

Title: Lecture - Beautiful Papers

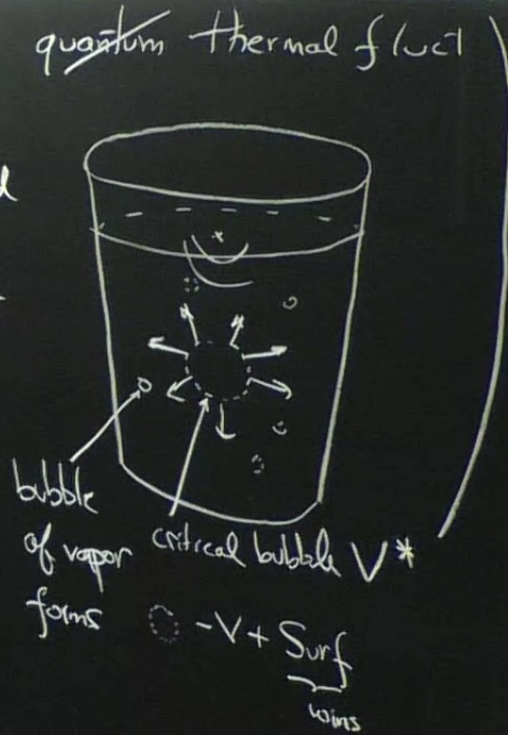
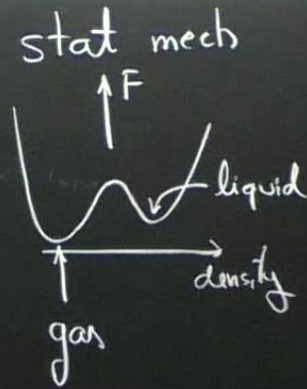
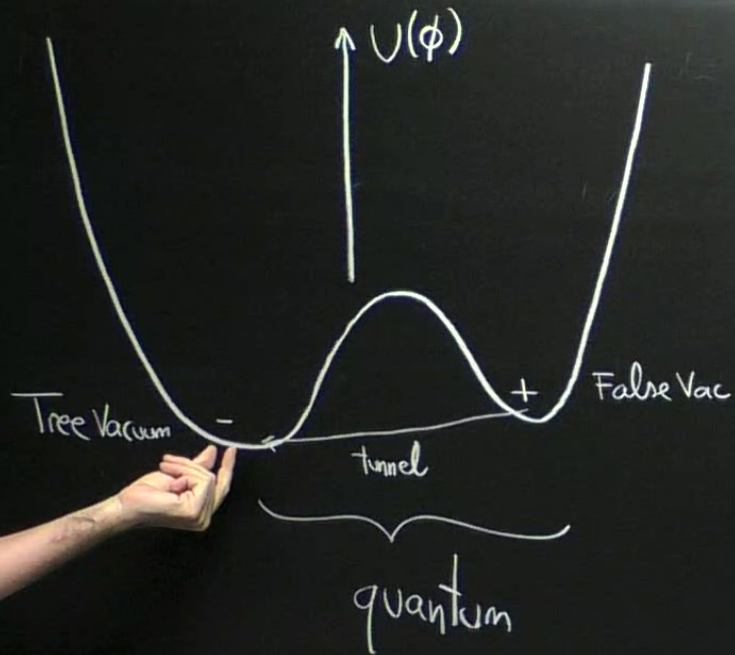
Speakers: Pedro Vieira

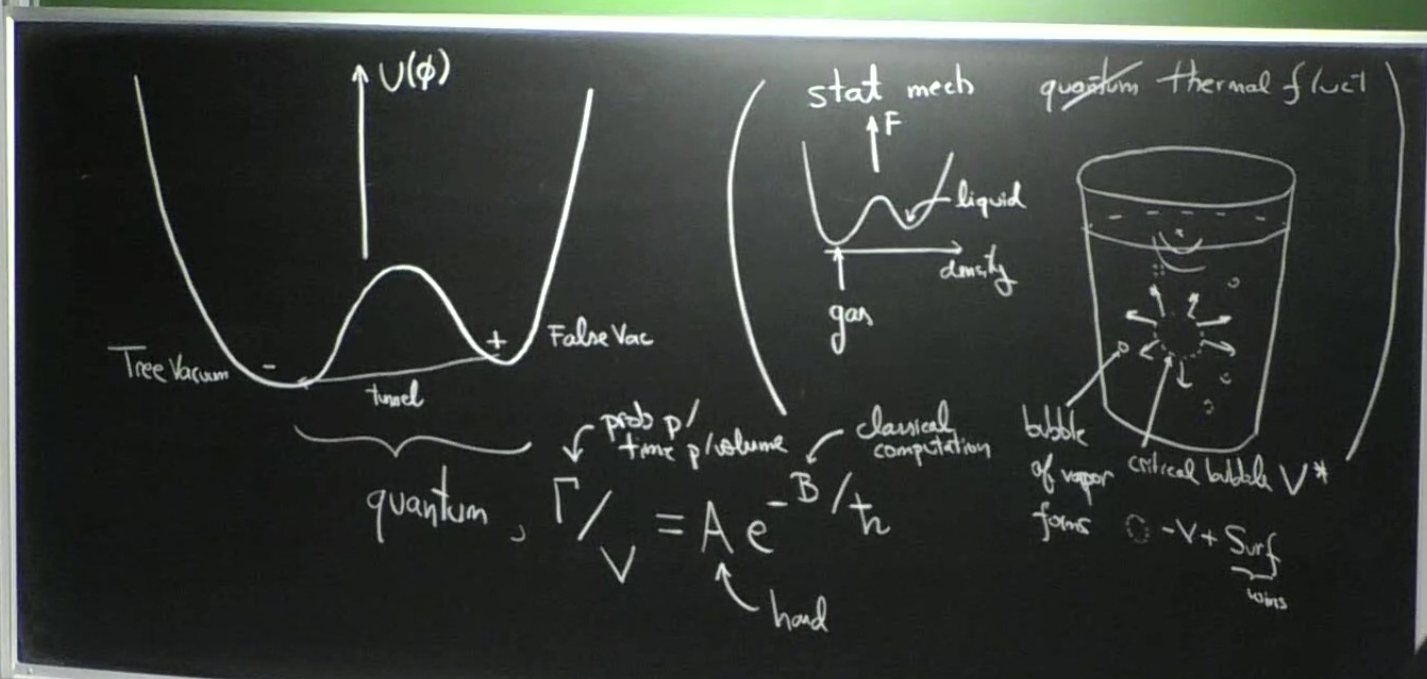
Collection/Series: Beautiful Papers - October 7, 2024 - January 31, 2025

