Title: Lecture - Relativity, PHYS 604

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Subject: Cosmology, Strong Gravity

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Abstract:

$$S_{EH} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,R \longrightarrow \delta_g S = \frac{1}{16\pi G}\int \sqrt{-g}\,G_{\mu\nu}\,\delta g^{\mu\nu}$$

EOM $\dfrac{\delta_g S}{\delta g^{\mu\nu}}$: $G_{\mu\nu} = 0$ Vacuum solutions

(1) $g^{\mu\nu}G_{\mu\nu} = g^{\mu\nu}R_{\mu\nu} - \frac{1}{2}g^{\mu\nu}g_{\mu\nu}R = 0$

$R - \frac{1}{2}dR = 0 \Rightarrow (1-\frac{1}{2}d)R = 0$

$d \neq 2 \Rightarrow$ in vacuum $R = 0$

2. $G_{\mu\nu}(g, \partial g, \partial^2 g) = 0$ → 2nd order

Lovelock theorem:

in 4D E-H action is the only <u>local</u> action for <u>metric</u> $(g,)$

$\Rightarrow$ 2nd order PDEs for g

Horndeski: $g_{\mu\nu} + \phi$ ⇒ Horndeski gravity

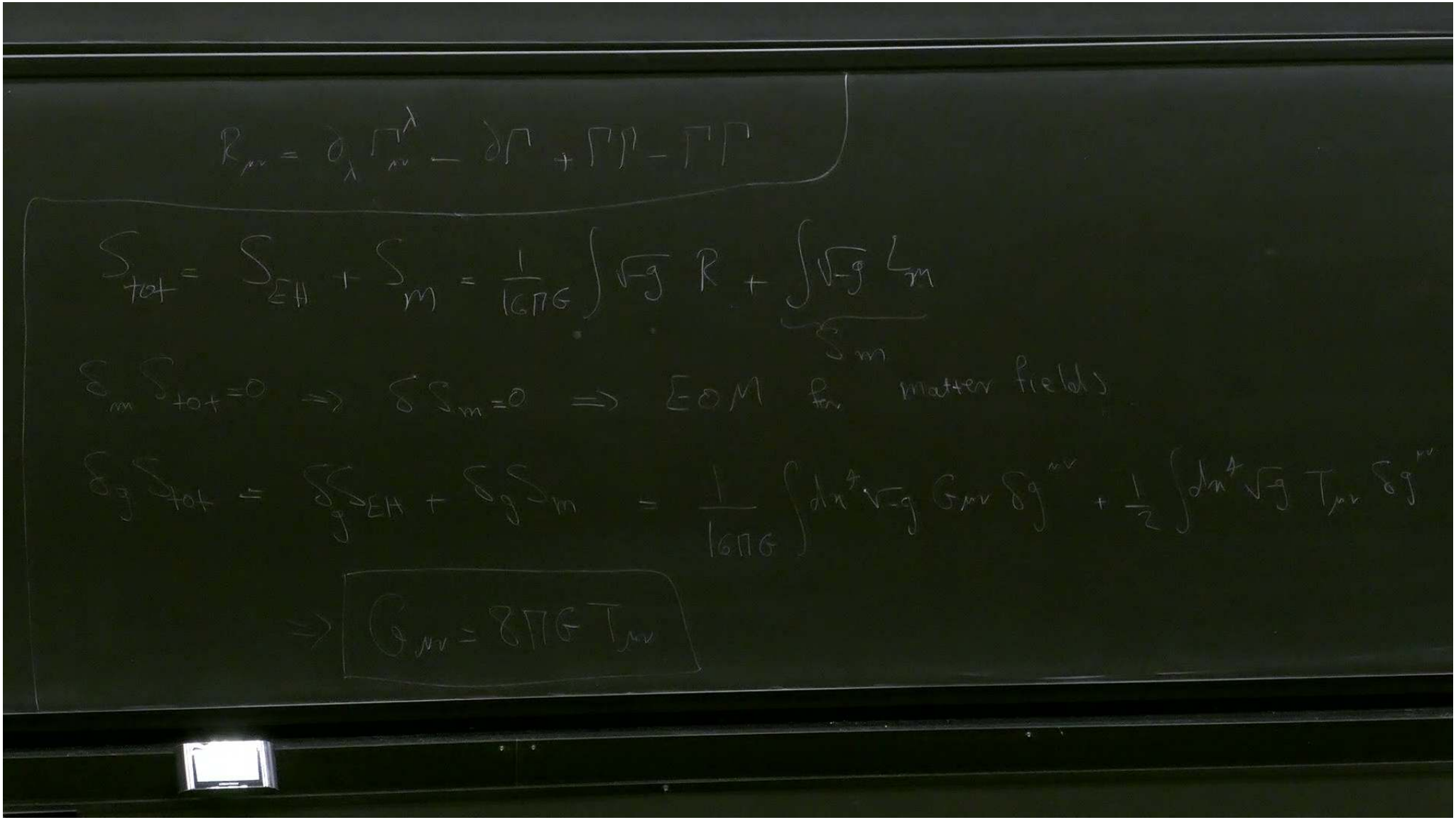
$f(R)$ → $g_{\mu\nu} + \phi$

$S_{Palatini}(g, \Gamma) \Rightarrow S_P(g, \Gamma) = \frac{1}{16\pi G} \int d^4x \sqrt{-g}\, g^{\mu\nu} R_{\mu\nu}$

$\delta_g S = \frac{1}{16\pi G} \int \sqrt{-g} \left[-\frac{1}{2} R\, g_{\mu\nu} + R_{\mu\nu} \right] \delta g^{\mu\nu}$

EoM $\quad G_{\mu\nu} = 0$

$\delta_\Gamma S \rightarrow$ EoM $\rightarrow$ Christoffel Symbols.


$R_{\mu\nu} = \partial_\lambda \Gamma^\lambda_{\mu\nu} - \partial\Gamma + \Gamma\Gamma - \Gamma\Gamma$
