Title: Lecture - Beautiful Papers

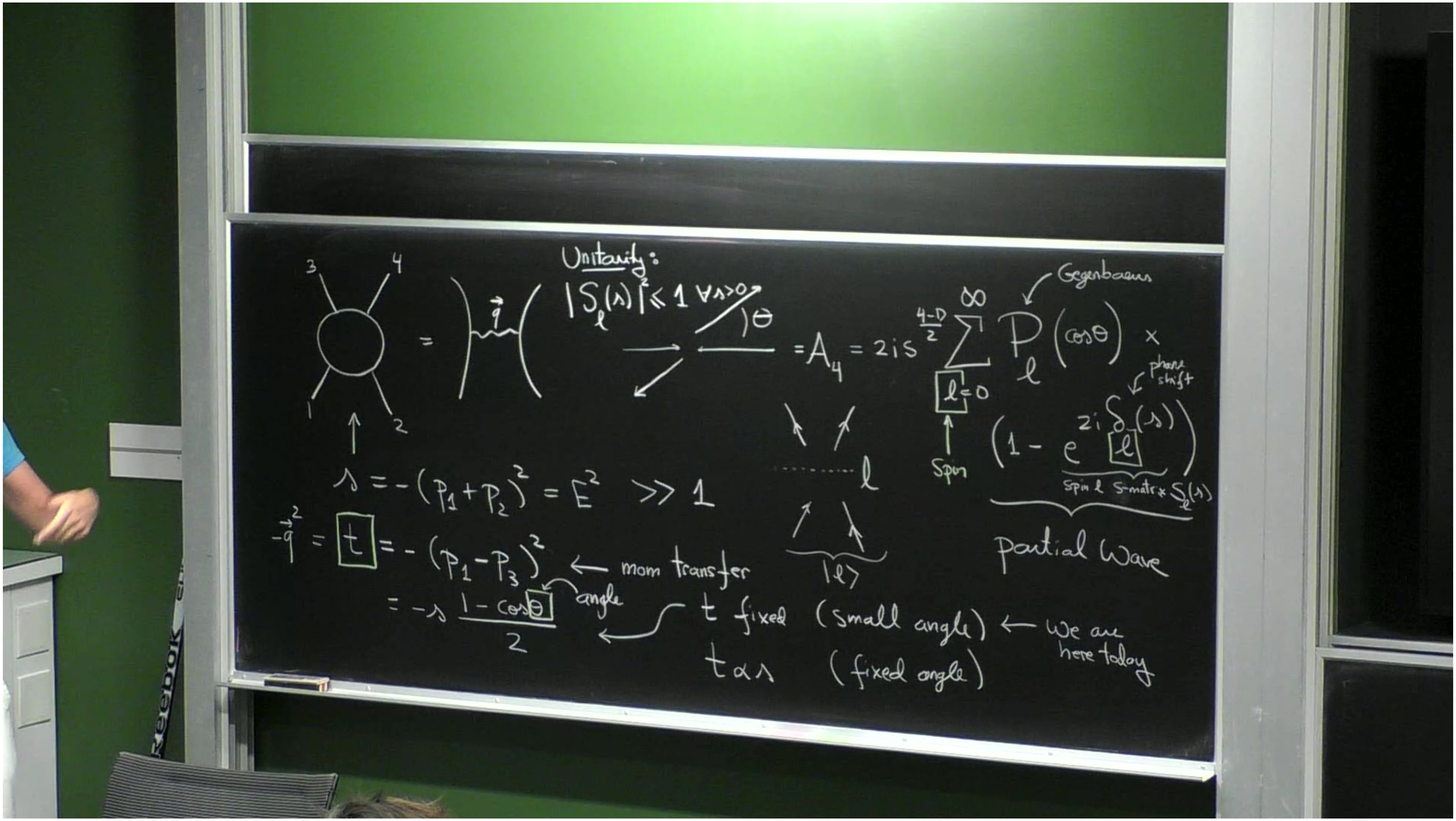
Speakers: Pedro Vieira

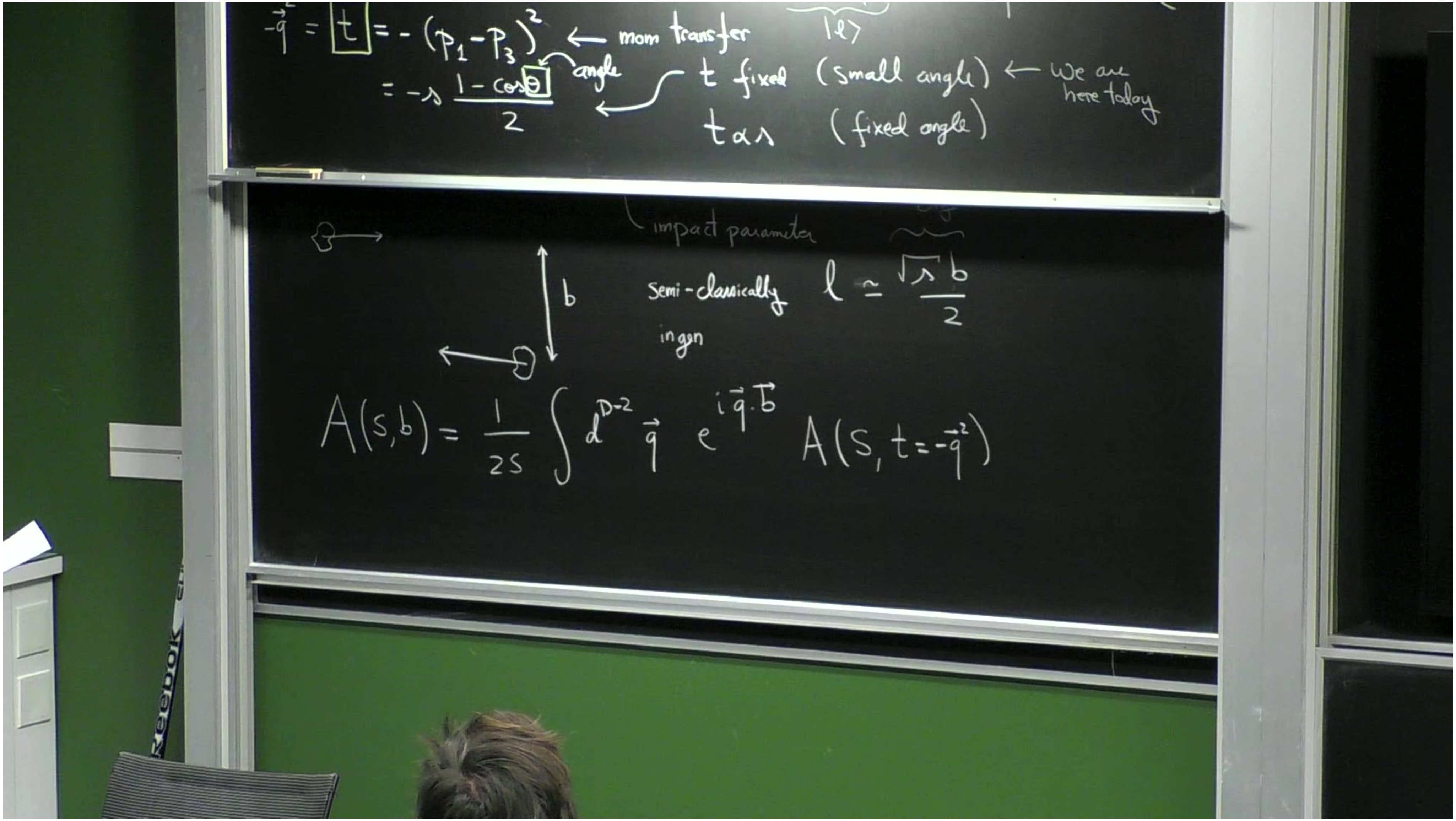
Collection/Series: Beautiful Papers - October 7, 2024 - January 31, 2025