Subject: Other

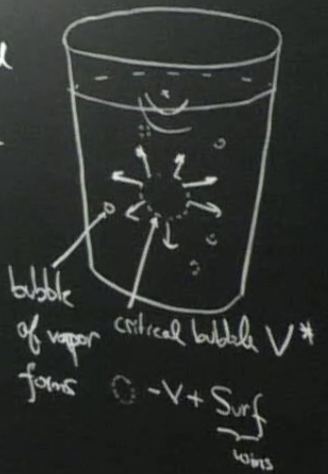
Date: November 18, 2024 - 9:15 AM

URL: <https://pirsa.org/24110035>





$\Gamma/V = A e^{-B/h}$
 quantum, hard
 prob p / time p / volume
 classical computation



$$S_E = \int \left[\frac{1}{2} (\partial_\mu \phi)^2 + U(\phi) \right] d^4x$$

$\underbrace{\hspace{10em}}_{2\pi^2 \rho^3 d\rho}$

$$B = S_E(\phi) - S_E(\phi_+)$$

where ϕ obeys S_E eom and

- * $\phi(\rho \rightarrow \infty) = \phi_+$
- * $\phi(\rho)$ is not a constant
- * $S_E(\phi)$ is maximal

$$S_E = \int \left[\frac{1}{2} (\partial_\mu \phi)^2 + U(\phi) \right] d^4x \quad + \underbrace{\text{Gravity}}_{\text{next}}$$

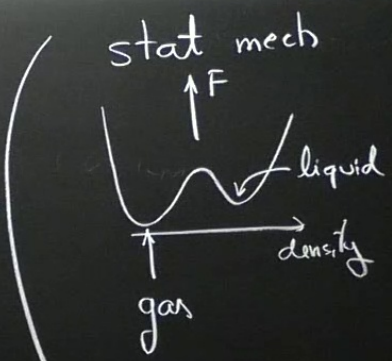
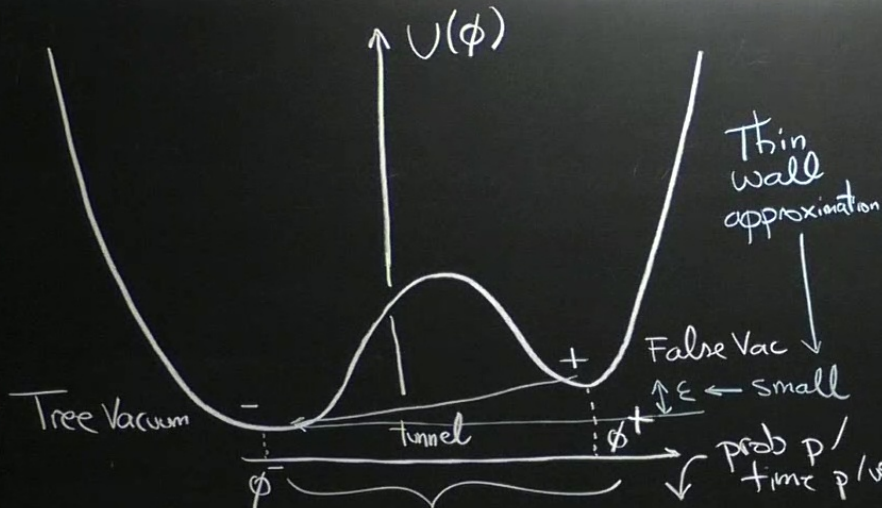
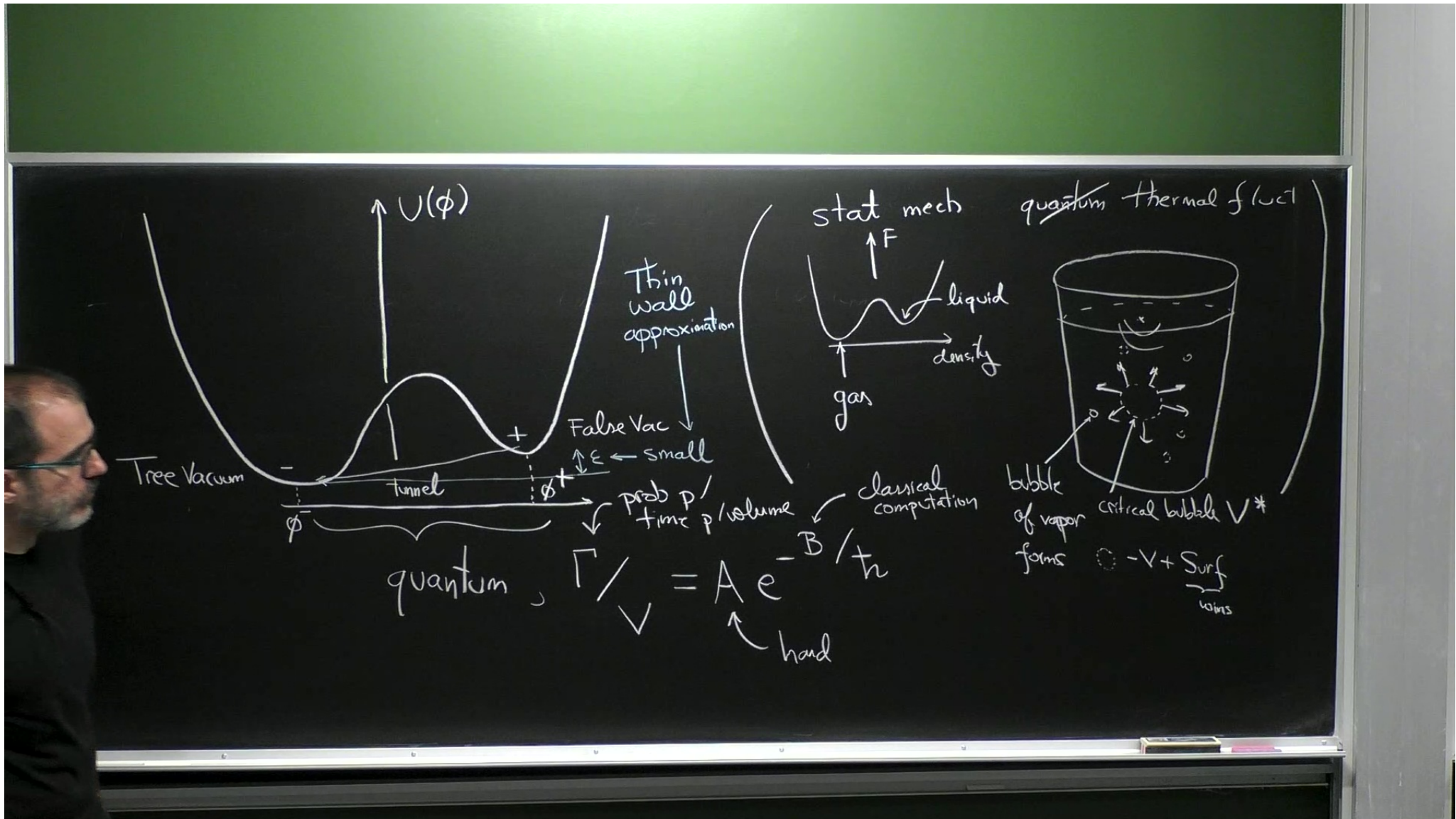
$\underbrace{d^4x}_{2\pi^2 \rho^3 d\rho}$

CLAIM:

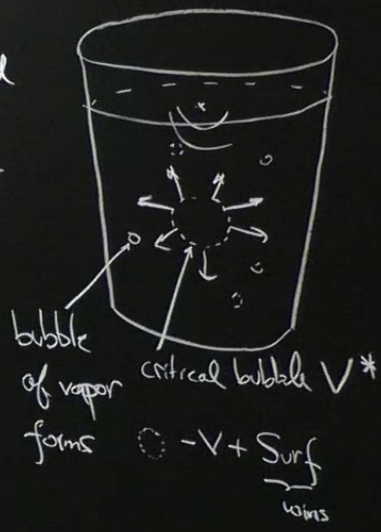
$$B = S_E(\phi) - S_E(\phi_+)$$

where ϕ obeys S_E eom and
 \uparrow "the bounce" solution

- * $\phi(\rho \rightarrow \infty) = \phi_+$
- * $\phi(\rho)$ is not a constant
- * $S_E(\phi)$ is maximal



quantum thermal fluct



quantum

prob p / time p / volume

classical computation

$$\frac{\Gamma}{V} = A e^{-B/h}$$

hard

$$S_E = \int \left[\frac{1}{2} (\partial_\mu \phi)^2 + U(\phi) \right] \underbrace{d^4x}_{2\pi^2 \rho^3 d\rho} + \underbrace{\text{Gravity}}_{\text{next}}$$

CLAIM :

$$B = S_E(\phi) - S_E(\phi_+)$$

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 "the bounce" solution

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$$S_E = \int \left[\frac{1}{2} (\partial_\mu \phi)^2 + U(\phi) \right] \underbrace{d^4x}_{2\pi^2 \rho^3 d\rho} + \underbrace{\text{Gravity}}_{\text{next}}$$

CLAIM :

$$B = S_E(\phi) - S_E(\phi_+)$$

where ϕ obeys S_E eom and
 ↑
 "the bounce" solution

* $\phi(\rho \rightarrow \infty) = \phi_+$

* $\phi(\rho)$ is not a constant

* $S_E(\phi)$ is maximal \Rightarrow spherical sym Without gravity

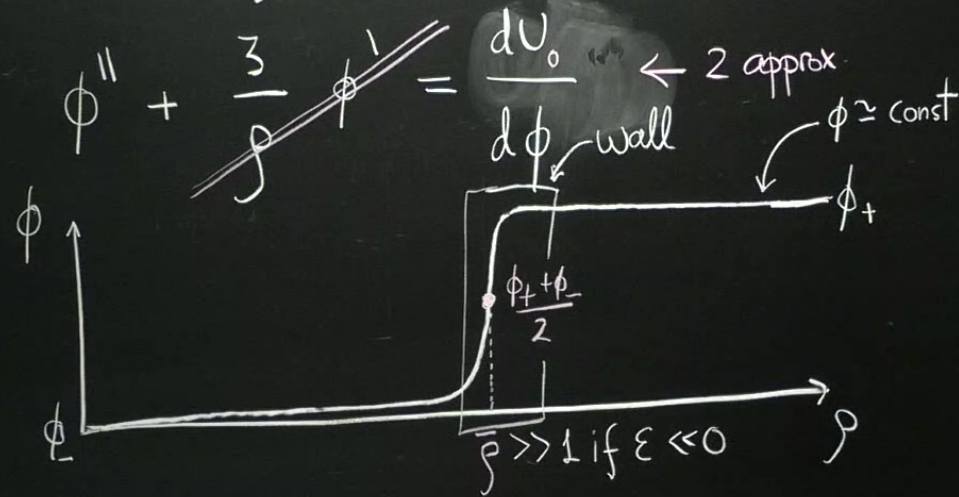
$$S_E = 2\pi^2 \int g^3 d\rho \left(\frac{1}{2} \phi'^2 - U(\phi) \right)$$

$$\phi'' + \frac{3}{g} \phi' = \frac{dU_0}{d\phi} \leftarrow 2 \text{ approx}$$

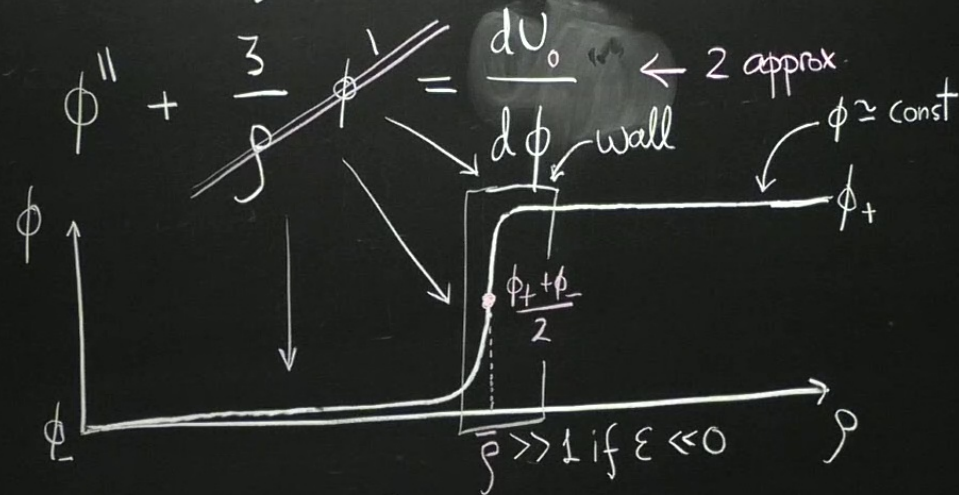
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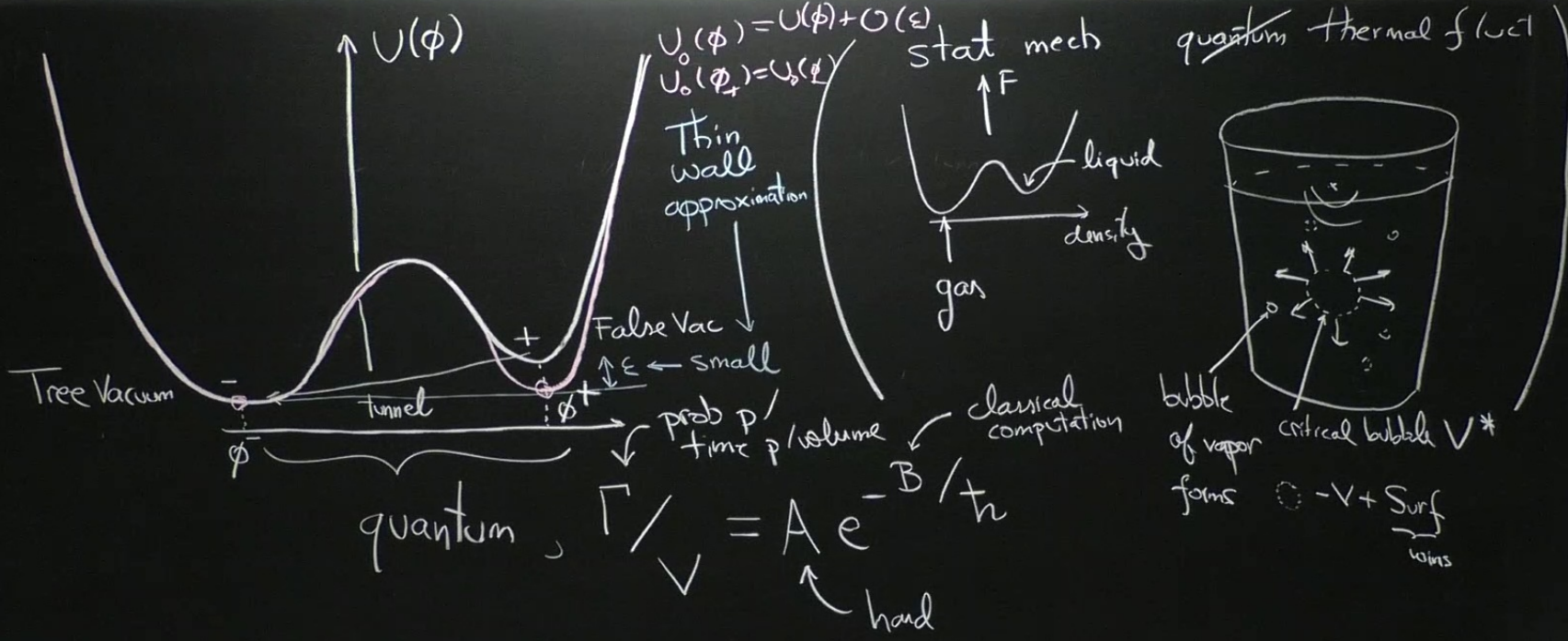


