Title: TBA - Quantum Fields & Strings Seminar

Speakers: Laura Donnay

Collection/Series: Quantum Fields and Strings

Subject: Quantum Fields and Strings

Date: November 05, 2024 - 2:00 PM

URL: https://pirsa.org/24110050

$w_{1+\infty}$ symmetries in Carrollian holography

w/ Laurent Freidel, Yannick Herfray

Laura DONNAY

Quantum Fields & Strings Seminar
Perimeter Institute
Nov 5 2024

SISSA

Motivation

The holographic principle → **How general is it?**

Quantum gravity is *encoded* in a different theory that lives in a *lower-dimensional* spacetime.

['t Hooft '93; Susskind '94; Maldacena '97]

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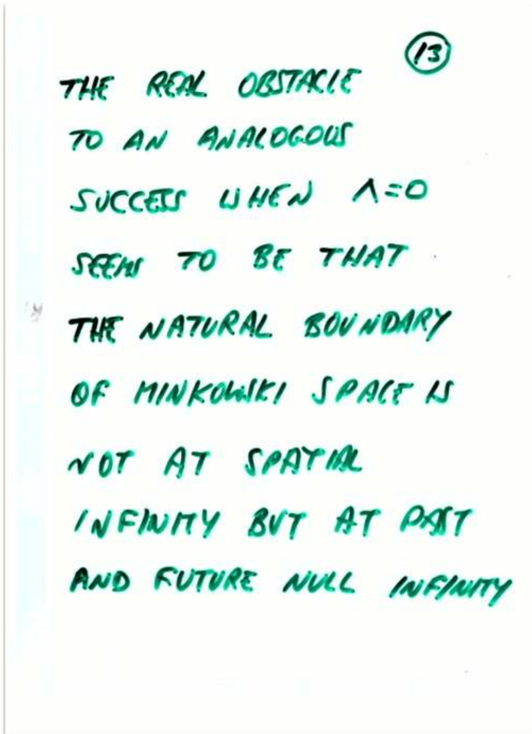
Flat space holography: a *structure X* ?



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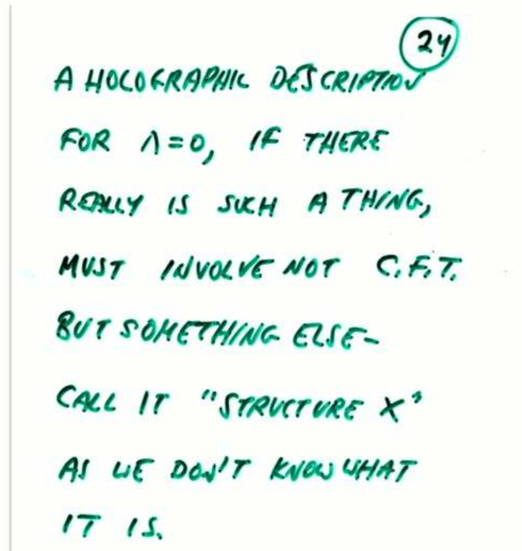
THE REAL OBSTACLE
TO AN ANALOGOUS
SUCCESS WHEN $\Lambda=0$
SEEMS TO BE THAT
THE NATURAL BOUNDARY
OF MINKOWSKI SPACE IS
NOT AT SPATIAL
INFINITY BUT AT PAST
AND FUTURE NULL INFINITY



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A HOLOGRAPHIC DESCRIPTION
FOR $\Lambda=0$, IF THERE
REALLY IS SUCH A THING,
MUST INVOLVE NOT C.F.T.
BUT SOMETHING ELSE –
CALL IT "STRUCTURE X"
AS WE DON'T KNOW WHAT
IT IS.

E. Witten's talk - *Strings* 1998

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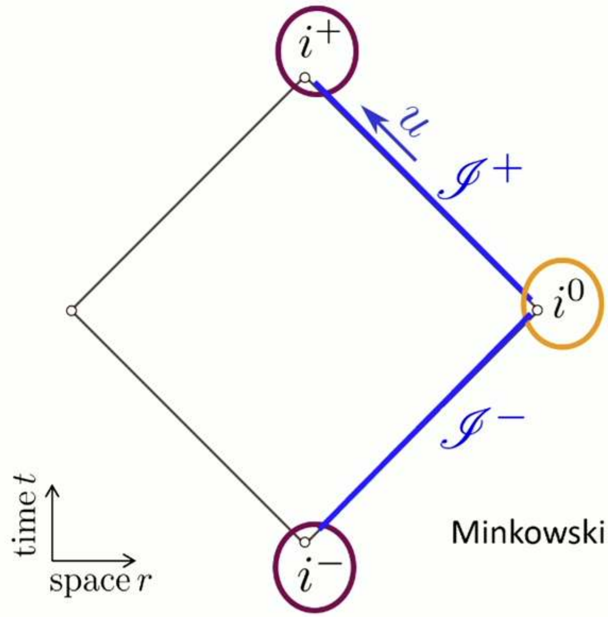
Flat space holography

Holographic description of quantum gravity in 4d asymptotically flat spacetimes?

