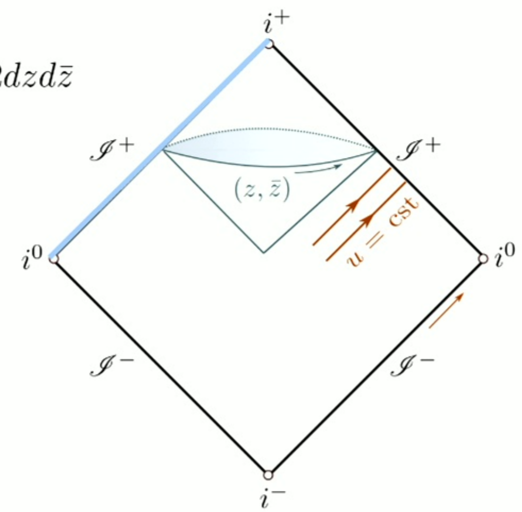
q_{ab} : a *degenerate* metric

a vector field satisfying $q_{ab}n^b = 0$

$\mathscr{I}^+$ $x^a = (u, z, \bar{z})$

$$q_{ab}dx^a dx^b = 0 \times du^2 + 2dzd\bar{z}$$

$$n = \partial_u$$



Conformal symmetries of this Carroll structure

$$\mathcal{L}_\xi q_{ab} = 2\alpha q_{ab} \qquad \mathcal{L}_\xi n^a = -\alpha n^a$$

$$\rightarrow \xi = \left[\mathcal{T} + \frac{u}{2}(\partial\mathcal{Y} + \bar{\partial}\bar{\mathcal{Y}})\right]\partial_u + \mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial}$$

span the algebra $\mathfrak{CCarr}_d = \mathfrak{bms}_{d+1}$

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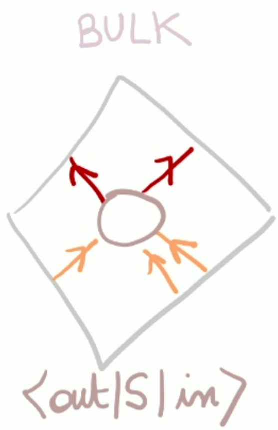
`**Carrollian** holography'?

Observables: S-matrix elements as correlators of a '**Carrollian**' field theory

[LD, Fiorucci, Herfray, Ruzziconi '22]



=

$$\left\langle \sigma_{k_1,\bar{k}_1}(u_1,z_1,\bar{z}_1)\dots\sigma_{k_m,\bar{k}_m}(u_m,z_m,\bar{z}_m)\right\rangle_{\text{Carr}}$$

Field-operator map
(for outgoing massless spin s field)

$$\Phi^{(s)}(X) \overset{\mathscr{I}^+}{\sim} r^{s-1}\,\sigma^{\text{out}}_{k,\bar{k}}(u,z,\bar{z})$$

transform as a 'conformal Carrollian primary' of weights $(k,\bar{k})$

$$\delta_{\bar{\xi}}\sigma_{k,\bar{k}} = \left[\left(\mathcal{T}+\tfrac{u}{2}(\partial\mathcal{Y}+\bar{\partial}\bar{\mathcal{Y}})\right)\partial_u + \mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial} + k\,\partial\mathcal{Y} + \bar{k}\,\bar{\partial}\bar{\mathcal{Y}}\right]\sigma_{k,\bar{k}}$$

From bulk to boundary operators (and back)

From **bulk** to **boundary**:

$$\Phi(X) = \int \frac{d^3p}{(2\pi)^3 2p^0} \left[a(p)e^{ip\cdot X} + a(p)^\dagger e^{-ip\cdot X}\right]$$

$$= \int \frac{d^2\vec{w}}{2(2\pi)^3}\omega d\omega \left[a(\omega,\vec{w})e^{i\omega q\cdot X} + a(\omega,\vec{w})^\dagger e^{-i\omega q\cdot X}\right]$$