Subject: Other

Date: November 29, 2024 - 9:15 AM

URL: https://pirsa.org/24110037

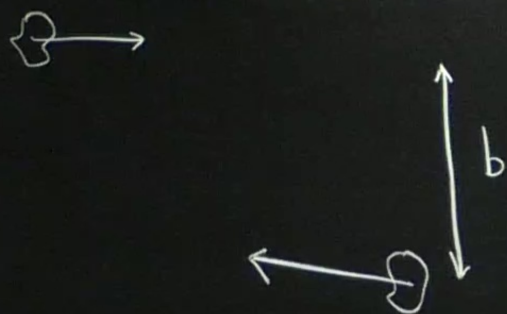

Unitarity:
|S_ℓ(s)|² ≤ 1 ∀ s>0
θ
= A_4 = 2is^((4-D)/2) Σ_{ℓ=0}^∞ P_ℓ(cosθ) ×
Gegenbauers
phase shift
(1 − e^(2iδ_ℓ(s)))
Spin
spin ℓ S-matrix S_ℓ(s)
Partial Wave
ℓ
|ℓ⟩
s = −(p_1 + p_2)² = E² >> 1
−q⃗² = t = −(p_1 − p_3)² ← mom transfer
= −s (1 − cosθ)/2
angle
t fixed (small angle) ← We are here today
t ∝ s (fixed angle)


$-\vec{q}^{\,2} = \boxed{t} = -(p_1 - p_3)^2$ ← mom transfer
$= -s\,\frac{1-\cos\theta}{2}$ angle
$|\ell\rangle$
t fixed (small angle) ← we are here today
$t \propto s$ (fixed angle)
impact parameter
b
semi-classically $\ell \simeq \frac{\sqrt{s}\,b}{2}$
in gen
$A(s,b) = \frac{1}{2s}\int d^{D-2}\vec{q}\; e^{i\vec{q}\cdot\vec{b}}\, A(s, t=-\vec{q}^{\,2})$

2 $\quad t \propto s$ (fixed angle)

s + t, θ, l and b

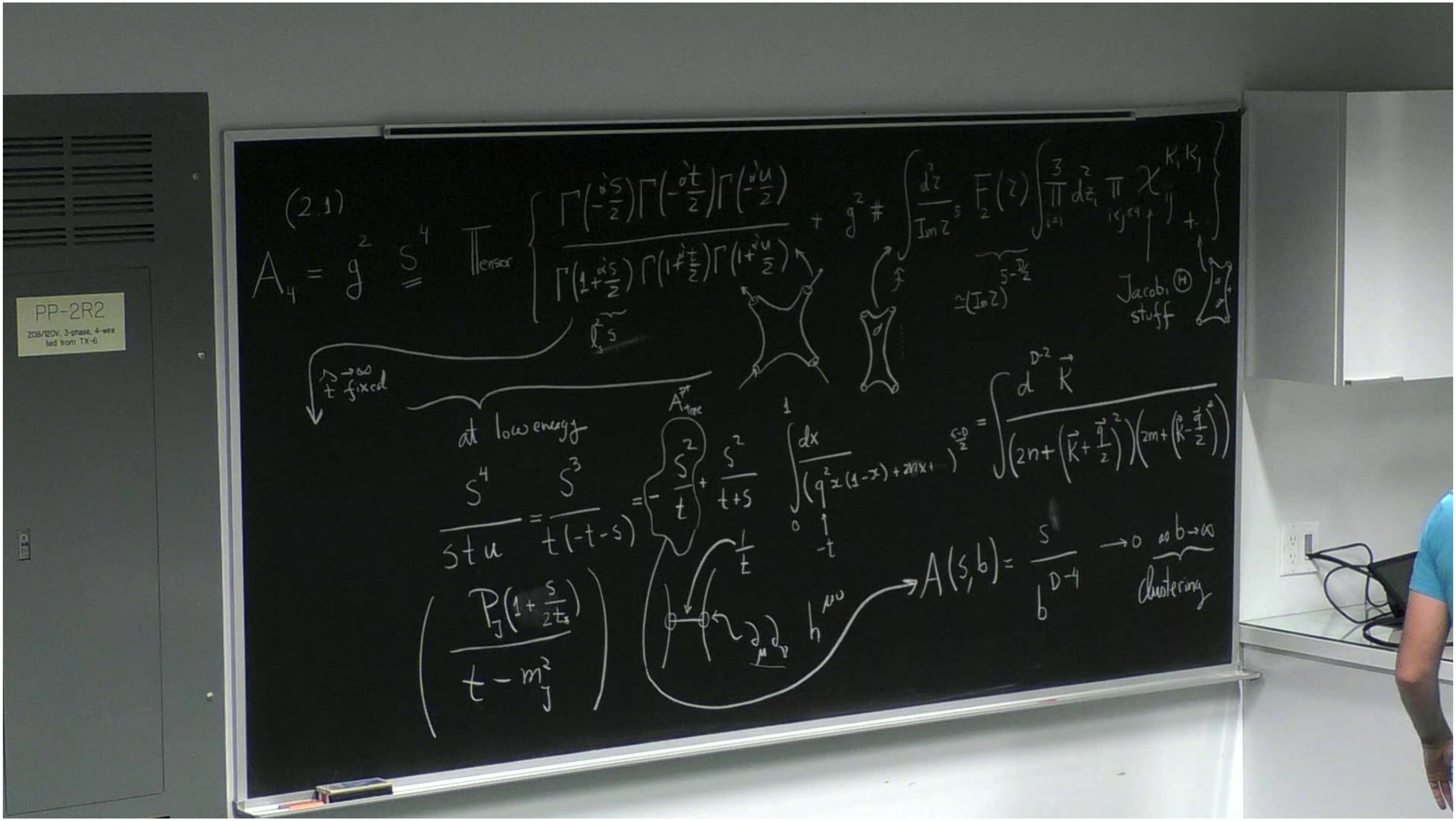
impact parameter



Semi-classically $l \simeq \dfrac{\overbrace{\sqrt{s}\,\boxed{b}}^{\text{big}}}{2}$

in gen

$$A(s,\boxed{b}) = \frac{1}{2s}\int d^{D-2}\vec{q}\; e^{i\vec{q}\cdot\vec{b}}\, A(s, t=-\vec{q}^{\,2})$$


