Title: Lecture - Beautiful Papers

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Subject: Other

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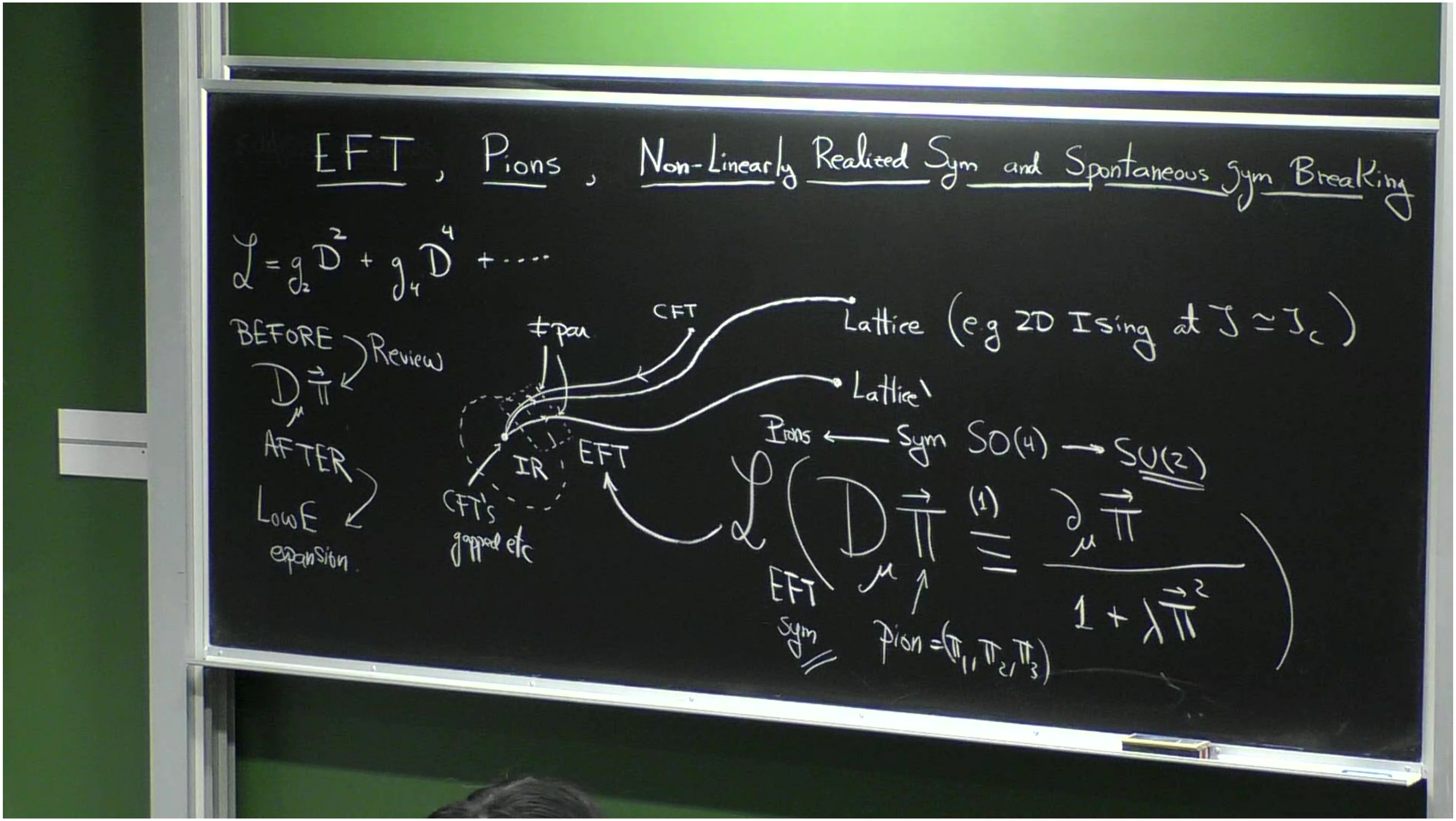