$p^\mu = \omega q^\mu(\vec{w})$

momentum of a massless particle heading towards the celestial sphere

Go to Bondi coordinates $X^\mu = (u, r, z, \bar{z})$ and make a large *r* expansion

Scalar field: $\Phi \sim \frac{1}{r}\int_0^{+\infty} d\omega \left[a(\omega,z,\bar{z})e^{-i\omega u} - a(\omega,z,\bar{z})^\dagger e^{+i\omega u}\right]$

Graviton: $h_{zz} \sim rC_{zz}(u,z,\bar{z})$

gravitational shear = 'Carrollian primary' of weights $\left(\frac{3}{2}, -\frac{1}{2}\right)$

$$C_{zz} = \int_0^{+\infty} d\omega \left[a_+(\omega,z,\bar{z})e^{-i\omega u} - a_-(\omega,z,\bar{z})^\dagger e^{+i\omega u}\right]$$

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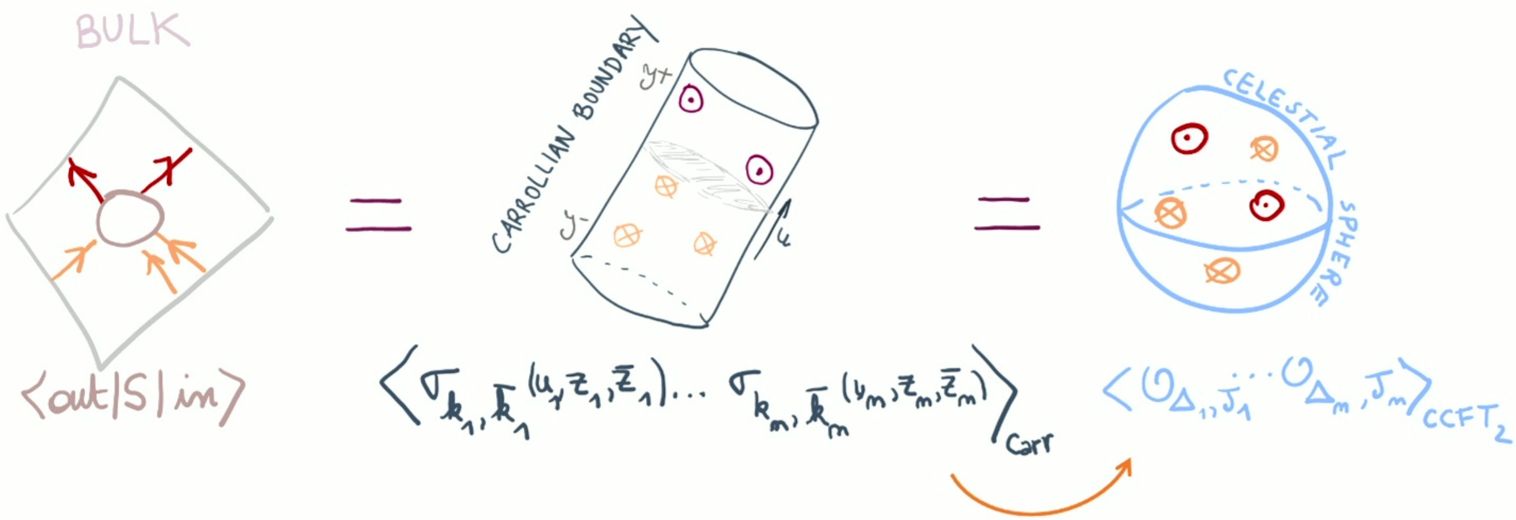
From **Carrollian** to **celestial** ‘dictionary’

Observables: S-matrix elements as correlators of a ‘**Carrollian**’ field theory



$$\mathcal{O}^{\text{out}}_{(\Delta,J)}(z,\bar{z}) = \lim_{\epsilon\to 0^+}\int_{-\infty}^{+\infty}\frac{du}{(u+i\epsilon)^{\Delta}}\,\sigma^{\text{out}}_{(k,\bar{k})}(u,z,\bar{z})$$