$S_{tot} = S_{EH} + S_m = \frac{1}{16\pi G}\int \sqrt{-g}\, R + \int \sqrt{-g}\, \mathcal{L}_m$
$\mathcal{L}_m$
$\mathcal{E}_m S_{tot} = 0 \Rightarrow \delta S_m = 0 \Rightarrow$ EOM for matter fields
$\delta_g S_{tot} = \delta_g S_{EH} + \delta_g S_m = \frac{1}{16\pi G}\int d^4x \sqrt{-g}\, G_{\mu\nu}\, \delta g^{\mu\nu} + \frac{1}{2}\int d^4x \sqrt{-g}\, T_{\mu\nu}\, \delta g^{\mu\nu}$
$\Rightarrow G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$

$\nabla_\mu G^{\mu\nu} = 0$ Bianchi identity

$\nabla_\mu T^{\mu\nu} = 0$ Conservation of energy-momentum

diffeos + EOM $\Rightarrow \nabla_\mu T^{\mu\nu} = 0$

$\Leftarrow$?

$x^\mu \longrightarrow x^\mu + \xi^\mu$

$\delta_\xi S_{tot} = \delta_\xi S_{EH} + \delta_\xi S_m = 0 \Rightarrow$

$\delta_\xi S_{EH}$: diffs

$\frac{\delta S_m}{\delta g}\delta_\xi g + \frac{\delta S_m}{\delta X}\delta_\xi X$

$\frac{\delta S_m}{\delta X}\underbrace{\delta_\xi X}_{f(X,\partial X)\xi}$

- how many dynamical dof one GR?

$$\nabla_\mu G^{\mu\nu} = 0$$

$$\nabla_0 G^{0\nu} + \nabla_i G^{i\nu} = 0$$

∇_i: at most ∂_t^2

$$\partial_t G^{0\nu} + \Gamma^0_{0\alpha} G^{\alpha\nu} + \Gamma^\nu_{0\alpha} G^{0\alpha} = -\nabla_i G^{i\nu}$$

∂_t + (1) → ∂_t^2; $\Gamma^\nu_{0\alpha} G^{0\alpha}$: ∂_t^2; $-\nabla_i G^{i\nu}$: at most ∂_t^2