Main obstructions/difficulties:

(1) The conformal **boundary** includes

- future/past timelike infinity
- future/past null infinity
- spatial infinity



Flat space holography

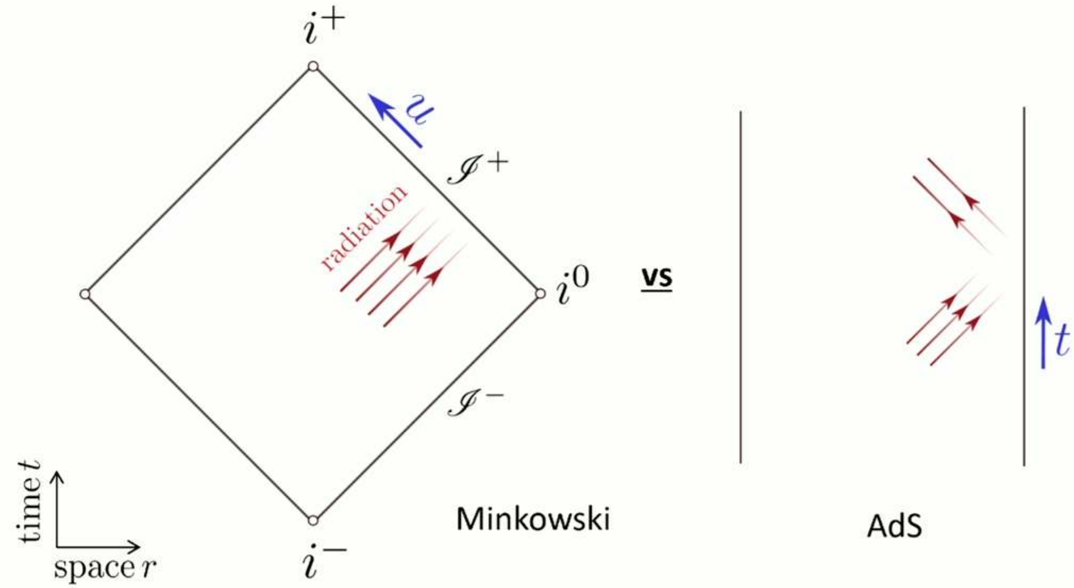
Holographic description of quantum gravity in 4d asymptotically flat spacetimes?

Main obstructions/difficulties:

① The conformal **boundary** includes

future/past timelike infinity
future/past null infinity
spatial infinity

② There are **fluxes** leaking out the boundary



Flat space holography

Holographic description of quantum gravity in 4d asymptotically flat spacetimes?

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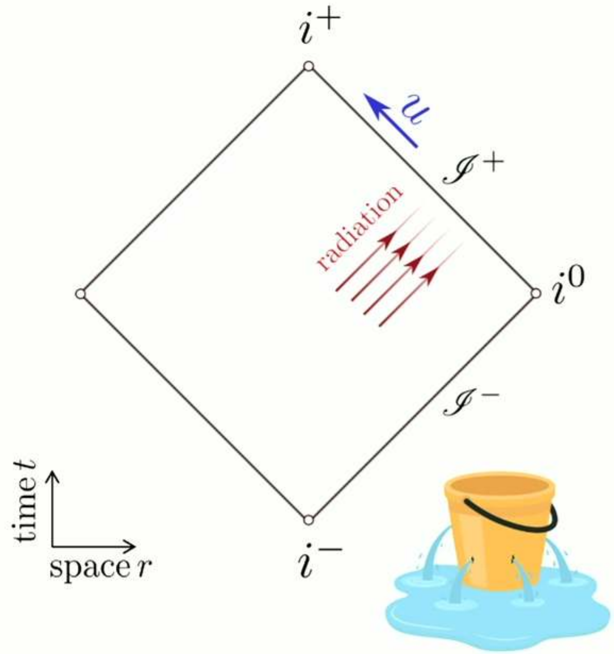
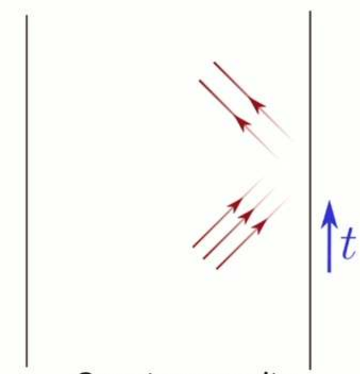


vs

AdS



Quantum gravity 'in a **box**'

Flat space holography

--> Road map: symmetries

- The phenomenon of **symmetry enhancement** is a key feature of asymptotically flat spacetimes, due to the presence of gravitational radiation

other example: AdS_3 / CFT_2

asymptotic symmetry group of AdS_3

$\updownarrow$

the conformal group in $d = 2$ gets enhanced to Virasoro

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Flat space holography

--> Road map: symmetries

- The phenomenon of **symmetry enhancement** is a key feature of asymptotically flat spacetimes, due to the presence of gravitational radiation

4 Poincaré translations

↓ *Symmetry enhancement*

∞ BMS supertranslations

what was expected

Poincaré

what was found

Bondi-Metzner-Sachs (BMS) ('62)

+ van der Burg

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Flat space holography

--> Road map: symmetries

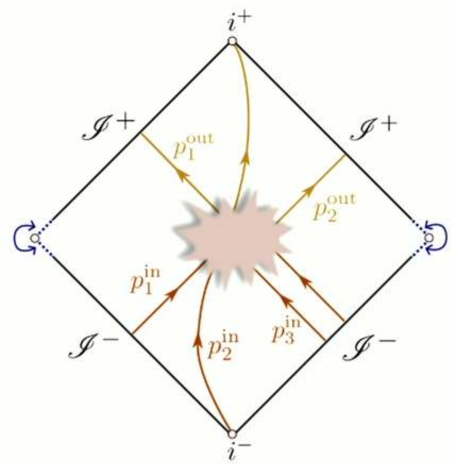
- The phenomenon of symmetry enhancement is a key feature of asymptotically flat spacetimes, due to the presence of gravitational radiation

While BMS symmetries were originally disregarded, it was realized (50 years later, [Strominger '13]) that they