PP-2R2
(2.1)
Jacobi stuff
t fixed
at low energy
A(s,b) =
clustering

$s \to \infty$, b large $\Rightarrow$ easy

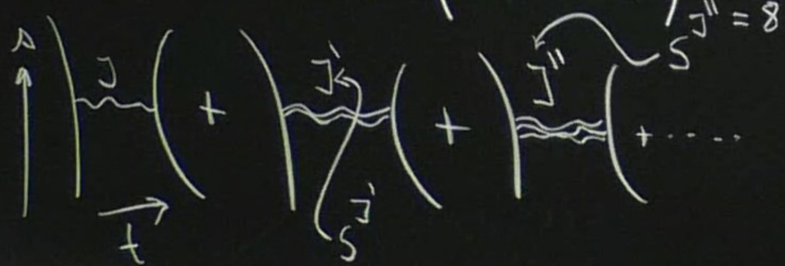
$\to \infty$, b small $\Rightarrow$ hard, we need all loops (genus) to unitarize A.

$$\mathcal{A}_{\text{TREE ST}} = g^2 \frac{\Gamma\left(-\frac{t}{2}\right)}{\Gamma\left(1+\frac{t}{2}\right)} e^{-i\frac{\pi}{2}t} \left(\frac{s}{2}\right)^{2+t} = f(t)\, s^{\overbrace{2+t}^{\alpha(t)}} \longrightarrow \infty$$

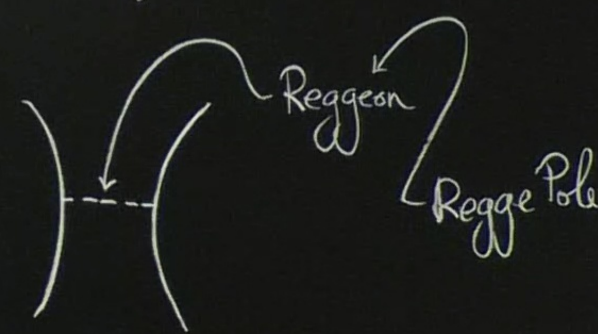
$s \to \infty$, b large $\Rightarrow$ easy

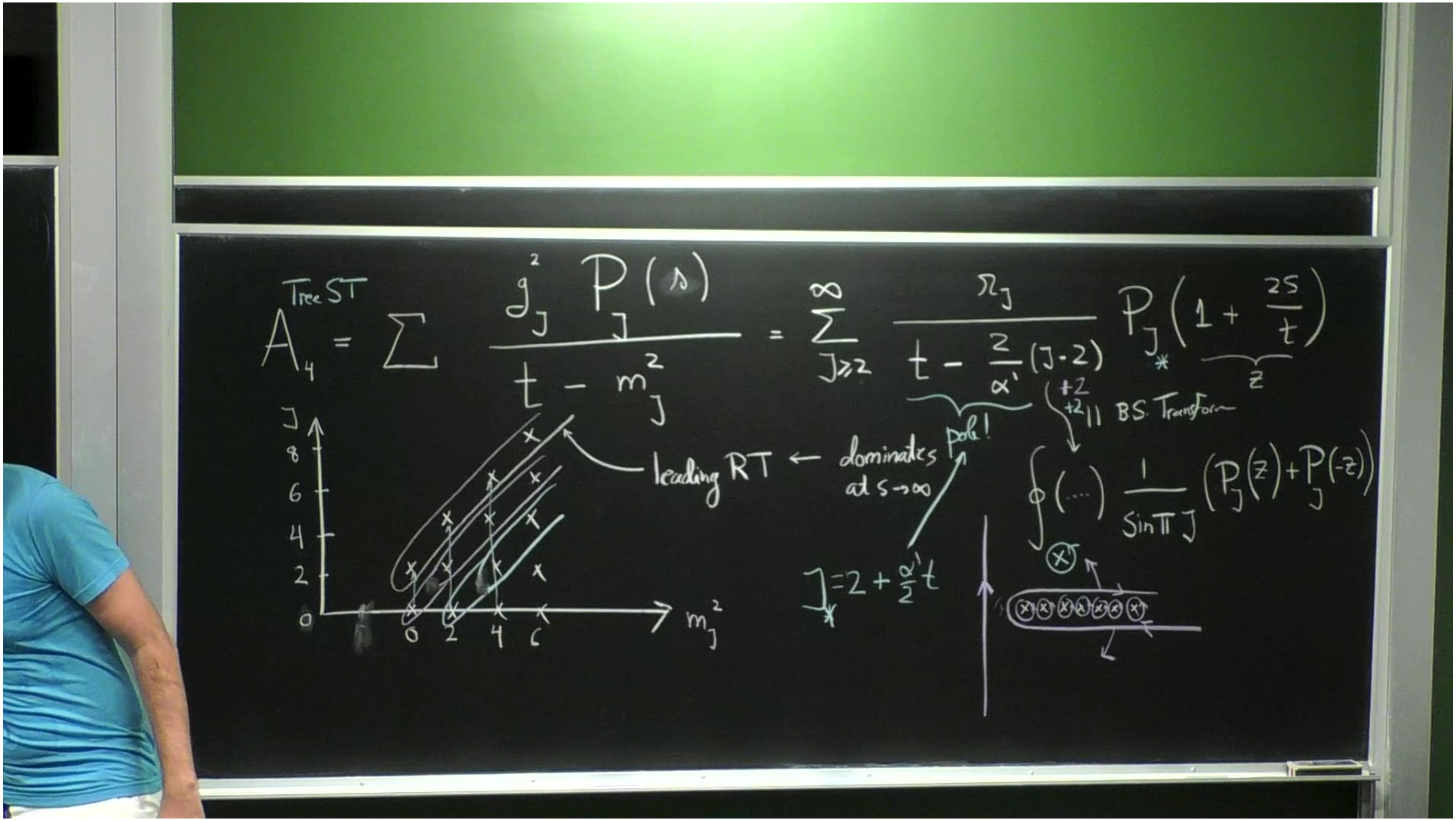
$\to \infty$, b small $\Rightarrow$ hard, we need all loops (genus) to unitarize A.

