Title: Lecture - Beautiful Papers

Speakers: Pedro Vieira

Collection/Series: Beautiful Papers - October 7, 2024 - January 31, 2025

Subject: Other

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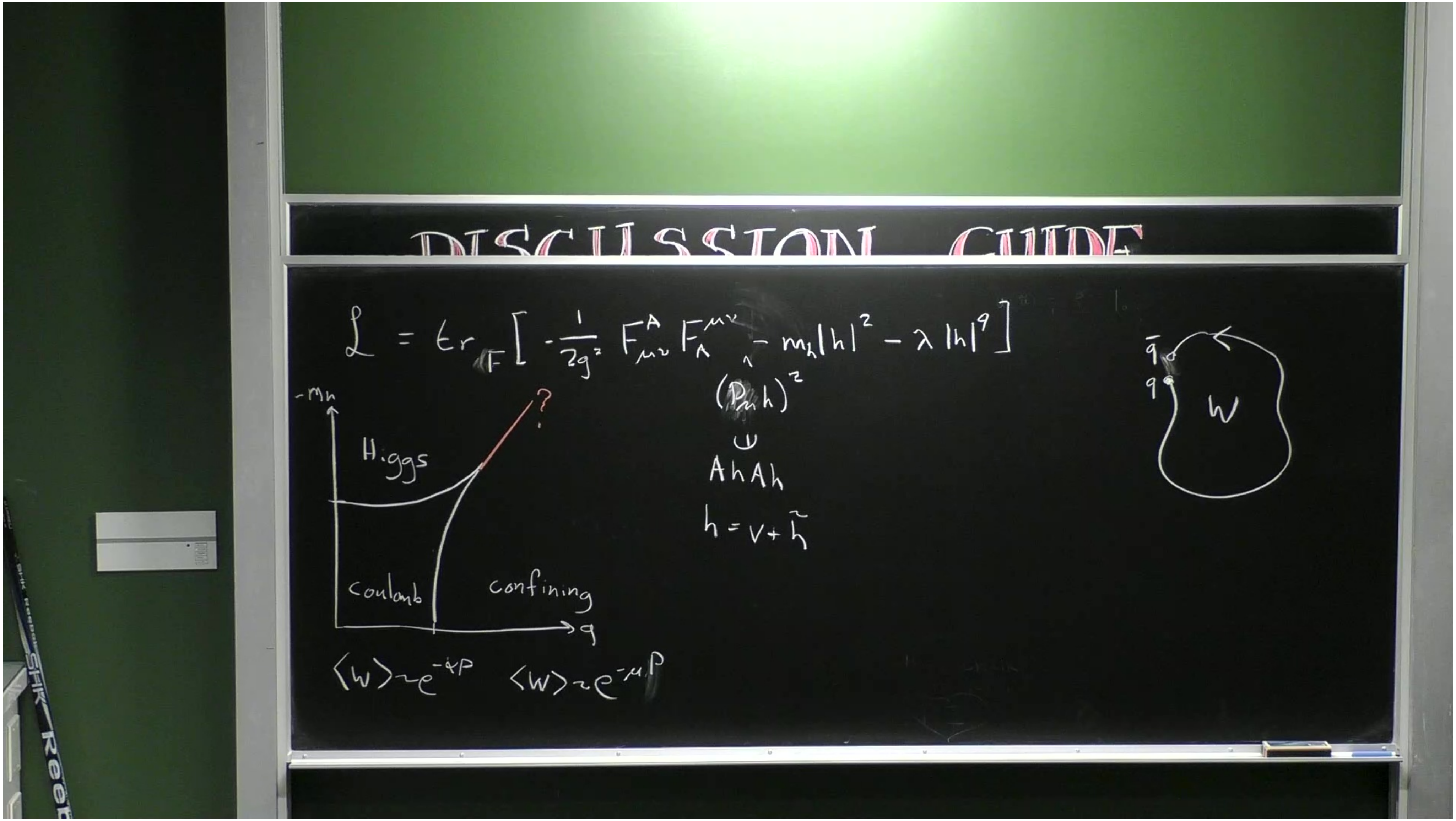

