Title: Image Reconstruction From Intensity Interferometry

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Collection/Series: Future Prospects of Intensity Interferometry

Subject: Cosmology

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Abstract:

Even with dense sampling of the uv plane intensity correlations only contain half the information required to reconstruct an image. Intensity correlations do contain the full information of the image power spectrum and therefore of the image 2-point correlation function. With some practice one can gain intuitive understanding in interpreting 2-point correlation function "images". This is illustrated with both toy examples and modeling of real astronomical images. In some assumptions one can even interpret these 2-point correlation function "images" with only a few baselines.

Intensity Interferometry Image Reconstruction

Future Prospects of Intensity Interferometry
Perimeter Institute
Waterloo, Canada
2024-11-01

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Fermilab

Why Image Reconstruction?

For illustrative purposes

lite enlightenment of yourself / others - nothing serious

For analysis

value added data representation - serious

Visualization

different representation of the data

Intensity Interferometry

$$r_{12} = \delta t\, r_1\, r_2 \left(1 + 2\frac{\Phi}{m}\right)$$ "Gaussian" Radiation

- $\mathbf{r_{12}}$ rate of "simultaneous" counts
- $\mathbf{r_i}$ count rate
- $\mathbf{m}$ number of independent modes

$m \cong 2\,\delta t\,\delta\nu$ polarized counters

$m \cong 4\,\delta t\,\delta\nu$ unpolarized counters

$\delta t\,\delta\nu \le \frac{1}{2}$ Schwarz Inequality

$\Phi[\vec{\ell},\nu] \equiv |\phi[\vec{\ell},\nu]|^2$ coherence function = intensity power spectrum

$\phi[\vec{\ell},\nu] \equiv \tilde{I}_\nu[\vec{\ell}]]/f_\nu$ correlation coefficient

$$\frac{r_{12}}{r_{12}^{\text{poisson}}} \le 3$$

$f_\nu = \tilde{I}_\nu[\vec{0}]$ flux density (i.e. in Janskies)

$$\tilde{I}_\nu[\vec{\ell}] = \int d^2\hat{\mathbf{n}}\, e^{-i\vec{\ell}\cdot\hat{\mathbf{n}}} \sqrt{B_1[\hat{\mathbf{n}},\nu]\, B_2[\hat{\mathbf{n}},\nu]}\; I_\nu[\hat{\mathbf{n}}]$$ Beam-weighted Fourier transform of intensity pattern

$\vec{\ell} = 2\pi \frac{\nu}{c}\mathbf{b}_\perp$ angular wavenumber

$\mathbf{b} \equiv \mathbf{x}_1 - \mathbf{x}_2$ telescope baseline

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suggestion 1
Linear Image Reconstruction

In many types of measurement what is measured is, to some approximation, linear in the quantity whose image one wants to reconstruct

Usually for the dominate source of noise is additive.

suggest constructing image estimators which are linear in the measured quantities.

- a good feature: for linear reconstructions the noise is additive

example: astronomical intensity interferometry of incoherent light

$$\Phi[\vec{\ell},\nu] = g^{(2)} - 1 = |\phi[\vec{\ell},\nu]|^2$$

Φ is linear in $|\phi[\vec{\ell},\nu]|^2$

suggest constructing linear image estimator

$$\hat{I}_\nu[\vec{\theta}] \propto \Phi \propto |\phi[\vec{\ell},\nu]|^2$$

suggestion 2

Coherence Correlation Function

- For intensity interferometry there is a linear relation between $\Phi \propto I_\nu[\hat{\mathbf{n}}_1]\, I_\nu[\hat{\mathbf{n}}_2]$ so the observable is **not** linear in I_ν nor localized in $\hat{\mathbf{n}}$.
- Rather