EFT, Pions, Non-Linearly Realized Sym and Spontaneous Sym Breaking
L = g2 D^2 + g4 D^4 + ...
BEFORE
Review
AFTER
Low E expansion
CFT
Lattice (e.g 2D Ising at J ≃ Jc)
Lattice'
IR
EFT
CFT's gapped etc
Pions ← Sym SO(4) → SU(2)
EFT Sym
pion = (π1, π2, π3)
1 + λπ^2

$$\vec{\pi} \leftsquigarrow SO(4) = SU(2) \times SU(2) \longrightarrow SU(2)$$

$$\mathcal{L}_{QCD} = -\bar{u}(\gamma^{\mu} D_{\mu})u - \bar{d}(\gamma^{\mu} D_{\mu})d + \underbrace{\cdots}_{\text{mass of } u,d \,\leftarrow\, \text{very light}}$$

$$\begin{pmatrix} u \\ d \end{pmatrix} \longrightarrow \exp\left(\underbrace{i\,\vec{\Theta}_V \cdot \vec{\sigma}}_{SU(2)} + \underbrace{i\,\gamma_5 \vec{\Theta}_A \cdot \vec{\sigma}}_{SU(2)} \right) \begin{pmatrix} u \\ d \end{pmatrix}$$

$$\vec{t}_L = \frac{1-\gamma_5}{2}\vec{\sigma}$$

$$\vec{t}_R = \frac{1+\gamma_5}{2}\vec{\sigma}$$

$$[t_L, t_L] = \varepsilon\, t_L$$

$$[\ \ _R\ \ \ _R\ ] = \ \ \ \ _R$$

$$[\ \ _R\ \ \ _L\ ] = 0$$

$$t = t_L + t_R$$

$$x = t_L - t_R$$

$$[t, t] = t$$ duh

$$[t_a, x_b] = i\varepsilon_{abc} x_c$$ duh

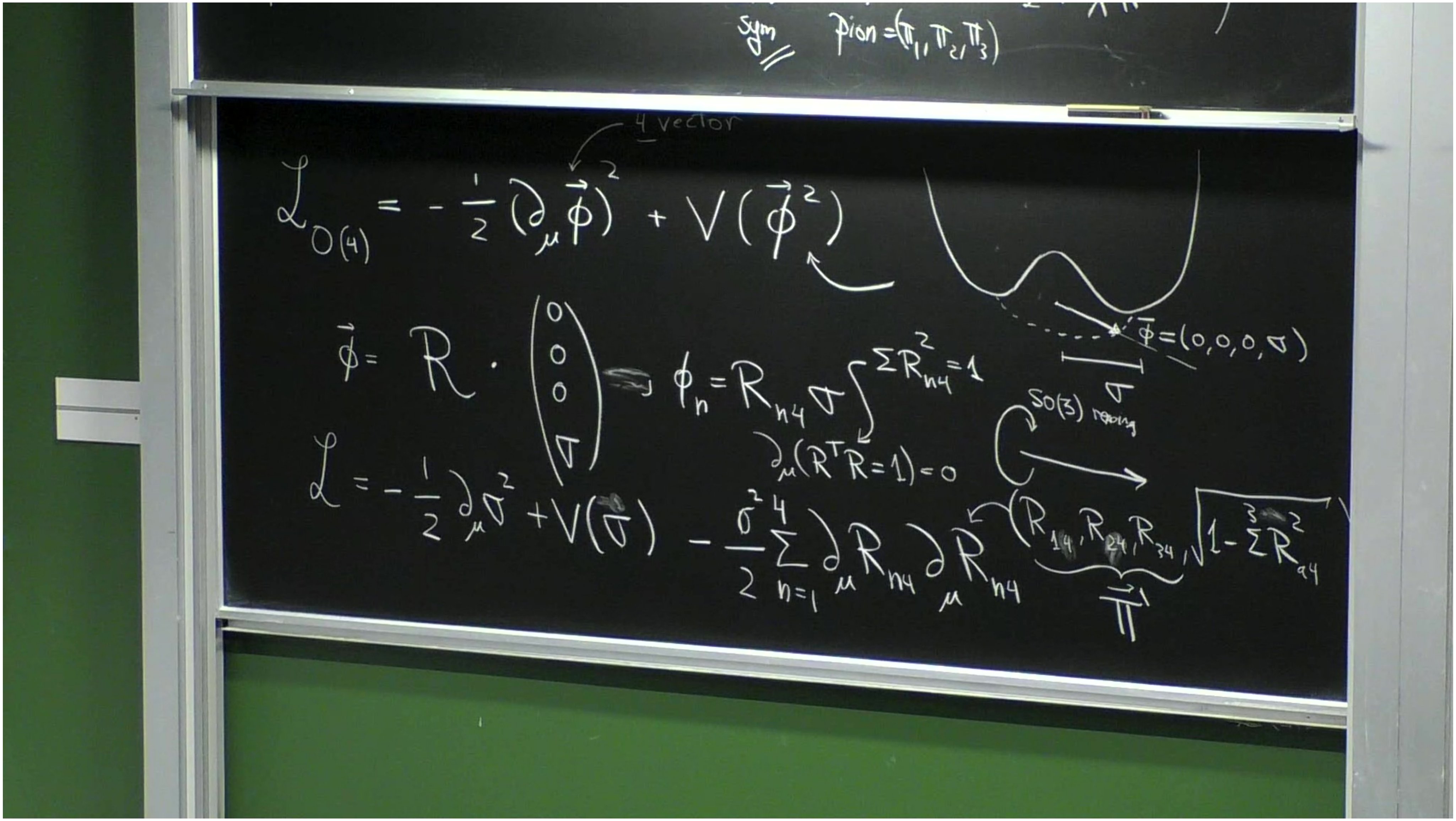
$$[x_a, x_b] = i\varepsilon_{abc} t_c$$ ~~duh~~