$\mathcal{L} = tr_F [-\frac{1}{2g^2} F^A_{\mu\nu} F_A^{\mu\nu} - m_h |h|^2 - \lambda |h|^4]$
$(D_\mu h)^2$
$AhAh$
$h = v + \tilde{h}$
$-m_h$
Higgs
Coulomb
confining
g
$\langle W \rangle \sim e^{-\mu P}$
$\langle W \rangle \sim e^{-\mu P}$
$\bar{q}$
q
W

$$\mathcal{L}_f = \bar{\Psi} \not{\partial} \Psi + m(\bar{\Psi}\bar{\Psi} + \Psi\Psi)$$

$$Z[e^{i\theta} m] = Z[m]\, e^{-\frac{i\theta}{8\pi^2}\int_M R\wedge R}$$

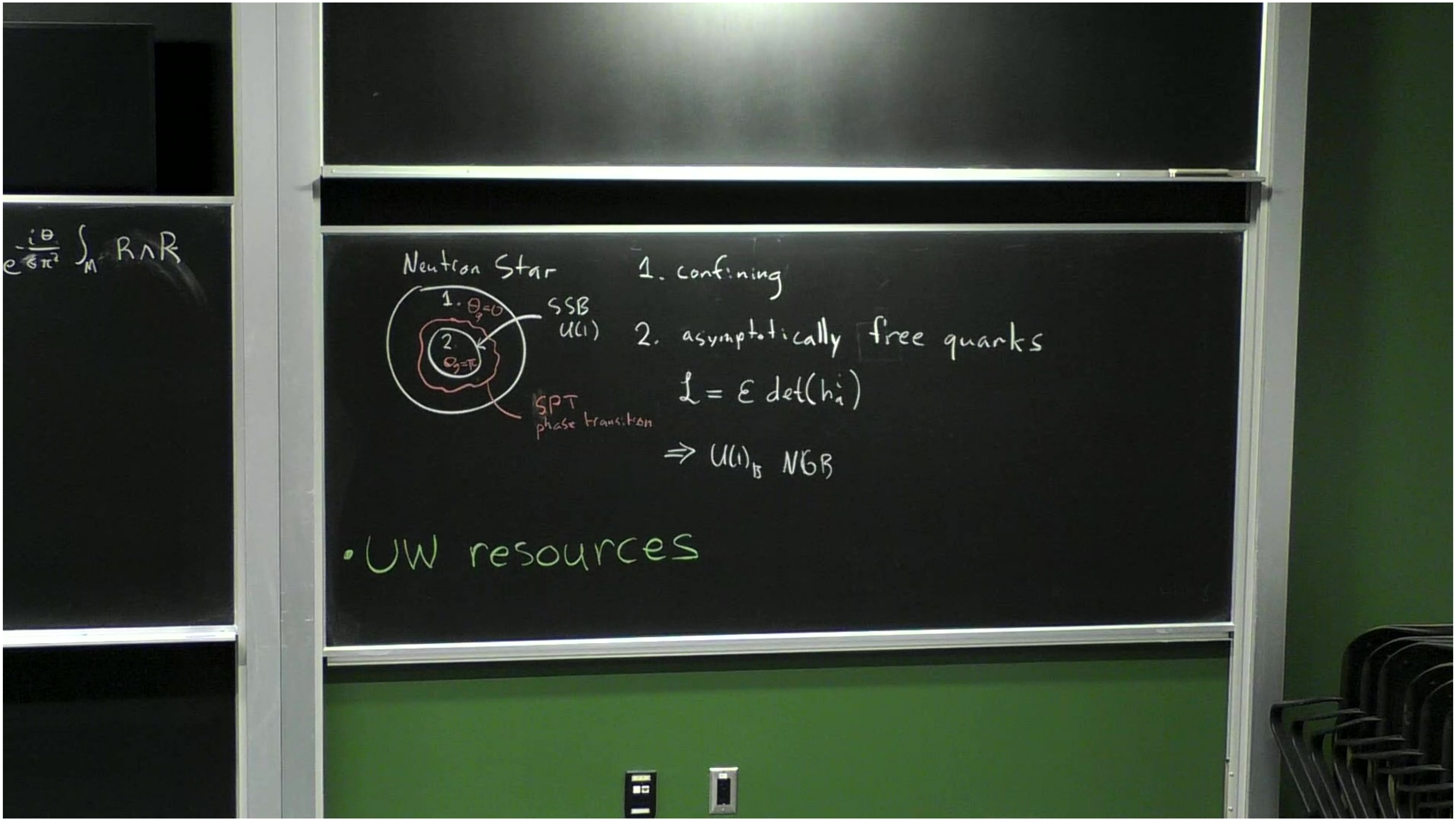
$U(1)^A$: $\Psi \to e^{i\theta/2}\Psi$

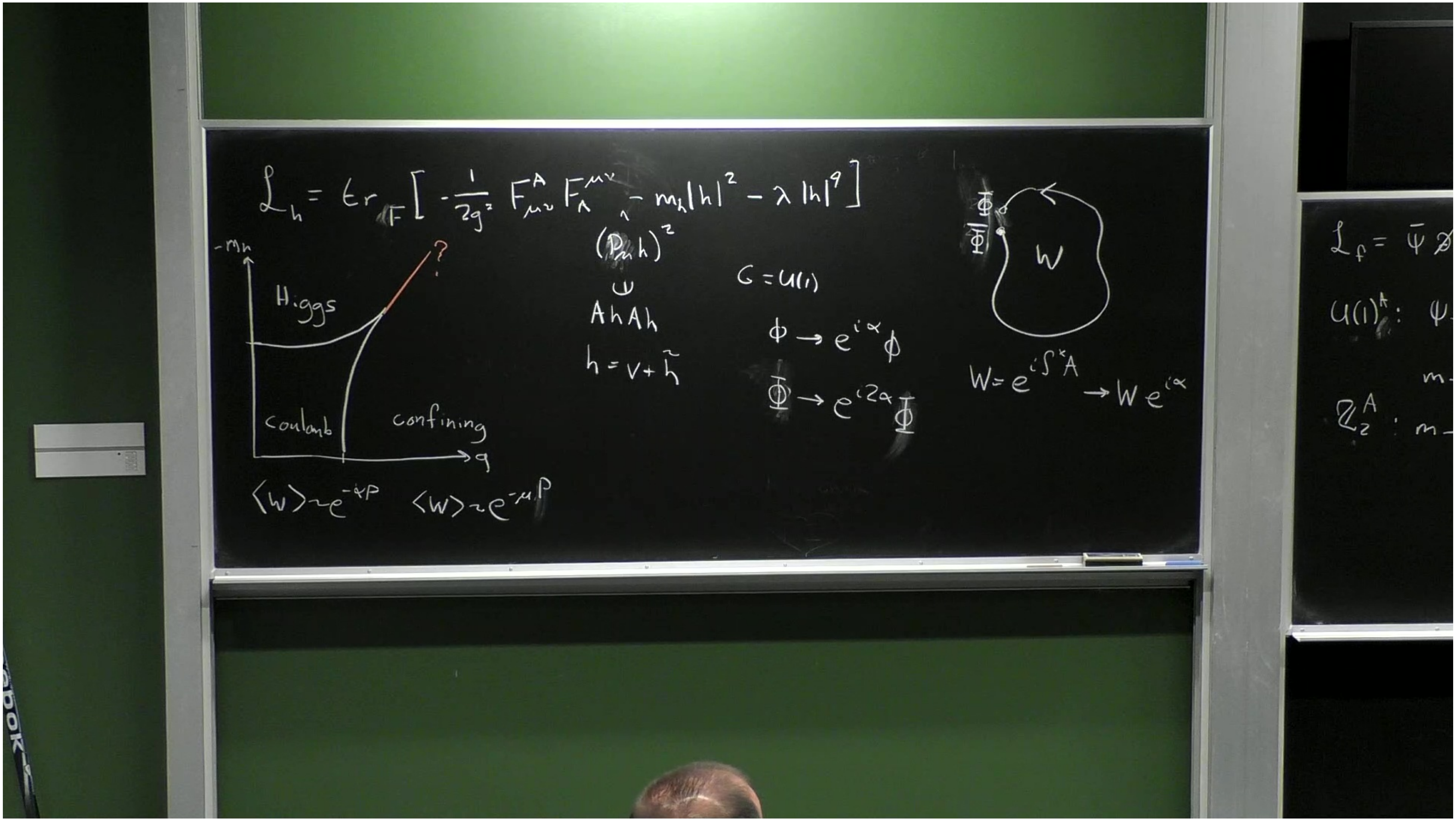
$m \to e^{i\theta} m$

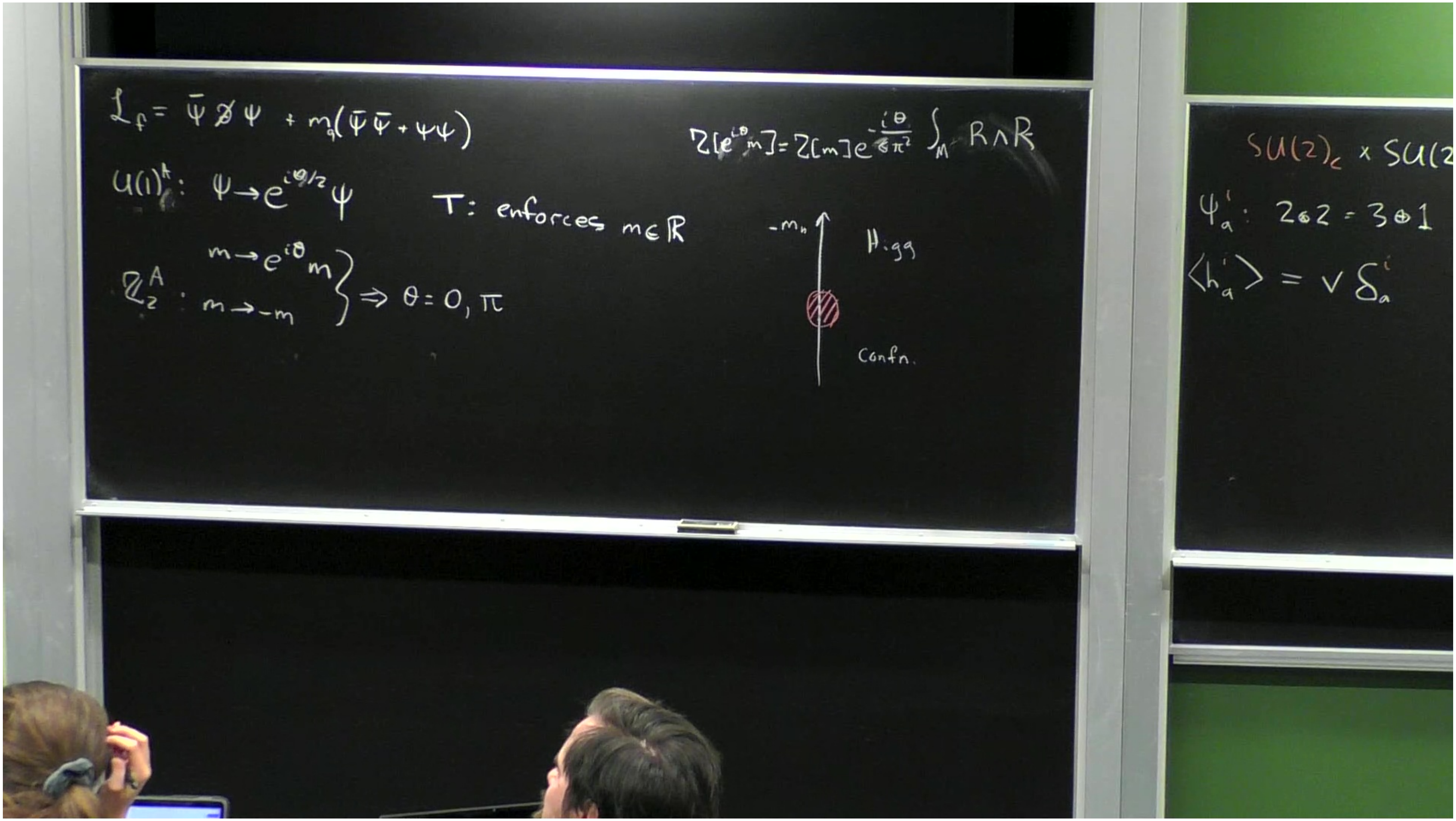