- constrain the gravitational **S-matrix**
- have associated low-energy **observables** (memory effects)
- allow further extensions, including the local **conformal** group

↓

revival of **flat holography**



$\langle \text{out}|S|\text{in}\rangle$

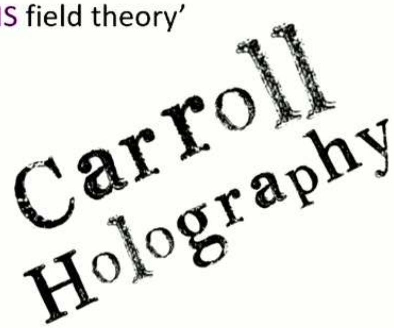
Which boundary?

null infinity

lighlike 3d hypersurface

$$\mathscr{I} = \mathbb{R} \times S^2$$

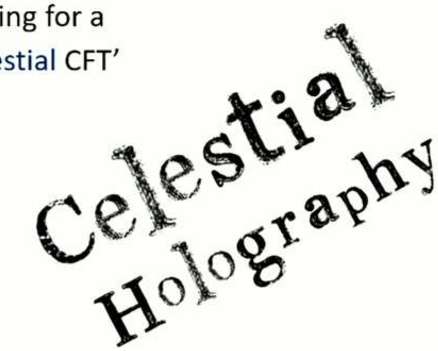
Looking for a
3d 'BMS field theory'



celestial sphere

Euclidean 2d-sphere

Looking for a
2d 'celestial CFT'



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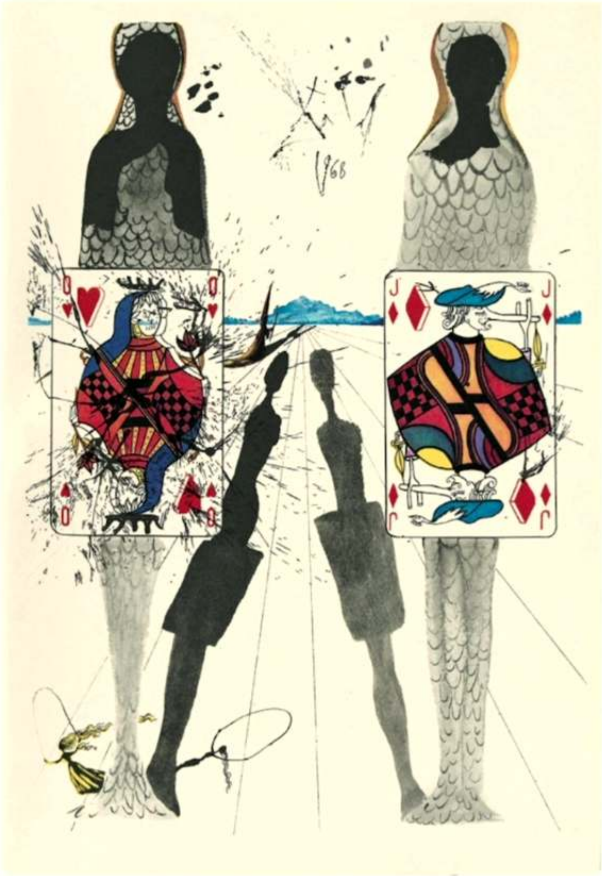
Salvador Dalí, illustrations for *Alice's Adventures in Wonderland*, 1969:



Outline

Carrollian holography

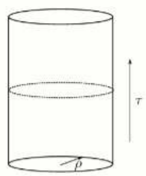
$\mathscr{L}w_{1+\infty}$ symmetries

Final remarks

Carroll symmetries

Holographic duality: step 0

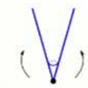
- AdS$_{d+1}$ / CFT$_d$



$SO(d,2)$ conformal group $=$ isometry group of $\boxed{\mathrm{AdS}}$ $=$ conformal symmetries of $\boxed{\partial\mathrm{AdS}}$

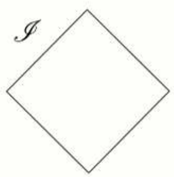
$\downarrow$ $\Lambda \to 0$

$c \to 0$ $\downarrow$ Carrollian limit



- Mink$_{d+1}$ / ?$_d$

$\mathscr{I}$



$\mathrm{CCarr}(d)$ conformal Carroll group $=$ isometry group of $\boxed{\text{Minkowski}}$ $=$ conformal symmetries of $\boxed{\mathscr{I}}$

$\mathrm{ISO}(d,1)$

gets enhanced in presence of radiation

"Alice's Adventures in Wonderland"
Lewis Carroll (1865)

BMS = conformal Carroll

- Null infinity is endowed with a `Carrollian structure'

[Geroch][Penrose][Henneaux][Duval, Gibbons, Horvathy][Hartong][Ciambelli, Leigh, Marteau, Petropoulos][Bekaert, Morand][Herfray]...