$$S_E = 2\pi^2 \int \rho^3 d\rho \left(\frac{1}{2} \phi'^2 - U(\phi) \right)$$

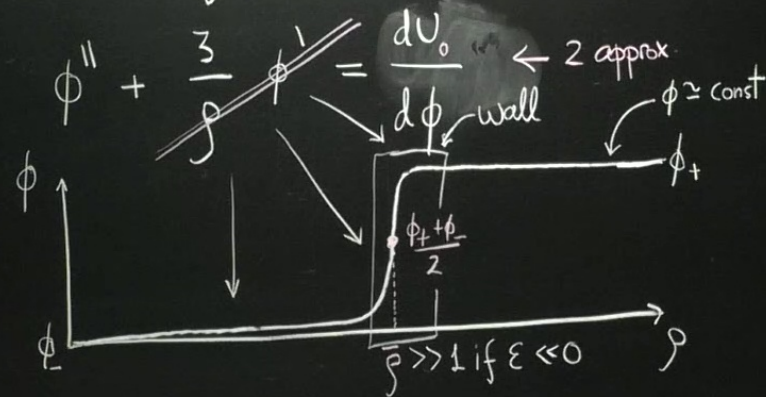


$$S_E = 2\pi^2 \int \rho^3 d\rho \left(\frac{1}{2} \phi'^2 - U(\phi) \right)$$





$$S_E = 2\pi^2 \int \bar{\rho}^3 d\bar{\rho} \left(\frac{1}{2} \phi'^2 - U(\phi) \right)$$



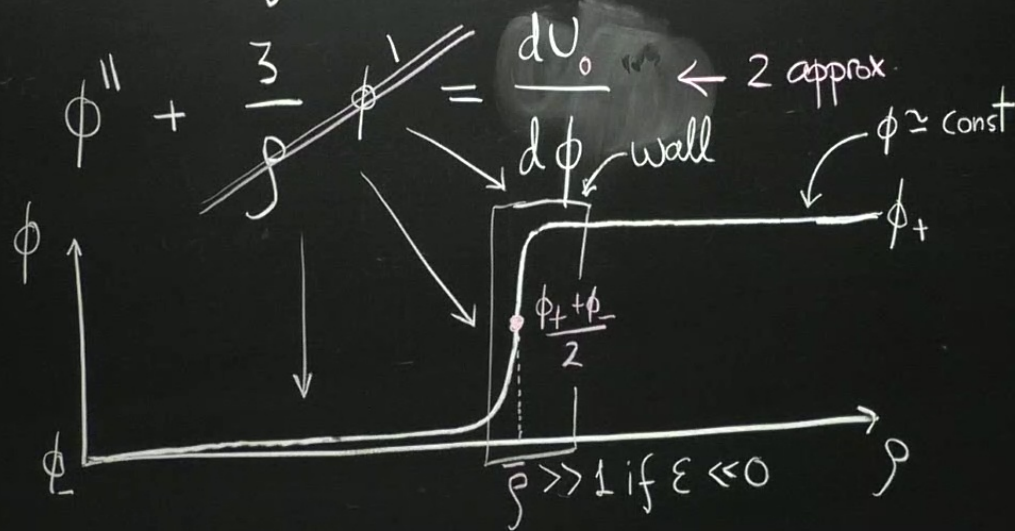
vity
 ext

eom and

unce" solution

$$S_E = 2\pi^2 \int \rho^3 d\rho \left(\frac{1}{2} \phi'^2 - U(\phi) \right)$$

$$\phi_{\pm} \approx \frac{\mu}{\sqrt{\lambda}}$$



Example:

$$U_0 = \frac{\lambda}{8} \left(\phi^2 - \frac{\mu^2}{\lambda} \right)^2$$

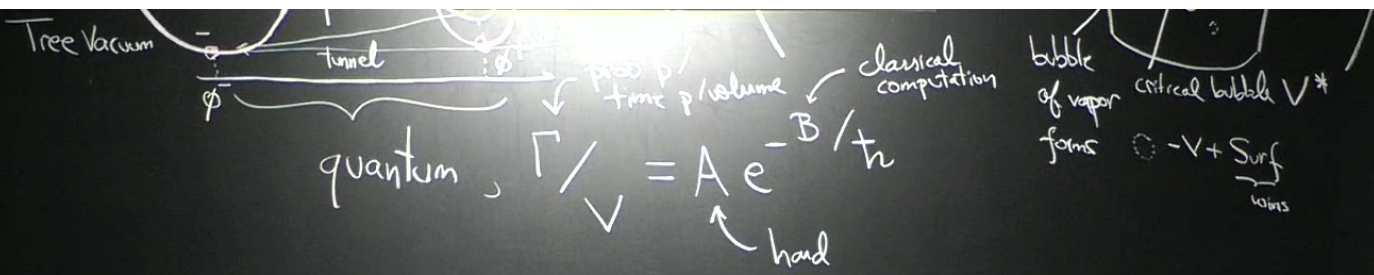
$$\phi = \frac{\mu}{\sqrt{\lambda}} \tanh\left(\frac{\mu}{2}(\rho - \bar{\rho})\right)$$

↑
not yet fixed

tunnel ϕ^- ϕ^+ ϕ^- ϕ^+
 quantum, $\Gamma/V = A e^{-B/\hbar}$
 classical computation
 bubble of vapor forms ϕ^- ϕ^+
 critical bubble V^*
 $-V + \text{Surf}$
 wins
 hard

ϕ^-
 bubble of true vac.
 $\frac{1}{2} \phi'^2 - U_0(\phi) = -U_0(\phi_+)$, $\bar{\phi} - \bar{\phi} = \int_{\frac{\phi^+ + \phi^-}{2}} \sqrt{2(U_0(\phi) - U_0(\phi_+))}$
 $\bar{\phi}$ is given by $\max S_E!$