$$\Phi[\vec{\ell},\nu] = |\phi[\vec{\ell},\nu]|^2 = \frac{(\int d^2\hat{\mathbf{n}}_1\, e^{-i\,\vec{\ell}\cdot\hat{\mathbf{n}}_1} B_1[\hat{\mathbf{n}}_1,\nu]\, I_\nu[\hat{\mathbf{n}}_1])(\int d^2\hat{\mathbf{n}}_2\, e^{+i\,\vec{\ell}\cdot\hat{\mathbf{n}}_2} B_2[\hat{\mathbf{n}}_2,\nu]\, I_\nu[\hat{\mathbf{n}}_2])}{(\int d^2\hat{\mathbf{n}}_1\, B_1[\hat{\mathbf{n}}_1,\nu]\, I_\nu[\hat{\mathbf{n}}])\,(\int d^2\hat{\mathbf{n}}_2\, B_2[\hat{\mathbf{n}}_2,\nu]\, I_\nu[\hat{\mathbf{n}}])}$$

- One could take the **image space** to be the normalized intensity spatial power spectra: $|\phi[\vec{\ell},\nu]|^2$.
  - power spectrum **misses half the information**: $|\phi[\vec{\ell},\nu]|$ but not $\arg[\phi[\vec{\ell},\nu]]$.
- However it is **suggested** to instead use the **coherence correlation function**, which is the Fourier transform of the power spectrum,, as the image space

$$w_\Phi[\Delta\hat{\mathbf{n}},\nu] \equiv \frac{\int d^2\vec{\ell}\, e^{i\vec{\ell}\cdot\vec{\Delta\theta}}\, \Phi[\vec{\ell},\nu]}{\int d^2\vec{\ell}\, \Phi[\vec{\ell},\nu]} = \frac{\int d^2\hat{\mathbf{n}}\, B_1[\hat{\mathbf{n}},\nu]\, B_2[\hat{\mathbf{n}}+\Delta\hat{\mathbf{n}},\nu]\, I_\nu[\hat{\mathbf{n}}]\, I_\nu[\hat{\mathbf{n}}+\Delta\hat{\mathbf{n}}]}{\int d^2\hat{\mathbf{n}}\, B_1[\hat{\mathbf{n}},\nu]\, B_2[\hat{\mathbf{n}},\nu]\, I_\nu[\hat{\mathbf{n}}]^2}$$

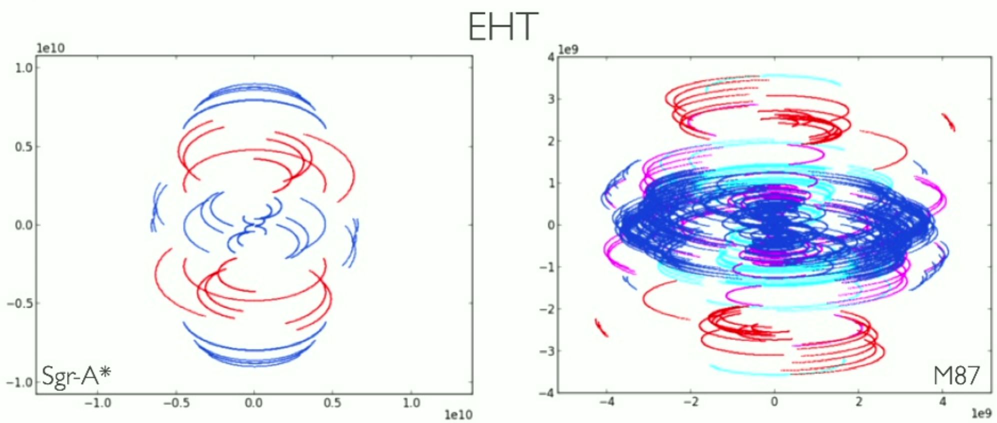
  with properties

  - $w_\Phi[\Delta\hat{\mathbf{n}},\nu]$ contains **all** the information from intensity interferometry
  - $0 \le w_\Phi \le 1$
  - $w_\Phi[\mathbf{0},\nu] = 1$
  - $w_\Phi[\Delta\hat{\mathbf{n}},\nu]$ invariant under $\Delta\hat{\mathbf{n}} \to -\Delta\hat{\mathbf{n}}$ and $I_\nu \to -I_\nu$
  - if the support, $|\hat{\mathbf{n}}_1 - \hat{\mathbf{n}}_2|$, of I_ν is compact then so is the support of $w_\Phi[\Delta\hat{\mathbf{n}},\nu]$
- No attempt to reconstruct $I_\nu[\hat{\mathbf{n}}]$

suggestion 3
Restrict to "Space of Beams"

For linear measurements there is (usually) an infinite image (Hilbert) space but only a finite set of measurements.