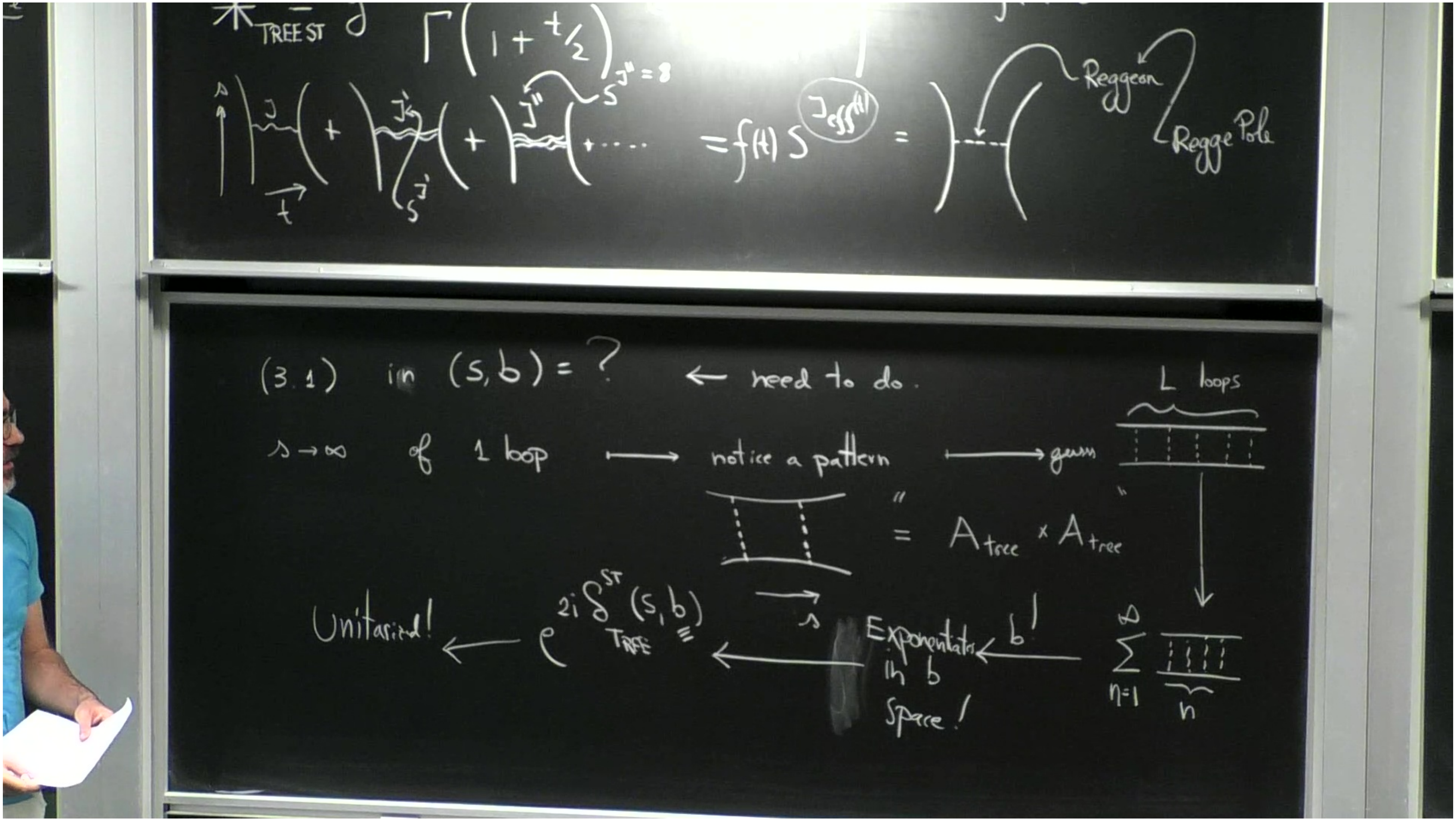
$$*_{\text{TREE ST}} = g^2 \frac{\Gamma\left(-t/2\right)}{\Gamma\left(1+t/2\right)} e^{-i\frac{\pi}{2}t} \left(\frac{s}{2}\right)^{2+t} = f(t)\, s^{\overbrace{2+t}^{\alpha(t)}} \longrightarrow \infty$$

