Title: Lecture - Relativity, PHYS 604

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Collection/Series: Relativity (Core), PHYS 604, November 12 - December 11, 2024

Subject: Cosmology, Strong Gravity

Date: November 27, 2024 - 10:45 AM

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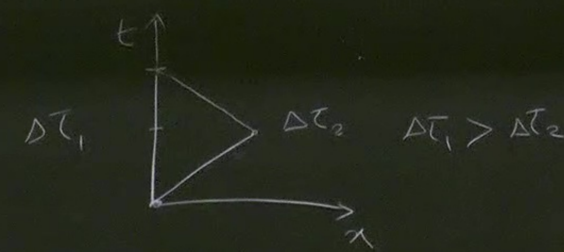
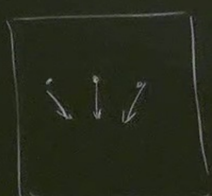
Abstract:

$$\frac{Du^\mu}{d\tau} = 0 \qquad u^\mu = \frac{dx^\mu}{d\tau} \qquad u^\alpha u^\mu{}_{;\alpha} = 0$$

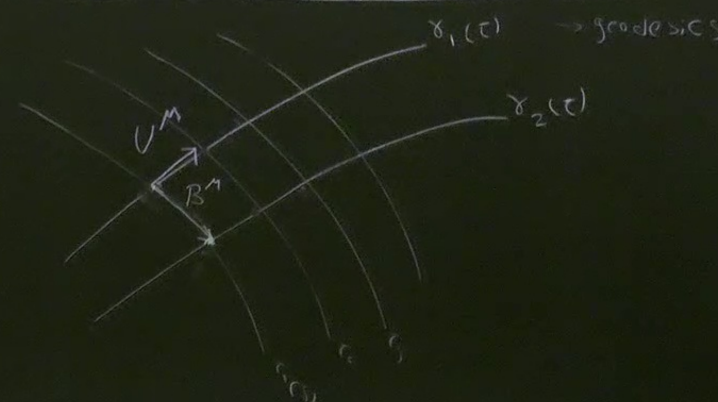


- geodesic deviation Equation.



$\nexists$ zero size Elevator.

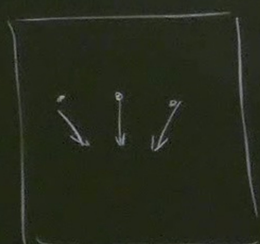
$$\partial_i \partial_j \phi$$



$$\frac{Du^\mu}{d\tau} = 0 \qquad u^\mu = \frac{dx^\mu}{d\tau} \qquad u^\alpha u^\mu{}_{;\alpha} = 0$$

$\Delta\tau_1$

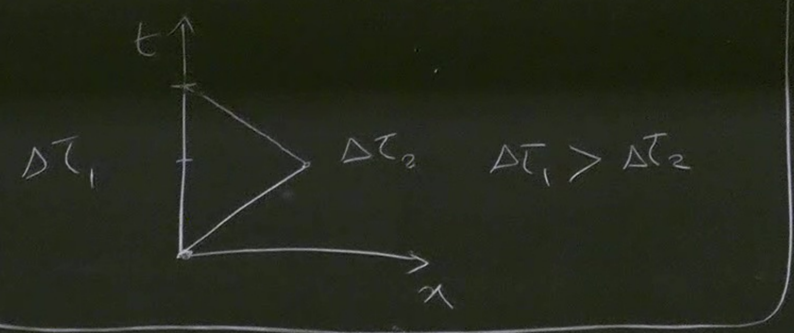
- geodesic deviation Equation.