Carrollian – **celestial** operator map

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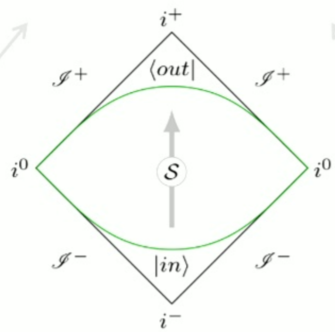
In summary

Momentum basis

$$\mathcal{A}_N = \langle \text{out}|\mathcal{S}|\text{in}\rangle_{\text{momentum}}$$

$$\int_0^{+\infty} d\omega\, \omega^{\Delta-1}$$

Mellin



Fourier

$$\int_0^{+\infty} d\omega\, e^{-i\omega u}$$

Celestial basis

$$\mathcal{M}_N = \langle \text{out}|\mathcal{S}|\text{in}\rangle_{\text{boost}}$$

Mellin + Fourier

$$\int_{-\infty}^{+\infty} du\, u^{-\Delta}$$

Carrollian basis

$$\mathcal{C}_N = \langle \text{out}|\mathcal{S}|\text{in}\rangle_{\text{position}}$$

Just a change of basis?

Is this really holography?

Is this useful?

Can we learn something we did not know already?

Salvador Dalí, illustrations for *Alice's Adventures in Wonderland*, 1969:

$\mathscr{L}w_{1+\infty}$ symmetries

$\mathscr{L}w_{1+\infty}$ symmetries in celestial CFT

- Celestial operators of integer conformal dimension give rise to 2d currents

$$H^k(z,\bar z) := \lim_{\varepsilon\to 0} \varepsilon \mathcal{O}_{k+\varepsilon,+2} \qquad k = 2, 1, 0, -1, \ldots$$

(Weinberg's) leading soft graviton theorem ($k=2$)

subleading soft theorem ($k=1$)

tower of sub^n-leading soft theorem [Hamada, Shiu '18][Li, Lin, Zhang '18]

(Remind Beniamino & Shreyansh's talks)

soft theorem	Ward identity	2d current
leading ω^{-1}	supertranslations $\delta C_{zz} = \partial_z^2 f$	$P(z,\bar z)\mathcal{O}_{h,\bar h}(w,\bar w) \sim \frac{1}{(z-w)}\mathcal{O}_{h+\frac{1}{2},\bar h+\frac{1}{2}}(w,\bar w)$ P: $(\frac{3}{2},\frac{1}{2})$ primary $\quad \Delta = 1$
subleading ω^0	superrotations $\delta C_{zz} = u\partial_z^3 Y^z$	$T(z)\mathcal{O}_{h,\bar h}(w,\bar w) \sim \frac{h}{(z-w)^2}\mathcal{O}_{h,\bar h}(w,\bar w) + \frac{\partial\mathcal{O}_{h,\bar h}(w,\bar w)}{z-w}$ T: $(2,0)$ primary $\quad$ Shadow of $\Delta = 0$

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$\mathscr{L}w_{1+\infty}$ symmetries in celestial CFT

- Celestial operators of integer conformal dimension give rise to 2d currents

$$H^k(z,\bar{z}) := \lim_{\varepsilon\to 0} \varepsilon \mathcal{O}_{k+\varepsilon,+2} \qquad k = 2, 1, 0, -1, \dots$$

Celestial graviton OPE (helicity +)

- $$\mathcal{O}_{\Delta_1,+2}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,+2}(z_2,\bar{z}_2) \sim -\frac{\kappa}{2}\frac{1}{z_{12}}\sum_{n=0}^{\infty} B(\Delta_1+n-1, \Delta_2-1)\frac{(\bar{z}_{12})^{n+1}}{n!}\bar{\partial}^n \mathcal{O}_{\Delta_1+\Delta_2,+2}(z_2,\bar{z}_2)$$