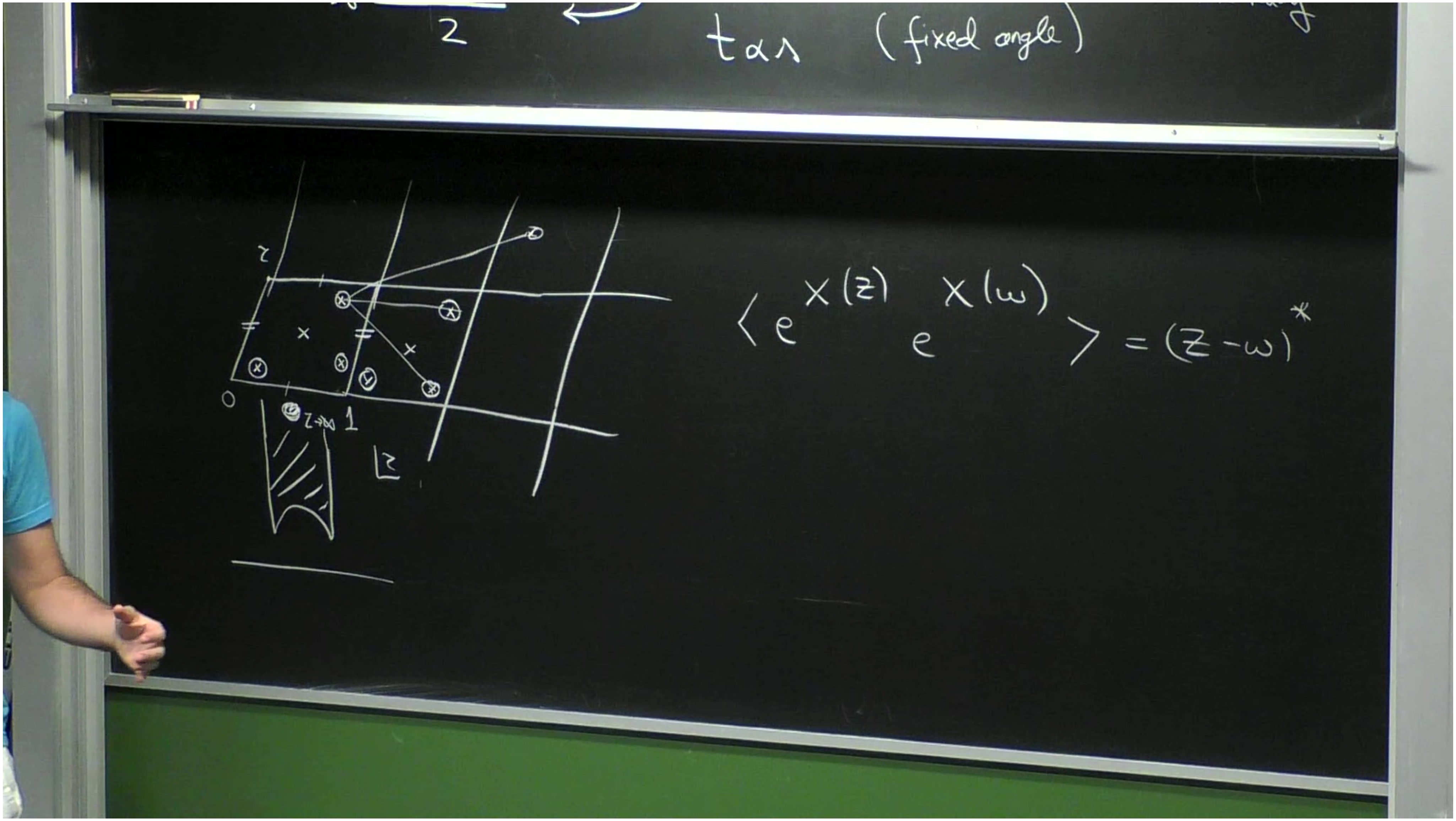


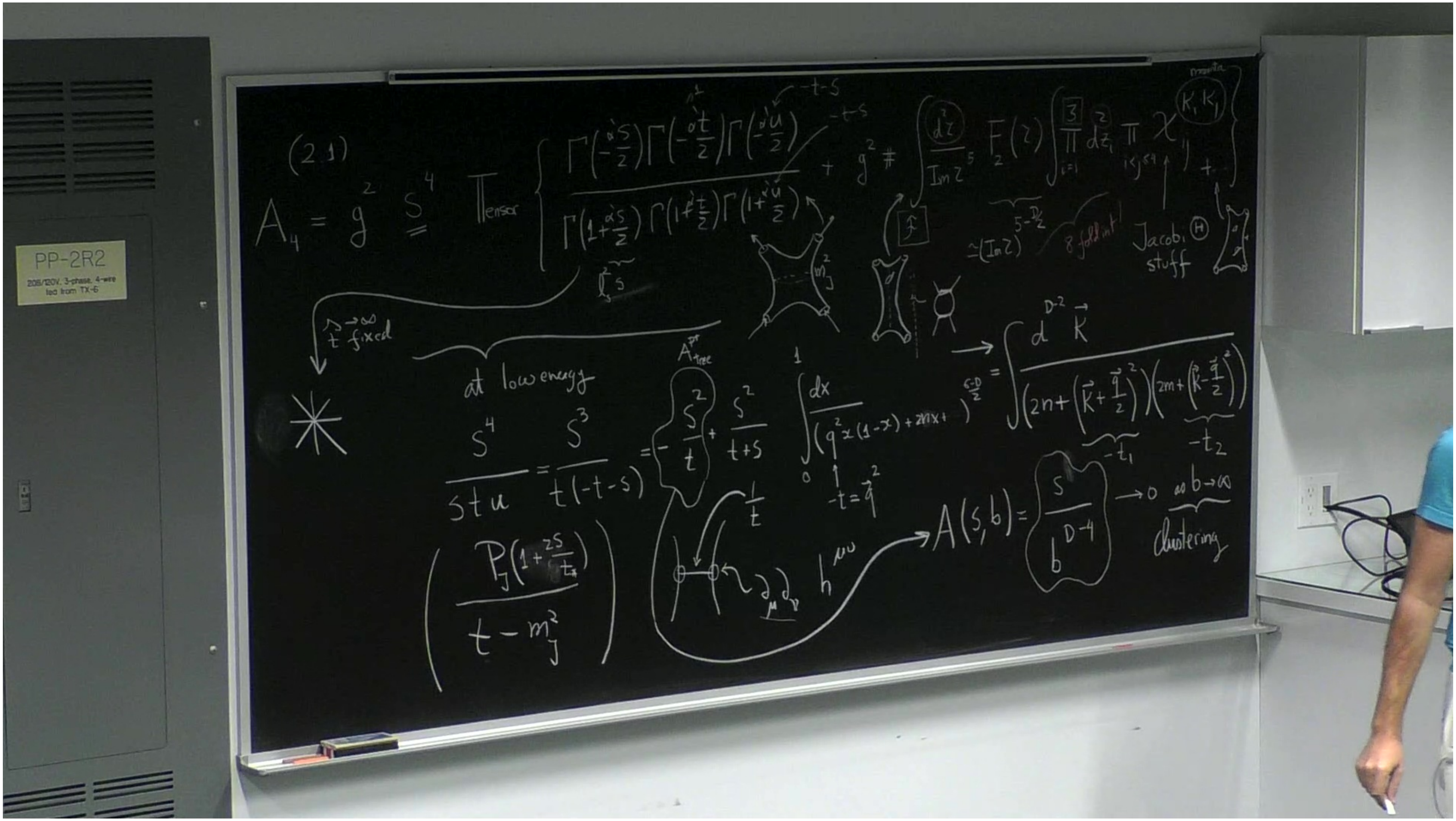
$$= f(t)\, s^{J_{eff}(t)} =$$

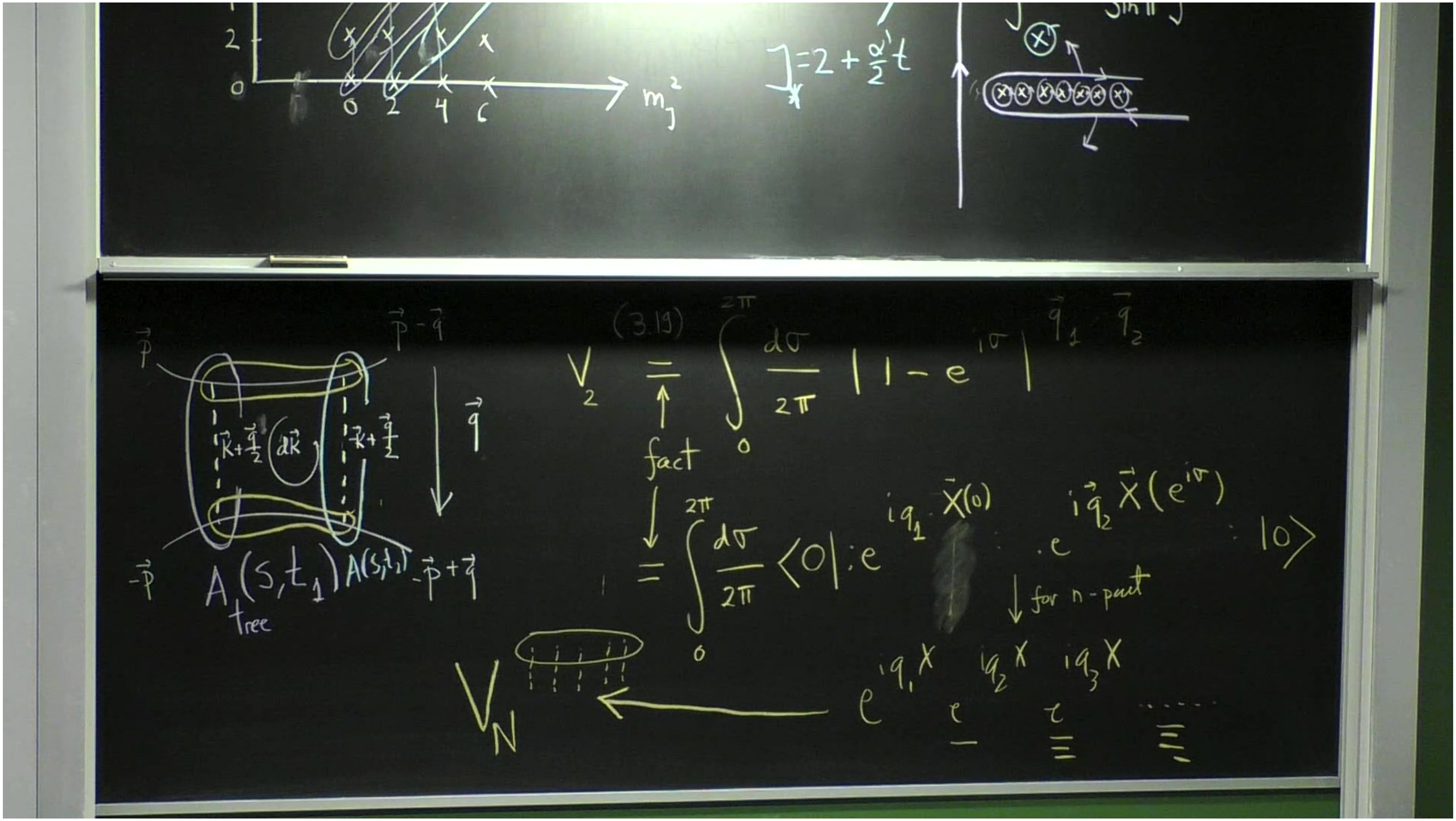



Tree ST
$A_4 = \sum \frac{g_J^2 P_J(s)}{t - m_J^2} = \sum_{J \geq 2}^{\infty} \frac{r_J}{t - \frac{2}{\alpha'}(J-2)} P_J\Big(\underbrace{1 + \frac{2s}{t}}_{z}\Big)$
BS Transform
leading RT ← dominates at $s \to \infty$
pole!
$J = 2 + \frac{\alpha'}{2} t$
$\oint (\dots) \frac{1}{\sin \pi J} \left(P_J(z) + P_J(-z)\right)$
m_J^2


TREE ST
Γ(1 + t/2)
J'' = 8
s^{J''}
= f(t) s^{J_eff(t)} =
Reggeon
Regge Pole
(3.1) in (s, b) = ? ← need to do.
s → ∞ of 1 loop ⟶ notice a pattern ⟶ guess
L loops
" = A_tree × A_tree "
Unitarized! ← e^{2i δ^{ST}_{TREE}(s, b)} ≡ ⟵ Exponentiates in b space! ← b
∑_{n=1}^{∞}
n


tan (fixed angle)
⟨e^{X(z)} e^{X(w)}⟩ = (z − w)^*
0
1
z


PP-2R2
208/120V, 3-phase, 4-wire
fed from TX-6
(2.1)
Tensor
at low energy
t fixed
Jacobi stuff
8 fold int!
clustering
A(s,b) =