T: enforces $m \in \mathbb{R}$

$\mathbb{Z}_2^A$: $m \to -m$

$SU(2)$


$e^{-\frac{i\theta}{8\pi^2}\int_M R\wedge R}$
Neutron Star
1. $\theta_q=0$
2 $\theta_q=\pi$
SSB
U(1)
SPT
phase transition
1. confining
2. asymptotically free quarks
$\mathcal{L} = \varepsilon \det(h^i_a)$
$\Rightarrow U(1)_B$ NGB
• UW resources


$\mathcal{L}_h = tr_F\left[-\frac{1}{2g^2}F^A_{\mu\nu}F^{\mu\nu}_A - m_h|h|^2 - \lambda|h|^4\right]$
$(D_\mu h)^2$
$AhAh$
$h = v + \tilde{h}$
$G = U(1)$
$\phi \to e^{i\alpha}\phi$
$\Phi \to e^{i2\alpha}\Phi$
W
$W = e^{i\int A} \to W e^{i\alpha}$
$-m_h$
Higgs
Coulomb
confining
?
g
$\langle W \rangle \sim e^{-\#P}$
$\langle W \rangle \sim e^{-\mu P}$
$\mathcal{L}_f = \bar{\Psi}$
$U(1)^A$:
$\mathbb{Z}_2^A$:


$\mathcal{L}_f = \bar{\Psi} \not{D} \Psi + m_q(\bar{\Psi}\bar{\Psi} + \Psi\Psi)$
$U(1)^A: \psi \to e^{i\theta/2}\psi$
T: enforces $m \in \mathbb{R}$
$m \to e^{i\theta} m$
$\mathbb{Z}_2^A: m \to -m$
$\Rightarrow \theta = 0, \pi$
$Z[e^{i\theta} m] = Z[m] e^{-\frac{i\theta}{8\pi^2}\int_M R \wedge R}$
$-m_H$
Higgs
Confn.
$SU(2)_c \times SU(2$
$\Psi_a^i: 2 \otimes 2 = 3 \oplus 1$
$\langle h_a^i \rangle = v\,\delta_a^i$

Vacuum Structure of

Charge-K 2D QCD

Based on:

[1912.10064] Armoni, Sugimoto.