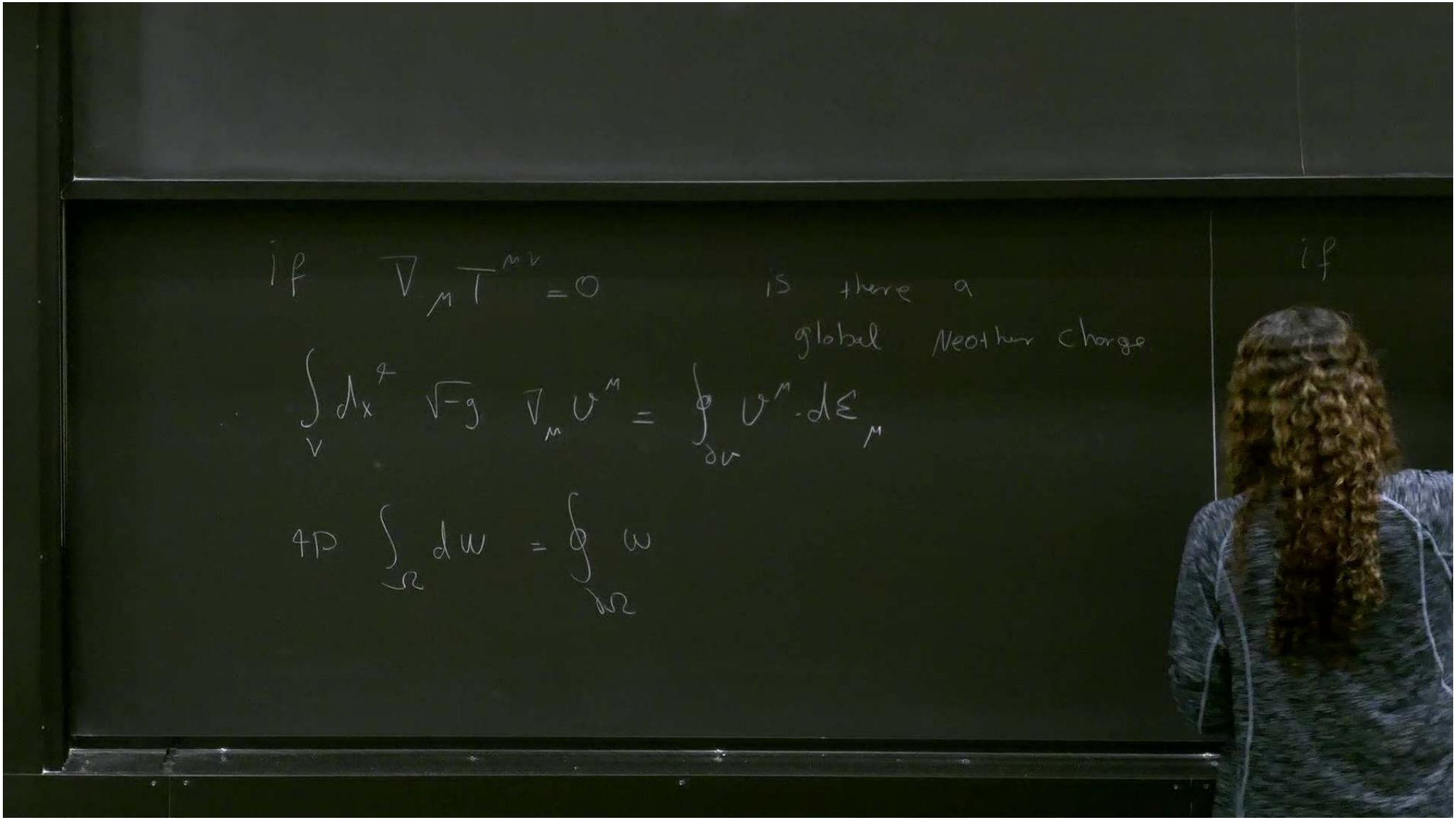
$(\partial_t \partial_t)$

$G^{0\nu} = 0 \to \partial_t \Rightarrow$ 10 equations

-4 constraints (not dynamical)

-4 coordinate trans

2 DOF

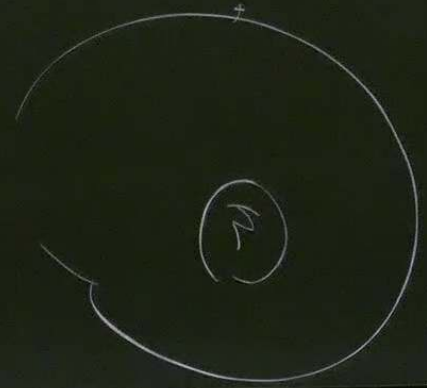

if $\nabla_\mu T^{\mu\nu} = 0$
is there a global Neother charge
$\int_V d^4x \sqrt{-g}\, \nabla_\mu v^\mu = \oint_{\partial V} v^\mu \cdot d\Sigma_\mu$
4D $\int_\Omega d\omega = \oint_{\partial\Omega} \omega$
if