Carrollian geometry

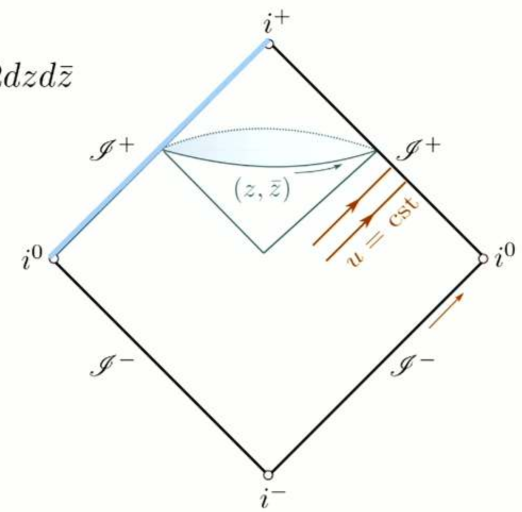
q_{ab} : a *degenerate* metric

a vector field satisfying $q_{ab}n^b = 0$

$\mathscr{I}^+$ $x^a = (u, z, \bar{z})$

$$q_{ab}dx^a dx^b = 0 \times du^2 + 2dzd\bar{z}$$

$$n = \partial_u$$



Conformal symmetries of this Carroll structure

$$\mathcal{L}_\xi q_{ab} = 2\alpha q_{ab} \qquad \mathcal{L}_\xi n^a = -\alpha n^a$$

$$\rightarrow \xi = \left[\mathcal{T} + \frac{u}{2}(\partial\mathcal{Y} + \bar{\partial}\bar{\mathcal{Y}})\right]\partial_u + \mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial}$$

span the algebra $\mathfrak{CCarr}_d = \mathfrak{bms}_{d+1}$

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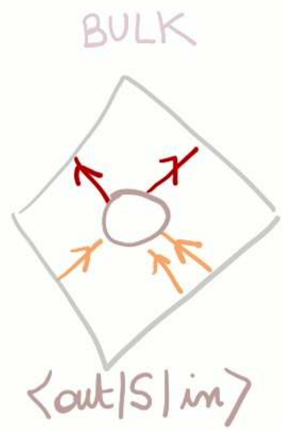
`**Carrollian** holography'?

Observables: S-matrix elements as correlators of a '**Carrollian**' field theory

[LD, Fiorucci, Herfray, Ruzziconi '22]



=

$$\left\langle \sigma_{k_1,\bar{k}_1}(u_1,z_1,\bar{z}_1)\dots\sigma_{k_m,\bar{k}_m}(u_m,z_m,\bar{z}_m)\right\rangle_{Carr}$$

Field-operator map
(for outgoing massless spin s field)

$$\Phi^{(s)}(X) \overset{\mathscr{I}^+}{\sim} r^{s-1}\,\sigma^{\text{out}}_{k,\bar{k}}(u,z,\bar{z})$$

transform as a 'conformal Carrollian primary' of weights $(k,\bar{k})$

$$\delta_{\bar{\xi}}\sigma_{k,\bar{k}} = \left[\left(\mathcal{T}+\tfrac{u}{2}(\partial\mathcal{Y}+\bar{\partial}\bar{\mathcal{Y}})\right)\partial_u + \mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial} + k\,\partial\mathcal{Y} + \bar{k}\,\bar{\partial}\bar{\mathcal{Y}}\right]\sigma_{k,\bar{k}}$$

From bulk to boundary operators (and back)

From **bulk** to **boundary**:

$$\Phi(X) = \int \frac{d^3p}{(2\pi)^3 2p^0} \left[a(p)e^{ip\cdot X} + a(p)^\dagger e^{-ip\cdot X}\right]$$

$$= \int \frac{d^2\vec{w}}{2(2\pi)^3}\omega d\omega \left[a(\omega,\vec{w})e^{i\omega q\cdot X} + a(\omega,\vec{w})^\dagger e^{-i\omega q\cdot X}\right]$$

$p^\mu = \omega q^\mu(\vec{w})$

momentum of a massless particle heading towards the celestial sphere

Go to Bondi coordinates $X^\mu = (u, r, z, \bar{z})$ and make a large *r* expansion

Scalar field: $\Phi \sim \frac{1}{r}\int_0^{+\infty} d\omega \left[a(\omega,z,\bar{z})e^{-i\omega u} - a(\omega,z,\bar{z})^\dagger e^{+i\omega u}\right]$

Graviton: $h_{zz} \sim rC_{zz}(u,z,\bar{z})$

gravitational shear = 'Carrollian primary' of weights $\left(\frac{3}{2}, -\frac{1}{2}\right)$

$$C_{zz} = \int_0^{+\infty} d\omega \left[a_+(\omega,z,\bar{z})e^{-i\omega u} - a_-(\omega,z,\bar{z})^\dagger e^{+i\omega u}\right]$$

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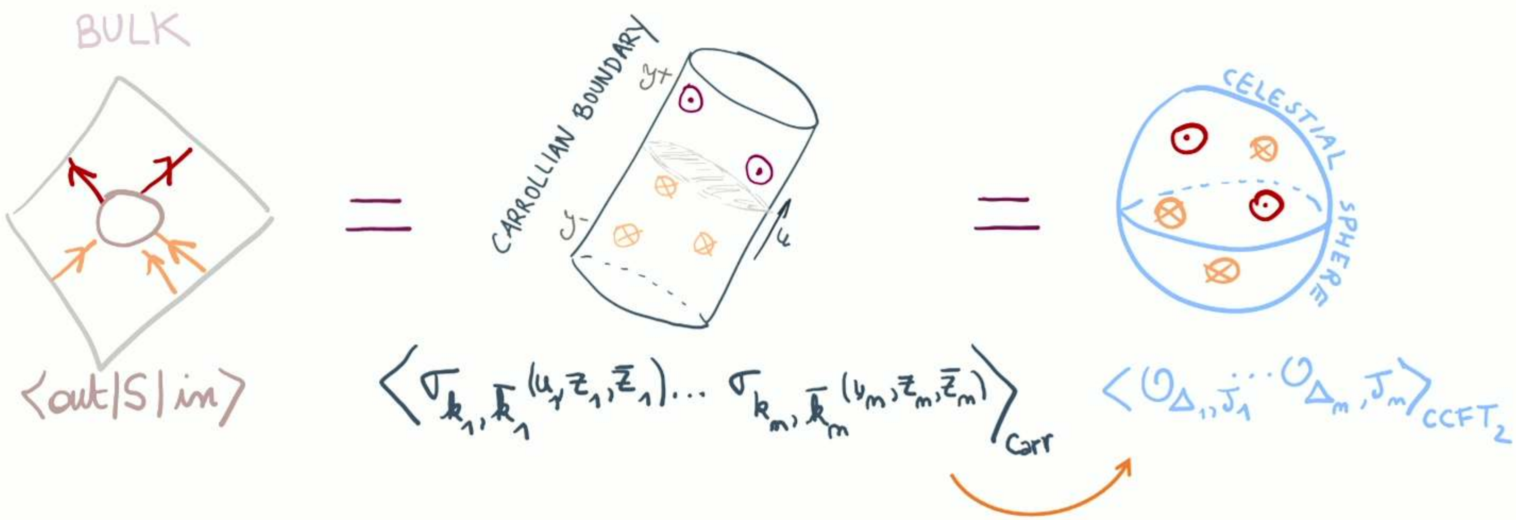
From **Carrollian** to **celestial** ‘dictionary’