[2008.07567] Komargodski et al

Part I : The Model

$$S_{QED} = \int d^2x \left\{ \frac{-1}{4e^2} F^2 + i\bar{\psi}\gamma^\mu(\partial_\mu + ikA_\mu)\psi \right.$$

$k \in \mathbb{Z}_{>0}$

Reps: $\gamma^0 = \sigma^1$, $\gamma^1 = i\sigma^2$ and $\psi = \begin{pmatrix}\psi_R \\ \psi_L\end{pmatrix}$

With $\psi_{L/R}$ handed Spinors.

U(1) Gauge Trans: $A \to A + d\lambda$, $\psi \to e^{-ik\lambda}\psi$

$\lambda \sim \lambda + 2\pi$

flux Quantization : $\frac{1}{2\pi}\int F \in \mathbb{Z}$.

Classical Symmetries:

Reps: $\gamma^0 = \sigma^1$, $\gamma^1 = i\sigma^2$ and $\psi = (\psi_L)$

With $\psi_{L/R}$ handed spinors.

U(1) Gauge Trans: $A \to A + d\lambda$, $\psi \to e^{-ik\lambda}\psi$

$\lambda \sim \lambda + 2\pi$

flux Quantization : $\frac{1}{2\pi}\int F \in \mathbb{Z}$.

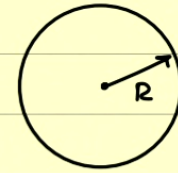
Classical Symmetries:

$\boxed{1}$ Axial for $e^{i\alpha} \in U(1)_A / \mathbb{Z}_2$.

$$\psi_L \to e^{-i\alpha}\psi_L \ , \quad \psi_R \to e^{i\alpha}\psi_R$$

Anomaly free symmetry: $\boxed{\mathbb{Z}_k^{\text{Axial}}}$

$\boxed{2}$ 1-form Symmetry : $\boxed{\mathbb{Z}_K^{1\text{-form}}}$



$$A \to A + \frac{1}{k} dz \quad , \quad \psi \to e^{-iz}\psi$$

with : $z(x^0, x^1 + 2\pi R) = z(x^0, x^1) + 2\pi \ell$

and : $\ell = 1, 2, \dots, k.$

NOT a gauge transformation $\forall\, \ell \neq k$.

e.g: $z(x) = \frac{\ell}{R} x^1 \implies A_1 \to A_1 + \frac{\ell}{kR}$

But $A_1 \to A_1 + \frac{1}{R}$ is a gauge transformation.

Action on Wilson Loops

$\mathbb{Z}_k^{1\text{-form}}$: $\boxed{W \to e^{i2\pi\ell/k} W}$

Working in $A_0 = 0$ gauge:

$$S_\varphi = \int d^2x \left\{ \frac{1}{2e^2} (\partial_0 A_1)^2 + \frac{1}{8\pi} (\partial_\mu \varphi)^2 + \frac{K}{2\pi} \varphi A_1 \right\}$$

Canonical Momenta:

$$\pi_{A_1} = \frac{1}{e^2} \partial_0 A_1 + \frac{K}{2\pi} \varphi \quad ; \quad \partial_1 \pi_{A_1} = 0$$

$$\pi_\varphi = \frac{1}{4\pi} \partial_0 \varphi$$

Hamiltonian:

$$H = \int dx^1 \left\{ \frac{e^2}{2} \left(\pi_{A_1} - \frac{K}{2\pi} \varphi \right)^2 + 2\pi \pi_\varphi^2 + \frac{1}{8\pi} (\partial_1 \varphi)^2 \right\}$$

Algebra: $[A_1, \pi_{A_1}] \propto \delta$

$[\varphi, \pi_\varphi] \propto \delta$

Energies of $|\theta\rangle$

Indep of (θ, M_0)

$$\mathcal{E}(\theta) = \langle\theta|\mathcal{H}|\theta\rangle \propto \tilde{f}(\theta) M_0 + (\cdots)$$

where M_0 is a mass perturbation :

$$M_0 \bar{\psi}\psi.$$

$\boxed{1}$ $M_0 = 0 \implies |\theta + 2\pi i\rangle$ all degenerate

$\boxed{2}$ $M_0 \neq 0$ but small. , lifted degeneracy

For $\theta \neq \pi$, Single lowest energy state

Part II: Confinement of Wilson Loops

Focusing on $M_0 = 0$:

$\boxed{1}$ $\hat{U}$ order parameter:

$$\langle \theta | \hat{U} | \theta \rangle = e^{i\theta/k} \neq 0 \quad \text{but}$$

mixed anomaly $\Rightarrow$ $\hat{V}\hat{U}\hat{V}^{-1} = e^{-2\pi i/k}\,\hat{U}$.

Thus, $\langle \theta | \hat{U} | \theta \rangle$ allows us to identify the vacuum.