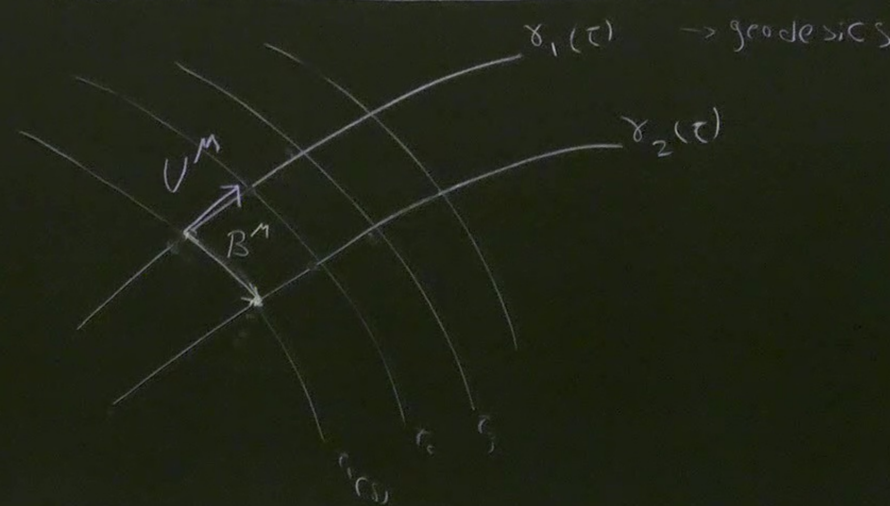
$\nexists$ zero size Elevator.

$$\partial_i \partial_j \phi$$

$\frac{x^{\mu}}{\tau}$ $\qquad U^{\alpha} U^{\mu}{}_{;\alpha} = 0$



n Equation.



zero Size Elevator.

$\partial_i \partial_j \phi$

sics

$$v^\mu = \frac{DB^\mu}{d\tau} \qquad A^\mu = \frac{D^2 B^\mu}{d\tau^2} = \frac{Dv^\mu}{d\tau}$$

$$= u^\nu \nabla_\nu B^\mu$$

$$= \frac{dB^\mu}{d\tau} + \Gamma^\mu_{\alpha\nu} B^\alpha u^\nu$$

$$A^\mu = \frac{D}{d\tau} v^\mu = \frac{dv^\mu}{d\tau} + \Gamma^\mu_{\alpha\nu} v^\alpha u^\nu = \frac{d^2}{d\tau^2} B^\mu + \frac{d\Gamma^\mu_{\alpha\nu}}{d\tau} B^\alpha u^\alpha + \cdots$$

$$\cdots \Rightarrow \frac{D^2 B^\mu}{d\tau^2} =$$

sics

$$= U^\nu \nabla_\nu B^\mu$$

$$= \frac{dB^\mu}{d\tau} + \Gamma^\mu_{\alpha\nu} B^\alpha U^\nu$$

$$A^\mu = \frac{D}{d\tau} U^\mu = \frac{dU^\mu}{d\tau} + \Gamma^\mu_{\alpha\nu} U^\alpha U^\nu = \frac{d^2}{d\tau^2} B^\mu + \frac{d\Gamma^\mu_{\alpha\nu}}{d\tau} B^\alpha U^\alpha + \dots$$

$$\dots \Rightarrow \frac{D^2 B^\mu}{d\tau^2} = -R^\mu{}_{\nu\rho\sigma} U^\nu U^\sigma B^\rho$$

Including Matter in curve space time (3.2 D.K.)

Action for Matter fields $S = \int dx^4 \; L_M(\phi, f^{\mu\nu}, \dots)$

$\eta_{\mu\nu} \to g_{\mu\nu} \quad \partial_\mu$

Ex Scalar field

$\partial_\mu \phi \, \partial^\mu \phi$

$$S_m = \int dx^4 \sqrt{-g} \; L_m(\phi, \nabla\phi, g)$$

$$\delta_\phi S_m = \int dx^d \frac{\delta L_M}{\delta \phi} \delta\phi = \int dx^d \sqrt{-g} \left(\frac{\partial L_M}{\partial \phi} \delta\phi + \frac{\partial L_M}{\partial \nabla_\mu \phi} \delta \nabla_\mu \phi \right) =$$

in curve space time (3.2 D.K.)

Matter fields $\quad S = \int dx^4 \; L_M(\phi, f^{\mu\nu}, \ldots)$

$$\eta_{\mu\nu} \longrightarrow g_{\mu\nu} \qquad \partial_\mu \longrightarrow \nabla_\mu \phi$$

$$\partial_\mu \phi \, \partial^\mu \phi \qquad d\tilde{v} = \sqrt{-g}\, dx^d$$

$$L_m(\phi, \nabla\phi, g)$$

$$\frac{\delta L_M}{\delta \phi}\delta\phi = \int dx^d \sqrt{-g}\left(\frac{\partial L_M}{\partial \phi}\delta\phi + \frac{\partial L_M}{\partial \nabla_\mu \phi}\overbrace{\delta \nabla_\mu \phi}^{\nabla_\mu \delta\phi}\right) = \int dx^d \sqrt{-g}$$