m_J^2
J = 2 + α'/2 t
(3.19)
V_2 = ∫_0^{2π} dσ/2π |1 − e^{iσ}|
fact
= ∫_0^{2π} dσ/2π ⟨0| :e^{i q_1 · X(0)}: :e^{i q_2 · X(e^{iσ})}: |0⟩
for n-part
e^{i q_1 X} e^{i q_2 X} e^{i q_3 X}
V_N
A_tree (s, t_1) A(s, t_1)
p − q
−p + q
q_1 q_2

$\boxed{t} = -(p_1 - p_3)^2 \leftarrow$ mom transfer

$= -s \, \dfrac{1 - \cos\boxed{\theta}}{2}$ angle

t fixed (small angle) $\leftarrow$ We are here today

$t \propto s$ (fixed angle)

$$\prod_{i=1}^{h+1} a^{\text{Tree ST}}(s, t_i) \left\langle 0 \right| \int \prod_j d\sigma_j^u \, d\sigma_j^d \; e^{i \vec{q}_j \cdot \left(\vec{X}^u(\sigma_j^u) - \vec{X}^d(\sigma_j^d) \right)} \left| 0 \right\rangle$$

$$\frac{1}{s} A(s,t) = \int d^{D-2} \vec{b} \; e^{i \vec{q} \cdot \vec{b}} \; a(s, \vec{b})$$

$$\left\langle 0 \right| \frac{1}{2i} \left(e^{2i\hat{\delta}} - 1 \right) \left| 0 \right\rangle$$

$$\times \left(\frac{\Gamma\left(1+t_1+t_2-t\right)}{\Gamma^2\left(1+\frac{t_1+t_2-t}{2}\right)} \right)^2$$