Observables: S-matrix elements as correlators of a ‘**Carrollian**’ field theory



$$\mathcal{O}^{\text{out}}_{(\Delta,J)}(z,\bar z) = \lim_{\epsilon\to 0^+}\int_{-\infty}^{+\infty}\frac{du}{(u+i\epsilon)^{\Delta}}\,\sigma^{\text{out}}_{(k,\bar k)}(u,z,\bar z)$$

Carrollian – **celestial** operator map

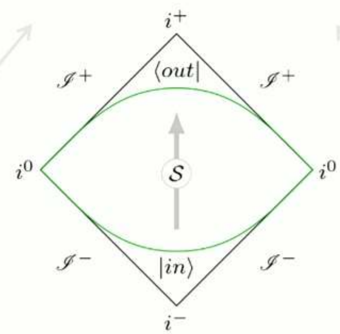
In summary

Momentum basis

$$\mathcal{A}_N = \langle \text{out}|\mathcal{S}|\text{in}\rangle_{\text{momentum}}$$

Mellin

$$\int_0^{+\infty} d\omega\, \omega^{\Delta-1}$$



Fourier

$$\int_0^{+\infty} d\omega\, e^{-i\omega u}$$

Celestial basis

$$\mathcal{M}_N = \langle \text{out}|\mathcal{S}|\text{in}\rangle_{\text{boost}}$$

Mellin + Fourier

$$\int_{-\infty}^{+\infty} du\, u^{-\Delta}$$

Carrollian basis

$$\mathcal{C}_N = \langle \text{out}|\mathcal{S}|\text{in}\rangle_{\text{position}}$$

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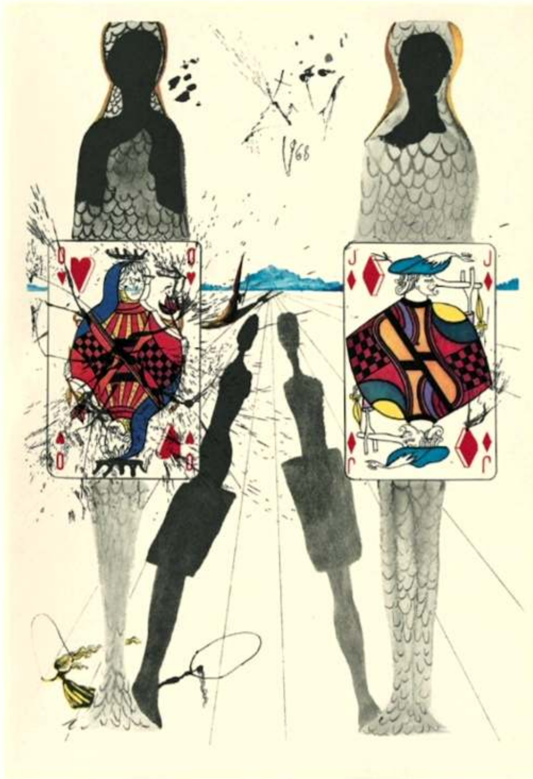
Just a change of basis?

Is this really holography?

Is this useful?

Can we learn something we did not know already?

Salvador Dali, illustrations for *Alice's Adventures in Wonderland*, 1969:



$\mathscr{L}w_{1+\infty}$ symmetries

$\mathscr{L}w_{1+\infty}$ symmetries in celestial CFT

- Celestial operators of integer conformal dimension give rise to 2d currents

$$H^k(z,\bar{z}) := \lim_{\varepsilon\to 0} \varepsilon \mathcal{O}_{k+\varepsilon,+2} \qquad k = 2, 1, 0, -1, \ldots$$

(Weinberg's) leading soft graviton theorem

subleading soft theorem

tower of sub^n-leading soft theorem [Hamada, Shiu '18][Li, Lin, Zhang '18]

(Remind Beniamino & Shreyansh's talks)

soft theorem	Ward identity	2d current
leading ω^{-1}	supertranslations $\delta C_{zz} = \partial_z^2 f$	$P(z,\bar{z})\mathcal{O}_{h,\bar{h}}(w,\bar{w}) \sim \frac{1}{(z-w)}\mathcal{O}_{h+\frac{1}{2},\bar{h}+\frac{1}{2}}(w,\bar{w})$ ↓ $(\frac{3}{2},\frac{1}{2})$ primary $\Delta = 1$
subleading ω^{0}	superrotations $\delta C_{zz} = u\partial_z^3 Y^z$	$T(z)\mathcal{O}_{h,\bar{h}}(w,\bar{w}) \sim \frac{h}{(z-w)^2}\mathcal{O}_{h,\bar{h}}(w,\bar{w}) + \frac{\partial\mathcal{O}_{h,\bar{h}}(w,\bar{w})}{z-w}$ ↓ $(2,0)$ primary Shadow of $\Delta = 0$