[Guevara, Himwich, Pate, Strominger '21]

$$H^k(z,\bar{z}) = \sum_{n=\frac{k-2}{2}}^{\frac{2-k}{2}} \frac{H_n^k(z)}{\bar{z}^{n+\frac{k-2}{2}}}$$ the holomorphic modes close the algebra

$$\left[H_m^k, H_n^l\right] = -\frac{\kappa}{2}\left[n(2-k) - m(2-l)\right]\frac{(\frac{2-k}{2} - m + \frac{2-l}{2} - n - 1)!}{(\frac{2-k}{2} - m)!(\frac{2-l}{2} - n)!}\frac{(\frac{2-k}{2} + m + \frac{2-l}{2} + n - 1)!}{(\frac{2-k}{2} + m)!(\frac{2-l}{2} + n)!} H_{m+n}^{k+l},$$

$\mathscr{L}w_{1+\infty}$ symmetries in celestial CFT

- Celestial operators of integer conformal dimension give rise to 2d currents

$$H^k(z,\bar{z}) := \lim_{\varepsilon\to 0} \varepsilon \mathcal{O}_{k+\varepsilon,+2} \qquad k = 2,1,0,-1,\dots$$

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$$H^k(z,\bar{z}) = \sum_{n=\frac{k-2}{2}}^{\frac{2-k}{2}} \frac{H_n^k(z)}{\bar{z}^{n+\frac{k-2}{2}}}$$

redefining $w_n^p = \frac{1}{\kappa}(p-n-1)!(p+n-1)!\, H_n^{-2p+4}$

$$[w_m^p, w_n^q] = [m(q-1) - n(p-1)]\, w_{m+n}^{p+q-2}$$

$p = 1, \frac{3}{2}, 2, \frac{5}{2}, \dots$ — $p=1$: (super)translations; $p=2$: Virasoro

$1-p \leq m \leq p-1$ 'wedge'

The infinite tower of celestial currents organizes into a single $\mathscr{L}w_{1+\infty}$ algebra ! [Strominger '21]

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$\mathscr{L}w_{1+\infty}$ symmetries seen from null infinity

- ... but how do these symmetries act on the Carrollian fields ?

➡ Go to twistor space ! [LD, Herfray, Freidel '24]

- The $\mathscr{L}w_{1+\infty}$ algebra has a very natural implementation in twistor space [Penrose '76] [Boyer, Plebanski '85][Adamo, Mason, Sharma '22]

$$\mathbb{PT} = \mathbb{C}P^3$$

coordinates $\quad \alpha = \{0,1\} \quad \dot{\alpha} \in \{\dot{0},\dot{1}\}$

$$[Z^A] = \begin{bmatrix} \mu^{\dot{\alpha}} \\ \lambda_\alpha \end{bmatrix} \qquad [\lambda_\alpha] = \begin{bmatrix} 1 \\ z \end{bmatrix} \in S^2$$

The algebra is given by the Poisson structure

$$\{g_1, g_2\} = \epsilon^{\dot{\alpha}\dot{\beta}} \frac{\partial g_1}{\partial \mu^{\dot{\alpha}}} \frac{\partial g_2}{\partial \mu^{\dot{\beta}}}$$

The generators g of the $\mathscr{L}w_{1+\infty}$ algebra are functions of homogeneity of degree 2 in Z^A of the form

$$g = g_0(z) + g_{\dot{\alpha}}(z)\mu^{\dot{\alpha}} + g_{\dot{\alpha}\dot{\beta}}(z)\mu^{\dot{\alpha}}\mu^{\dot{\beta}} + \dots$$