2 reggeon vertex

$V_2(t_1,t_2)^2$

FT

RT of TL

ST

Drop

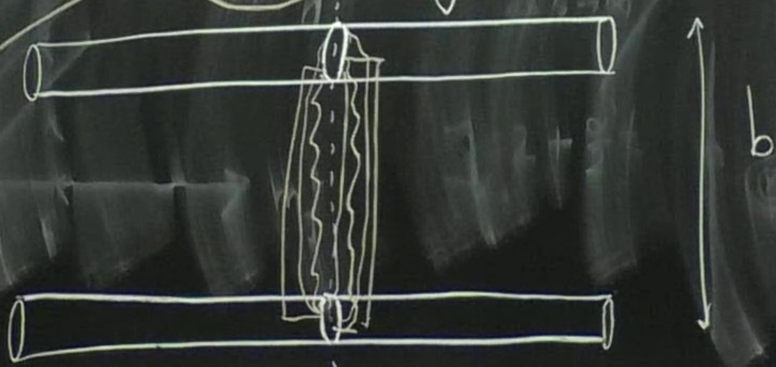
Where

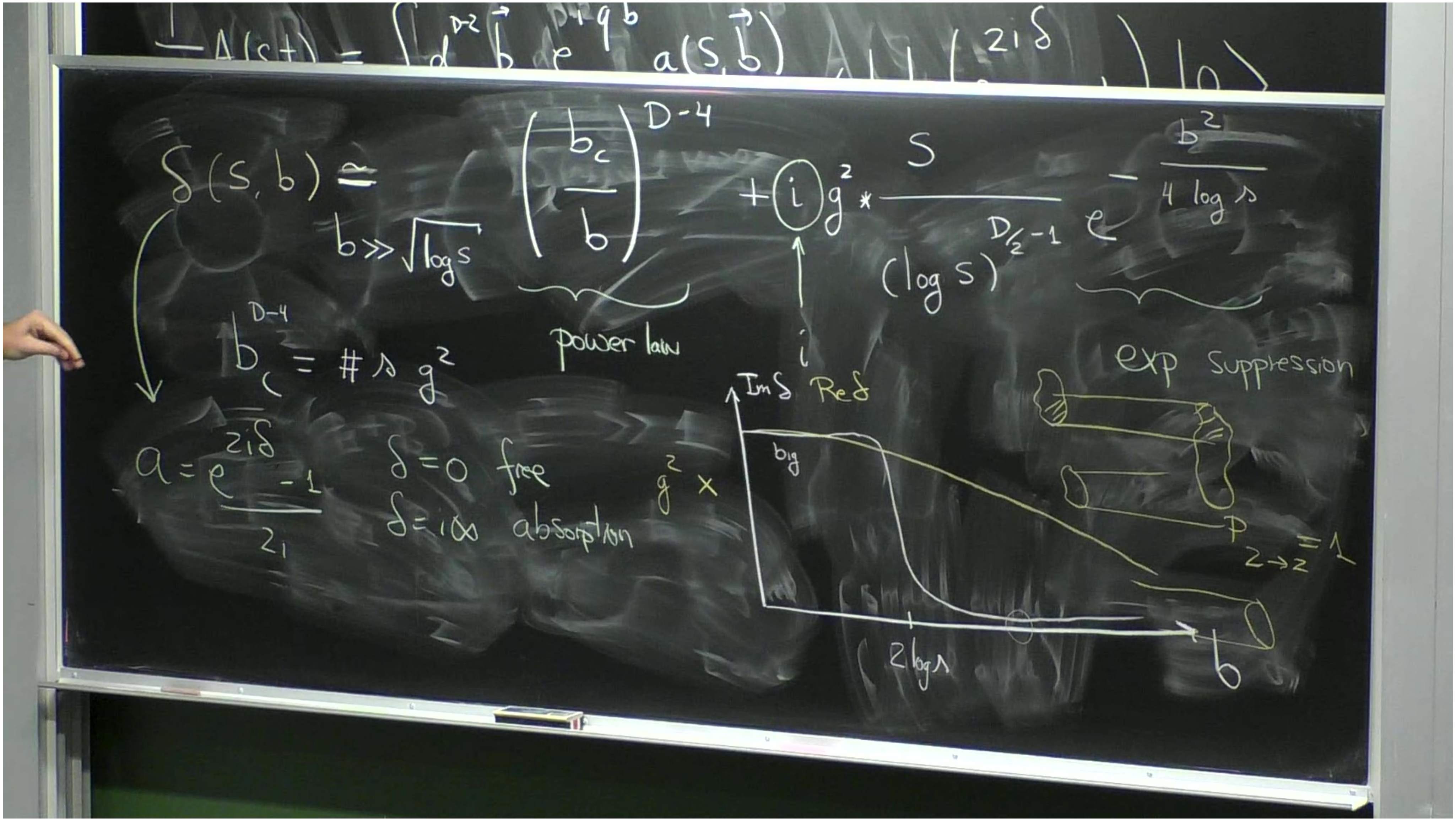
$$\hat{\delta} = \int \frac{d^{D-2}\vec{q}}{(2\pi)^{D-2}} \frac{a^{\text{Tree}}(s,t)}{s} \int d\sigma^u d\sigma^d \, e^{i\vec{q}\cdot\left(\vec{b} + \underbrace{\vec{X}^u - \vec{X}^d}\right)}$$

We can set to zero in many limits

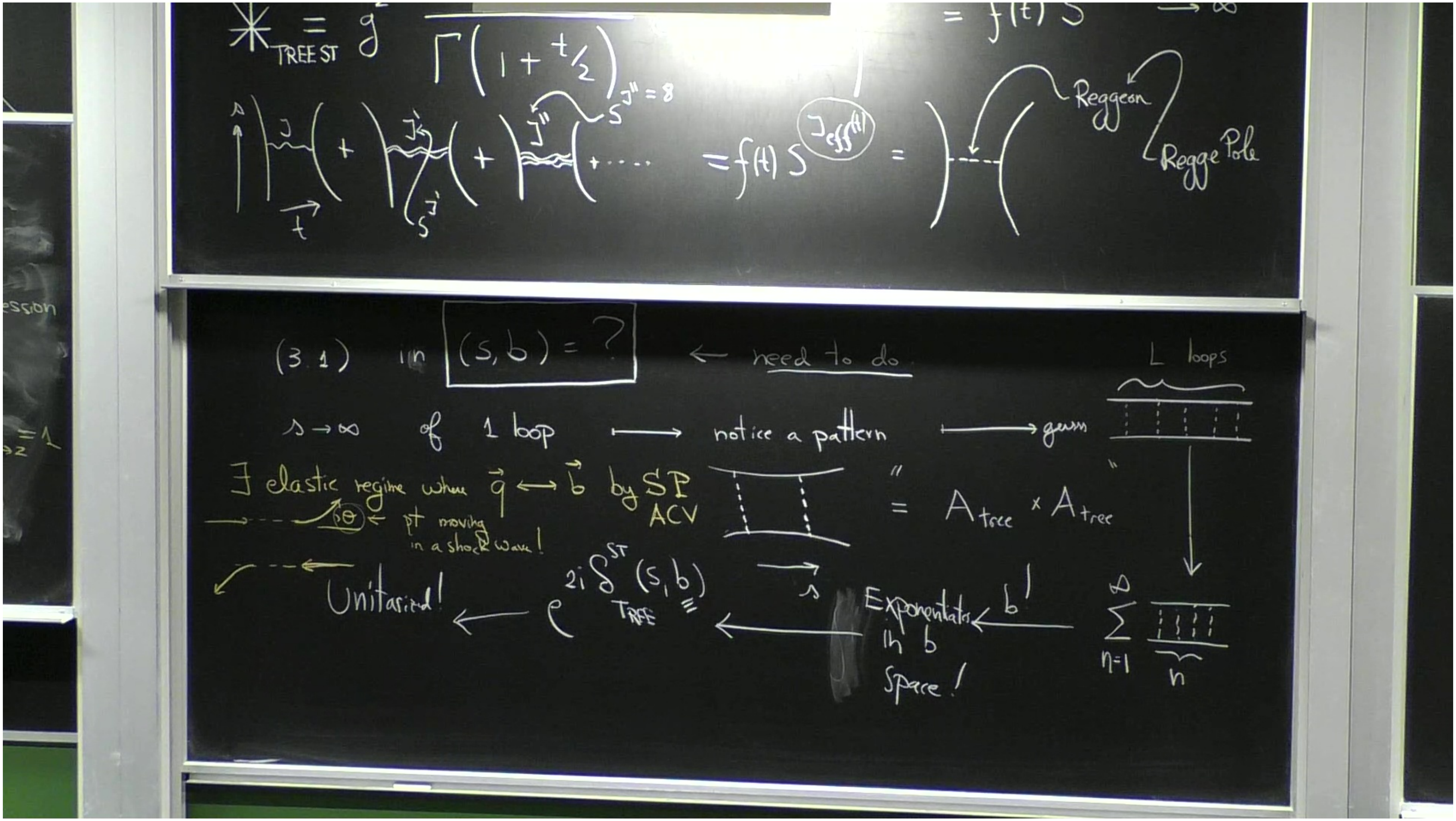
$$a(s,b) = e^{2i\,\delta^{\text{Tree}}(s,b)}$$