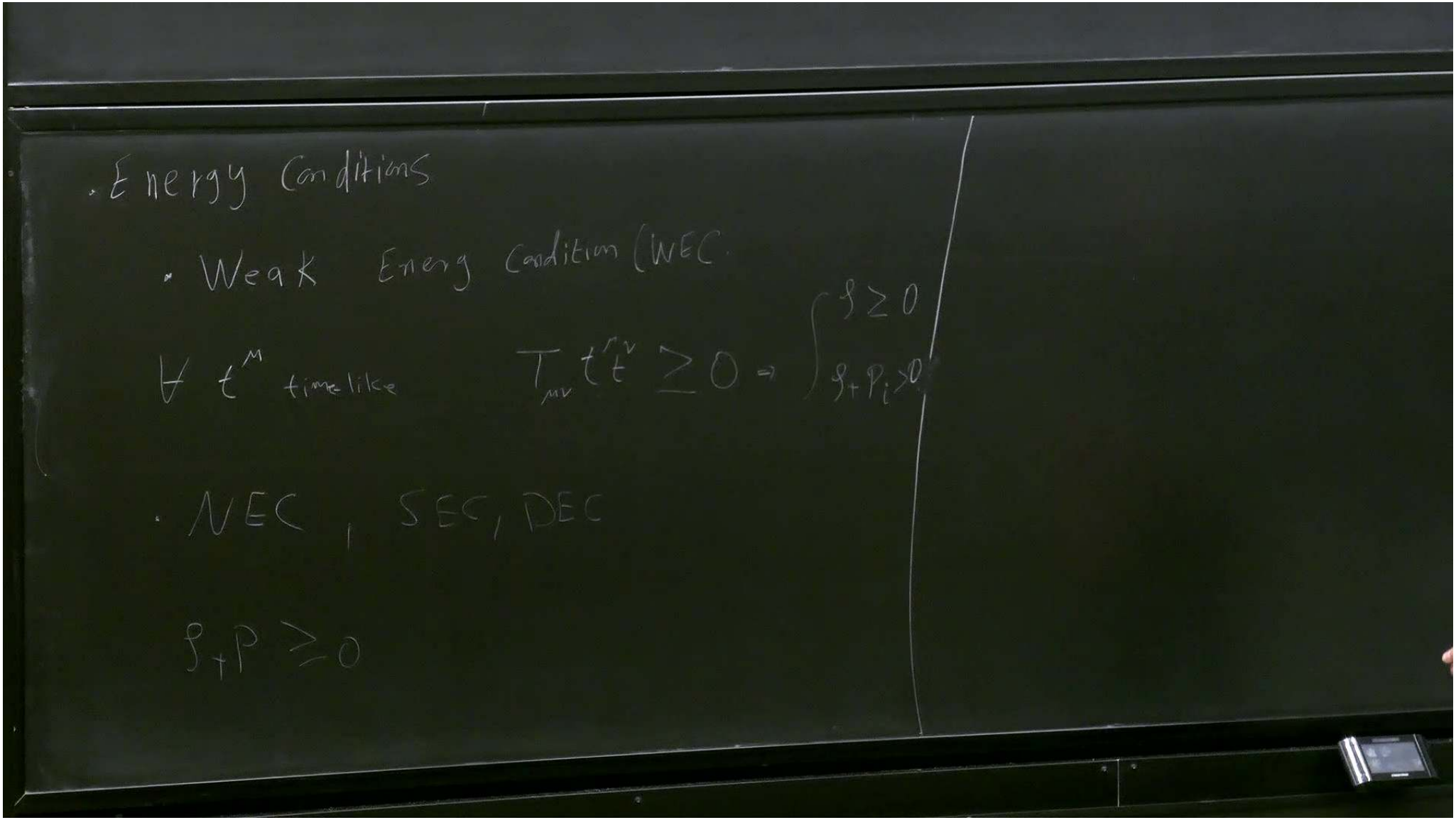
$\Rightarrow \boxed{G_{\mu\nu} = 8\pi G\, T_{\mu\nu}}$