- $B_{\text{outside}} = 0$
- $B_{\text{wall}} =$
- $B_{\text{inside}} =$



Wall

false vac

bubble of true vac

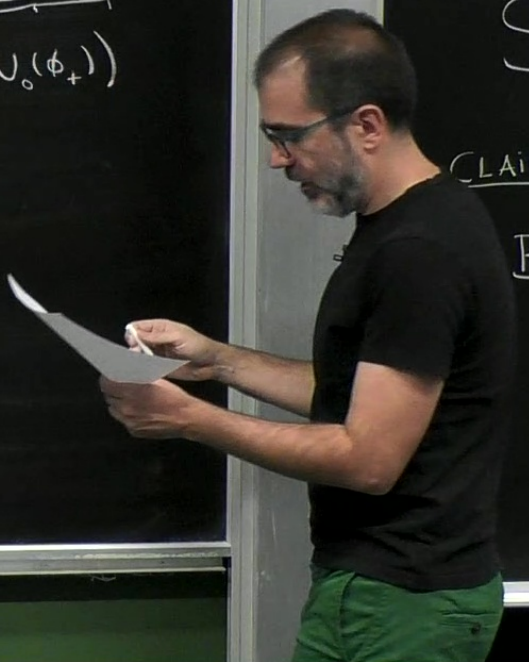
$\frac{1}{2} \phi'^2 - U_0(\phi) = -U_0(\phi_+)$, $\bar{\phi} - \bar{\phi} = \int_{\frac{\phi_+ + \phi_-}{2}}^{\phi} \frac{d\phi}{\sqrt{2(U_0(\phi) - U_0(\phi_+))}}$

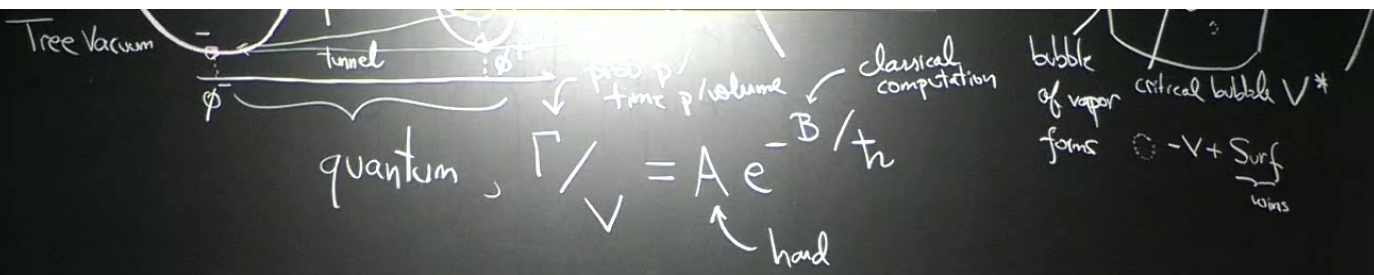
$\bar{\phi}$ is given by $\max S_E!$

$B_{\text{outside}} = 0$

$B_{\text{wall}} = 2\pi^2 \bar{\phi}^2 \int_a^b \left(\frac{\phi'^2}{2} + U(\phi) - U(\phi_+) \right) =$

$B_{\text{inside}} = -\epsilon \frac{\pi^2}{2} \bar{\phi}^4$





wall

false vac

bubble of true vac

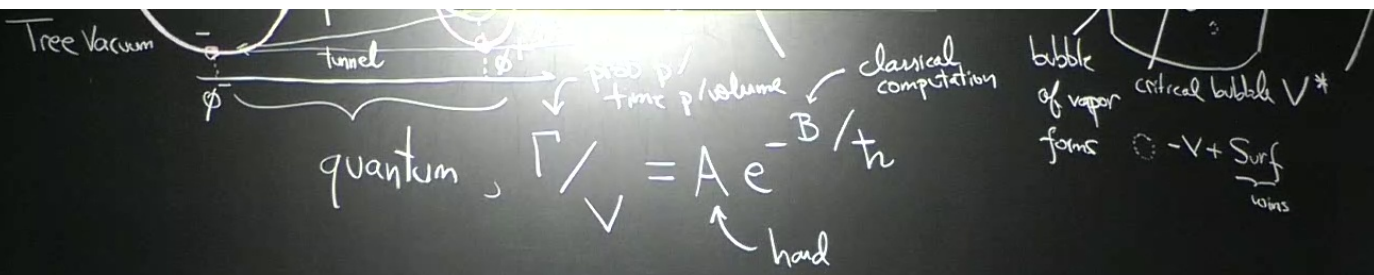
$\frac{1}{2} \phi^2 - U_0(\phi) = -U_0(\phi_+)$, $\bar{\rho} - \bar{\rho} = \int_{\frac{\phi_+ + \phi_-}{2}}^{\phi} \frac{d\phi}{\sqrt{2(U_0(\phi) - U_0(\phi_+))}}$

$\bar{\rho}$ is given by $\max S_E!$

$B_{\text{outside}} = 0$

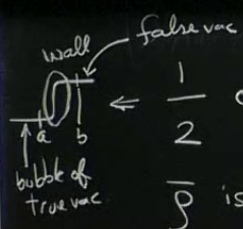
$B_{\text{wall}} = 2\pi^2 \bar{\rho}^3 \int_a^b d\rho \left(\frac{\phi^2}{2} + U(\phi) - U(\phi_+) \right) = 2\pi^2 \bar{\rho}^3 \left[\int_{\phi_-}^{\phi_+} d\phi \sqrt{U(\phi) - U(\phi_+)} \right]$

$B_{\text{inside}} = -\epsilon \frac{\pi^2}{2} \bar{\rho}^4$



$$\Gamma/V = A e^{-B/\hbar}$$

↑ hard



$$\frac{1}{2} \phi'^2 - U_0(\phi) = -U_0(\phi_+)$$

$$\rho - \bar{\rho} = \int_{\frac{\phi_+ + \phi_-}{2}}^{\phi} \frac{d\phi}{\sqrt{2(U_0(\phi) - U_0(\phi_+))}}$$

$\bar{\rho}$ is given by $\max S_E$!

$$B_{\text{outside}} = 0$$

$$B_{\text{wall}} = 2\pi^2 \bar{\rho}^3 \int_a^b d\rho \left(\frac{\phi'^2}{2} + U(\phi) - U(\phi_+) \right) = 2\pi^2 \bar{\rho}^3 \left[\int_{\phi_-}^{\phi_+} d\phi \sqrt{U(\phi) - U(\phi_+)} \right]$$

$$B_{\text{wall}}$$

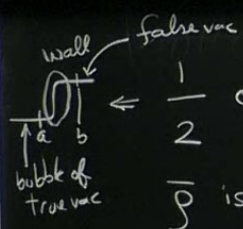
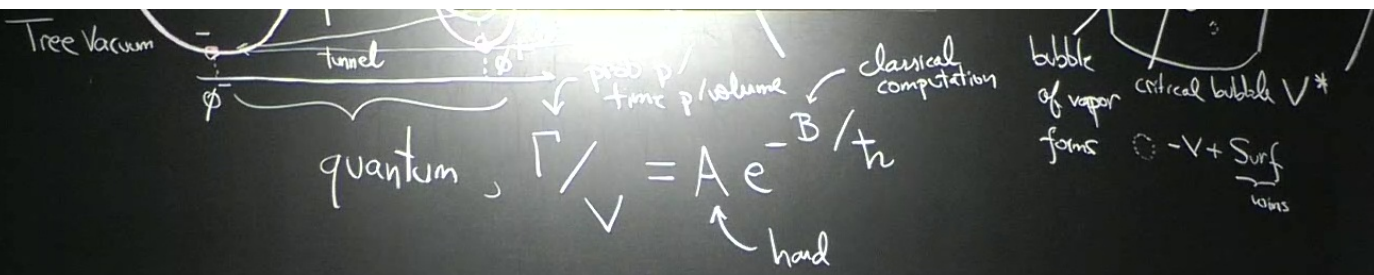
$$B_{\text{inside}} = -\epsilon \frac{\pi^2}{2} \bar{\rho}^4$$