Using the modes $w^p_m := (\mu^{\dot{0}})^{p+m-1}(\mu^{\dot{1}})^{p-m-1} \quad , |m| \leq p-1\,,$ we recover

$$\{w^p_m, w^q_n\} = (m(q-1) - n(p-1))\, w^{p+q-2}_{m+n}$$

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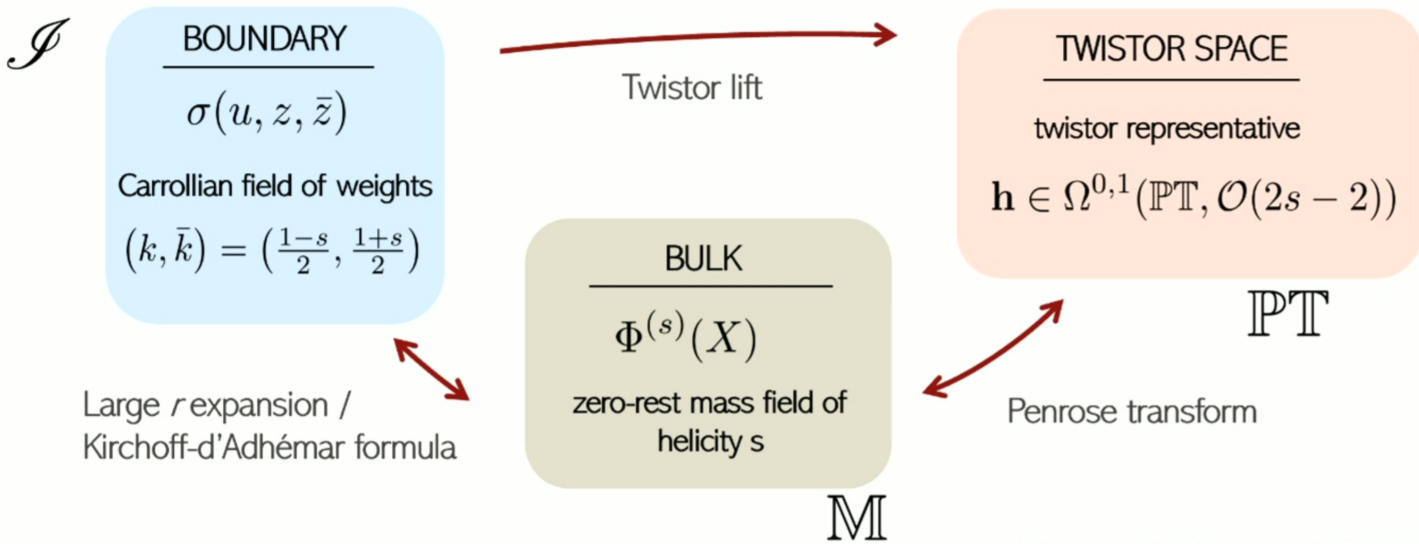
$\mathscr{L}w_{1+\infty}$ symmetries seen from null infinity

- The journey we took in [LD, Herfray, Freidel '24]



[Penrose '69] [Eastwood, Tod '82]

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$\mathscr{L}w_{1+\infty}$ symmetries seen from null infinity

JOURNAL OF MATHEMATICAL PHYSICS VOLUME 10, NUMBER 1 JANUARY 1969

Solutions of the Zero-Rest-Mass Equations

ROGER PENROSE
Birkbeck College, London, England

- The Penrose transform [Penrose '69]

BULK

$\Phi^{(s)}(X)$

zero-rest mass field of helicity s

↔ Penrose transform

TWISTOR SPACE

twistor representative

$\mathbf{h} \in \Omega^{0,1}(\mathbb{PT}, \mathcal{O}(2s-2))$

$$x^{\alpha\dot{\alpha}} = u\, n^{\alpha\dot{\alpha}} + r\, \lambda^{\alpha}\, \bar{\lambda}^{\dot{\alpha}} \quad \in \mathbb{M}$$

$$ds^2 = dx^{\alpha\dot{\alpha}} dx_{\alpha\dot{\alpha}} = 2dudr - 2r^2 dz d\bar{z}\,.$$

$(0,1)$-form $\quad \bar{\partial}\mathbf{h} = 0$

homogeneous degree $2s-2$

For positive helicity spin 2

$$h_{\mu\nu}(x) dx^{\mu} dx^{\nu} = \left(\frac{1}{2\pi i} \int_{\mathbb{CP}^1} \langle \zeta d\zeta \rangle \wedge \frac{\iota_{\alpha} \iota_{\beta}}{\langle \iota \zeta \rangle^2} \frac{\partial^2 \mathbf{h}}{\partial \mu^{\dot{\alpha}} \partial \mu^{\dot{\beta}}} (\mu^{\dot{\alpha}} = x^{\alpha\dot{\alpha}} \zeta_{\alpha}, \zeta_{\alpha}) \right) dx^{\alpha\dot{\alpha}} dx^{\beta\dot{\beta}}$$

