Title: Lecture - Beautiful Papers

Speakers: Pedro Vieira

Collection/Series: Beautiful Papers - October 7, 2024 - January 31, 2025

Subject: Other

Date: November 18, 2024 - 9:15 AM

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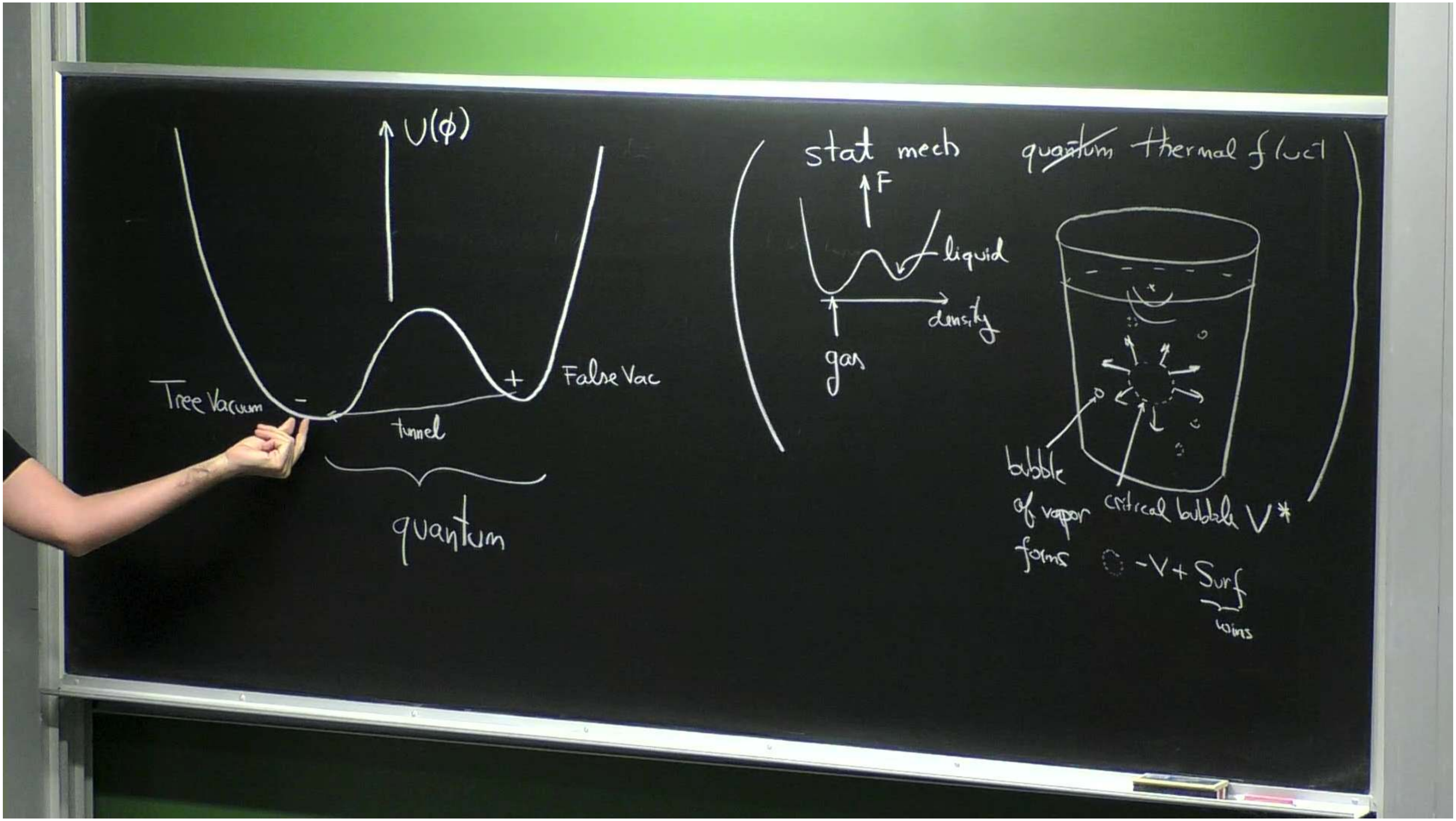
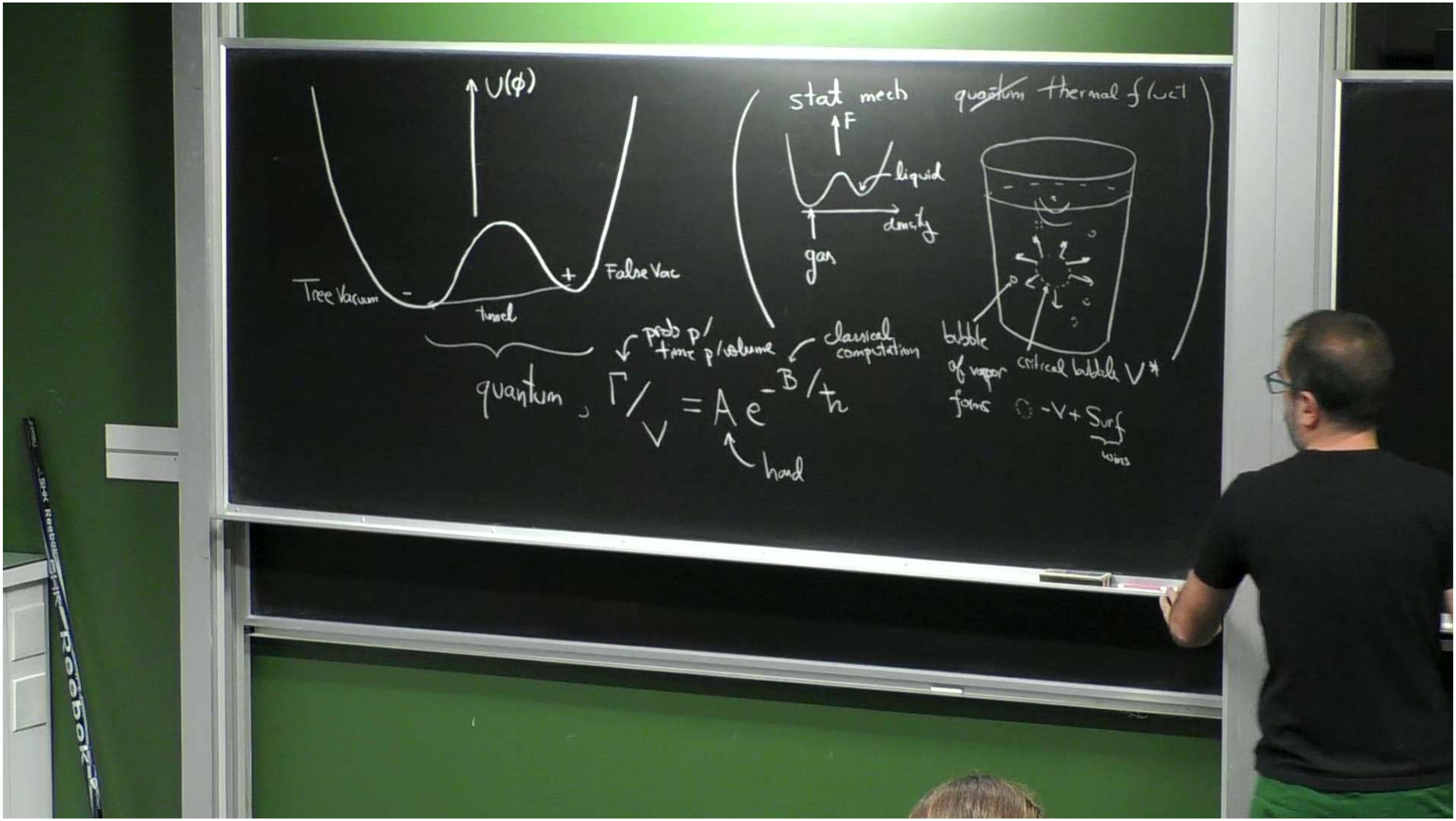


U(ϕ)
Tree Vacuum
False Vac
tunnel
quantum
stat mech
~~quantum~~ thermal fluct
F
liquid
density
gas
bubble of vapor forms
critical bubble V*
-V + Surf
wins


U(φ)
Tree Vacuum
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Γ/V = A e^{-B/ħ}
hard
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critical bubble V*
-V + Surf
wins

$$S_E = \int \left[\frac{1}{2} (\partial_\mu \phi)^2 \pm U(\phi) \right] \underbrace{d^4x}_{2\pi^2 \rho^3 d\rho}$$

$$B = S_E(\phi) - S_E(\phi_+)$$ where ϕ obeys S_E eom and

* $\phi(\rho \to \infty) = \phi_+$
* $\phi(\rho)$ is not a constant
* $S_E(\phi)$ is maximal

$$S_E = \int \left[\frac{1}{2}(\partial_\mu \phi)^2 \pm U(\phi)\right] \underbrace{d^4x}_{2\pi^2 \rho^3 d\rho} + \underbrace{\text{Gravity}}_{\text{next}}$$

CLAIM:

$$B = S_E(\phi) - S_E(\phi_+)$$

where ϕ obeys S_E eom and

"the bounce" solution

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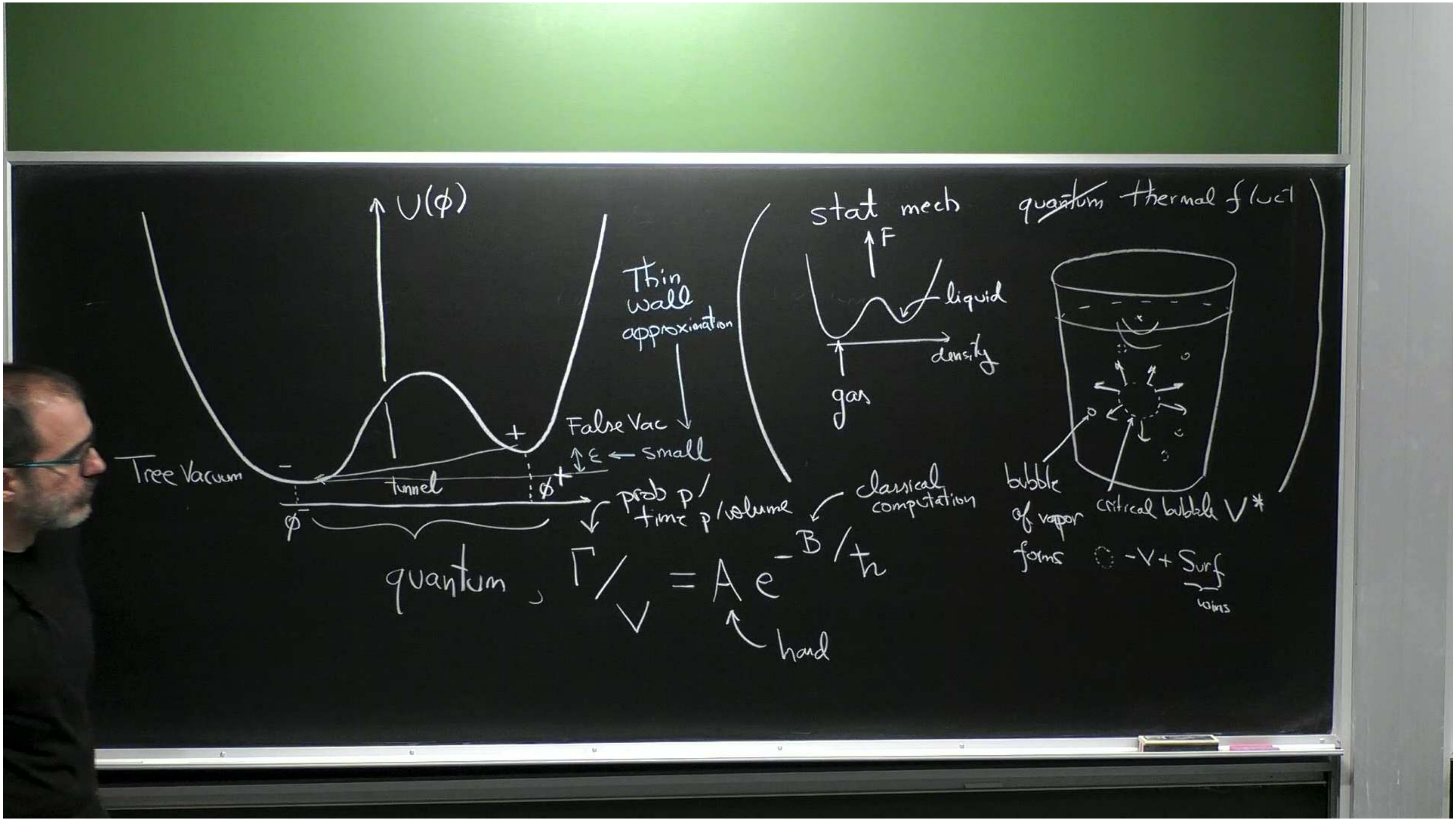
U(ϕ)
Tree Vacuum
tunnel
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ε ← small
Thin wall approximation
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$$S_E = \int \left[\frac{1}{2}(\partial_\mu \phi)^2 \pm U(\phi) \right] \underbrace{d^4x}_{2\pi^2 \rho^3 d\rho} \quad \underbrace{+ \text{ Gravity}}_{\text{next}}$$

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↑ "the bounce" solution

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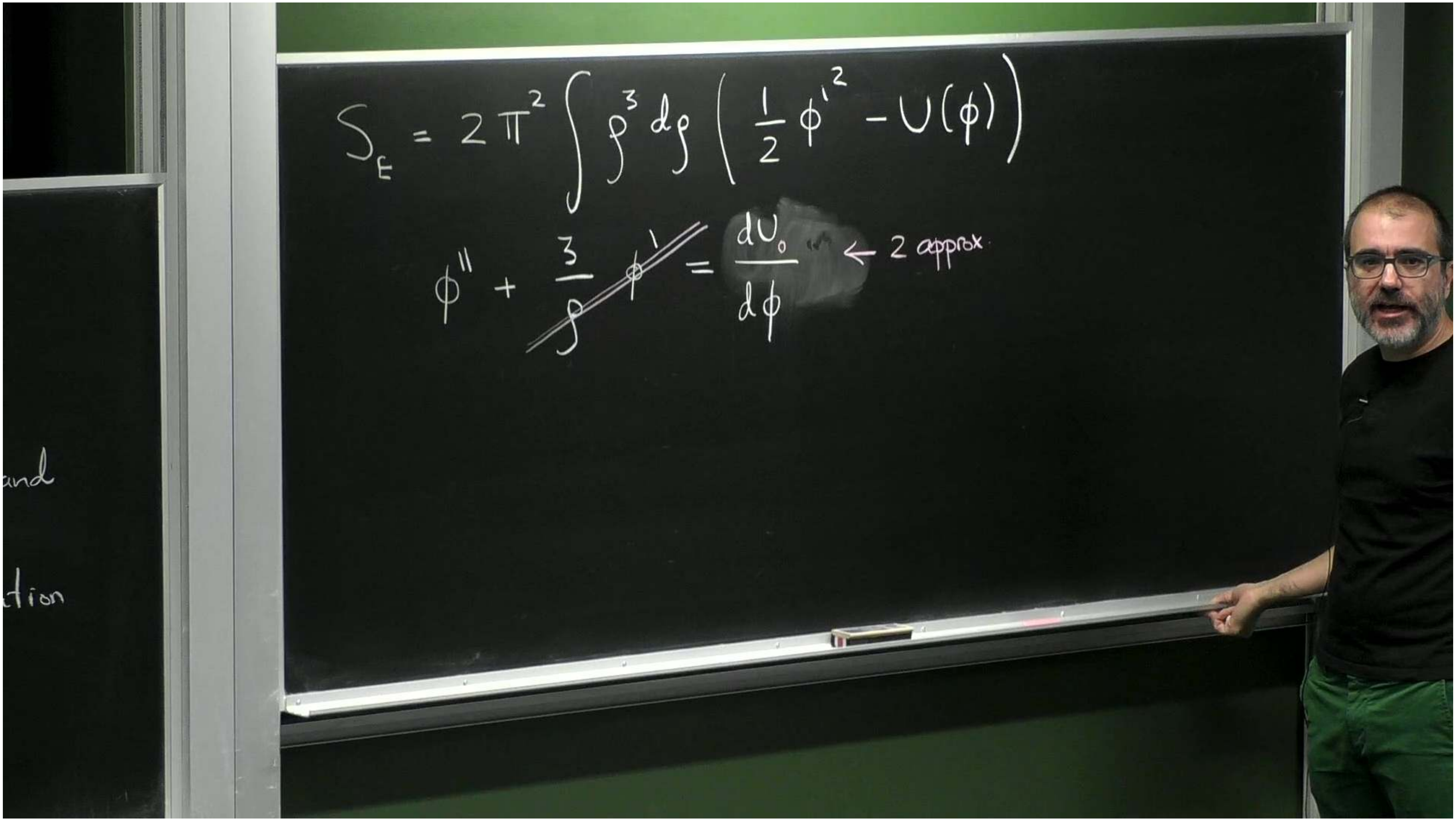
CLAIM:

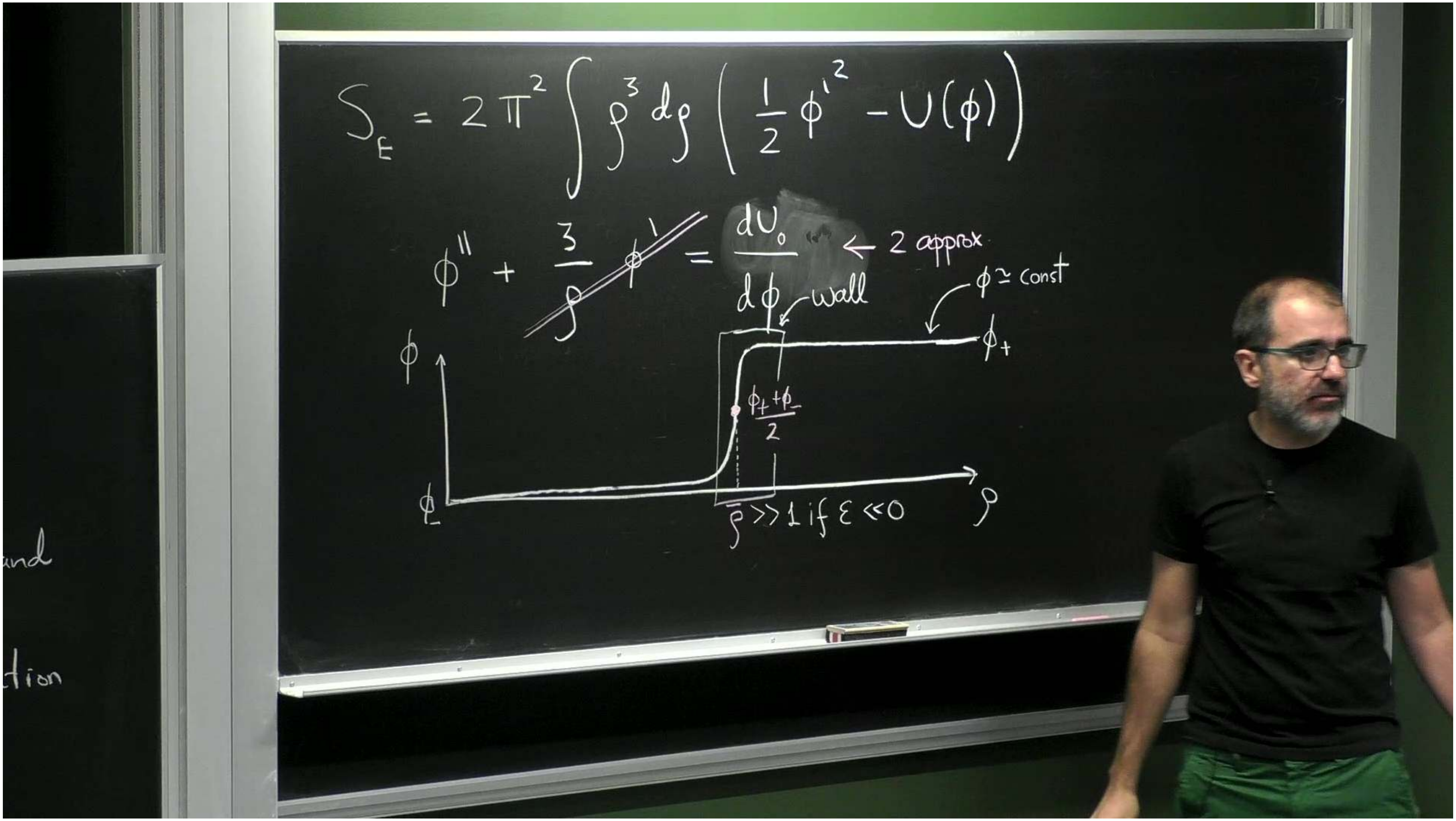
$$B = S_E(\phi) - S_E(\phi_+)$$

where ϕ obeys S_E eom and

"the bounce" solution

* $\phi(\rho \to \infty) = \phi_+$
* $\phi(\rho)$ is not a constant
* $S_E(\phi)$ is maximal $\Rightarrow$ spherical sym without gravity