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$\mathscr{L}w_{1+\infty}$ symmetries in celestial CFT

- Celestial operators of integer conformal dimension give rise to 2d currents

$$H^k(z,\bar z) := \lim_{\varepsilon\to 0} \varepsilon \mathcal{O}_{k+\varepsilon,+2} \qquad k = 2,1,0,-1,\dots$$

Celestial graviton OPE (helicity +)

- $$\mathcal{O}_{\Delta_1,+2}(z_1,\bar z_1)\mathcal{O}_{\Delta_2,+2}(z_2,\bar z_2) \sim -\frac{\kappa}{2}\frac{1}{z_{12}}\sum_{n=0}^{\infty} B(\Delta_1+n-1,\Delta_2-1)\frac{(\bar z_{12})^{n+1}}{n!}\bar\partial^n \mathcal{O}_{\Delta_1+\Delta_2,+2}(z_2,\bar z_2)$$

[Guevara, Himwich, Pate, Strominger '21]

$$H^k(z,\bar z) = \sum_{n=\frac{k-2}{2}}^{\frac{2-k}{2}} \frac{H^k_n(z)}{\bar z^{n+\frac{k-2}{2}}}$$ the holomorphic modes close the algebra

$$\left[H^k_m, H^l_n\right] = -\frac{\kappa}{2}\left[n(2-k)-m(2-l)\right]\frac{(\frac{2-k}{2}-m+\frac{2-l}{2}-n-1)!}{(\frac{2-k}{2}-m)!(\frac{2-l}{2}-n)!}\frac{(\frac{2-k}{2}+m+\frac{2-l}{2}+n-1)!}{(\frac{2-k}{2}+m)!(\frac{2-l}{2}+n)!}H^{k+l}_{m+n},$$

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$\mathscr{L}w_{1+\infty}$ symmetries in celestial CFT

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$$H^k(z,\bar{z}) = \sum_{n=\frac{k-2}{2}}^{\frac{2-k}{2}} \frac{H^k_n(z)}{\bar{z}^{n+\frac{k-2}{2}}}$$

redefining $w^p_n = \frac{1}{\kappa}(p-n-1)!(p+n-1)!\,H_n^{-2p+4}$

→ $$[w^p_m, w^q_n] = [m(q-1)-n(p-1)]\,w^{p+q-2}_{m+n}$$

$p = 1, \frac{3}{2}, 2, \frac{5}{2}, \dots$ — 1: (super)translations; $\frac{3}{2}$: Virasoro

$1-p \le m \le p-1$ 'wedge'

The infinite tower of celestial currents organizes into a single $\mathscr{L}w_{1+\infty}$ algebra ! [Strominger '21]

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$\mathscr{L}w_{1+\infty}$ symmetries seen from null infinity

- ... but how do these symmetries act on the Carrollian fields ?

  → Go to twistor space ! [LD, Herfray, Freidel '24]

- The $\mathscr{L}w_{1+\infty}$ algebra has a very natural implementation in twistor space [Penrose '76] [Boyer, Plebanski '85][Adamo, Mason, Sharma '22]

$\mathbb{PT} = \mathbb{C}P^3$

coordinates $\alpha = \{0,1\}$ $\dot{\alpha} \in \{\dot{0},\dot{1}\}$

$$[Z^A] = \begin{bmatrix} \mu^{\dot{\alpha}} \\ \lambda_\alpha \end{bmatrix} \qquad [\lambda_\alpha] = \begin{bmatrix} 1 \\ z \end{bmatrix} \in S^2$$

The algebra is given by the Poisson structure

$$\{g_1, g_2\} = \epsilon^{\dot{\alpha}\dot{\beta}} \frac{\partial g_1}{\partial \mu^{\dot{\alpha}}} \frac{\partial g_2}{\partial \mu^{\dot{\beta}}}$$

The generators g of the $\mathscr{L}w_{1+\infty}$ algebra are functions of homogeneity of degree 2 in Z^A of the form

$$g = g_0(z) + g_{\dot{\alpha}}(z)\mu^{\dot{\alpha}} + g_{\dot{\alpha}\dot{\beta}}(z)\mu^{\dot{\alpha}}\mu^{\dot{\beta}} + \ldots$$

Using the modes $w^p_m := (\mu^{\dot{0}})^{p+m-1}(\mu^{\dot{1}})^{p-m-1}$, $|m| \leq p-1$, we recover