Broken

$\vec{\pi}$ are pseudo-goldstone bosons of $SU(2) \times SU(2) \longrightarrow SU(2)$

$$SO(4) \longrightarrow SU(2) = SO(3)$$

NLSM


sym pion $= (\pi_1, \pi_2, \pi_3)$
4 vector
$\mathcal{L}_{O(4)} = -\frac{1}{2}(\partial_\mu \vec{\phi})^2 + V(\vec{\phi}^2)$
$\vec{\phi} = R \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ \sigma \end{pmatrix} \Rightarrow \phi_n = R_{n4}\sigma$
$\Sigma R_{n4}^2 = 1$
$\partial_\mu(R^T R = 1) = 0$
$\vec{\phi} = (0,0,0,\sigma)$
σ
SO(3) rotating
$\mathcal{L} = -\frac{1}{2}\partial_\mu \sigma^2 + V(\sigma) - \frac{\sigma^2}{2}\sum_{n=1}^{4} \partial_\mu R_{n4} \partial_\mu R_{n4}$
$(\underbrace{R_{14}, R_{24}, R_{34}}_{\vec{\pi}}, \sqrt{1 - \sum^{3} R_{a4}^2})$

$$-R_{4n} = \underset{\substack{\shortparallel \\ 1\ldots3}}{R_{n4}} = \frac{2\pi_n}{1+\vec{\pi}^2}$$

$$R_{44} = \frac{1-\vec{\pi}^2}{1+\vec{\pi}^2}$$

$$\left(\underset{\substack{\shortparallel \\ 1\ldots3}}{R_{ab}} = \delta_{ab} - \frac{2\pi_a\pi_b}{1+\pi^2} \right)$$

$$\mathcal{L} = \mathcal{L}_\sigma - \frac{1}{2}\left(D_\mu \vec{\pi}\right)^2$$

$$*$$

$$-\tfrac{1}{2}\,\partial_\mu \pi^2 + \lambda\pi^2\,\partial_\mu \pi^2$$

$$+ \ldots$$

SO(4) sym in $\vec{\pi}$ language

$$\vec{\pi} \to R_{SO(3)} \cdot \vec{\pi} \iff \begin{array}{l} \delta\vec{\pi} = \vec{\Theta} \times \vec{\pi} \\ \delta\sigma = 0 \end{array} \quad \checkmark$$

un-broken
(isospin)

broken gen

$$\delta\phi_4 = -2\vec{\varepsilon}\cdot\vec{\phi}$$

$$\delta\vec{\phi} = +2\vec{\varepsilon}\,\phi_4$$

$$\underbrace{\left\{ \begin{array}{l} \delta\sigma = 0 \\ \delta\vec{\pi} = \vec{\epsilon}\,(1-\vec{\pi}^{\,2}) + 2\vec{\pi}\,(\vec{\epsilon}\cdot\vec{\pi}) \end{array} \right.}_{\text{NLRSym}}$$

