Title: Lecture - Relativity, PHYS 604

Speakers: Ghazal Geshnizjani

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Subject: Cosmology, Strong Gravity

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Abstract:

$$S_{EH} = \frac{1}{16\pi G}\int dx^4 \sqrt{-g}\, R \longrightarrow \delta_g S = \frac{1}{16\pi G}\int \sqrt{-g}\, G_{\mu\nu}\,\delta g^{\mu\nu}$$

EoM $\frac{\delta_g S}{\delta g^{\mu\nu}}$: $G_{\mu\nu} = 0$ Vacuum solutions

(1) $g^{\mu\nu}G_{\mu\nu} = g^{\mu\nu}R_{\mu\nu} - \frac{1}{2}g^{\mu\nu}g_{\mu\nu}R = 0$

$$R - \frac{1}{2}dR = 0 \Rightarrow (1 - \tfrac{1}{2}d)R = 0$$

$d \neq 2 \Rightarrow$ in vacuum $R = 0$

2. $G_{\mu\nu}(g, \partial g, \partial^2 g) = 0 \Rightarrow$ 2nd order

Lovelock theorem:

in 4D E-H action is the only local action for metric $(g, \ldots)$

$\Rightarrow$ 2nd order PDEs for g

Horndeski $g_{\mu\nu} + \phi \Rightarrow$ Horndeski gravity

$f(R) \to g_{\mu\nu} + \phi$

$$S_{\text{Palatini}}(g, \Gamma) \Rightarrow S_P(g,\Gamma) = \frac{1}{16\pi G}\int d^4x \sqrt{-g}\, g^{\mu\nu} R_{\mu\nu}$$

$$\delta_g S = \frac{1}{16\pi G}\int \sqrt{-g}\left[\underbrace{-\frac{1}{2} R g_{\mu\nu} + R_{\mu\nu}}\right]\delta g^{\mu\nu}$$

EOM $\quad G_{\mu\nu} = 0$

$\delta_\Gamma S \Rightarrow$ EOM $\rightarrow$ Christoffel symbols.

$$R_{\mu\nu} = \partial_\lambda \Gamma^\lambda_{\mu\nu} - \partial\Gamma + \Gamma\Gamma - \Gamma\Gamma$$

$$S_{tot} = S_{EH} + S_m = \frac{1}{16\pi G}\int \sqrt{-g}\, R + \underbrace{\int \sqrt{-g}\, L_m}_{S_m}$$

$\delta_m S_{tot} = 0 \Rightarrow \delta S_m = 0 \Rightarrow$ EOM for matter fields

$$\delta_g S_{tot} = \delta_g S_{EH} + \delta_g S_m = \frac{1}{16\pi G}\int dx^4 \sqrt{-g}\, G_{\mu\nu}\,\delta g^{\mu\nu} + \frac{1}{2}\int dx^4 \sqrt{-g}\, T_{\mu\nu}\,\delta g^{\mu\nu}$$

$$\Rightarrow \boxed{G_{\mu\nu} = 8\pi G\, T_{\mu\nu}}$$

$\nabla_\mu G^{\mu\nu} = 0$ Bianchi Identity

$\nabla_\mu T^{\mu\nu} = 0$ Conservation of energy-momentum

diffeo S_m + EOM $\Rightarrow \nabla_\mu T^{\mu\nu} = 0$

$\overset{?}{\Longleftarrow}$

$x^\mu \longrightarrow x^\mu + \xi^\mu$

$$\delta_\xi S_{tot} = \delta_\xi \cancel{S}_{EH} + \underbrace{\delta_\xi S_m}_{\cancel{\frac{\delta S_m}{\delta g}\delta_\xi g} + \frac{\delta S_m}{\delta X}\delta_\xi X} = 0 \Rightarrow \underbrace{\frac{\delta S_m}{\delta X}\,\delta_\xi X}_{f(X, \partial X)\,\delta\xi}$$

diffs

- how many dynamical dof are GR?

$$\nabla_\mu G^{\mu\nu} = 0 \qquad \nabla_0 G^{0\nu} + \nabla_i G^{i\nu} = 0$$

$\nabla_i \to \partial_i$ $\qquad$ $G^{i\nu} \to$ at most ∂_t^2

$$\partial_t G^{0\nu} + \underbrace{\Gamma^0_{0\alpha} G^{\alpha\nu}}_{\to \partial_t^2} + \underbrace{\Gamma^\nu_{0\alpha} G^{0\alpha}}_{\partial_t^2} = -\underbrace{\nabla_i G^{i\nu}}_{\text{at most } \partial_t^2}$$

$1 + 1$

$\partial_t \; \partial_t$

$G^{0\nu} = 0 \to \partial_t \Rightarrow$

10 equations
-4 constraints (not-dynamical)
-4 coordinate tran
2 DOF

if $\nabla_\mu T^{\mu\nu} = 0$ is there a global Neother charge

$$\int_V dx^4 \sqrt{-g}\, \nabla_\mu v^\mu = \oint_{\partial V} v^\mu \cdot d\Sigma_\mu$$