Gauss - Stokes theorem

$$\int_V dx^d \sqrt{-g}\, \nabla_\mu v^\mu = \oint_{\partial V} v^\mu d\Sigma_\mu$$

∂V

V

$$v^\mu = \frac{\partial L_M}{\partial \nabla_\mu \phi} \delta\phi$$

$\Rightarrow \nabla_\mu \phi$

$d\tilde{v} = \sqrt{-g}\, dx^d$

$$\int_V dx^d \sqrt{-g} \left(\nabla_\mu \frac{\partial L_M}{\partial \nabla_\mu \phi} \right) \delta\phi + \int_V dx^d \sqrt{-g} \left(\frac{\partial L_M}{\partial \nabla_\mu \phi} \right) \nabla_\mu \delta\phi$$

$= B.T$

EOM

$$\int dx^d \sqrt{-g} \left[\frac{\partial L_M}{\partial \phi} - \nabla_\mu \frac{\partial L_M}{\partial \nabla_\mu \phi} \right] \delta\phi \Rightarrow \frac{\partial L_M}{\partial \phi} - \nabla_\mu \frac{\partial L_M}{\partial \nabla_\mu \phi} = 0$$

Energy - Momentum Tensor

$$\delta_g S_M = -\frac{1}{2} \int dx^d \sqrt{-g}\; T_{\mu\nu}\, \delta g^{\mu\nu} \qquad \text{or} \qquad T_{\mu\nu} \equiv \frac{-2}{\sqrt{-g}} \frac{\delta}{\delta$$

↳ symmetric

S_M diffeomorphiss Inv + EOM for Matter $\Longrightarrow$ Conserva

$$\nabla_\mu$$

$$x'^\mu = x^\mu - \xi^\mu$$

$$g'^{\mu\nu}(x') = g^{\mu\nu}(x') + \delta g^{\mu\nu}(x')$$

$$\Rightarrow \delta g^{\mu\nu} = 2 \nabla^{(\mu} \xi^{\nu)} = L_\xi g^\mu$$

nsor

$\sqrt{-g}\; T_{\mu\nu}\, \delta g^{\mu\nu}$ or $T_{\mu\nu} \equiv \dfrac{-2}{\sqrt{-g}} \dfrac{\delta S_M}{\delta g^{\mu\nu}}$

↳ symmetric

+ EOM for Matter $\Longrightarrow$ Conservation of Energy-Momentum

$$\nabla_\mu T^{\mu\nu} = 0$$

(x')

$$\Rightarrow \delta g^{\mu\nu} = 2 \nabla^{(\mu} \xi^{\nu)} = L_\xi g^{\mu}$$

- Momentum

$$\delta_\xi S_m = 0 \qquad \forall \xi^\mu$$

$$\Rightarrow \delta_\xi S_m = \int \underbrace{\frac{\delta S_m}{\delta g^{\mu\nu}}}_{-\frac{1}{2}\sqrt{-g}\,T_{\mu\nu}} \underbrace{\delta_\xi g^{\mu\nu}}_{2\nabla^{(\mu}\xi^{\nu)}} + \overset{\text{EOM}}{\cancel{\frac{\delta S_m}{\delta \phi}}}\, \delta_\xi \phi = 0$$

$$= \int d^d x \sqrt{-g}\; T_{\mu\nu} \nabla^{(\mu}\xi^{\nu)} = \int d^d x \sqrt{-g}\; T_{\mu\nu} \nabla^{\mu}\xi^{\nu} = 0$$

$$= -\int \sqrt{-g}\left(\nabla^\mu T_{\mu\nu}\right)\xi^\nu = 0 \Rightarrow \boxed{\nabla^\mu T_{\mu\nu} = 0}$$

$$\delta_\phi S_M = \int d^dx \sqrt{-g}\, \frac{\delta L_M}{\delta \phi}\, \delta\phi = \int d^dx \sqrt{-g}\left(\frac{\partial L_M}{\partial \phi}\delta\phi + \frac{\partial L_M}{\partial \nabla_\mu \phi}\delta \nabla_\mu \phi\right) = \int d^dx \sqrt{\ }$$

Ex 1. Scalar field

$$L(\phi) = -\frac{1}{2}\underbrace{\nabla_\mu\phi\nabla^\mu\phi}_{g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi} - V(\phi)$$