$$\{w^p_m, w^q_n\} = (m(q-1) - n(p-1))\, w^{p+q-2}_{m+n}$$

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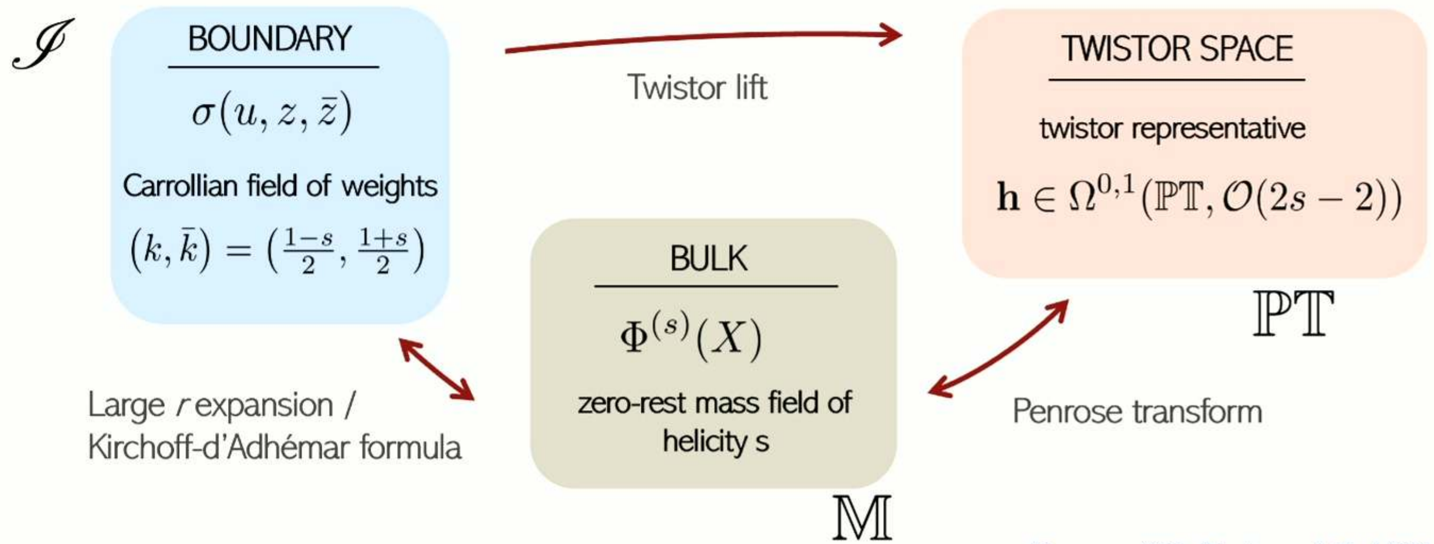
$\mathscr{L}w_{1+\infty}$ symmetries seen from null infinity

- The journey we took in [LD, Herfray, Freidel '24]



[Penrose '69] [Eastwood, Tod '82]

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$\mathscr{L}w_{1+\infty}$ symmetries seen from null infinity

JOURNAL OF MATHEMATICAL PHYSICS VOLUME 10, NUMBER 1 JANUARY 1969

Solutions of the Zero-Rest-Mass Equations

ROGER PENROSE
Birkbeck College, London, England

- The Penrose transform [Penrose '69]

BULK

$\Phi^{(s)}(X)$

zero-rest mass field of helicity s

Penrose transform

TWISTOR SPACE

twistor representative

$\mathbf{h} \in \Omega^{0,1}(\mathbb{PT}, \mathcal{O}(2s-2))$

$$x^{\alpha\dot{\alpha}} = u\, n^{\alpha\dot{\alpha}} + r\, \lambda^{\alpha}\, \bar{\lambda}^{\dot{\alpha}} \quad \in \mathbb{M}$$

$$ds^2 = dx^{\alpha\dot{\alpha}} dx_{\alpha\dot{\alpha}} = 2dudr - 2r^2 dz d\bar{z}\,.$$

$(0,1)$-form $\quad \bar{\mathrm{d}}\mathbf{h} = 0$

homogeneous degree $2s-2$

For positive helicity spin 2

$$h_{\mu\nu}(x)dx^{\mu}dx^{\nu} = \left(\frac{1}{2\pi i}\int_{\mathbb{CP}^1} \langle\zeta d\zeta\rangle \wedge \frac{\iota_\alpha \iota_\beta}{\langle\iota\zeta\rangle^2}\, \frac{\partial^2 \mathbf{h}}{\partial\mu^{\dot{\alpha}}\partial\mu^{\dot{\beta}}}(\mu^{\dot{\alpha}} = x^{\alpha\dot{\alpha}}\zeta_\alpha, \zeta_\alpha)\right) dx^{\alpha\dot{\alpha}} dx^{\beta\dot{\beta}}$$

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$\mathscr{L}w_{1+\infty}$ symmetries seen from null infinity

- What is the action of $\mathscr{L}w_{1+\infty}$ on the boundary Carrollian fields ? [LD, Herfray, Freidel '24]

$\mathscr{I}$ — $\mathbb{PT}$

$$\bar{\sigma}(u,\lambda) \longrightarrow h(u,\lambda) = \partial_u^{-1}\,\bar{\sigma} \xrightarrow[\text{twistor lift}]{(i)} \mathbf{h} = h(u=\mu\lambda,\lambda)\, D\bar{\lambda}$$

$$\downarrow Lw_{1+\infty} \qquad\qquad \downarrow Lw_{1+\infty}$$

$$\delta\bar{\sigma}(u,\lambda) \xleftarrow[(iii)]{\text{large } r} \delta\Phi(x) = \int_{\mathbb{CP}^1} \frac{\partial^2 \delta\mathbf{h}}{\partial\mu\partial\mu} \xleftarrow[(ii)]{\text{Penrose transform}} \delta\mathbf{h} = \{g,\mathbf{h}\}$$

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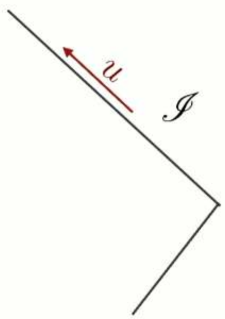
$\mathscr{L}w_{1+\infty}$ symmetries seen from null infinity

- What is the action of $\mathscr{L}w_{1+\infty}$ on the boundary Carrollian fields ? [LD, Herfray, Freidel '24]