$$\int_{\phi_-}^{\phi_+} d\phi \sqrt{U(\phi) - U(\phi_+)} \equiv S_1$$

S

CLAIM

B



$$\frac{1}{2} \phi'^2 - U_0(\phi) = -U_0(\phi_+), \quad \rho - \bar{\rho} = \int_{\frac{\phi^+ + \phi^-}{2}}^{\phi} \frac{d\phi}{\sqrt{2(U_0(\phi) - U_0(\phi_+))}}$$

$\bar{\rho}$ is given by $\max S_E$!

$$B_{\text{outside}} = 0$$

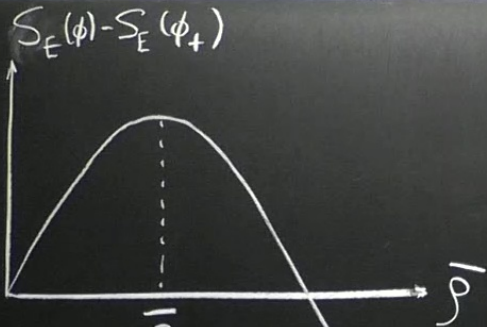
$$B_{\text{wall}} = 2\pi^2 \bar{\rho}^3 \int_a^b d\rho \left(\frac{\phi'^2}{2} + U(\phi) - U(\phi_+) \right) = 2\pi^2 \bar{\rho}^3 \left[\int_{\phi^-}^{\phi^+} d\phi \sqrt{U(\phi) - U(\phi_+)} \right]$$

$$B_{\text{inside}} = -\epsilon \frac{\pi^2}{2} \bar{\rho}^4$$

$(S_1 = \frac{2\pi^3}{3\lambda})$
// EXAMPLE

$\equiv S_1$

S
E
CLAIM
B



$$B = \frac{27 \pi^2 S_1^4}{2 \epsilon^3}$$

$$= \frac{8 \pi^2 \mu^{12}}{2 \epsilon^3 \lambda^4}$$

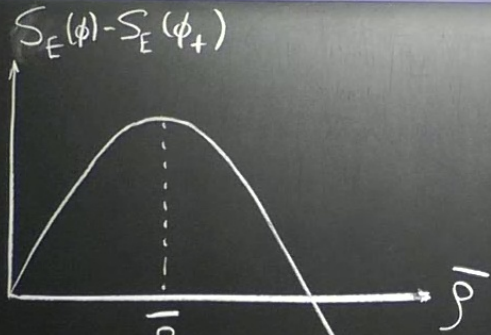
size of bubble

prob of bubbles
 $e^{-B/k}$

$$\bar{\phi} = \frac{3 S_1}{\epsilon}$$

big for ϵ small

$$S = \int d^4x \left[\frac{1}{2} (\partial_\mu \phi)^2 + U(\phi) \right] + \underbrace{\text{Gravity}}_{\text{next}}$$



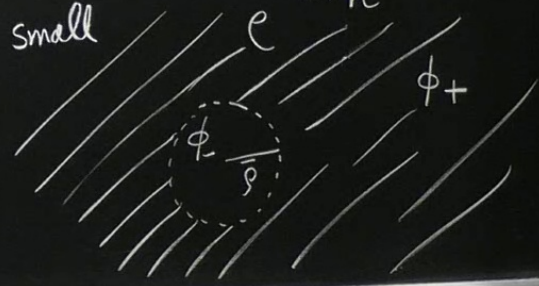
$$B = \frac{27 \pi^2 S_1^4}{2 \epsilon^3}$$

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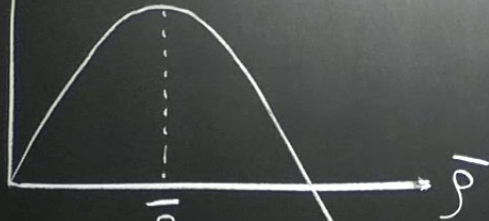
$$\bar{\rho} = \frac{3 S_1}{\epsilon}$$