$\boxed{2}$ Action on Wilson Loops

$$\hat{U}\,\mathcal{W}\,\hat{U}^{-1} = e^{i2\pi/k}\,\mathcal{W}.$$

$\Rightarrow$ $\mathcal{W}$ op shifts $\theta \to \theta \pm 2\pi$ depending on orientation.

Confinement Vs. Deconfinement

$\theta = 0$

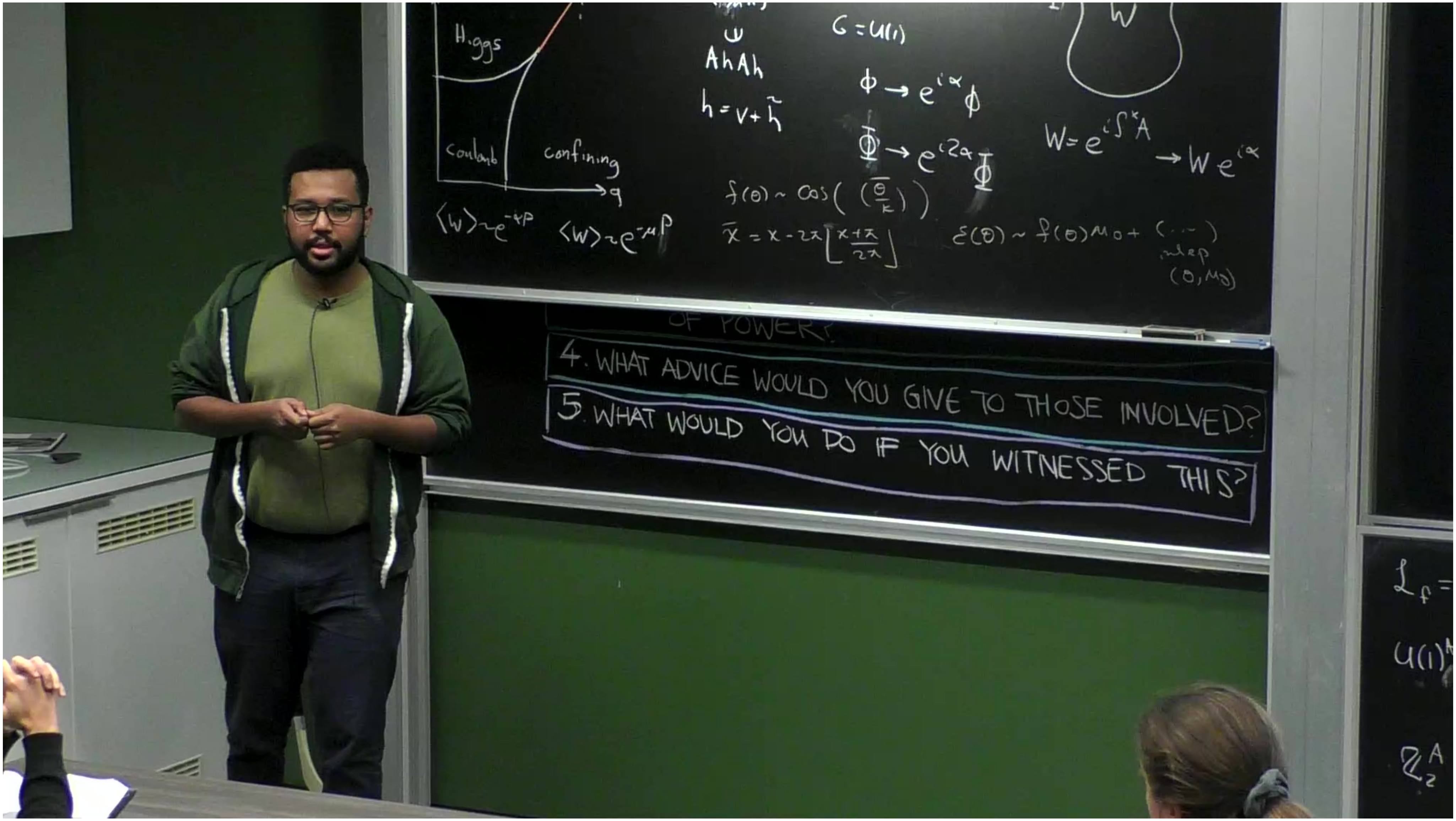
$|\theta \pm 2\pi\rangle$ $\quad |\theta\rangle$ $\quad$ with $\quad E_\theta < E_{\theta \pm 2\pi}$

$$\Rightarrow \quad \langle W_\square \rangle \sim e^{-\text{Area}} \quad \Rightarrow \text{Confined.}$$

$\theta = \pi$, Two degenerate vacua: $|\pm\pi\rangle$

$|-\pi\rangle$ or $|2\pi\rangle$ $\quad |\pi\rangle$ $\quad E_\pi = E_{-\pi}$ but

$E_\pi < E_{2\pi}$


Higgs
Coulomb
confining
G = U(1)
OF POWER?
4. WHAT ADVICE WOULD YOU GIVE TO THOSE INVOLVED?
5. WHAT WOULD YOU DO IF YOU WITNESSED THIS?