SU(2) SU(2)

$$-R_{4n} = R_{\underset{1\ldots 3}{n4}} = \frac{2\pi_n}{1+\vec{\pi}^2}$$

$$R_{44} = \frac{1-\vec{\pi}^2}{1+\vec{\pi}^2}$$

$$\left(R_{\underset{1\ldots 3}{ab}} = \delta_{ab} - \frac{2\pi_a\pi_b}{1+\pi^2} \right)$$

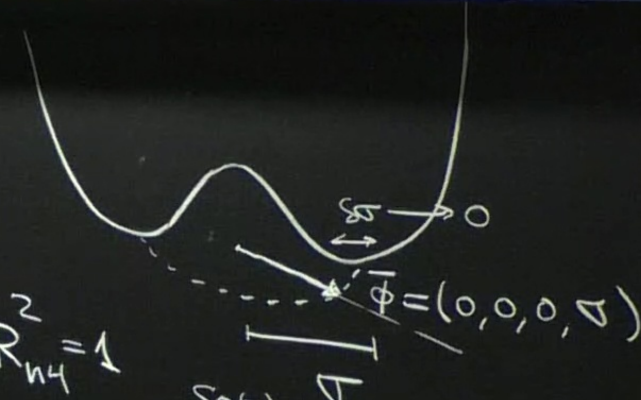
cov. der.

$$\mathcal{L} = \mathcal{L}_\sigma - \frac{1}{2}\Big(\underbrace{D_\mu}_{*}\,\vec{\pi}\Big)^2$$

$$- \frac{1}{2}(\partial_\mu \vec{\pi})^2 + \lambda \pi^2\, \partial_\mu \pi^2 + \ldots$$

sym pion $=(\pi_1, \pi_2, \pi_3)$

$$\mathcal{L}_{O(4)} = -\frac{1}{2}(\partial_\mu \vec{\phi})^2 + V(\vec{\phi}^2) + (\partial_\mu \vec{\phi})^4$$



$$\vec{\phi} = R \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ \sigma \end{pmatrix} \Rightarrow \phi_n = R_{n4}\,\sigma \qquad \Sigma R_{n4}^2 = 1$$

$$\partial_\mu (R^T R = 1) = 0$$

$$\mathcal{L} = \cancel{-\frac{1}{2}\partial_\mu \sigma^2} + \cancel{V(\sigma)} - \frac{\sigma^2}{2}\sum_{n=1}^{4} \partial_\mu R_{n4}\, \partial_\mu R_{n4}$$

$$\Big(\underbrace{R_{14}, R_{24}, R_{34}}_{\vec{\pi}}, \sqrt{1 - \textstyle\sum_{a=1}^{3} R_{a4}^2}\Big)$$

sym // pion $= (\pi_1, \pi_2, \pi_3)$

$$\delta\left(\frac{\partial_\mu \vec{\pi}}{1+\vec{\pi}^2}\right) = 2\,\underbrace{\left(\vec{\pi}\times\vec{\epsilon}\right)}_{\vec{\Theta}\text{ that is field dep.}} \times D_\mu \vec{\pi} + O$$

nice d

$(\vec{D}\pi)^2$ is inv

fix $f(\pi)$

$$\delta\left(f(\vec{\pi}^2)\,\partial_\mu \pi\right) = \underset{\perp}{\ldots} + \underset{/\!/}{\underbrace{\ldots}_{=0}}$$

$\perp D_\mu \vec{\pi}$

$[t_L, t_L] = \varepsilon\, t_L$

$[\ \ _R\ ,\ \ _R] = \ \ _R$

$[\ \ _R\ ,\ \ _L] = 0$

$t = t_L + t_R$ ✓

$x = t_L - t_R$ <u>NL</u>

$[t, t] = t$ duh

$[t_a, x_b] = i\varepsilon_{abc} x_c$ duh

$[x_a, x_b] = i\varepsilon_{abc} t_c$ ~~duh~~

Broken

$\vec{\pi}$ are pseudo-goldstone bosons of $SU(2) \times SU(2) \longrightarrow SU(2)$