size of bubble
 ← big for ϵ small

prob of bubbles
 $e^{-B/t}$



$$S_E(\phi) - S_E(\phi_+)$$



$$B = \frac{27 \pi^2 S_1^4}{2 \epsilon^3}$$

$$= \frac{8 \pi^2 \mu^{12}}{2 \epsilon^3 \lambda^4}$$

size of bubble

prob of bubbles

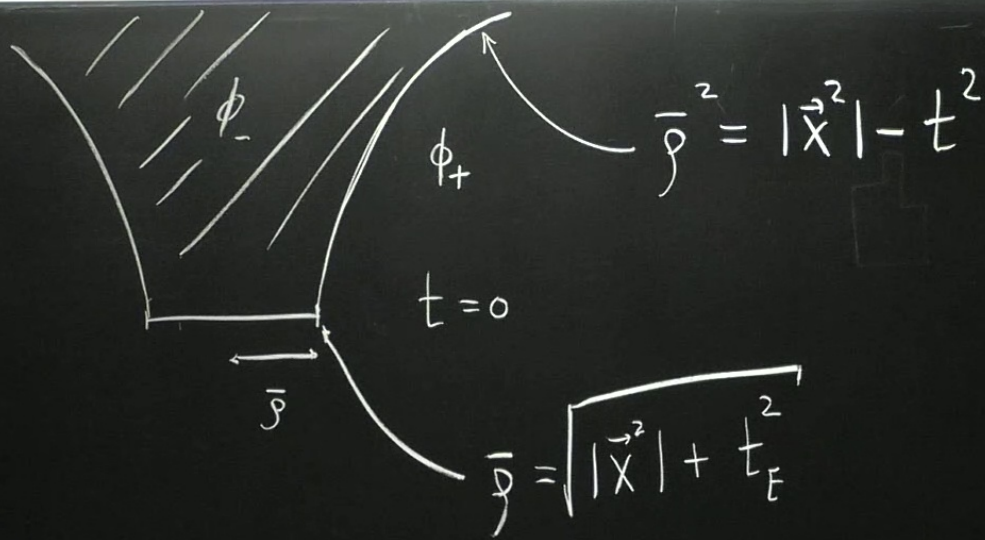
$$\bar{\rho} = \frac{3 S_1}{\epsilon}$$

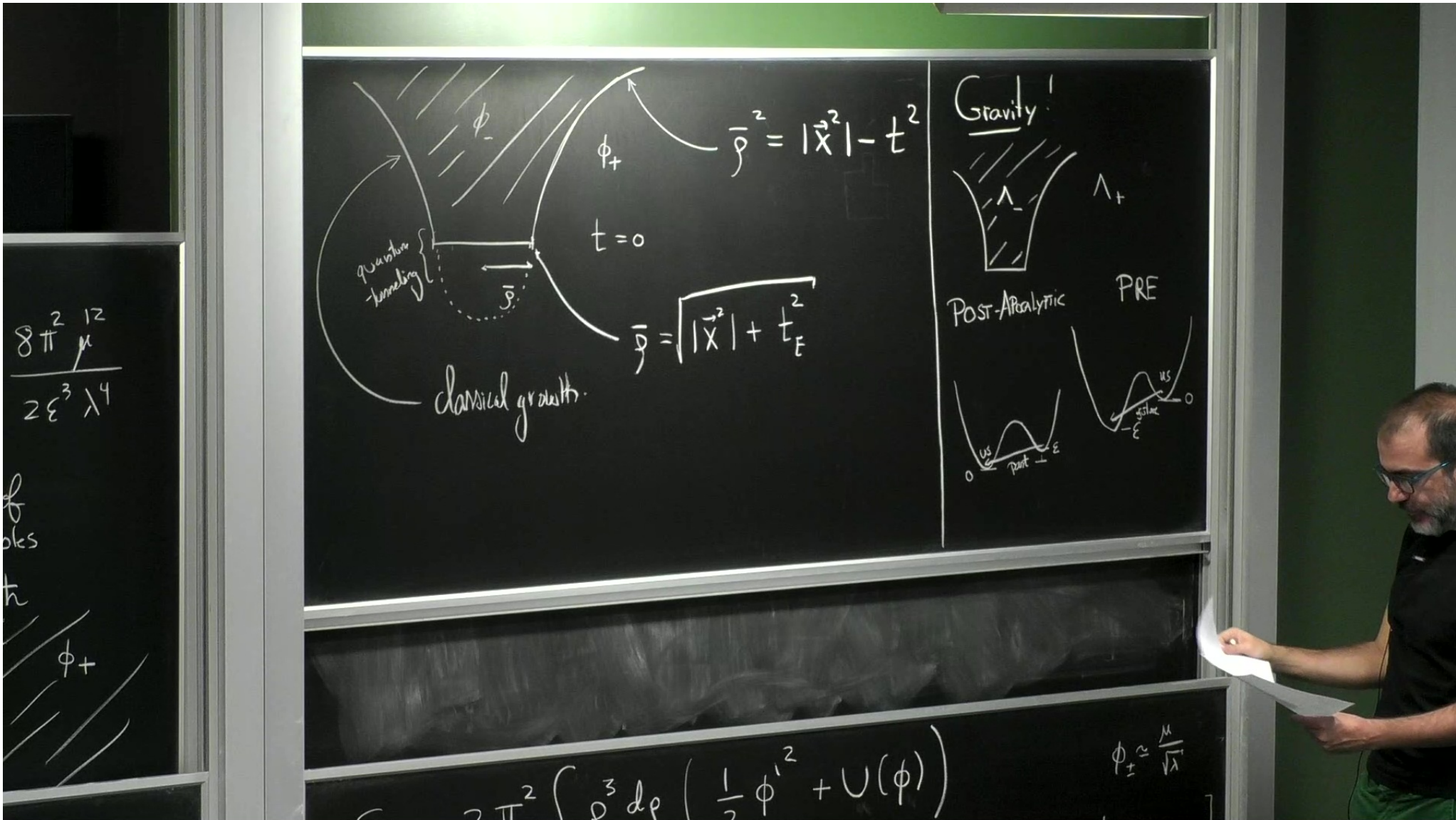
big for ϵ small

$$e^{-B/t}$$

at $t=0$







quantum, $\Gamma/V = A e^{-D/\hbar}$

↑
hard

forms $\circ -V + \underbrace{\text{Surf}}_{\text{wins}}$

$$S_E = 2\pi^2 \int d\xi \left(\rho^3 \left(\frac{\phi'^2}{2} + U(\phi) \right) + \frac{3}{8\pi G} \left(\rho^2 \rho'' + \rho \rho' - \rho \right) \right)$$

Coordinate

$$ds^2 = d\xi^2 + \rho(\xi)^2 \underbrace{d\Omega^2}_{S^3}$$

ρ, ϕ extremize B

$B_{\text{Gravity}} = \dots$

$$\begin{cases} \phi'' + \frac{3\rho'}{\rho} \phi' = \frac{dU}{d\phi} \\ \rho'^2 = 1 + \frac{3}{16\pi G} \rho^2 \left(\frac{\phi'^2}{2} - U \right) \end{cases}$$

quantum, $\Gamma/V = A e^{-D/t\hbar}$

↑
hard

forms $\circ -V + \underbrace{\text{Surf}}_{\text{wins}}$

$$S_E = 2\pi^2 \int d\xi \left(\rho^3 \left(\frac{\phi'^2}{2} + U(\phi) \right) + \frac{3}{8\pi G} \left(\rho^2 \rho'' + \rho \rho' - \rho \right) \right)$$

Coordinate

$$ds^2 = d\xi^2 + \rho(\xi)^2 d\Omega^2$$

$\underbrace{\quad}_{S^3}$

$\frac{3R}{R \rightarrow 0}$
turnoff
gravity

$$\left\{ \begin{aligned} \phi'' + \frac{3\rho'}{\rho} \phi' &= \frac{dU}{d\phi} \\ \rho'^2 &= 1 + \frac{3}{16\pi G} \rho^2 \left(\frac{\phi'^2}{2} - U \right) \end{aligned} \right.$$

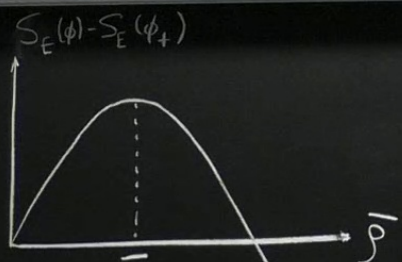
$\rho\phi, \rho$ extremize B

$B_{\text{Gravity}} = \dots$

$$B = S_E(\phi) - \dots$$

- * $\phi(\rho \rightarrow \infty) = \phi_+$
- * $\phi(\rho)$ is not a constant
- * $S_E(\phi)$ is maximal \Rightarrow spherical sym without gravity

"the bounce" solution



$$B_0 = \frac{27 \pi^2 S_1^4}{2 \epsilon^3} = \frac{8 \pi^2 \mu^{12}}{2 \epsilon^3 \lambda^4}$$

no grav

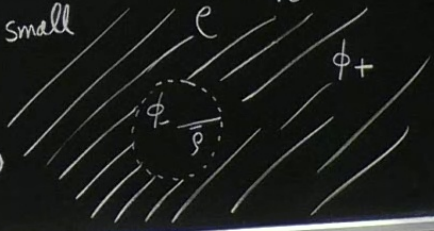
$$\bar{\rho}_0 = \frac{3 S_1}{\epsilon}$$

size of bubble

prob of bubbles $e^{-B/t}$

big for ϵ small

at $t=0$



quantum, $\Gamma/V = A e^{-D/\hbar}$
 ↗ hand

forms $\circ -V + \underbrace{\text{Surf}}_{\text{wins}}$

$$S_E = 2\pi^2 \int d\xi \left(\rho^3 \left(\frac{\phi'^2}{2} + U(\phi) \right) + \frac{3}{8\pi G} \left(\rho^2 \rho'' + \rho \rho' - \rho \right) \right)$$