4D $\int_\Omega d\omega = \oint_{\partial\Omega} \omega$

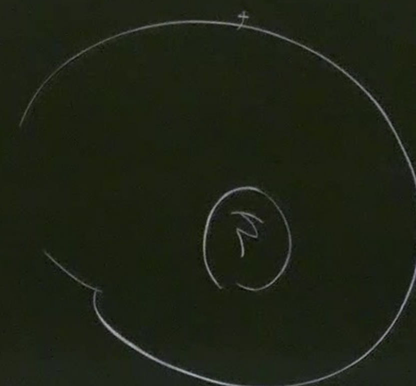
if

$$\Rightarrow \boxed{G_{\mu\nu} = 8\pi G\, T_{\mu\nu}}$$

if $\exists$ a killing vector $\xi^\mu \longrightarrow \nabla_{(\nu}\xi_{\mu)} = 0$

$$j^\mu = T^{\mu\nu}\xi_\nu \Rightarrow \nabla_\mu j^\mu = (\cancel{\nabla_\mu T^{\mu\nu}})\,\xi_\nu + T^{\mu\nu}\cancel{\nabla_{(\mu}}\,\xi_{\nu)} = 0$$

$$\int_V \sqrt{-g}\;\nabla_\mu j^\mu = \oint j^\mu \cdot d\Sigma_\mu = 0$$



· Energy Conditions

· Weak Energy Condition (WEC)

$\forall\, t^\mu$ timelike $\quad T_{\mu\nu} t^\mu t^\nu \geq 0 \Rightarrow \begin{cases} \rho \geq 0 \\ \rho + p_i \geq 0 \end{cases}$

· NEC , SEC, DEC

$\rho + p \geq 0$

Linearized Gravity 3.5 DK

$\swarrow$ GWs

Here linear around Minkowski

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + O(h^2) \quad |h_{\mu\nu}| \ll 1$$

$$\Gamma^{\sigma}_{\mu\nu} = \frac{1}{2}\eta^{\sigma\lambda}\left(h_{\nu\lambda,\mu} + h_{\mu\lambda,\nu} - h_{\mu\nu,\lambda}\right)$$

$$R^{\sigma}{}_{\mu\rho\nu} = \Gamma^{\sigma}_{\mu\nu,\rho} - \Gamma^{\sigma}_{\mu\rho,\nu} + \underbrace{\cancel{\Gamma\Gamma}}_{O(h^2)} - \underbrace{\cancel{\Gamma\Gamma}}_{O(h^2)}$$

$$= \frac{1}{2}\eta^{\sigma\lambda}\left(h_{\lambda\nu,\mu\rho} + h_{\mu\rho,\lambda\nu} - h_{\mu\nu,\lambda\rho} - h_{\lambda\rho,\mu\nu}\right)$$

$$h = h^{\mu}{}_{\mu} \ (\text{Trace}) \qquad \Box = \partial^{\mu}\partial_{\mu}$$

$$R_{\mu\nu} = \frac{1}{2}\left(\partial^{\lambda}\partial_{\mu}h_{\nu\lambda} - \Box h_{\mu\nu} + \partial^{\lambda}\partial_{\nu}h_{\mu\lambda} - \partial_{\mu}\partial_{\nu}h\right)$$

$$R = \partial^{\mu}\partial^{\nu}h_{\mu\nu} - \Box h$$

$$G_{\mu\nu} - \frac{1}{2}\left(\; \right)^{\lambda}\partial_{\lambda} h_{\mu\nu} + \dots \;) = 8\pi G\, T_{\mu\nu}$$

$$\partial^{\mu}\Big(\underbrace{h_{\mu\nu} - \frac{1}{2} h\, \eta_{\mu\nu}}_{\bar{h}_{\mu\nu}}\Big) \overset{?}{=} 0 \quad \Rightarrow \Box\, \bar{h}_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

$$\rightarrow \partial^{\mu} \bar{h}_{\mu\nu} \overset{?}{=} 0$$

under small diffs $\quad x^{\mu} \longrightarrow x^{\mu} - \xi^{\mu}(x)$

$$\delta_{\xi}\, g_{\mu\nu} = \nabla_{\mu} \xi_{\nu} + \nabla_{\nu} \xi_{\mu} \simeq \partial_{\mu} \xi_{\nu} + \partial_{\nu} \xi_{\mu}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \to g'_{\mu\nu} = \eta_{\mu\nu} + h'_{\mu\nu}$$

$$h'_{\mu\nu} = h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

$$h' = h + 2\partial^\mu \xi_\mu$$

$$\partial^\mu \bar{h}'^{(new)}_{\mu\nu} = \partial^\mu \bar{h}^{(old)}_{\mu\nu} + \overbrace{\partial^\mu \partial_\mu \xi_\nu}^{\Box \xi_\nu} + \cancel{\partial^\mu \partial_\nu \xi_\mu} - \cancel{\partial_\nu \partial^\mu \xi_\mu}$$