- there are many image patterns to which the data is completely insensitive - **suggestion** do not include these in the reconstruction
- caveat: there are a variety of ways to this which corresponds to different weighting schemes



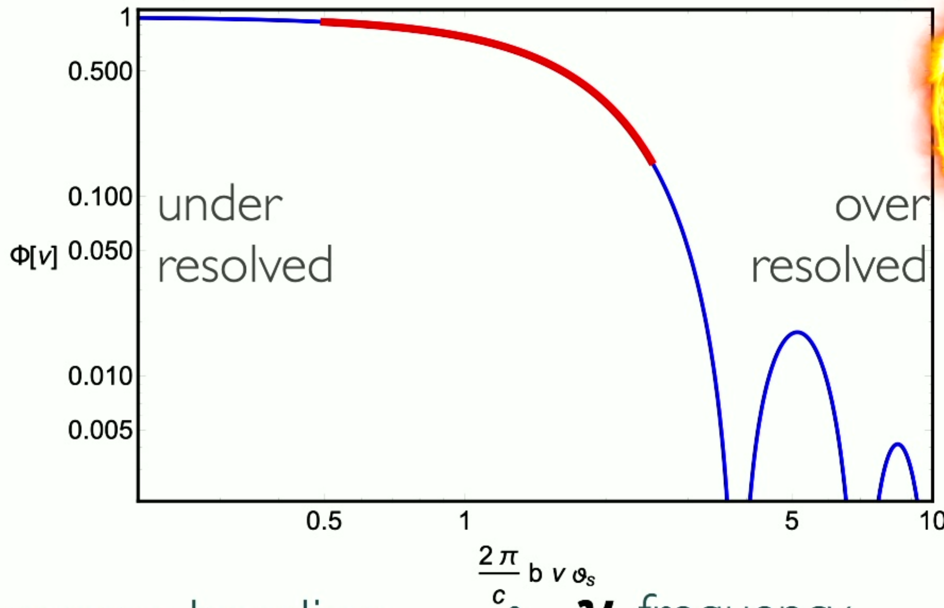
$$uv = \overrightarrow{\ell} = 2\pi \vec{b}_{\perp} \nu / c$$

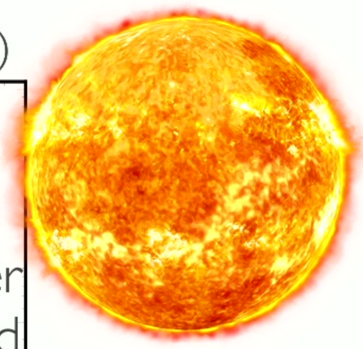
Φ vs BASELINE or frequency

- **b** for moveable telescopes may be varied almost arbitrarily
  - for fixed telescopes **b** only varies with Earth rotation
  - many telescopes give many simultaneous baselines
    - interferometric lingo: "fill the uv plane"
- ν single photon optical detectors vary by small factor ≲4
  - depends on detector technology
  - dense spectral sampling expensive
  - degeneracy of spectral and spatial information
- **b** + ν
  - expensive!