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\partial \sqrt{-g}\,L}{\partial g^{\mu\nu}} = \frac{-2}{\sqrt{-g}}\left(\frac{\partial\sqrt{-g}}{\partial g^{\mu\nu}}\right)L - 2\frac{\partial L}{\partial g^{\mu\nu}} = -\frac{\cancel{2}}{\cancel{\sqrt{-g}}}\left(\frac{1}{\cancel{2}}\cancel{\sqrt{-g}}\, g_{\mu\nu}\right)L + \nabla_\mu\phi$$

$$\delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g}\, g_{\mu\nu}\,\delta g^{\mu\nu}$$

$$L = -\frac{1}{2}F^{\mu\nu}F_{\mu\nu} - eJ^\mu A_\mu$$

$$+\frac{\partial L_M}{\partial \nabla_\mu \phi}\delta \nabla_\mu \phi \Big) = \int d x^d \sqrt{-g}\left[\frac{\partial L_M}{\partial \phi} - \nabla_\mu \frac{\partial L_M}{\partial \nabla_\mu \phi}\right]\delta\phi \Rightarrow \frac{\partial L_M}{\partial \phi} - \nabla_\mu \frac{\partial L_M}{\partial \nabla_\mu \phi} = 0$$

$$- V(\phi) \qquad \text{EOM: } -\frac{dV}{d\phi} + \nabla_\mu\left(g^{\mu\nu}\nabla_\nu \phi\right) = 0 \qquad \nabla^\mu \nabla_\mu \phi - V'(\phi) = 0$$

$$\nabla_\mu \phi = \partial_\mu \phi$$

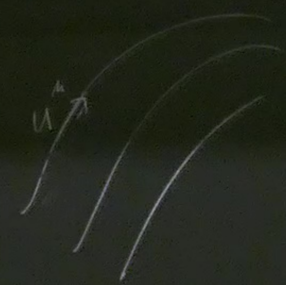
ϕ

$${}_{\mu\nu} = -\frac{\cancel{2}}{\cancel{\sqrt{-g}}}\left(\frac{1}{\cancel{2}}\cancel{\sqrt{-g}}\, g_{\mu\nu}\right)L + \nabla_\mu \phi \nabla_\nu \phi = -g_{\mu\nu}L + \nabla_\mu\phi\nabla_\nu\phi = -g_{\mu\nu}\left(-\tfrac{1}{2}\nabla_\alpha\phi\nabla^\alpha\phi - V(\phi)\right) + \nabla_\mu\phi\nabla_\nu\phi$$

$$= -\tfrac{1}{2}F^{\mu\nu}F_{\mu\nu} - eJ^\mu A_\nu \qquad F_{\mu\nu} = 2\nabla_{[\mu}A_{\nu]}$$

perfect fluid

$$T^{\mu\nu} \equiv (\rho + P)\, u^\mu u^\nu + P g^{\mu\nu}$$



$$\nabla_\mu T^{\mu\nu} = 0$$

relativistic Navier_Stokes $\quad (\rho + P)\, \nabla_u u^\alpha = -(g^{\mu\alpha} + u^\mu u^\alpha)\, \nabla_\mu P$

" Continuity eq. $\quad \dfrac{d\rho}{d\tau} + (\rho + P)\, \nabla_\mu u^\mu = 0$