$$\partial^\mu h \, \eta_{\mu\nu} = \partial_\nu h$$

$+ \, h'_{\mu\nu}$

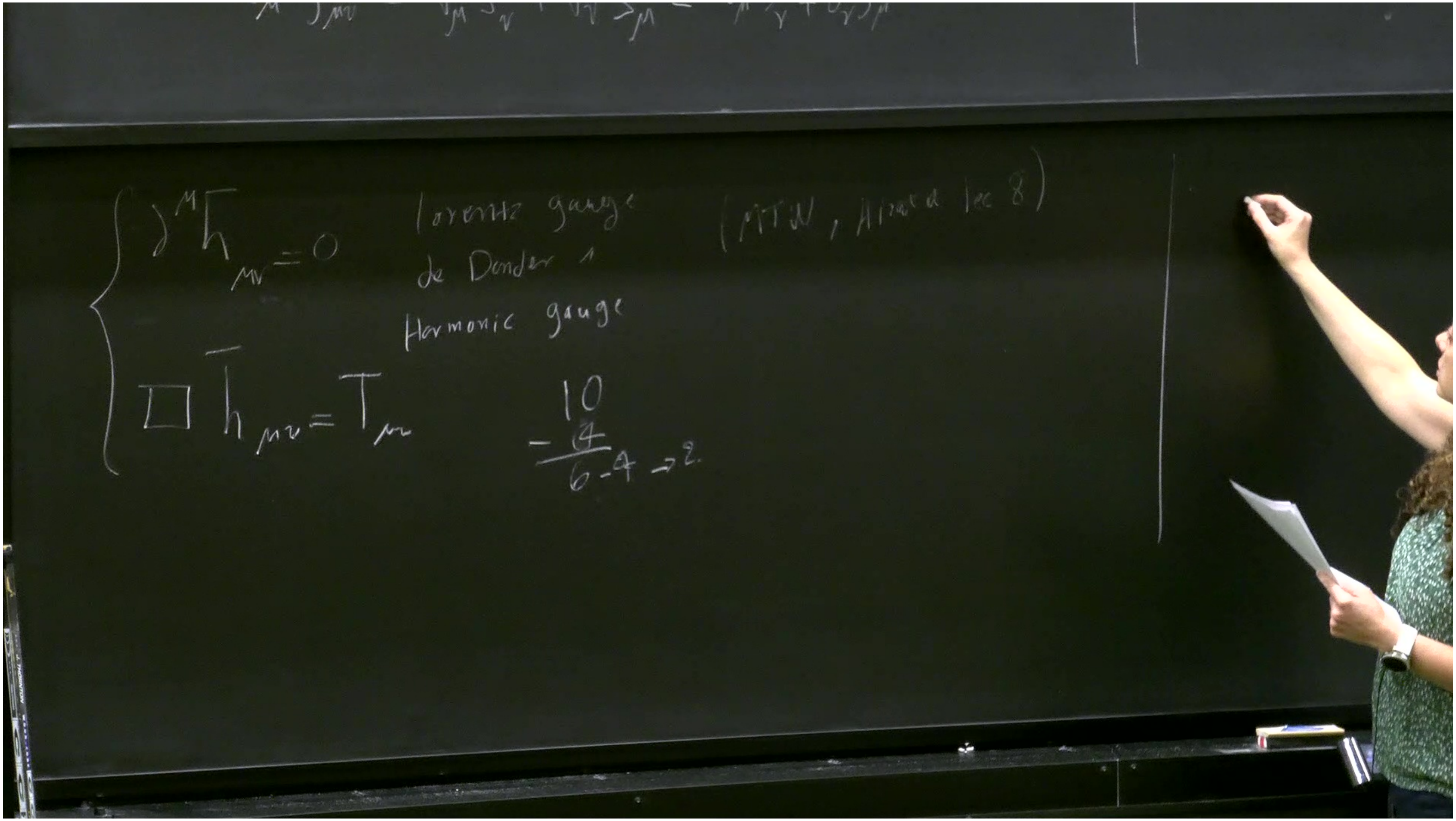
$\Box \xi_\nu$

$\partial_\mu \xi_\nu + \cancel{\partial^\mu \partial_\nu \xi_\mu} - \cancel{\partial_\nu \partial^\mu \xi_\mu}$

$$\Box \xi_\nu(x) = - \partial^\mu \bar{h}_{\mu\nu} \quad \text{(old)}$$

$$\xi_\nu(x) = \xi^{n.H}(x) + \alpha \xi^{H}(x)$$

$$\hookrightarrow \partial^\mu \bar{h}_{\mu\nu} = 0$$


$\partial^\mu \bar{h}_{\mu\nu} = 0$
Lorentz gauge
de Donder
Harmonic gauge
$\Box \bar{h}_{\mu\nu} = T_{\mu\nu}$
10
- 4
6 - 4 → 2

$$\partial_\mu \bar{h}^{\mu\nu} = 0 \Longleftrightarrow \Gamma^{\mu}_{\alpha\beta}\, g^{\alpha\beta} = 0 \Leftrightarrow \nabla_\alpha \nabla^\alpha x^\mu = 0$$