$S_E = 2\pi^2 \int \rho^3 d\rho \left(\frac{1}{2} \phi'^2 - U(\phi) \right)$
$\phi'' + \cancel{\frac{3}{\rho} \phi'} = \frac{dU_0}{d\phi}$ ← 2 approx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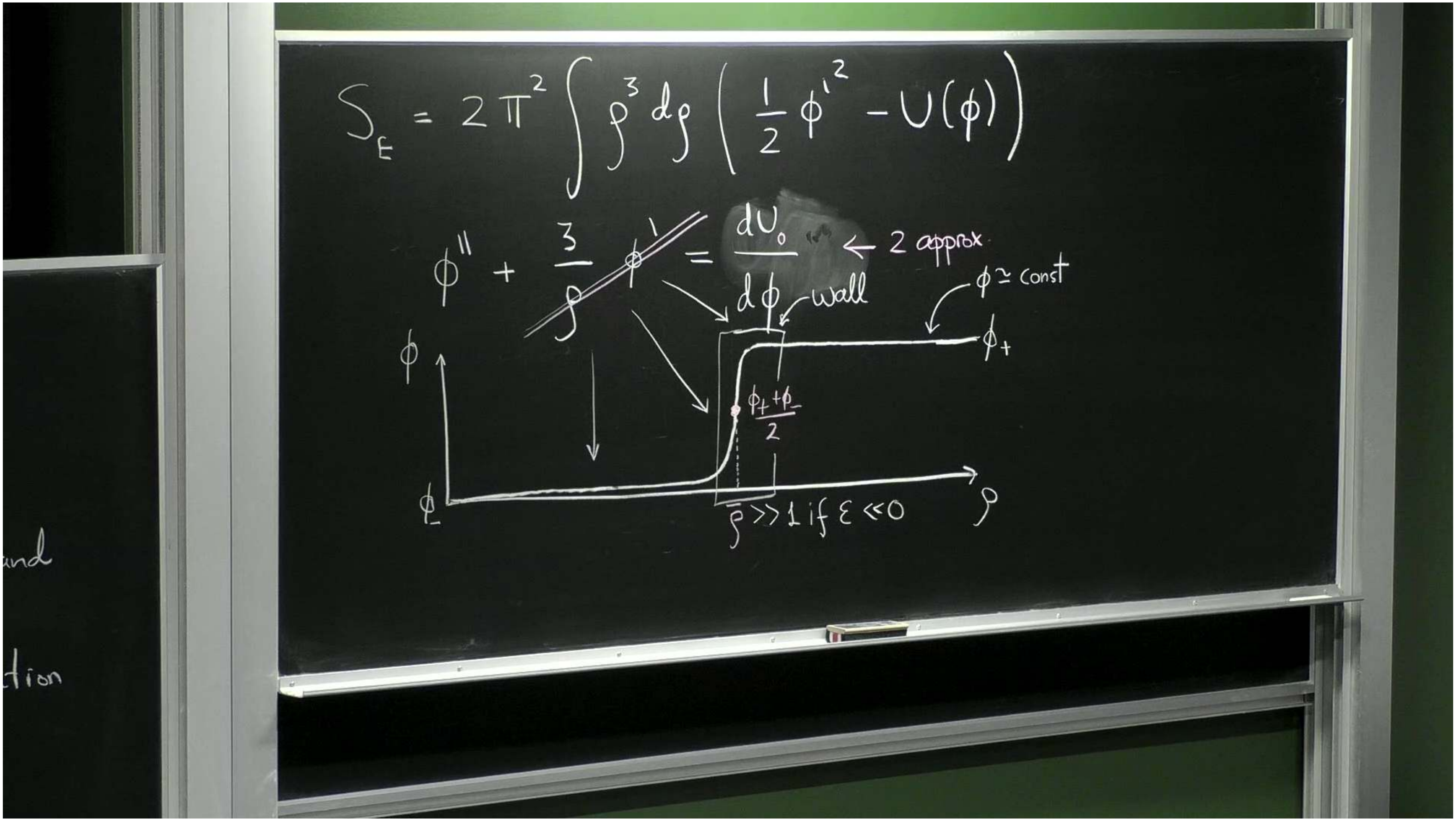
$\phi'' + \cancel{\frac{3}{\rho}\phi'} = \frac{dU_0}{d\phi}$
← 2 approx
wall
$\phi \simeq$ const
ϕ_+
ϕ
$\frac{\phi_+ + \phi_-}{2}$
ϕ_-
ρ
$\bar{\rho} \gg 1$ if $\varepsilon \ll 0$
and
tion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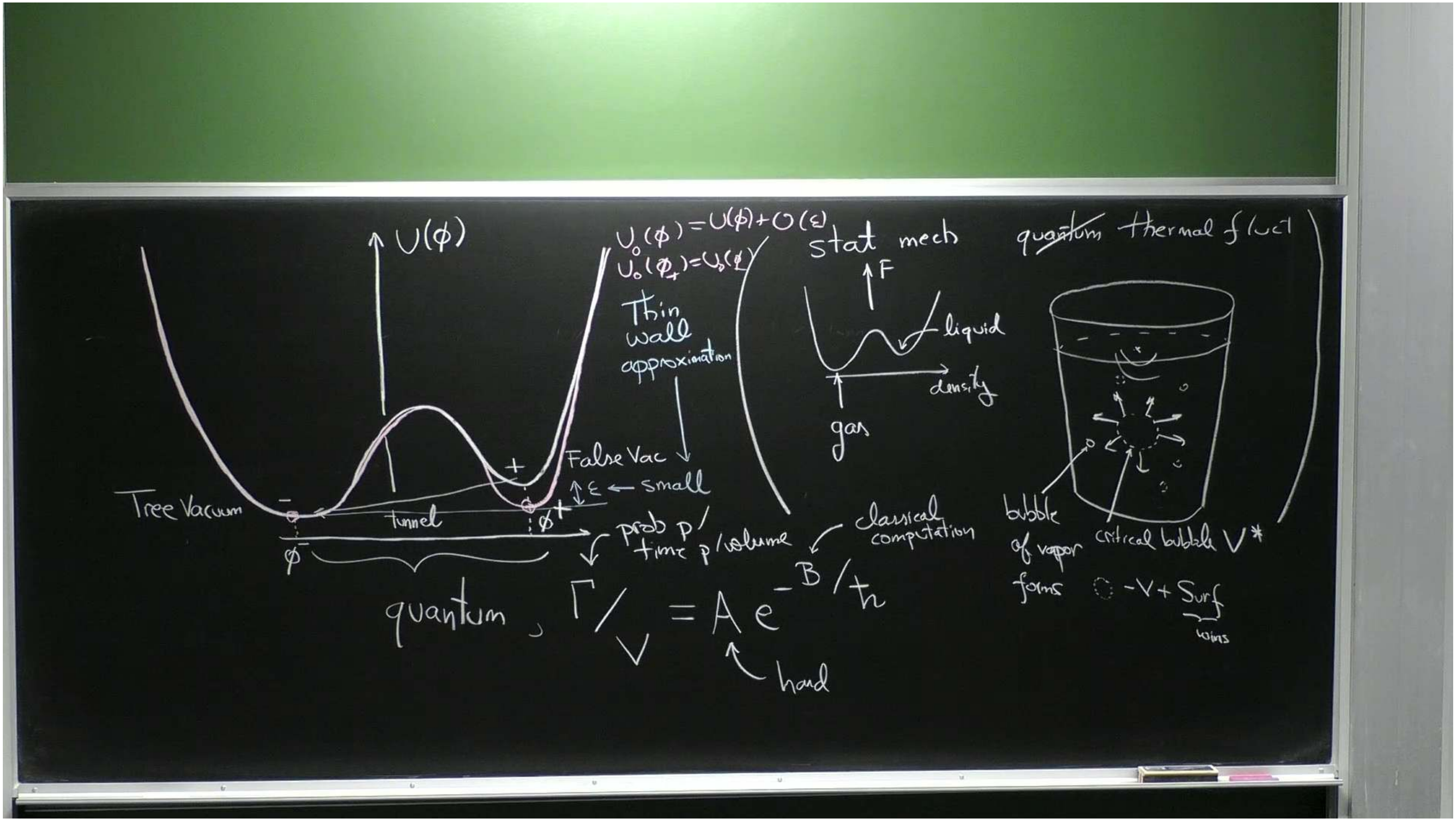
← 2 approx
wall
$\phi \simeq$ const
ϕ
ϕ_+
$\frac{\phi_+ + \phi_-}{2}$
ϕ_-
$\bar{\rho} >> 1$ if $\varepsilon << 0$
ρ
and
tion