if $\exists$ a killing Vector $\xi^\mu \longrightarrow \nabla_{(\mu}\xi_{\nu)} = 0$

$$j^\mu = T^{\mu\nu}\xi_\nu \Rightarrow \nabla_\mu j^\mu = (\nabla_\mu T^{\mu\nu})\xi_\nu + T^{\mu\nu}\nabla_{(\mu}\xi_{\nu)} = 0$$

$$\int_V \sqrt{-g}\, \nabla_\mu j^\mu = \oint j^\mu \cdot d\Sigma_\mu = 0$$




• Energy Conditions
• Weak Energy Condition (WEC
$\forall\ t^\mu$ timelike $\quad T_{\mu\nu} t^\mu t^\nu \geq 0 \Rightarrow \begin{cases} \rho \geq 0 \\ \rho + P_i \geq 0 \end{cases}$
• NEC, SEC, DEC
$\rho + P \geq 0$

Linearized Gravity 3.5 DK.

↙ GWs

Here linear around Minkowski

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + O(h^2) \qquad |h_{\mu\nu}| \ll 1$$

$$\Gamma^{\sigma}_{\mu\nu} = \frac{1}{2}\eta^{\sigma\lambda}\left(h_{\lambda\nu,\mu} + h_{\mu\lambda,\nu} - h_{\mu\nu,\lambda}\right)$$

$$R^{\sigma}_{\mu\rho\nu} = \Gamma^{\sigma}_{\mu\nu,\rho} - \Gamma^{\sigma}_{\mu\rho,\nu} + \underbrace{\Gamma\Gamma}_{O(h^2)} - \underbrace{\Gamma\Gamma}_{O(h^2)}$$

$$= \frac{1}{2}\eta^{\sigma\lambda}\left(h_{\lambda\nu,\mu\rho} + h_{\mu\rho,\lambda\nu} - h_{\mu\nu,\lambda\rho} - h_{\lambda\rho,\mu\nu}\right)$$

$h = h^{\mu}_{\ \mu}$ (Trace) $\qquad \Box = \partial^{\mu}\partial_{\mu}$

$$R_{\mu\nu} = \frac{1}{2}\left(\partial^{\lambda}\partial_{\mu}h_{\nu\lambda} - \Box h_{\mu\nu} + \partial^{\lambda}\partial_{\nu}h_{\mu\lambda} - \partial_{\mu}\partial_{\nu}h\right)$$

$$R = \partial^{\mu}\partial^{\nu}h_{\mu\nu} - \Box h$$

$G_{\mu\nu} - \frac{1}{2}(\;)^{\lambda}\partial_{\lambda} h_{\mu\nu} + \dots\;) = 8\pi G T_{\mu\nu}$

$\partial^{\mu}\left(\underbrace{h_{\mu\nu} - \frac{1}{2} h \eta_{\mu\nu}}_{\bar{h}_{\mu\nu}}\right) \overset{?}{=} 0 \quad \Rightarrow \Box \bar{h}_{\mu\nu} = 8\pi G T_{\mu\nu}$

$\rightarrow \partial^{\mu} \bar{h}_{\mu\nu} \overset{?}{=} 0$

under small diffs $x^{\mu} \longrightarrow x^{\mu} - \xi^{\mu}(x)$

$\delta_{\mu} g_{\mu\nu} = \nabla_{\mu} \xi_{\nu} + \nabla_{\nu} \xi_{\mu} \simeq \partial_{\mu} \xi_{\nu} + \partial_{\nu} \xi_{\mu}$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \rightarrow g'_{\mu\nu} = \eta_{\mu\nu} + h'_{\mu\nu}$$