1) Mcdonell 93';

2) Ambjorn 15'

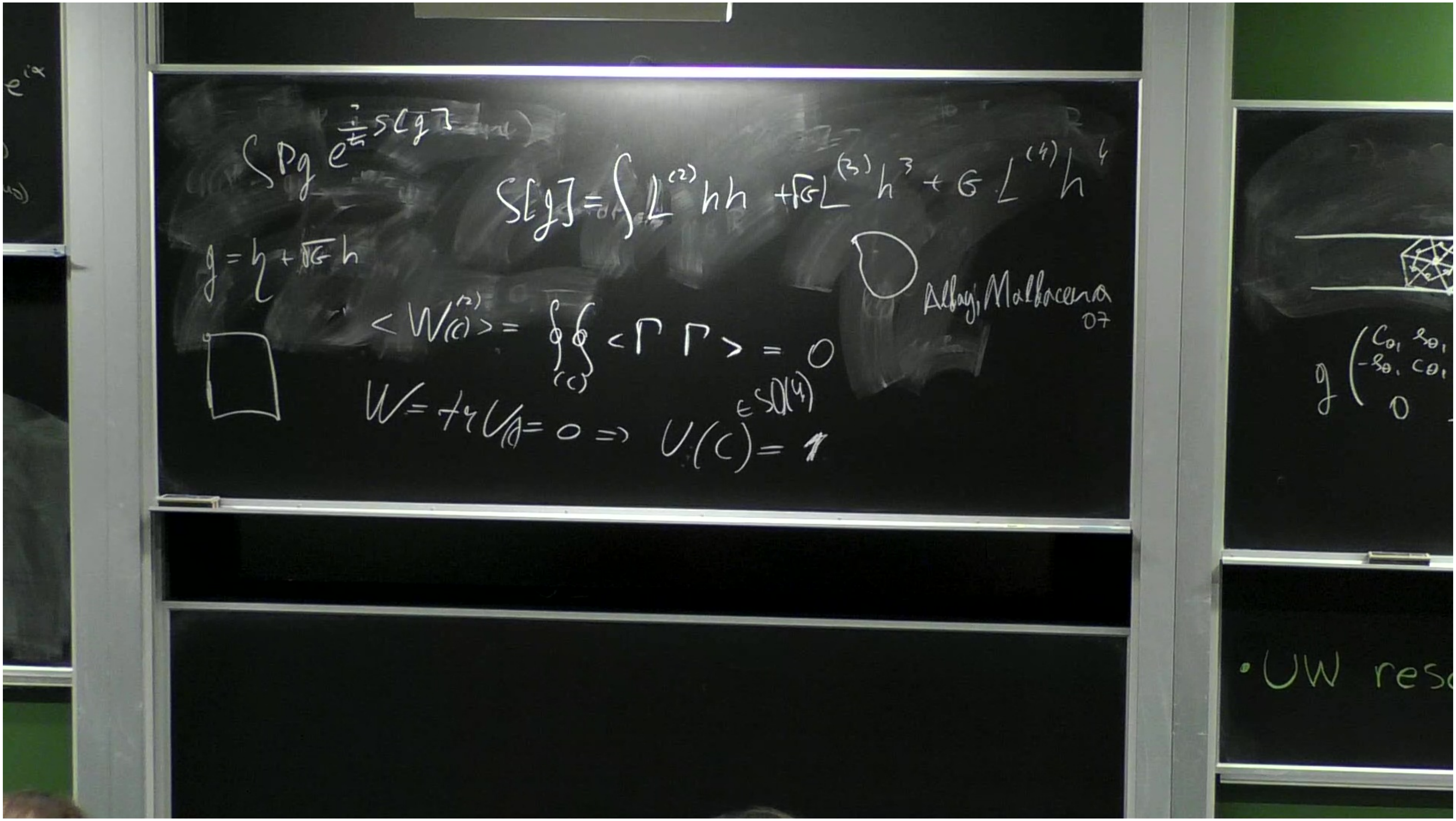
$$\Delta V^\alpha = -\Gamma^\alpha_{\mu\beta} V^\beta dx^\mu$$