U(φ)
U₀(φ) = U(φ) + O(ε)
U₀(φ₊) = U₀(φ₋)
Thin wall approximation
False Vac
ε ← small
True Vacuum
tunnel
φ⁻
φ⁺
prob p/ time p/ volume
quantum
Γ/V = A e^(−B/ħ)
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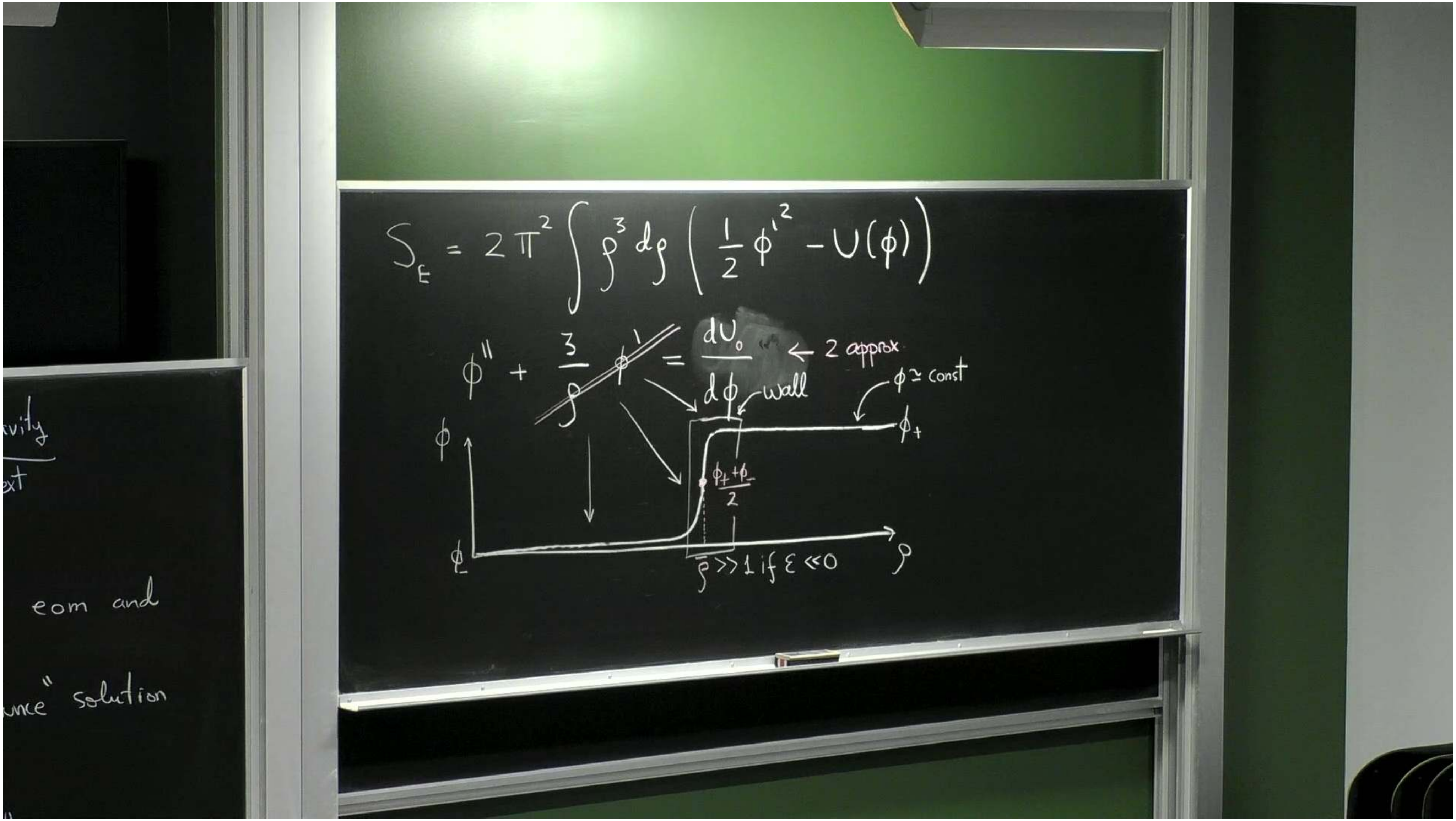
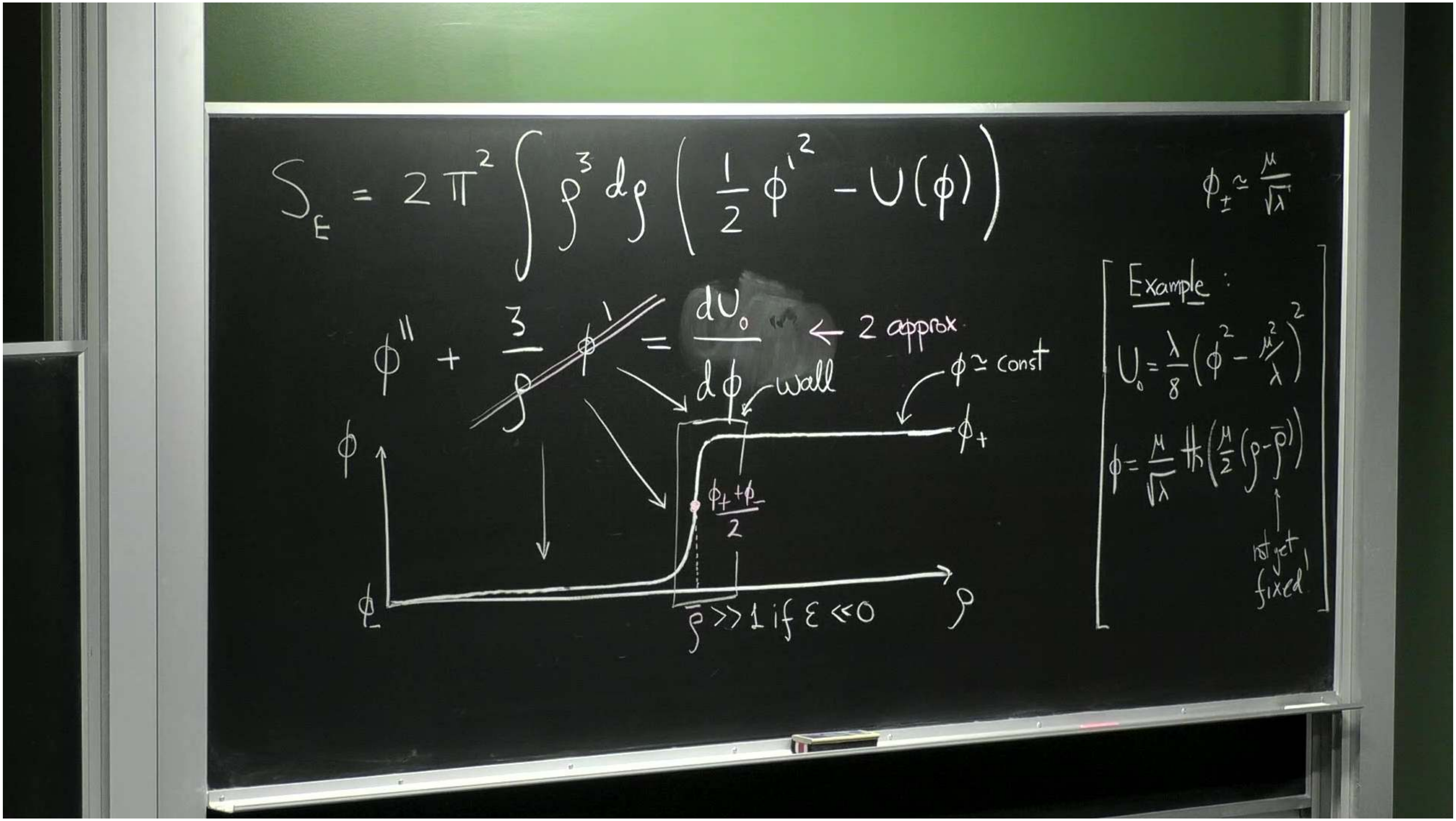

$S_E = 2\pi^2 \int \rho^3 d\rho \left(\frac{1}{2}\phi'^2 - U(\phi) \right)$
$\phi'' + \cancel{\frac{3}{\rho}\phi'} = \frac{dU_0}{d\phi}$
← 2 approx
wall
$\phi \simeq$ const
ϕ_+
$\frac{\phi_+ + \phi_-}{2}$
ϕ_-
$\bar{\rho} >> 1$ if $\varepsilon << 0$
eom and
solution



$S_E = 2\pi^2 \int \rho^3 d\rho \left(\frac{1}{2}\phi'^2 - U(\phi) \right)$
$\phi_\pm \simeq \frac{\mu}{\sqrt{\lambda}}$
$\phi'' + \cancel{\frac{3}{\rho}\phi'} = \frac{dU_0}{d\phi}$
← 2 approx.
wall
$\phi \simeq$ const
ϕ
ϕ_+
$\frac{\phi_+ + \phi_-}{2}$
ϕ_-
ρ
$\bar{\rho} \gg 1$ if $\varepsilon \ll 0$
Example:
$U_0 = \frac{\lambda}{8}\left(\phi^2 - \frac{\mu^2}{\lambda}\right)^2$
$\phi = \frac{\mu}{\sqrt{\lambda}} \tanh\left(\frac{\mu}{2}(\rho - \bar{\rho})\right)$
not yet fixed

tunnel

ϕ^-

prob p/ time p/volume

classical computation

$$\text{quantum}, \quad \Gamma/V = A e^{-B/\hbar}$$

hard

bubble of vapor forms

critical bubble V^*

$-V + \underbrace{Surf}_{\text{wins}}$

$$\frac{1}{2}\phi'^2 - U_0(\phi) = -U_0(\phi_+), \quad \rho - \bar{\rho} = \int_{\frac{\phi^+ + \phi^-}{2}} \sqrt{2(U_0(\phi) - U_0(\phi_+))}$$

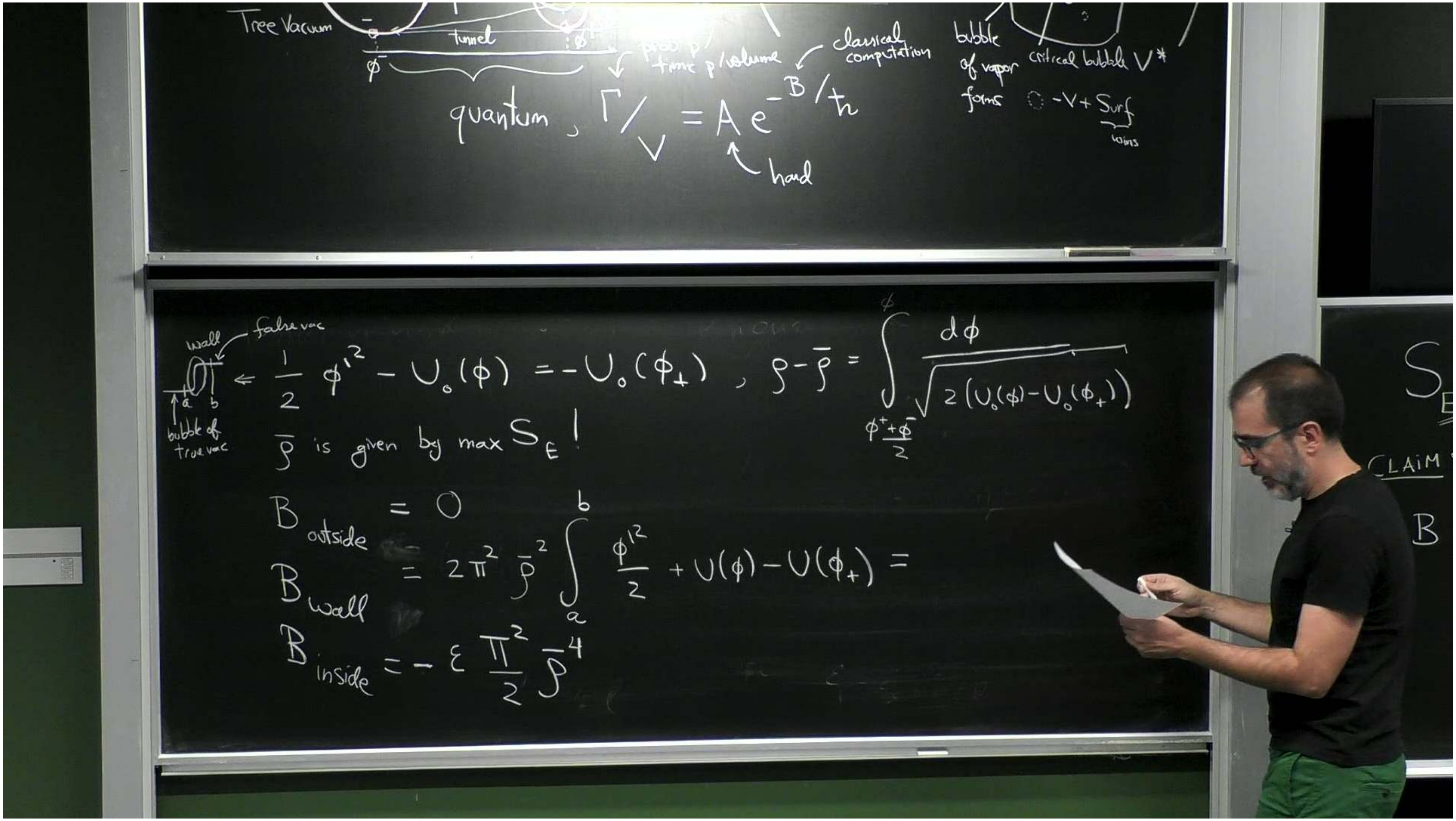
bubble of true vac

$\bar{\rho}$ is given by max S_E!