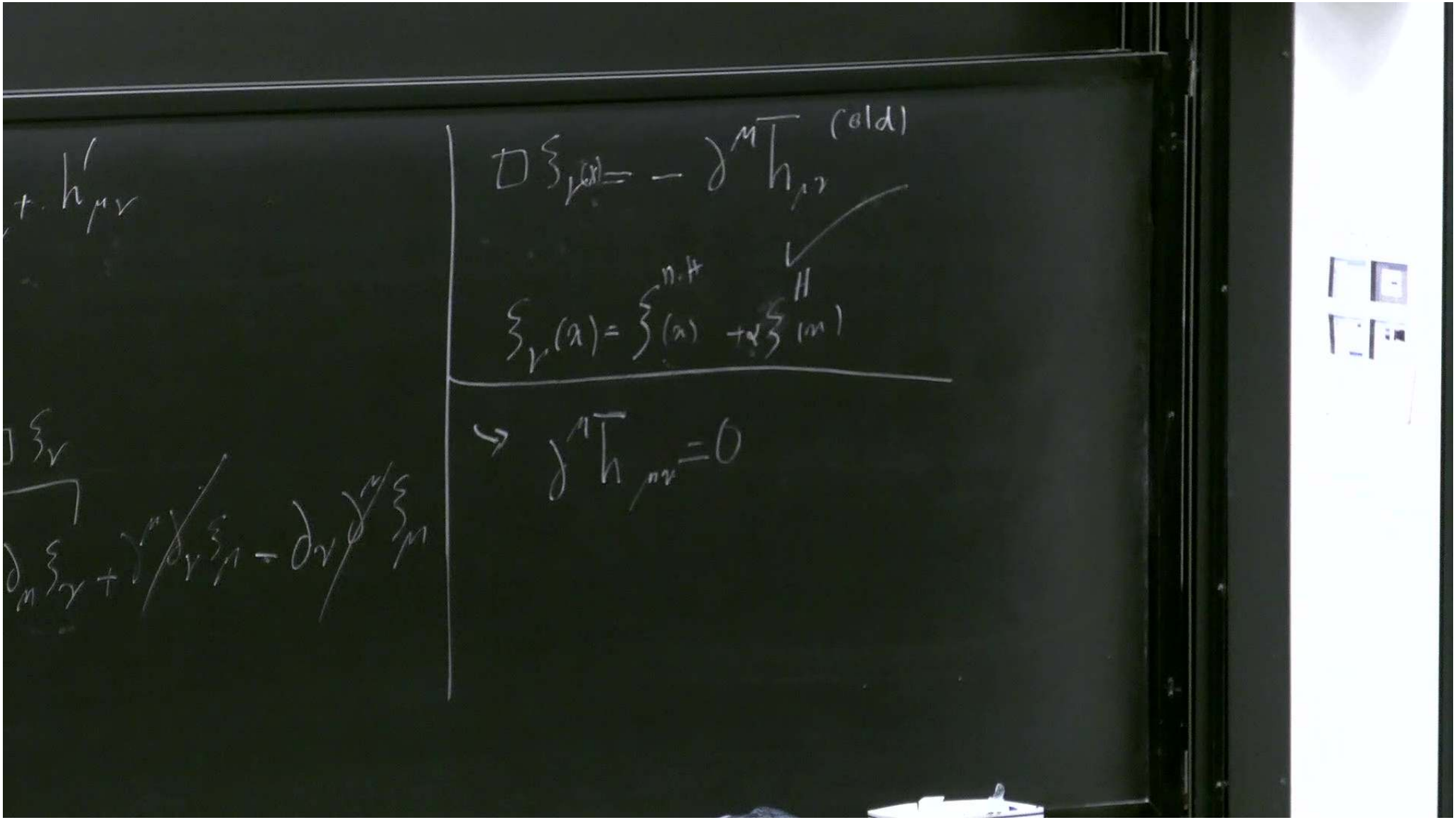
$$h'_{\mu\nu} = h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