COHERENCE FUNCTION

(UNIFORM BRIGHTNESS DISK E.G. STAR)





- b transverse baseline
- ν frequency
- ϑ_s angular radius of disk
- Φ coherence function $\in[0,1]$

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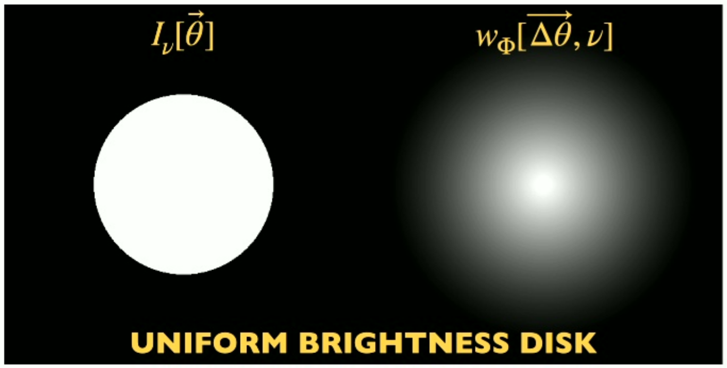
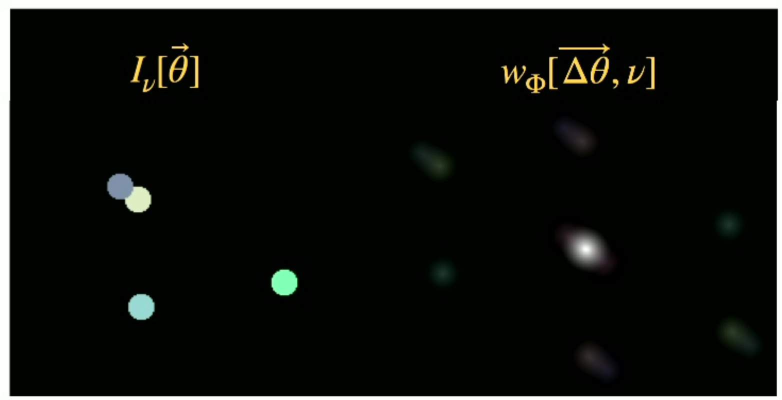
Complete uv Coverage

$$w_\Phi[\overrightarrow{\Delta\theta}, \nu] \equiv \frac{\int d^2\overrightarrow{\ell}\, e^{i\overrightarrow{\ell}\cdot\overrightarrow{\Delta\theta}}\, \Phi[\overrightarrow{\ell}, \nu]}{\int d^2\overrightarrow{\ell}\, \Phi[\overrightarrow{\ell}, \nu]}$$





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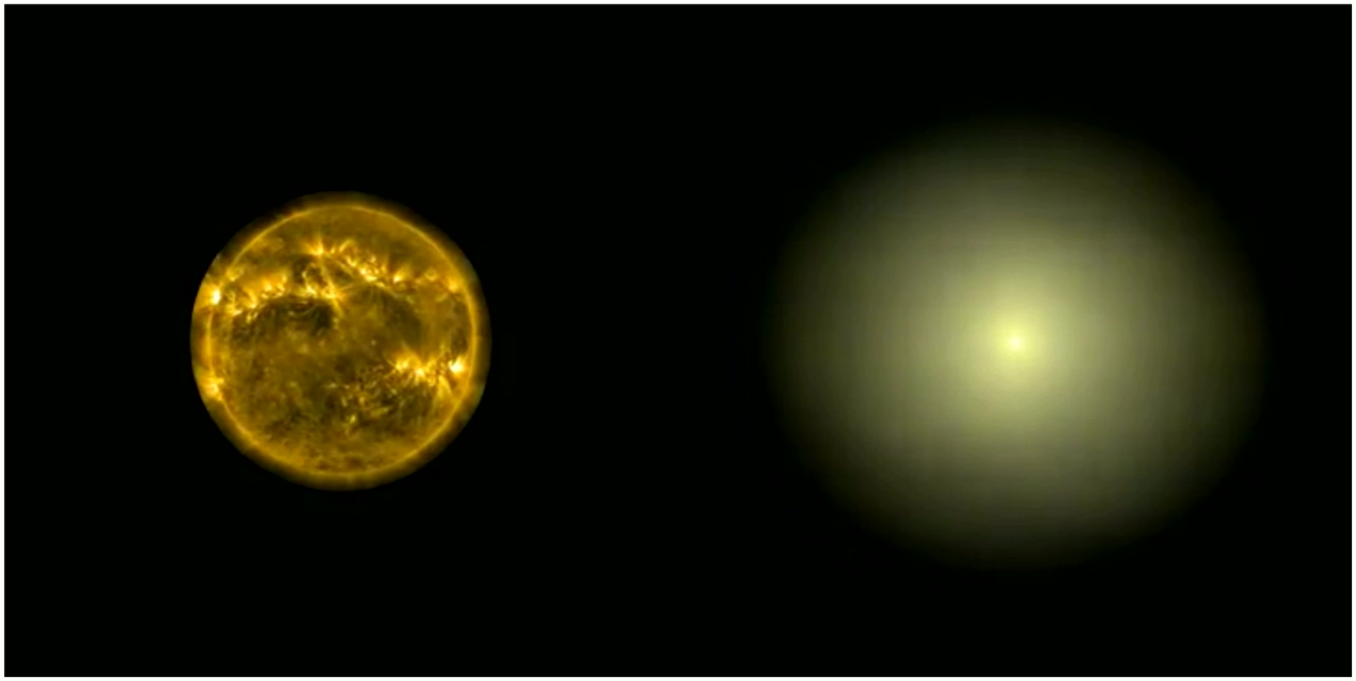
NON-UNIFORM DISK