Main result:

$$\delta_n \bar{\sigma} = \sum_{\ell=0}^{n} \bar{\partial}^{n-\ell}\left(g_{\dot{\alpha}(n)}\bar{\lambda}^{\dot{\alpha}(n)}\right)\frac{\ell}{(n-\ell)!}\partial_u^3\left(u^{n-\ell}\,\partial_u^{-1-\ell}\,\bar{\partial}^{\ell-1}\bar{\sigma}\right) \qquad s=+2$$

$$\delta_n \sigma = \sum_{\ell=0}^{n} \bar{\partial}^{n-\ell}\left(g_{\dot{\alpha}(n)}\bar{\lambda}^{\dot{\alpha}(n)}\right)\frac{\ell}{(n-\ell)!}\partial_u^{-1}\left(u^{n-\ell}\,\partial_u^{3-\ell}\,\bar{\partial}^{\ell-1}\sigma\right) \qquad s=-2$$

$\mathscr{L}w_{1+\infty}$ generators

$n = 0, 1, 2, \ldots$

non-local action
(vs local in twistor space)

✓ explicit match with the canonical analysis of[Freidel, Raclariu, Pranzetti '21]

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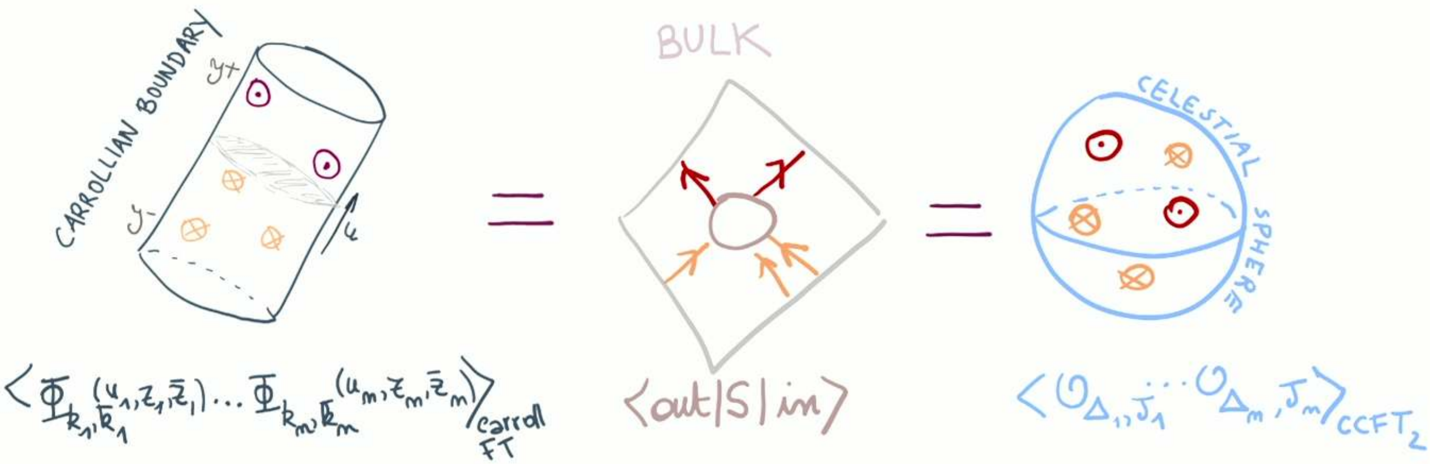
In summary

Celestial CFT living on the celestial sphere
↕
Conformal Carrollian field theory living at null infinity

⟷ quantum gravity in flat spacetime



In summary

- $\mathscr{L}w_{1+\infty}$ symmetries organize an infinite tower of celestial currents at tree level

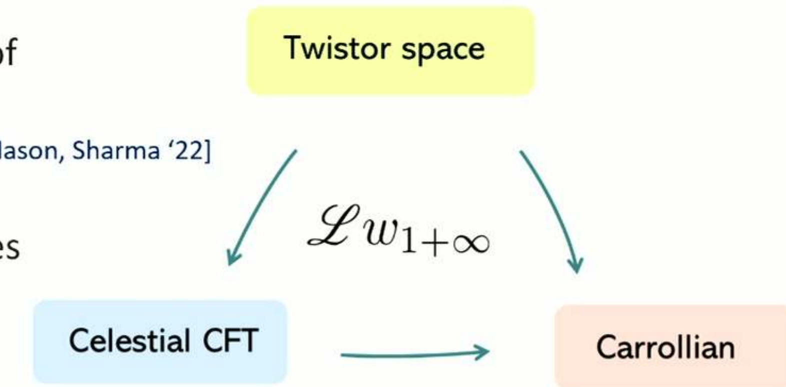
  [Guevara, Himwich, Pate, Strominger '21][Strominger '21][Adamo, Mason, Sharma '22]

- There is an explicit realization of these symmetries for Carrollian fields at null infinity.

  [Freidel, Raclariu, Pranzetti '21] [LD, Herfray, Freidel '24]
  see also [Geiller '24][Kmec, Mason, Ruzziconi, Srikant '24] [Cresto,Freidel '24]

  The representation of these symmetries is local in twistor space but non-local in spacetime.

$$\delta_n \bar{\sigma} = \sum_{\ell=0}^{n} \bar{\partial}^{n-\ell}\left(g_{\dot{\alpha}(n)}\bar{\lambda}^{\dot{\alpha}(n)}\right)\frac{\ell}{(n-\ell)!}\partial_u^3\left(u^{n-\ell}\,\partial_u^{-1-\ell}\,\bar{\partial}^{\ell-1}\bar{\sigma}\right)$$



at the intersection of...

amplitudes

gravitational waves observation

conformal field theory

twistor theory

asymptotic symmetries

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Thank you!