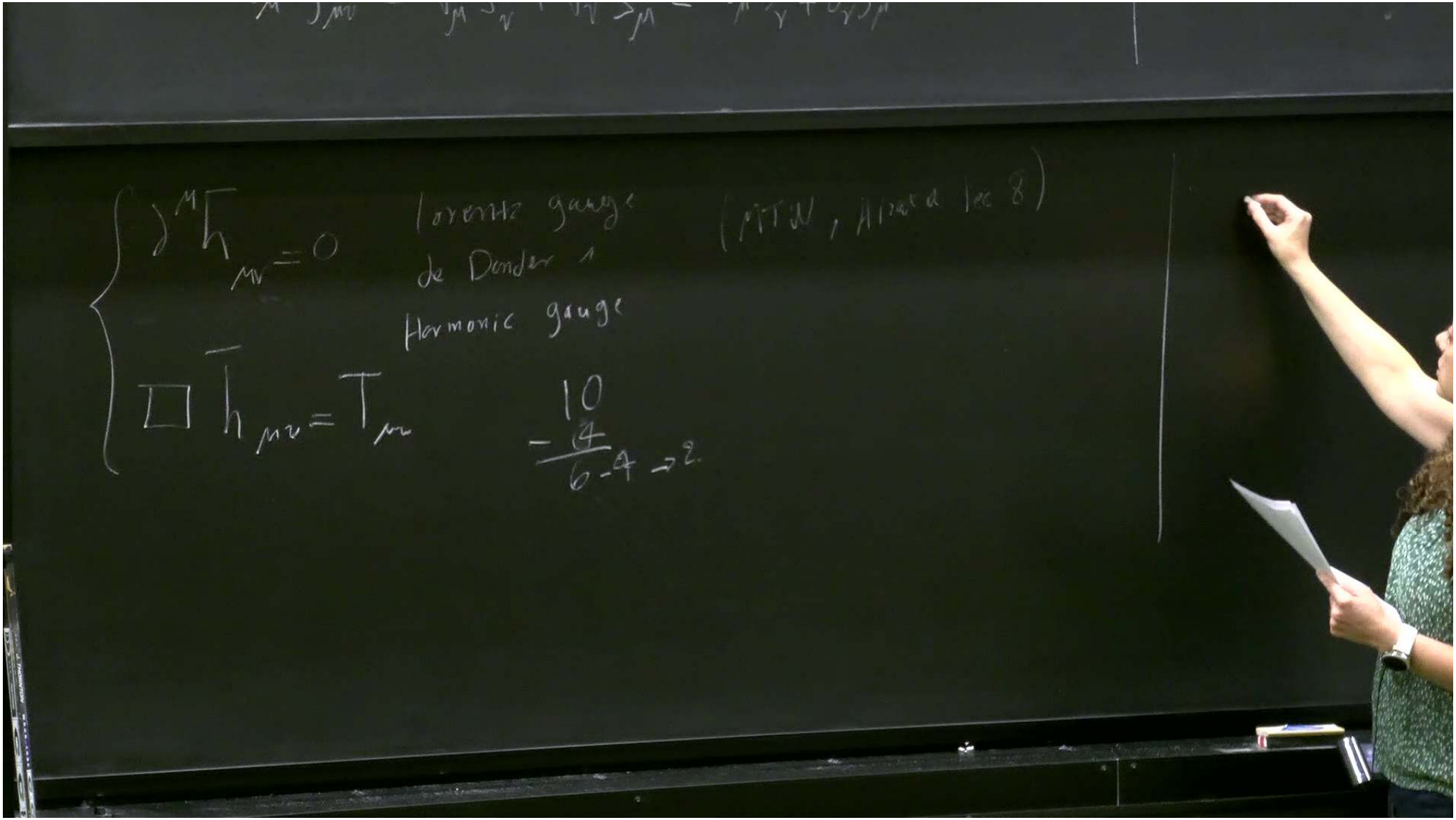
$$h' = h + 2\partial^\mu \xi_\mu$$

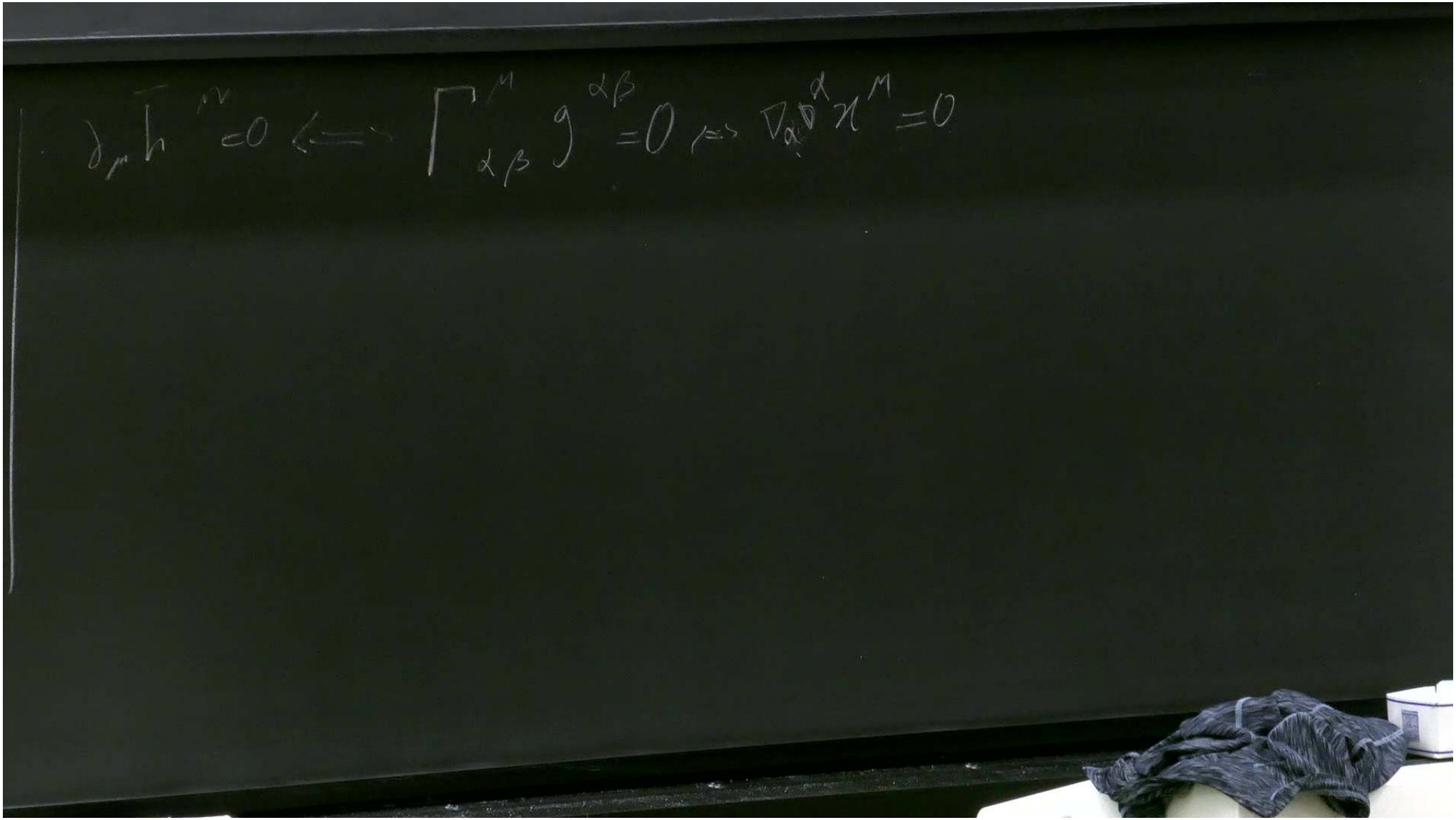
$$\partial^\mu \bar{h}'_{\mu\nu} = \partial^\mu \bar{h}_{\mu\nu} + \overbrace{\partial^\mu \partial_\mu \xi_\nu}^{\Box \xi_\nu} + \cancel{\partial^\mu \partial_\nu \xi_\mu} - \cancel{\partial_\nu \partial^\mu \xi_\mu}$$

(new) (old)

$$\partial^\mu h \eta_{\mu\nu} = \partial_\nu h$$


$+ h'_{\mu\nu}$
$\Box \xi_\nu$
$\partial_\mu \xi_\nu + \partial^\mu \partial_\nu \xi_\mu - \partial_\nu \partial^\mu \xi_\mu$
$\Box \xi_{\mu(old)} = -\partial^\mu T_{\mu\nu}$ (old)
$\xi_\nu(x) = \xi^{n.h}_{(x)} + \xi^{H}_{(m)}$
$\Rightarrow \partial^\mu T_{\mu\nu} = 0$


$\partial^\mu \bar{h}_{\mu\nu} = 0$
Lorentz gauge
de Donder
Harmonic gauge
(MTW, Alpert lec 8)
$\Box \bar{h}_{\mu\nu} = T_{\mu\nu}$
10
-4
$6 - 4 \to 2$.


$\partial_\mu h^{\mu\nu} = 0 \iff \Gamma^\mu_{\alpha\beta} g^{\alpha\beta} = 0 \iff D_\alpha D^\alpha x^\mu = 0$