- **rotating "dynamic" star [Sun from Solar Dynamics Observatory]**



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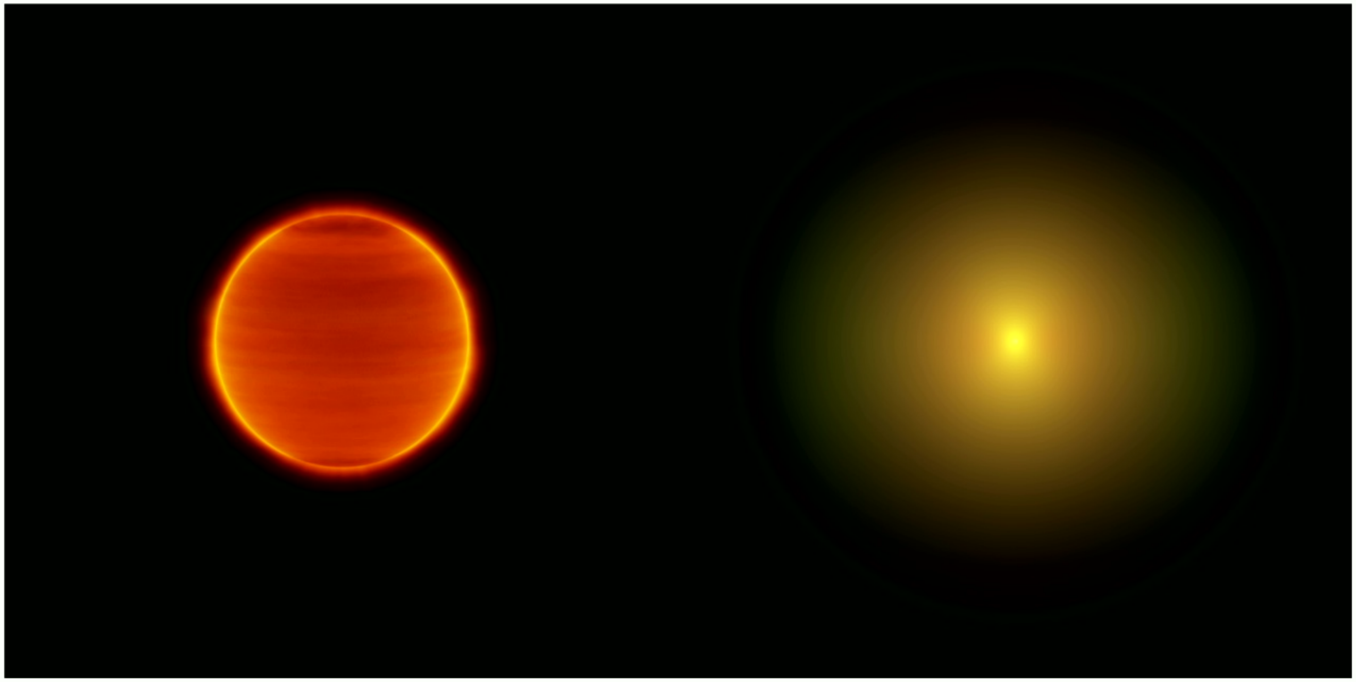
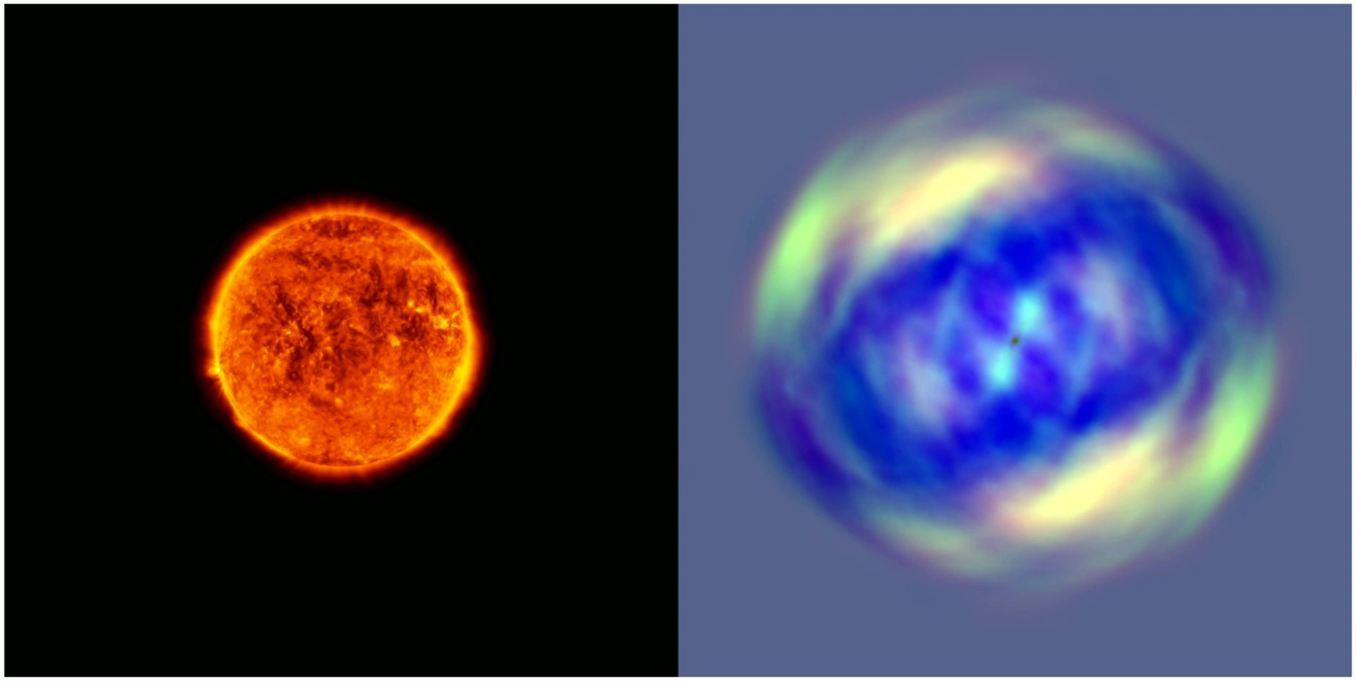
AXIS OF ROTATION