Coordinate $\rho(\xi)$

$$ds^2 = d\xi^2 + \rho(\xi)^2 d\Omega^2$$

$\frac{3K}{R \rightarrow 0}$ turnoff gravity

$$\left. \begin{aligned} \phi'' + \frac{3\rho'}{\rho} \phi' &= \frac{dU}{d\phi} \\ \rho'^2 &= 1 + \frac{3}{16\pi G} \rho^2 \left(\frac{\phi'^2}{2} - U \right) \end{aligned} \right\}$$

ρ, ϕ extremize B

$$B_{\text{gravity}} = \frac{B_0}{\left(1 \pm \left(\frac{\bar{\rho}_0}{2\Lambda} \right)^2 \right)^2}, \quad \bar{\rho}_{\text{grav}} = \frac{\bar{\rho}_0}{1 \pm \left(\frac{\bar{\rho}_0}{2\Lambda} \right)^2}, \quad \Lambda = \left(\frac{K\varepsilon}{3} \right)^{1/2}, \quad \pm \rho$$

quantum, $\frac{\Gamma}{V} = A e^{-D/\hbar}$

forms $\circ -V + \underbrace{\text{Surf}}_{\text{wins}}$

$$S_E = 2\pi^2 \int d\xi \left(\rho^3 \left(\frac{\phi'^2}{2} + U(\phi) \right) + \frac{3}{8\pi G} \left(\rho^2 \rho'' + \rho \rho' - \rho \right) \right)$$

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ρ, ρ' extremize B

$$B_{\text{gravity}} = \frac{B_0}{\left(1 \pm \left(\frac{\bar{\rho}_0}{2\Lambda} \right)^2 \right)^2}, \quad \bar{\rho}_{\text{grav}} = \frac{\bar{\rho}_0}{1 \pm \left(\frac{\bar{\rho}_0}{2\Lambda} \right)^2}, \quad \Lambda = \left(\frac{KE}{3} \right)^{-1/2} \text{ large } \pm \text{POST/PRE}$$

quantum, $\Gamma/V = A e^{-D/\hbar}$
 hand

forms $\circ -V + \underbrace{\text{Surf}}_{\text{wins}}$

$$S_E = 2\pi^2 \int d\xi \left(\rho^3 \left(\frac{\phi'^2}{2} + U(\phi) \right) + \frac{3}{8\pi G} \left(\rho^2 \rho'' + \rho \rho' - \rho \right) \right)$$

Coordinate $\rho(\xi)$

$$ds^2 = d\xi^2 + \rho(\xi)^2 d\Omega^2$$

$\frac{3K}{R \rightarrow 0}$ turnoff gravity

$$\phi'' + \frac{3\rho'}{\rho} \phi' = \frac{dU}{d\phi}$$

ρ, ρ' extremize B

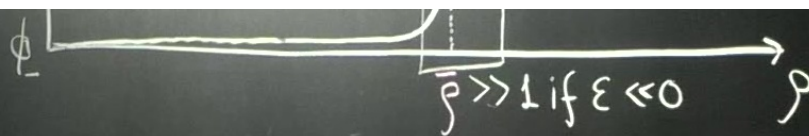
RATIO CAN BE SMALL OR LARGE

$$B_{\text{gravity}} = \frac{B_0}{\left(1 \pm \left(\frac{\bar{\rho}_0}{2\Lambda} \right)^2 \right)^2}, \quad \bar{\rho}_{\text{grav}} = \frac{\bar{\rho}_0}{1 \pm \left(\frac{\bar{\rho}_0}{2\Lambda} \right)^2}$$

$\bar{\rho}_0$ large

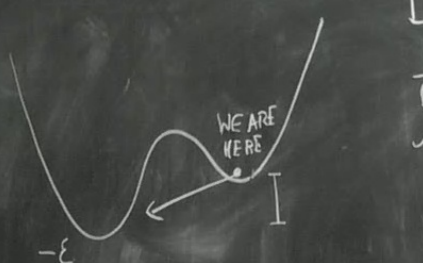
$$\rho'^2 = 1 + \frac{3}{16\pi G} \rho^2 \left(\frac{\phi'^2}{2} - U \right)$$

$\Lambda = \left(\frac{KE}{3} \right)^{-1/2}$ large, \pm POST/PRE



not yet fixed

(+) PRE



(-)

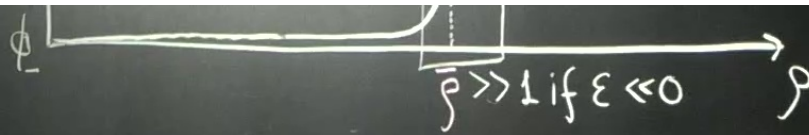


$B \downarrow$
 $\bar{p} \downarrow$
 more likely to form smaller bubbles

harder to tunnel -
 bubbles are bigger

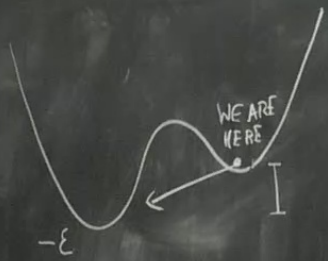
$$B = \frac{B_0}{\left(1 - \binom{I}{\epsilon}\right)^2}, \quad \bar{p} = \frac{\bar{p}_0}{\left(1 - \binom{I}{\epsilon}\right)^2}$$

if $\bar{p}_0 = 2\Lambda$



not yet fixed

(+) PRE



$B \downarrow$
 $\bar{p} \downarrow$
 more likely to form smaller bubbles

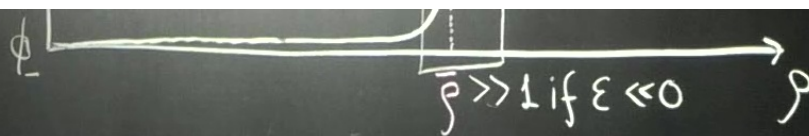


$$B = \frac{B_0}{\left(1 - \left(\frac{\bar{p}}{\bar{p}_0}\right)^2\right)^2}, \quad \bar{p} = \frac{\bar{p}_0}{\left(1 - \left(\frac{\bar{p}}{\bar{p}_0}\right)^2\right)}$$

harder to tunnel, bubbles are bigger

if $\bar{p}_0 = 2\lambda$
 $\leftrightarrow 2$
 $\epsilon = \frac{3kS_1}{4}$

$\bar{p} = \infty$



not yet fixed!

(+) PRE



$B \downarrow$ more likely to form smaller bubbles

$\bar{\rho} \downarrow$ Gravity can stabilize false vacuum!

$$B_0 = \frac{1}{(1 - (\dots)^2)^2}$$

$$\bar{\rho} = \frac{\bar{\rho}_0}{(1 - (\dots)^2)}$$

harder to tunnel bubbles outgrow

if $\bar{\rho}_0 = 2\Lambda$
 $\leftrightarrow 2$
 $\epsilon = \frac{3kS_1}{4}$

$\bar{\rho} = \infty$