$B_{\text{outside}} = 0$

$B_{\text{wall}} =$

$B_{\text{inside}} =$



Tree Vacuum
tunnel
prob p/ time p/volume
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bubble of vapor forms
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quantum , Γ/V = A e^{-B/ħ}
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wall
false vac
bubble of true vac
1/2 φ'^2 − U_0(φ) = −U_0(φ_+) , ρ − ρ̄ = ∫_{(φ^+ + φ^-)/2}^{φ} dφ / √(2(U_0(φ) − U_0(φ_+)))
ρ̄ is given by max S_E !
B_outside = 0
B_wall = 2π^2 ρ̄^2 ∫_a^b φ'^2/2 + U(φ) − U(φ_+) =
B_inside = − ε π^2/2 ρ̄^4
CLAIM

Tree Vacuum

tunnel

ϕ^- ϕ^+

prob p/ time p/ volume

classical computation

quantum , $\Gamma / V = A e^{-B/\hbar}$

hard

bubble of vapor forms

critical bubble V^*

$-V + \text{Surf}$ wins

wall

false vac

bubble of true vac

a b

$$\frac{1}{2}\phi'^2 - U_0(\phi) = -U_0(\phi_+) \quad , \quad \rho - \bar\rho = \int_{\frac{\phi^+ + \phi^-}{2}}^{\phi} \frac{d\phi}{\sqrt{2\left(U_0(\phi) - U_0(\phi_+)\right)}}$$

$\bar\rho$ is given by max S_E!

$$B_{\text{outside}} = 0$$

$$B_{\text{wall}} = 2\pi^2 \bar\rho^3 \int_a^b d\rho \left(\frac{\phi'^2}{2} + U(\phi) - U(\phi_+) \right) = 2\pi^2 \bar\rho^3 \left[\int_{\phi^-}^{\phi^+} d\phi \sqrt{U(\phi) - U(\phi_+)} \right]$$

$$B_{\text{inside}} = -\varepsilon \frac{\pi^2}{2} \bar\rho^4$$

S_E

True Vacuum

tunnel

ϕ^- ϕ^+

prob p/ time p/volume

classical computation

quantum , $\Gamma / V = A\, e^{-B/\hbar}$

hard

bubble of vapor forms

critical bubble V^*

$-V + \underbrace{\text{Surf}}_{\text{wins}}$

wall — false vac

bubble of true vac

$a \quad b$

$$\frac{1}{2}\phi'^2 - U_0(\phi) = -U_0(\phi_+) \quad , \quad \rho - \bar{\rho} = \int_{\frac{\phi^+ + \phi^-}{2}}^{\phi} \frac{d\phi}{\sqrt{2\left(U_0(\phi) - U_0(\phi_+)\right)}}$$

$\bar{\rho}$ is given by max S_E!

$$B_{\text{outside}} = 0$$

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$$B_{\text{inside}} = -\varepsilon \frac{\pi^2}{2} \bar{\rho}^4$$

S

CLAIM

B

Tree Vacuum

tunnel

ϕ^- ϕ^+

prob p/ time p/volume

classical computation

quantum , $\Gamma / V = A\, e^{-B/\hbar}$

hard

bubble of vapor forms

critical bubble V^*

$-V + \underbrace{\text{Surf}}_{\text{wins}}$

wall

false vac

a b

bubble of true vac

$$\frac{1}{2}\phi'^2 - U_0(\phi) = -U_0(\phi_+) \; , \quad \rho - \bar{\rho} = \int_{\frac{\phi^+ + \phi^-}{2}}^{\phi} \frac{d\phi}{\sqrt{2\left(U_0(\phi) - U_0(\phi_+)\right)}}$$

$\bar{\rho}$ is given by max S_E !

$$\left(S_1 = \frac{2}{3}\frac{\mu^3}{\lambda} \;\; // \text{Example} \right)$$

$$B_{\text{outside}} = 0$$

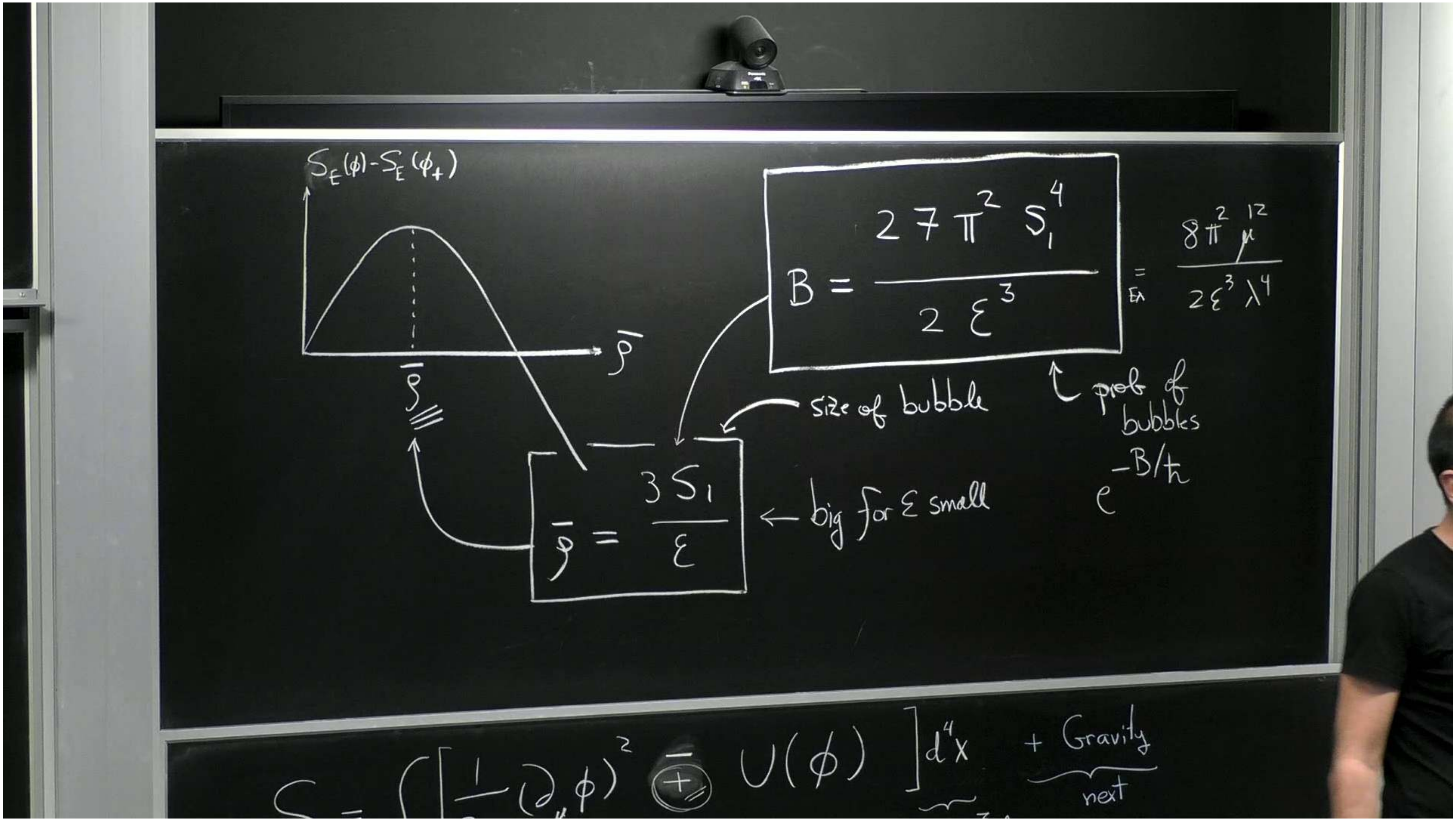
$$B_{\text{wall}} = 2\pi^2 \bar{\rho}^3 \int_a^b d\rho \left(\frac{\phi'^2}{2} + U(\phi) - U(\phi_+) \right) = 2\pi^2 \bar{\rho}^3 \underbrace{\left[\int_{\phi^-}^{\phi^+} d\phi \sqrt{U(\phi) - U(\phi_+)} \right]}_{= S_1}$$

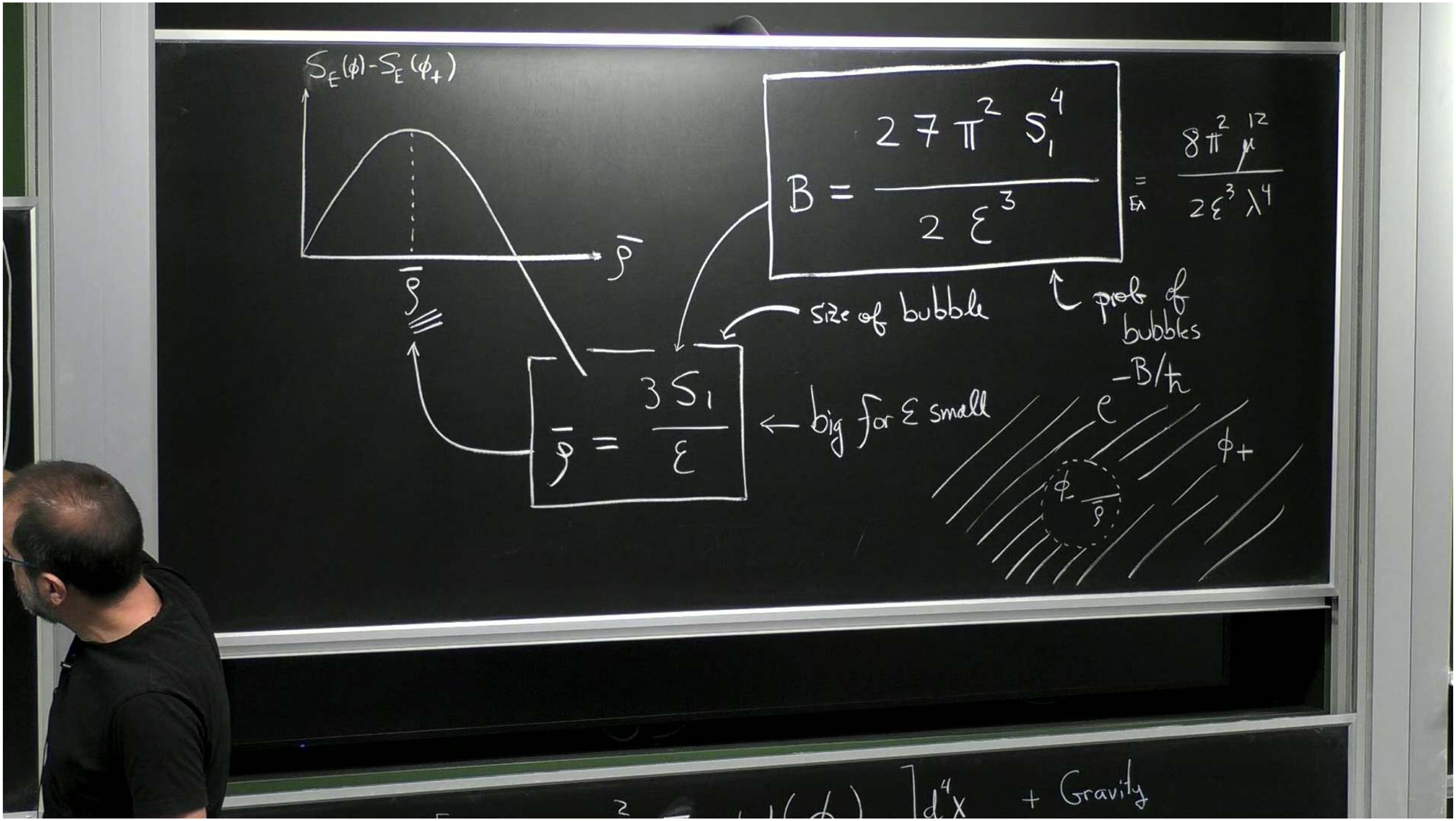
$$B_{\text{inside}} = -\varepsilon \frac{\pi^2}{2} \bar{\rho}^4$$