- **temporal time average**



COHERENCE VARIATIONS

SUBTRACT TEMPORAL MEAN



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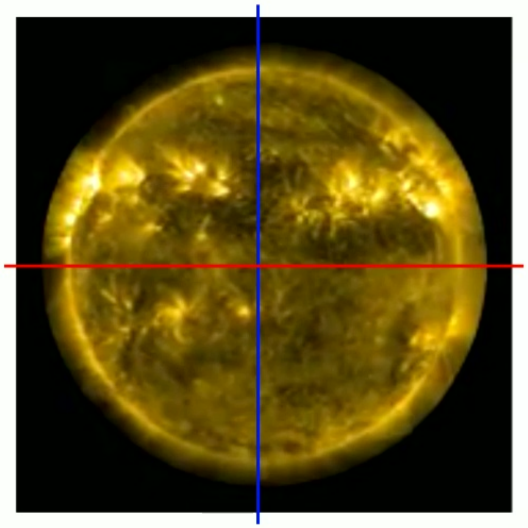
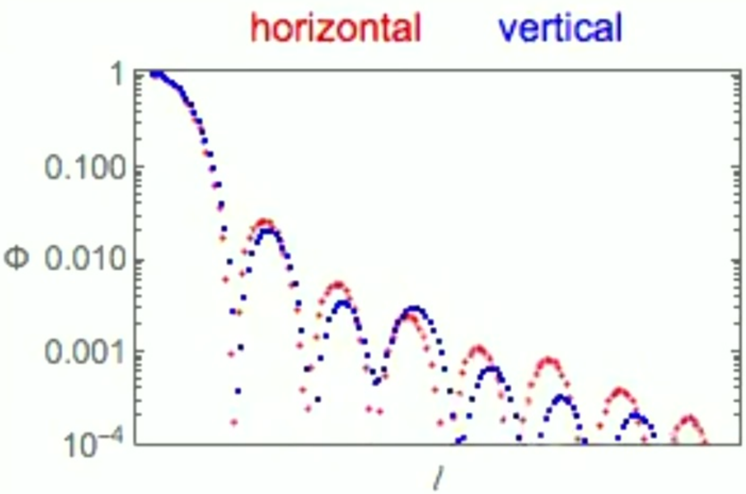
coherence Function for only 2 Baselines

ROTATING DYNAMIC STAR [SUN FROM SOLAR DYNAMICS OBSERVATORY]