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$\mathscr{L}w_{1+\infty}$ symmetries seen from null infinity

- What is the action of $\mathscr{L}w_{1+\infty}$ on the boundary Carrollian fields ? [LD, Herfray, Freidel '24]

$\mathscr{I}$ $\qquad\qquad\qquad\qquad$ $\mathbb{PT}$

$$\bar\sigma(u,\lambda) \longrightarrow h(u,\lambda) = \partial_u^{-1}\,\bar\sigma \xrightarrow[\text{twistor lift}]{(i)} \mathbf{h} = h(u=\mu\lambda,\lambda)\,D\bar\lambda$$

$\downarrow Lw_{1+\infty}$ $\qquad\qquad\qquad\qquad$ $\downarrow Lw_{1+\infty}$

$$\delta\bar\sigma(u,\lambda) \xleftarrow[(iii)]{\text{large } r} \delta\Phi(x) = \int_{\mathbb{CP}^1} \frac{\partial^2\delta\mathbf{h}}{\partial\mu\partial\mu} \xleftarrow[(ii)]{\text{Penrose transform}} \delta\mathbf{h} = \{g,\mathbf{h}\}$$

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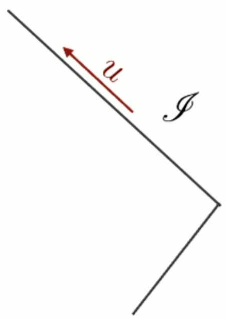
$\mathscr{L}w_{1+\infty}$ symmetries seen from null infinity

- What is the action of $\mathscr{L}w_{1+\infty}$ on the boundary Carrollian fields ? [LD, Herfray, Freidel '24]



Main result:

$$\delta_n \bar\sigma = \sum_{\ell=0}^{n} \bar\partial^{n-\ell}\left(g_{\dot\alpha(n)}\bar\lambda^{\dot\alpha(n)}\right)\frac{\ell}{(n-\ell)!}\partial_u^3\left(u^{n-\ell}\,\partial_u^{-1-\ell}\,\bar\partial^{\ell-1}\bar\sigma\right) \qquad s=+2$$

$$\delta_n \sigma = \sum_{\ell=0}^{n} \bar\partial^{n-\ell}\left(g_{\dot\alpha(n)}\bar\lambda^{\dot\alpha(n)}\right)\frac{\ell}{(n-\ell)!}\partial_u^{-1}\left(u^{n-\ell}\,\partial_u^{3-\ell}\,\bar\partial^{\ell-1}\sigma\right) \qquad s=-2$$

$\mathscr{L}w_{1+\infty}$ generators

$n = 0, 1, 2, \ldots$

non-local action
(vs local in twistor space)

✓ explicit match with the canonical analysis of [Freidel, Raclariu, Pranzetti '21]