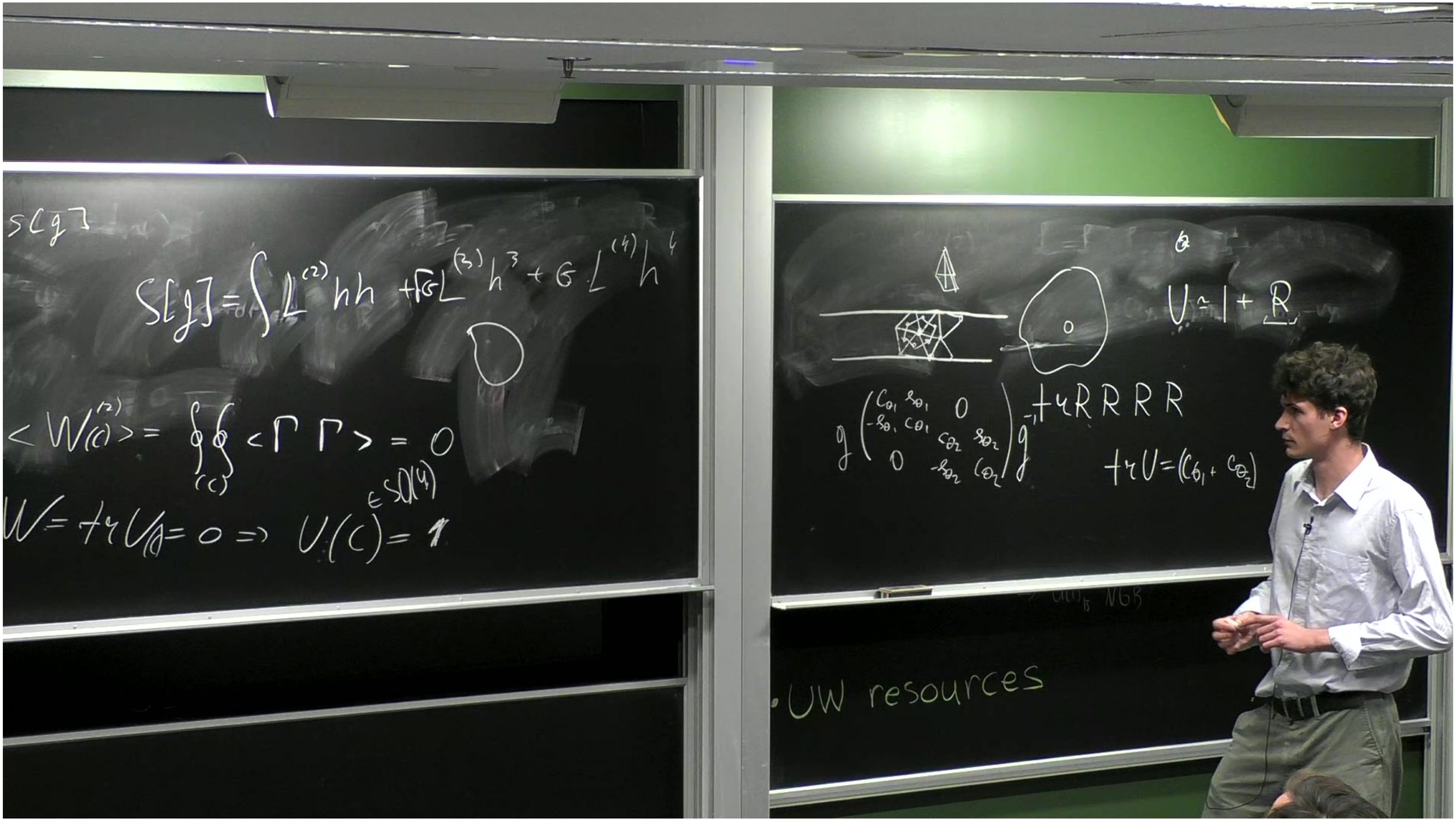
$$V'^\alpha = U^\alpha{}_\beta V^\beta$$

$$U^\alpha{}_\beta = P\exp \oint \Gamma^\alpha_{\mu\beta} dx^\mu$$

$$U \to \Lambda U \Lambda^{-1} \Rightarrow \mathrm{tr}\, U = \mathrm{inv}$$

$$W = -4 + \mathrm{tr}\, U$$


$\int \mathcal{D}g \, e^{\frac{i}{\hbar} S[g]}$
$S[g] = \int L^{(2)} hh + \sqrt{G} L^{(3)} h^3 + G L^{(4)} h^4$
$g = \eta + \sqrt{G} h$
Alday, Maldacena 07
$\langle W^{(2)}(C) \rangle = \oint\oint_{(C)} \langle \Gamma \Gamma \rangle = 0$
$\in SO(4)$
$W = \mathrm{tr}\, U(C) = 0 \Rightarrow U(C) = 1$
• UW res


S[g] = ∫ L^(2) hh + √G L^(3) h^3 + G L^(4) h^4
< W^(2)(c) > = ∮∮ <Γ Γ> = 0
W = tr U_g = 0 ⇒ U(C) = 1
U ≈ 1 + R
+ 4RRRR
tr U = (cθ1 + cθ2)
UW resources