ONLY TWO BASELINES FROM 3 TO 4 TELESCOPES

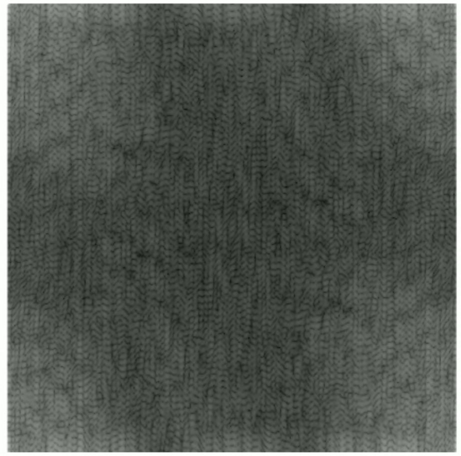


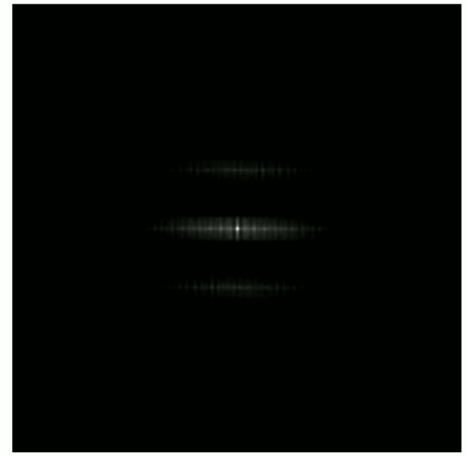


in foreseeable future most II observations will only have a few baselines

Perimeter

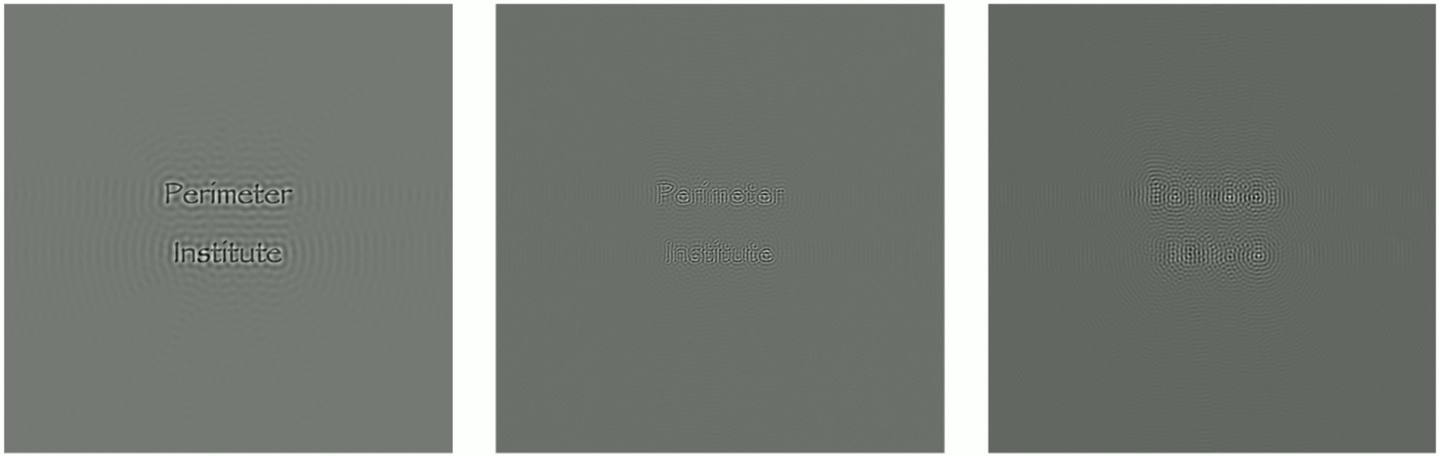
Institute



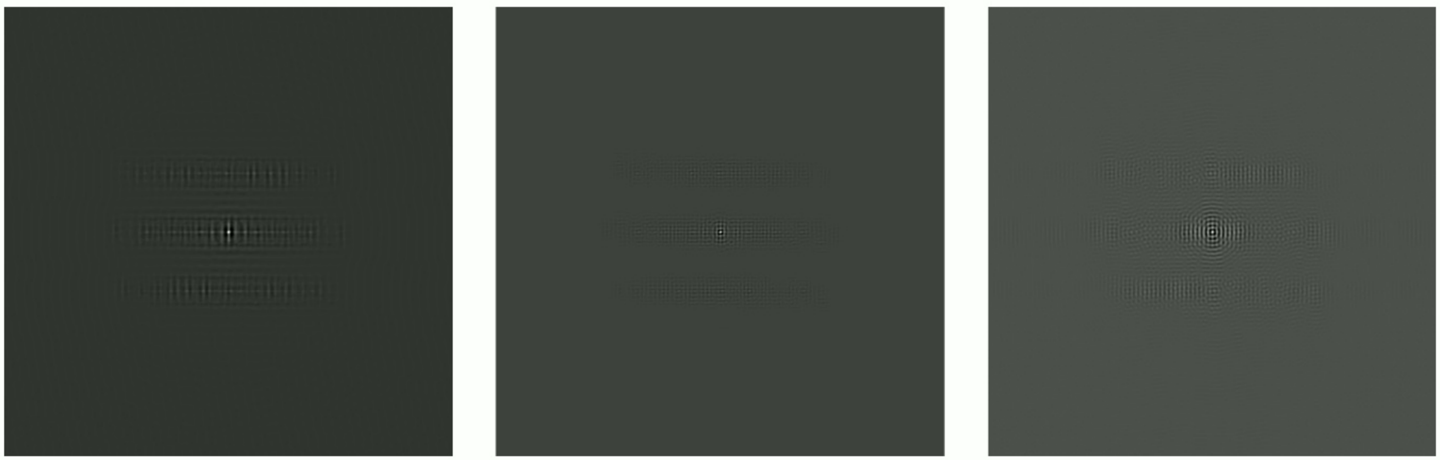


intensity pattern | power spectrum | correlation function

amplitude interferometry - finite uv coverage



x4 bandwidth | x2 bandwidth | 10% bandwidth



intensity interferometry

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Summary

visualization of the intensity 2-point correlation function from intensity interferometry measurements can be useful

even with limited uv coverage