$SO(4) \longrightarrow SU(2) = SO(3)$

NLSM

$$\vec{\pi} \to \Phi(\pi^2)\,\vec{\pi} = \tilde{\vec{\pi}}$$

$$[t_a, \pi_b] = i\,\varepsilon_{abc}\,\pi_c$$

$$[x_a, \pi_b] \equiv -i\,\underbrace{f_{ab}(\vec{\pi})}_{\delta_{ab} f(\pi^2) + \pi_a \pi_b\, g(\pi^2)}$$

MY FREEDOM

$$= \frac{1 + 2 f'(\pi^2) f(\pi^2)}{f(\pi^2) - 2\pi^2 f'(\pi^2)} \qquad f_x \qquad f'\,[x, \pi]$$

Dif Eq for f, g

$$0 = [X_a, [X_b, \pi_c]] - [X_b, [X_a, \pi_c]] - [\underbrace{[X_a, X_b]}_{t}, \pi_c]$$

π

$$\underbrace{[x_c, D_\mu \pi_a \equiv \boxed{d_{a\ell}(\vec{\pi})}\,\partial_\mu \pi_\ell]}_{=\,ooo} = -i\,\underbrace{v_{ab}(\vec{\pi})}_{\text{whatever}}\,\varepsilon_{bcd}\,\underbrace{D_\mu \pi_d}_{\perp}$$

$D\pi \leftarrow$ rot

$$D_\mu \vec{\pi} = \left(f^2(\pi^2) + \pi^2\right)^{-1/2} \boxed{\partial_\mu \vec{\pi}} + \left(f^2(\pi^2) + \pi^2\right)^{-1} \underbrace{\left(f'(\pi^2) - \frac{1/2}{f(\pi^2) + \sqrt{f(\pi^2) + \pi^2}}\right)}_{=0} \boxed{\vec{\pi}}\,\partial_\mu \pi^2$$

$$f(\vec{\pi}^2) = \frac{1 - \pi^2}{2}$$

$$\mathcal{L} = -\underbrace{\frac{g_2}{2} D_\mu \vec{\pi} D^\mu \vec{\pi}} + \frac{g_4^{(1)}}{4}\left(D_\mu \vec{\pi} \cdot D^\mu \vec{\pi}\right)^2$$

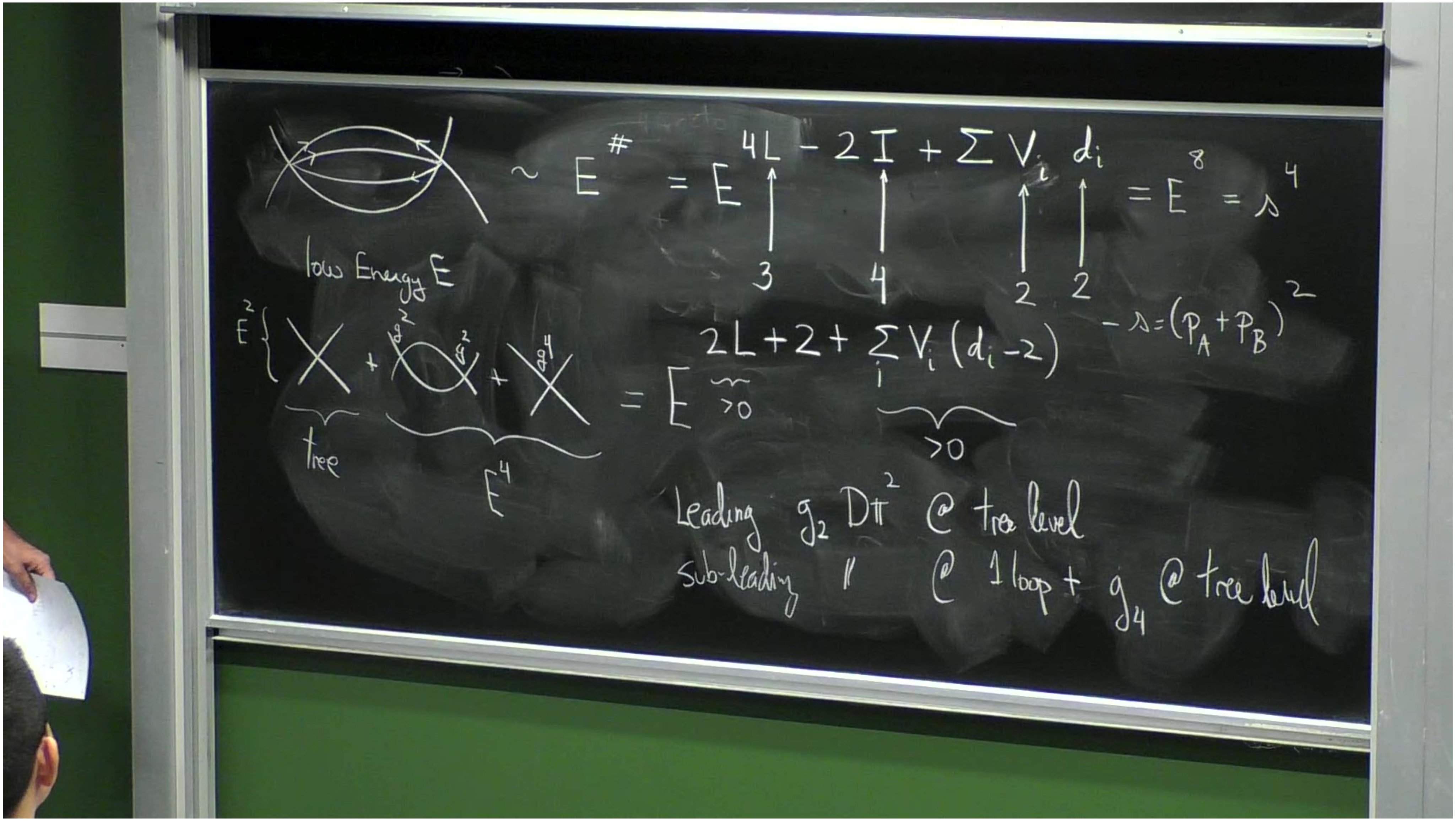
$$+ \frac{g_4^{(2)}}{4}\left(D_\mu \vec{\pi} D_\nu \vec{\pi}\right)\left(\mu\ \nu\right) + \ldots$$