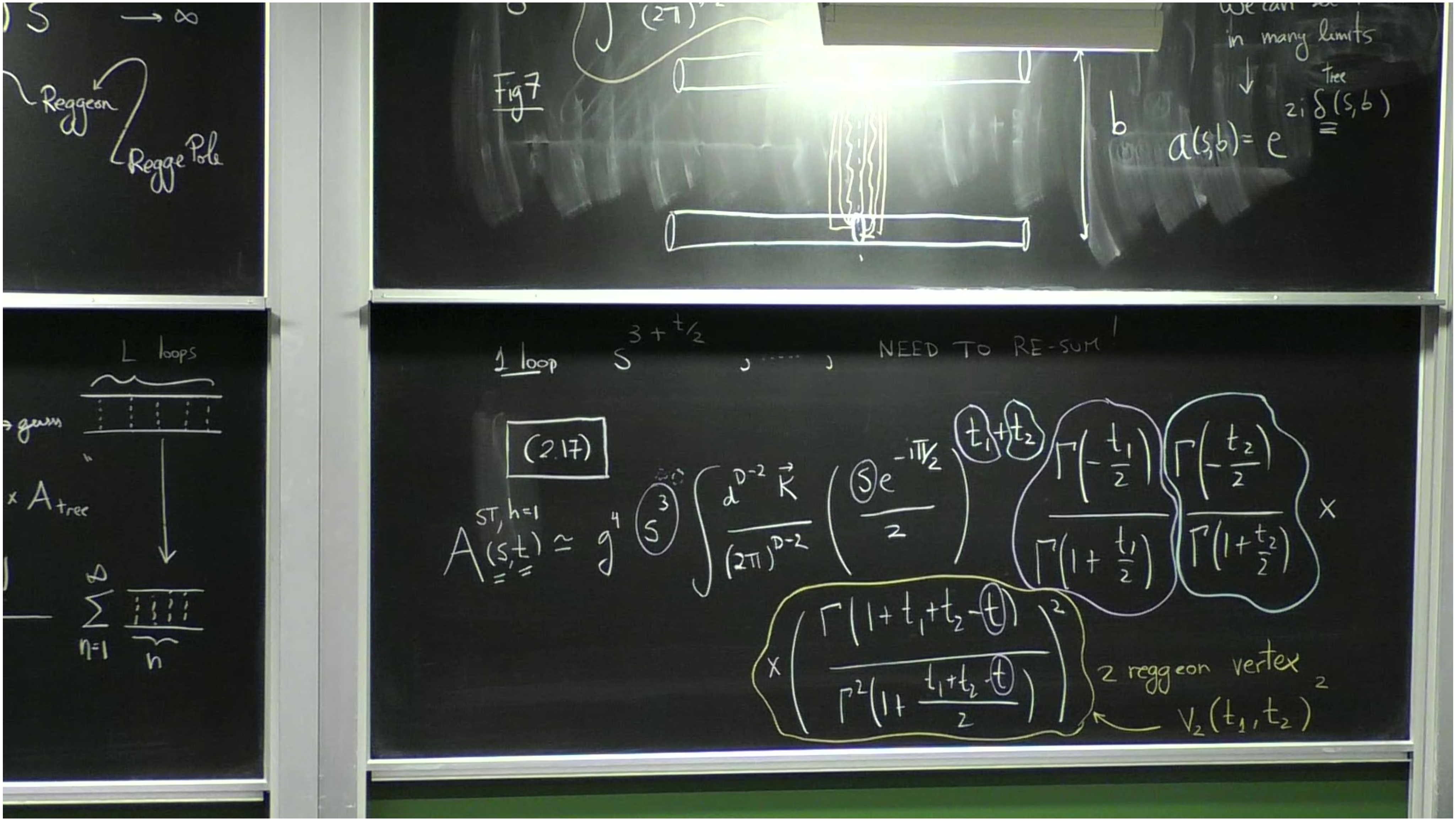
Fig 7





$\delta(s,b) \simeq \left(\frac{b_c}{b}\right)^{D-4} + i\, g_*^2 \frac{s}{(\log s)^{D/2-1}} e^{-\frac{b^2}{4\log s}}$
$b \gg \sqrt{\log s}$
power law
exp suppression
$b_c^{D-4} = \# s\, g^2$
$a = \frac{e^{2i\delta} - 1}{2i}$
$\delta = 0$ free
$\delta = i\infty$ absorption
$g^2 \times$
Im δ
Re δ
big
$2\log s$
b
P
$2 \to 2 = 1$


TREE ST
Reggeon
Regge Pole
(3.1) in (s,b) = ?
← need to do
L loops
s → ∞ of 1 loop ⟶ notice a pattern ⟶ guess
∃ elastic regime when q ⟷ b by SP
ACV
pt moving in a shock wave!
= A tree × A tree
Unitarized!
Exponentiates in b space!


s → ∞
Reggeon
Regge Pole
Fig 7
b
in many limits
tree
a(s,b) = e^{2i δ(s,b)}
L loops
× A_tree
Σ_{n=1}^{∞}
n
1 loop s^{3 + t/2}, ..., NEED TO RE-SUM!
(2.17)
A^{ST, h=1}(s,t) ≃ g^4 s^3 ∫ d^{D-2}K / (2π)^{D-2} (s e^{-iπ/2} / 2)^{t_1 + t_2} Γ(-t_1/2) / Γ(1 + t_1/2) Γ(-t_2/2) / Γ(1 + t_2/2) ×
× (Γ(1 + t_1 + t_2 - t) / Γ^2(1 + (t_1 + t_2 - t)/2))^2
2 reggeon vertex
V_2(t_1, t_2)^2