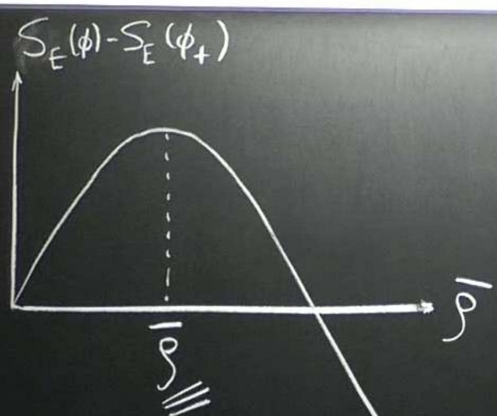
S_E

CLAIM

B


$S_E(\phi) - S_E(\phi_+)$
$\bar{\rho}$
$B = \frac{27\pi^2 S_1^4}{2\varepsilon^3}$
$\stackrel{=}{\text{Ex}} \frac{8\pi^2 \mu^{12}}{2\varepsilon^3\lambda^4}$
prob of bubbles
$e^{-B/\hbar}$
size of bubble
$\bar{\rho} = \frac{3S_1}{\varepsilon}$
big for ε small
$S = \int \left[\frac{1}{2}(\partial_\mu \phi)^2 \mp U(\phi)\right] d^4x$
+ Gravity
next


$S_E(\phi) - S_E(\phi_+)$
$\bar{\rho}$
$B = \frac{27\pi^2 S_1^4}{2\varepsilon^3} = \frac{8\pi^2\mu^{12}}{2\varepsilon^3\lambda^4}$
Ex
size of bubble
prob of bubbles
$e^{-B/\hbar}$
$\bar{\rho} = \frac{3S_1}{\varepsilon}$
big for ε small
ϕ_+
ϕ_-
d^4x + Gravity



$$\boxed{B = \frac{27\pi^2 S_1^4}{2\varepsilon^3}} \underset{E\lambda}{=} \frac{8\pi^2\mu^{12}}{2\varepsilon^3\lambda^4}$$

↑ prob of bubbles

$$e^{-B/\hbar}$$

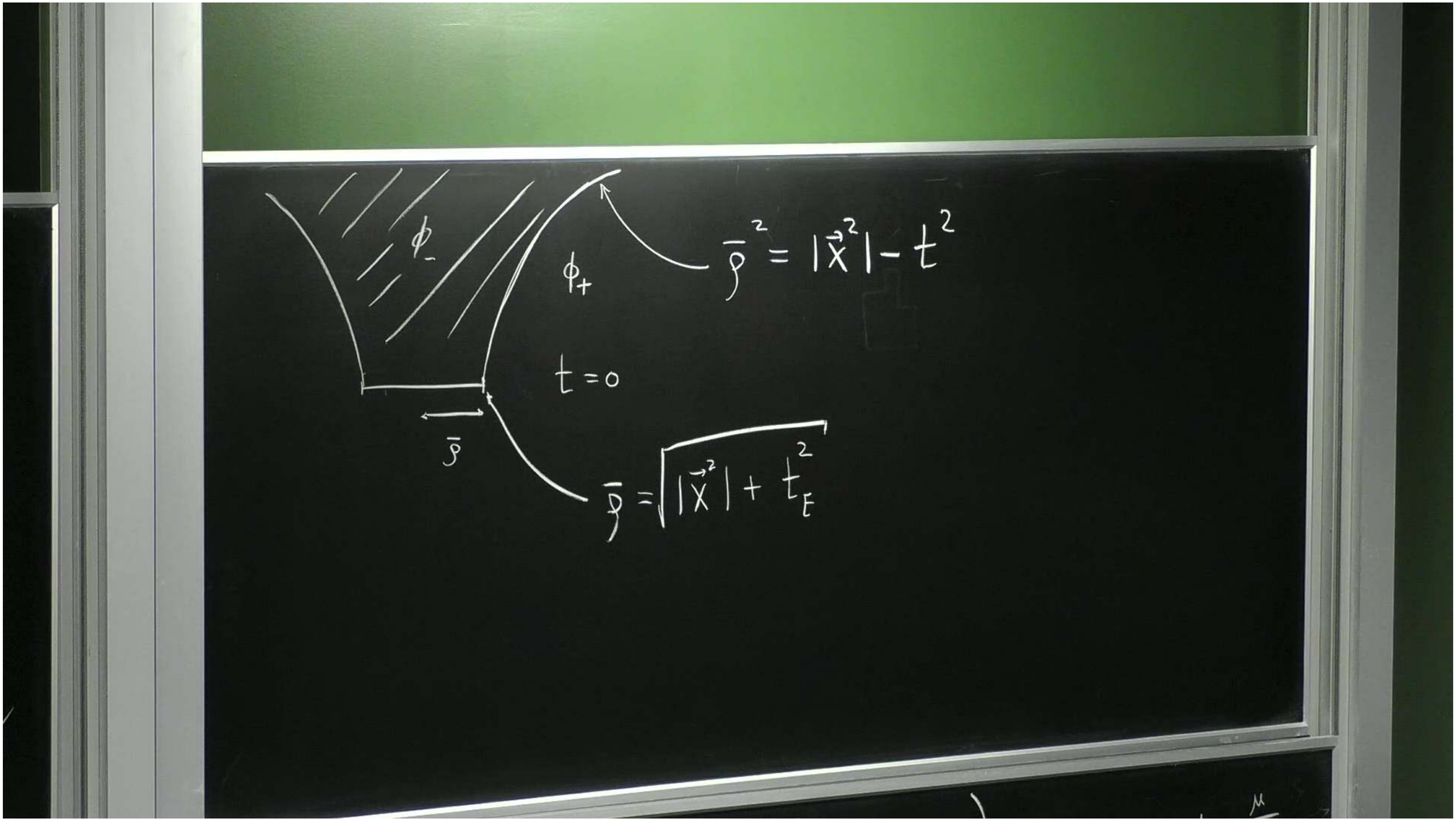
size of bubble

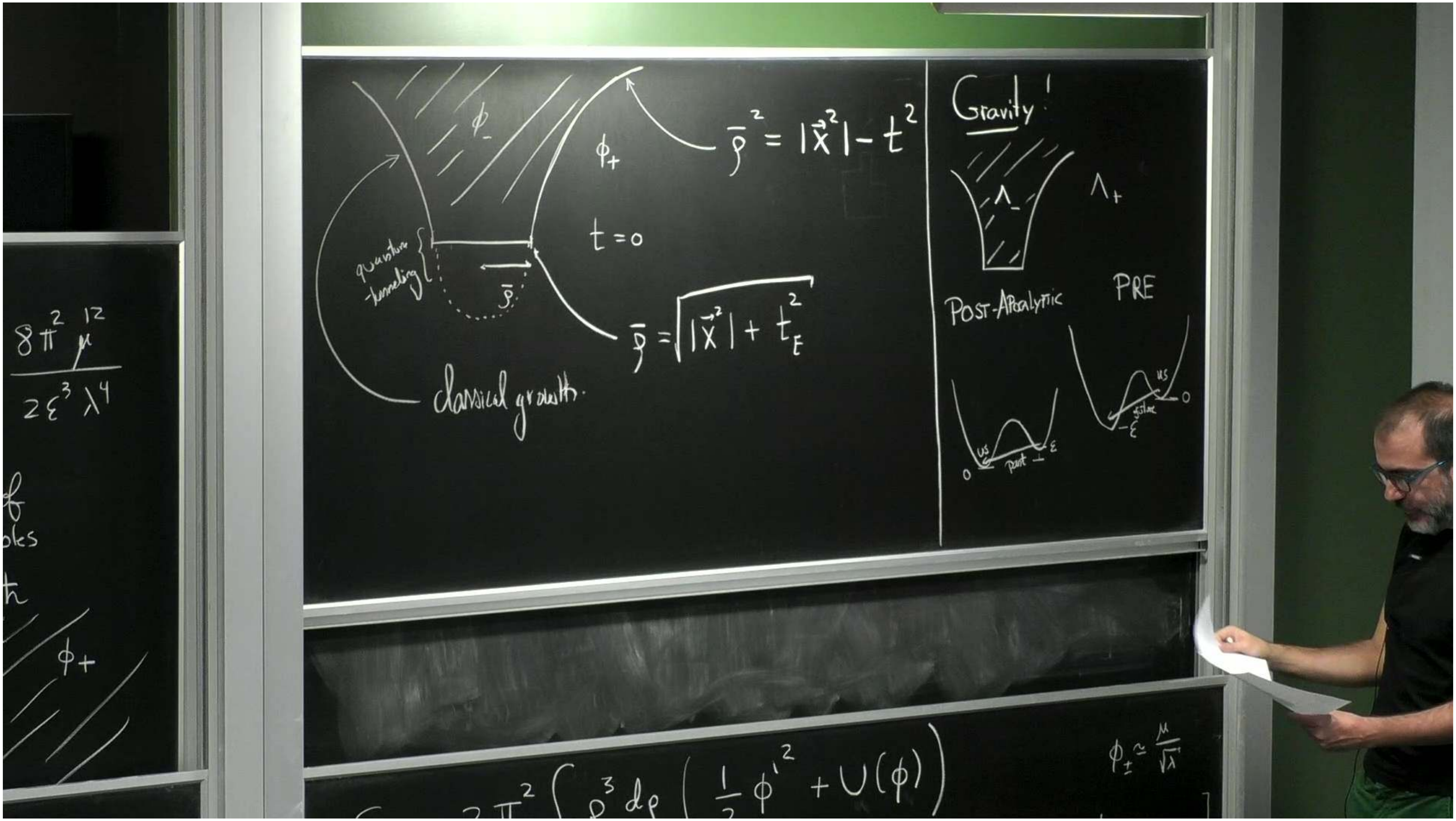
$$\boxed{\bar\rho = \frac{3S_1}{\varepsilon}}$$