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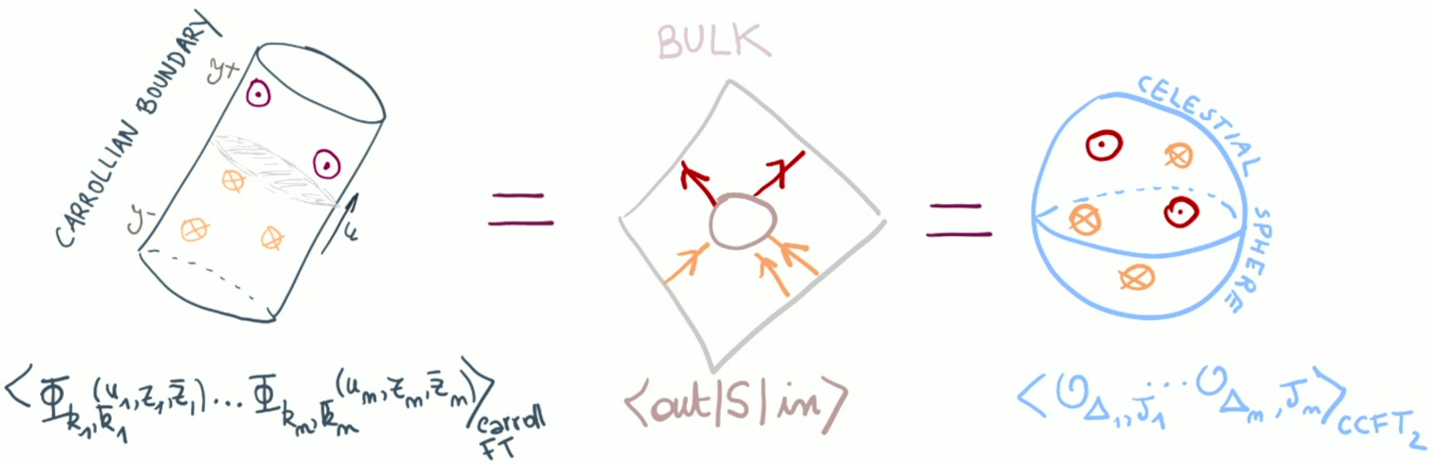
In summary

Celestial CFT living on the celestial sphere

↕ ⟷ quantum gravity in flat spacetime

Conformal Carrollian field theory living at null infinity



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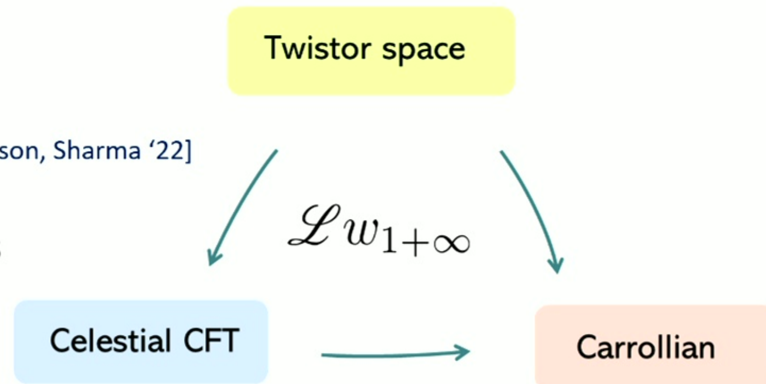
In summary

- $\mathscr{L}w_{1+\infty}$ symmetries organize an infinite tower of celestial currents at tree level

  [Guevara, Himwich, Pate, Strominger '21][Strominger '21][Adamo, Mason, Sharma '22]

- There is an explicit realization of these symmetries for Carrollian fields at null infinity.



[Freidel, Raclariu, Pranzetti '21] [LD, Herfray, Freidel '24]
see also [Geiller '24][Kmec, Mason, Ruzziconi, Srikant '24] [Cresto,Freidel '24]

The representation of these symmetries is local in twistor space but non-local in spacetime.

$$\delta_n \bar{\sigma} = \sum_{\ell=0}^{n} \bar{\partial}^{n-\ell}\left(g_{\dot{\alpha}(n)}\bar{\lambda}^{\dot{\alpha}(n)}\right)\frac{\ell}{(n-\ell)!}\partial_u^3\left(u^{n-\ell}\,\partial_u^{-1-\ell}\,\bar{\partial}^{\ell-1}\bar{\sigma}\right)$$

at the intersection of...

amplitudes
gravitational waves observation
conformal field theory
twistor theory
asymptotic symmetries
quantum field theory
hydrodynamics
string theory
mathematical GR

at the intersection of...

amplitudes

gravitational waves observation

conformal field theory

twistor theory

asymptotic symmetries

quantum field theory

hydrodynamics

string theory

mathematical GR

Thank you!