$$\left(\partial_\mu \vec{\pi} \cdot \partial_\mu \vec{\pi}\right) \vec{\pi}^{\,n}$$

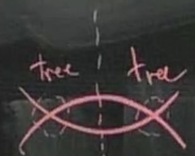
at leat 2 ∂

D^6 etc

more loops & more $\partial^2 \rightarrow$ less imp. @ low E.


$\sim E^{\#} = E^{4L - 2I + \sum V_i d_i} = E^8 = s^4$
3 4 2 2
low Energy E
E^2
g^2 g^2 g^4
$= E^{2L+2+\sum_i V_i (d_i - 2)}$
>0
>0
$-s = (p_A + p_B)^2$
tree
E^4
Leading $g_2 D\pi^2$ @ tree level
Sub-leading " @ 1 loop + g_4 @ tree level

$$(11)\quad \mathcal{M}^{\text{tree}}_{abcd} = \frac{4}{g_2}\Big(\underbrace{\delta_{ab}\delta_{cd}}_{\sim E^2}\, s + \delta_{ac}\delta_{bd}\, t + \delta_{ad}\delta_{bc}\, u \Big)$$



$$\begin{pmatrix} A(s\,|\,t,u) \\ s + \#(t+u) \\ \Downarrow \\ * s \end{pmatrix}$$

$A(t\,|\,s,u)$

$$s = -(p_a+p_b)^2$$
$$t = -(p_a-p_c)^2$$
$$+\; u = -(p_a-p_d)^2$$
$$\overline{\qquad 0 \qquad}$$

$$g_4^{(2)}(\mu) = g_4^{(2)} - \frac{4}{3\pi^2}\log\frac{\Lambda^2}{\mu^2}$$

$$(12)\quad \mathcal{M}^{1\,\text{loop}}(E) = \frac{\delta_{ab}\delta_{cd}}{g_2^2}\Big(-\frac{1}{2\pi^2} s^2 \log\Big(\frac{-s}{\mu}\Big) - \frac{1}{12\pi^2}(u^2-s^2-t^2)\log\Big(\frac{-t}{\mu}\Big) + \dots + \frac{1}{3\pi^2}(s^2+t^2+u^2)\cancel{\log(\Lambda^2)} - \frac{1}{2} g_4^{(1)}(\mu)\, s^2 - \frac{1}{4} g_4^{(2)}(\mu)(u^2+t^2)\Big)$$

$$E^* = \mu \lesssim E$$