← big for ε small

at $t=0$

$\ldots] d^4x$ + Gravity


ϕ_-
ϕ_+
$\bar{\rho}^2 = |\vec{x}^2| - t^2$
$t = 0$
$\bar{\rho}$
$\bar{\rho} = \sqrt{|\vec{x}^2| + t_E^2}$


Gravity!
Λ_-
Λ_+
POST-APOCALYPTIC
PRE
ϕ_-
ϕ_+
$\bar{\rho}^2 = |\vec{x}^2| - t^2$
$t = 0$
quantum tunneling
$\bar{\rho}$
$\bar{\rho} = \sqrt{|\vec{x}^2| + t_E^2}$
classical growth
us
past
ε
$-\varepsilon$
$\phi_\pm \simeq \frac{\mu}{\sqrt{\lambda}}$
$\frac{8\pi^2 \mu^{12}}{2\varepsilon^3 \lambda^4}$
$2\pi^2 \int \rho^3 d\rho \left(\frac{1}{2}\phi'^2 + U(\phi) \right)$

quantum, $\Gamma/V = \underbrace{A}_{\text{hard}}\, e^{-B/\hbar}$

forms $\bigcirc$ $-V + \underbrace{\text{Surf}}_{\text{wins}}$

$$S_E = 2\pi^2 \int d\xi \left(\rho^3 \left(\frac{\phi'^2}{2} + U(\phi) \right) + \frac{3}{8\pi G} \left(\rho^2 \rho'' + \rho \rho'^2 - \rho \right) \right)$$

ξ: Coordinate

$\rho(\xi)$

$$ds^2 = d\xi^2 + \rho(\xi)^2 \underbrace{d\Omega^2}_{S^3}$$

$$\begin{cases} \phi'' + \dfrac{3\rho'}{\rho} \phi' = \dfrac{dU}{d\phi} \\[2ex] \rho'^2 = 1 + \dfrac{3}{16\pi G} \rho^2 \left(\dfrac{\phi'^2}{2} - U \right) \end{cases}$$

$\rho, \phi, \bar{\rho}$ extremize, B

$B_{\text{Gravity}} = \ldots$

$$\text{quantum}, \quad \Gamma/V = A e^{-B/\hbar}$$

A — hard

forms: $-V + \underbrace{\text{Surf}}_{\text{wins}}$

$$S_E = 2\pi^2 \int d\xi \left(\rho^3 \left(\frac{\phi'^2}{2} + U(\phi) \right) + \underbrace{\frac{3}{8\pi G}}_{3R} \left(\rho^2 \rho'' + \rho \rho'^2 - \rho \right) \right)$$

ξ — Coordinate

$\rho(\xi)$

$$ds^2 = d\xi^2 + \rho(\xi)^2 \underbrace{d\Omega^2}_{S^3}$$

ϕ, ρ extremize, B

$R \to 0$ turn off gravity

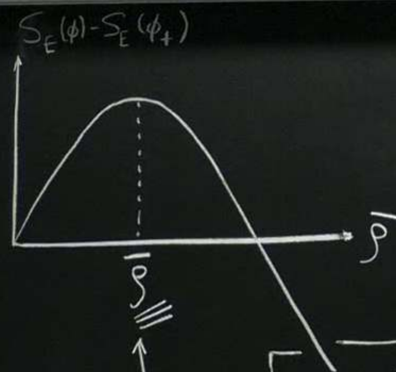
$$\begin{cases} \phi'' + \dfrac{3\rho'}{\rho}\phi' = \dfrac{dU}{d\phi} \\ \rho'^2 = 1 + \dfrac{3}{16\pi G}\rho^2\left(\dfrac{\phi'^2}{2} - U\right) \end{cases}$$

$$B_{\text{Gravity}} = \cdots$$

$B = S_E(\phi) - S_E(\phi_+)$ ... m and

"The bounce" solution

* $\phi(\rho \to \infty) = \phi_+$
* $\phi(\rho)$ is not a constant
* $S_E(\phi)$ is maximal $\Rightarrow$ spherical sym without gravity



$$\boxed{B_0 = \frac{27\pi^2 S_1^4}{2\varepsilon^3}} \underset{Ex}{=} \frac{8\pi^2 \mu^{12}}{2\varepsilon^3\lambda^4}$$

B_0 ← no grav

prob of bubbles $e^{-B/\hbar}$

size of bubble

$$\boxed{\bar\rho_0 = \frac{3S_1}{\varepsilon}}$$ ← big for ε small

at $t=0$

bubble V^*
+ Surf wins

$\frac{dU}{d\phi}$

$\left(\frac{\phi'^2}{2} - U\right)$

$S_E = 2$

ϕ''

quantum, $\Gamma / V = \underset{\nwarrow \text{hard}}{A}\, e^{-B/\hbar}$

forms : $-V + \underbrace{\text{Surf}}_{\text{wins}}$

$$S_E = 2\pi^2 \int \underset{\uparrow \text{coordinate}}{d\xi} \left(\underset{\nwarrow \rho(\xi)}{\rho^3} \left(\frac{\phi'^2}{2} + U(\phi) \right) + \frac{3}{\underbrace{8\pi G}_{3\kappa}} \left(\rho^2 \rho'' + \rho \rho'^2 - \rho \right) \right)$$

$\kappa \to 0$ turn off gravity

$$ds^2 = d\xi^2 + \rho(\xi)^2 \underbrace{d\Omega^2}_{S^3}$$

$$\begin{cases} \phi'' + \dfrac{3\rho'}{\rho}\phi' = \dfrac{dU}{d\phi} \\[2ex] \rho'^2 = 1 + \dfrac{3}{16\pi G}\rho^2 \left(\dfrac{\phi'^2}{2} - U \right) \end{cases}$$

$\rho, \phi, \bar\rho \Big\}$ extremize, B

$$B_{\text{Gravity}} = \frac{B_0}{\left(1 \pm \left(\frac{\bar\rho_0}{2\Lambda} \right)^2 \right)^2}, \quad \bar\rho_{\text{grav}} = \frac{\bar\rho_0}{1 \pm \left(\frac{\bar\rho_0}{2\Lambda} \right)^2}, \quad \Lambda = \left(\frac{\kappa \varepsilon}{3} \right)^{-1/2}, \quad \pm P$$

quantum , $\Gamma / V = A e^{-B/\hbar}$

A ← hard

forms ◌ $-V + \underbrace{\text{Surf}}_{\text{wins}}$

$$S_E = 2\pi^2 \int d\underset{\uparrow}{\xi} \left(\rho^3 \left(\frac{\phi'^2}{2} + U(\phi) \right) + \underbrace{\frac{3}{8\pi G}}_{3K} \left(\rho^2 \rho'' + \rho \rho'^2 - \rho \right) \right)$$

Coordinate

$\rho(\xi)$

$$ds^2 = d\xi^2 + \rho(\xi)^2 \underbrace{d\Omega^2}_{S^3}$$

$K \to 0$ turnoff gravity.

$$\phi'' + \frac{3\rho'}{\rho}\phi' = \frac{dU}{d\phi}$$

$$\rho'^2 = 1 + \frac{3}{16\pi G} \rho^2 \left(\frac{\phi'^2}{2} - U \right)$$

$\rho, \phi, \bar{\rho}$ extremize , B

$$B_{\text{Gravity}} = \frac{B_0}{\left(1 \pm \left(\frac{\bar{\rho}_0}{2\Lambda}\right)^2\right)^2}, \quad \bar{\rho}_{\text{grav}} = \frac{\bar{\rho}_0}{1 \pm \left(\frac{\bar{\rho}_0}{2\Lambda}\right)^2}, \quad \Lambda = \underbrace{\left(\frac{K\varepsilon}{3}\right)^{-1/2}}_{\text{large}}, \quad \pm \text{ POST/PRE}$$

quantum, $\Gamma / V = \underbrace{A}_{\text{hard}} e^{-B/\hbar}$

forms $-V + \underbrace{\text{Surf}}_{\text{wins}}$

$$S_E = 2\pi^2 \int d\xi \left(\rho^3 \left(\frac{\phi'^2}{2} + U(\phi) \right) + \frac{3}{8\pi G}\left(\rho^2 \rho'' + \rho \rho'^2 - \rho \right) \right)$$

ξ: Coordinate; $\rho(\xi)$

$\underbrace{8\pi G}_{3\kappa}$, $\kappa \to 0$ turnoff gravity.

$$ds^2 = d\xi^2 + \rho(\xi)^2 \underbrace{d\Omega^2}_{S^3}$$

$$\phi'' + \frac{3\rho'}{\rho}\phi' = \frac{dU}{d\phi}$$

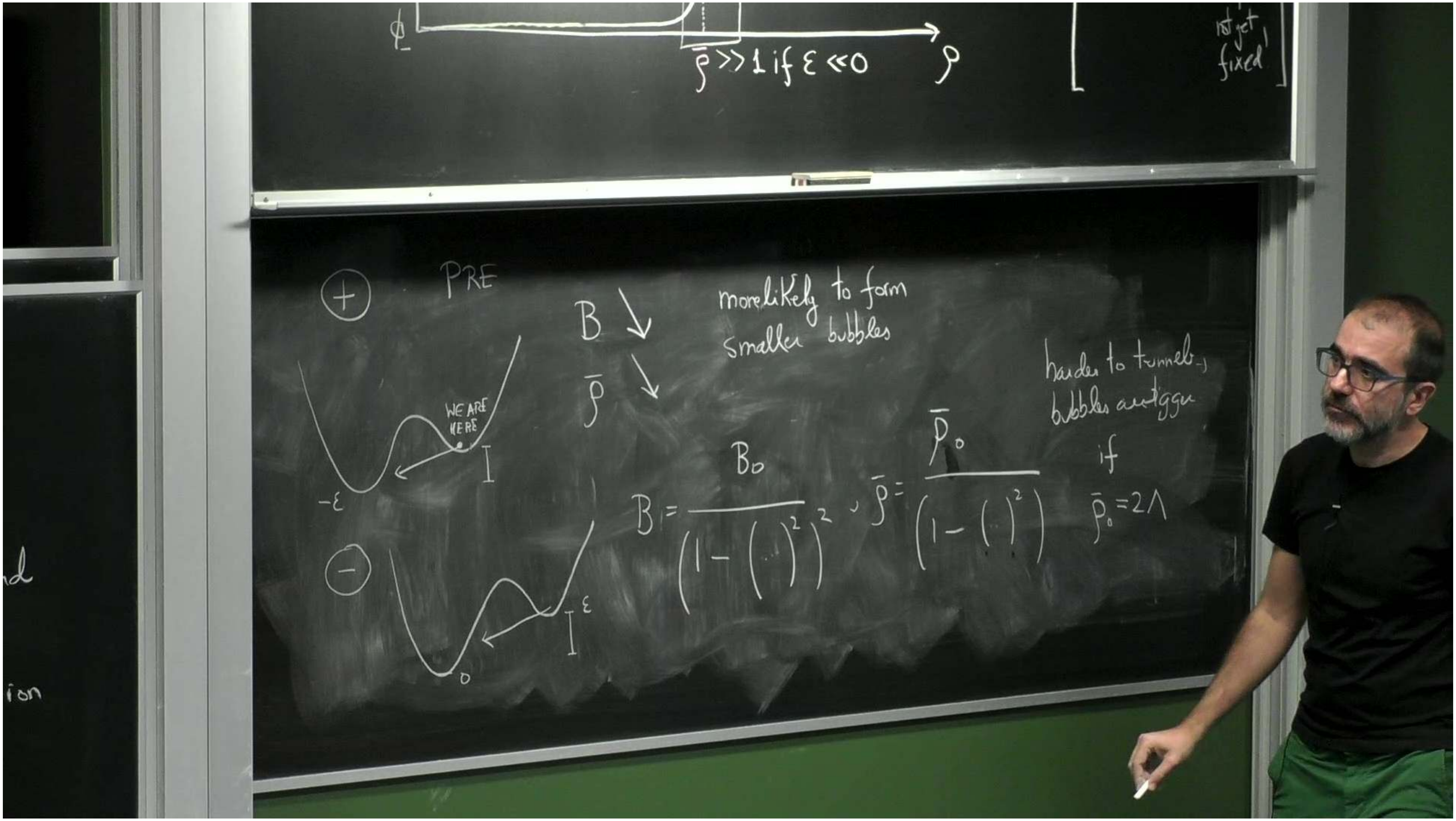
$$\rho'^2 = 1 + \frac{3}{16\pi G}\rho^2\left(\frac{\phi'^2}{2} - U\right)$$

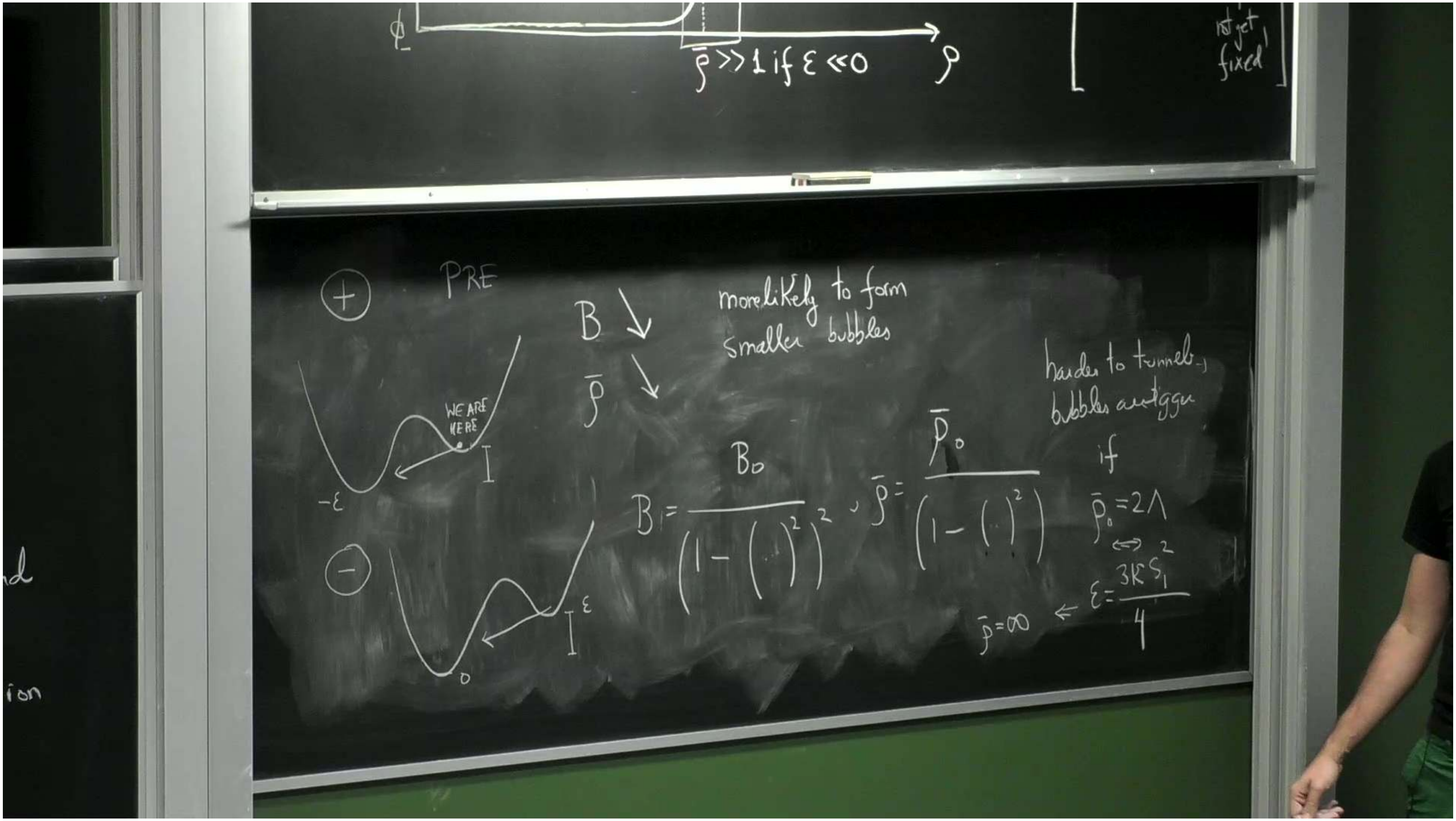
$\rho, \phi, \bar{\rho}$ extremize, B

RATIO CAN BE SMALL OR LARGE!

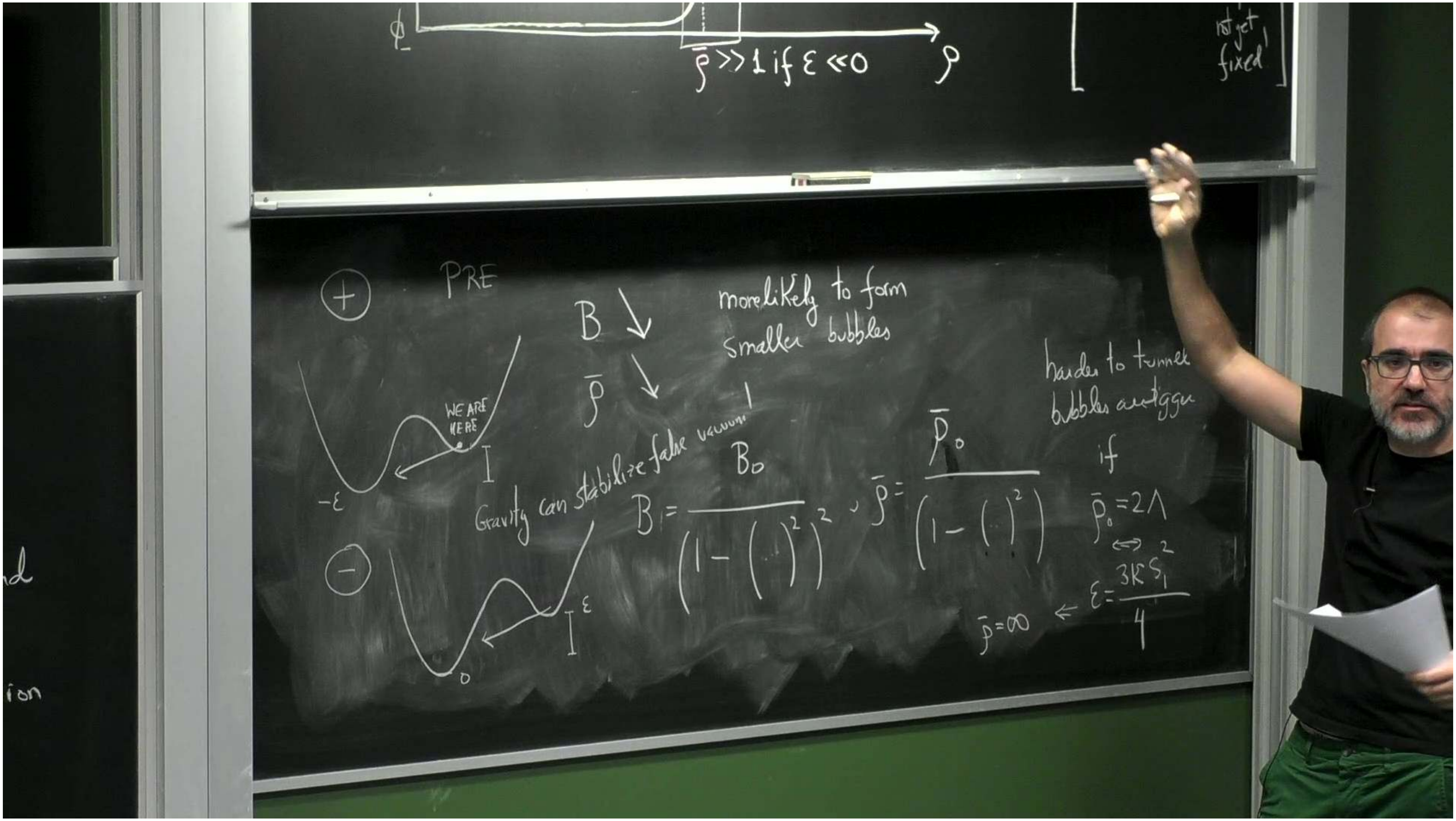
$$B_{Gravity} = \frac{B_0}{\left(1 \pm \left(\frac{\bar{\rho}_0}{2\Lambda}\right)^2\right)^2}, \quad \bar{\rho}_{grav} = \frac{\bar{\rho}_0}{1 \pm \left(\frac{\bar{\rho}_0}{2\Lambda}\right)^2}, \quad \Lambda = \underbrace{\left(\frac{\kappa \varepsilon}{3}\right)^{-1/2}}_{\text{large}}, \quad \pm \text{ POST/PRE}$$

large $\{ \bar{\rho}_0$


ρ̄ >> 1 if ε << 0
fixed
PRE
more likely to form
smaller bubbles
WE ARE HERE
harder to tunnel
if
ρ̄₀ = 2Λ



ρ̄ >> 1 if ε << 0
PRE
more likely to form
smaller bubbles
WE ARE HERE
harder to tunnel
if
ρ̄ = ∞


ρ̄ >> 1 if ε << 0
ρ
fixed
PRE
B ↘
ρ̄ ↘
more likely to form
smaller bubbles
WE ARE HERE
−ε
Gravity can stabilize false vacuum!
B = B₀ / (1 − ( )²)²
ρ̄ = ρ̄₀ / (1 − ( )²)
harder to tunnel
if
ρ̄₀ = 2Λ
ε = 3K S₁² / 4
ρ̄ = ∞
ε
0
ion