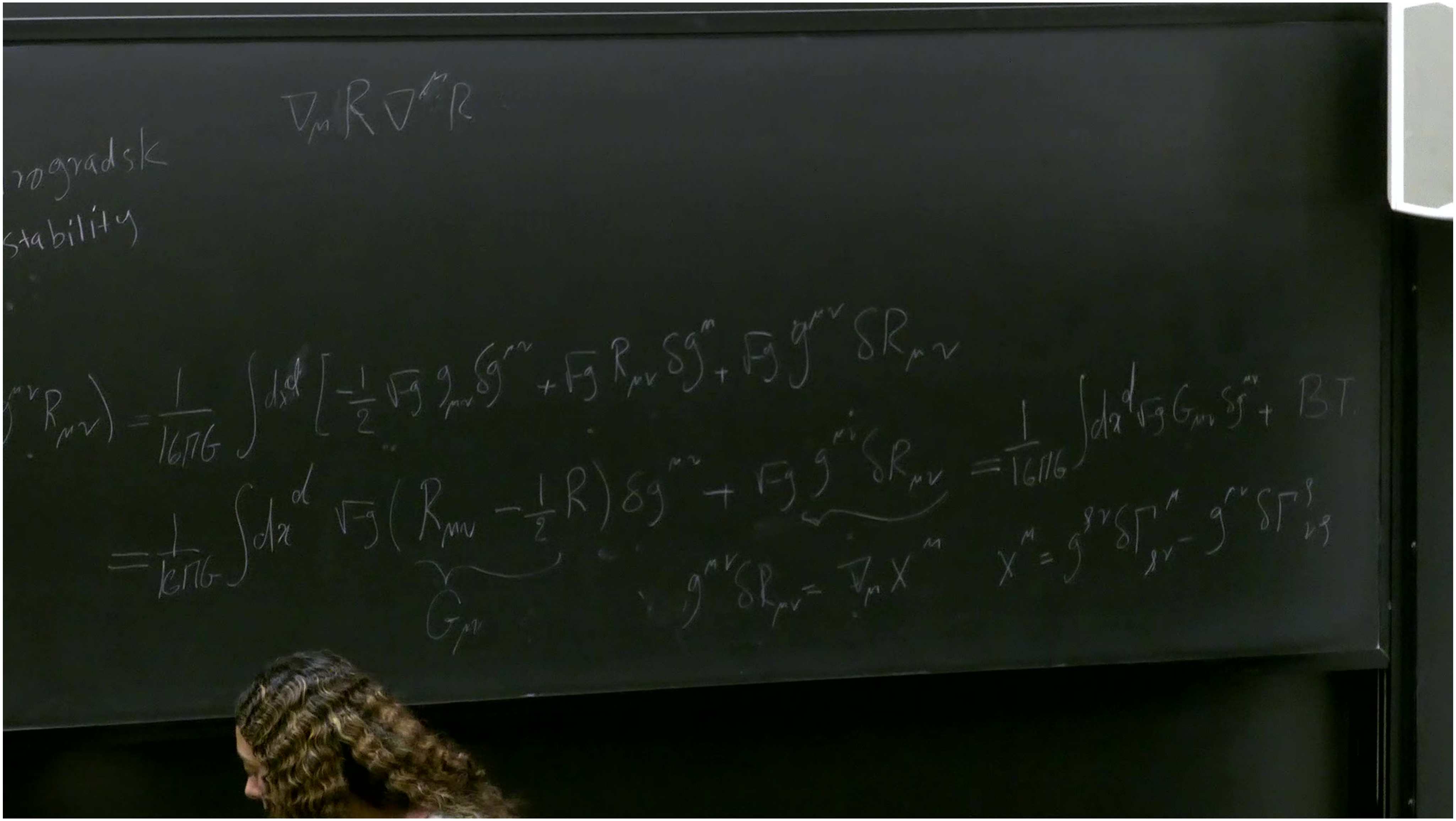
Einsten-Hilbert action.

$$\mathcal{L}(g, \partial g, \partial^2 g) \longrightarrow L(q, \dot{q}) \rightarrow \text{EoM } \ddot{q}$$

Ostrogradsk instability

$$L(q, \dot{q}, \ddot{q}) \rightarrow \frac{d^2}{dx^2}\frac{\partial L}{\partial \ddot{q}} \rightarrow \ddddot{q}$$

$$S_{SH} = \frac{1}{16\pi G}\int d^4x \sqrt{-g}\, R_{(g, \partial g, \partial^2 g)} \Rightarrow \delta_g S_{SH} = \frac{1}{16\pi G}\int$$


$\nabla_\mu R \nabla^\mu R$
rogradsk
stability
$g^{\mu\nu} R_{\mu\nu}) = \frac{1}{16\pi G} \int dx^d \left[-\frac{1}{2} \sqrt{-g}\, g_{\mu\nu} \delta g^{\mu\nu} + \sqrt{-g}\, R_{\mu\nu} \delta g^{\mu\nu} + \sqrt{-g}\, g^{\mu\nu} \delta R_{\mu\nu} \right.$
$= \frac{1}{16\pi G} \int dx^d \sqrt{-g} \left(R_{\mu\nu} - \frac{1}{2} R \right) \delta g^{\mu\nu} + \sqrt{-g}\, g^{\mu\nu} \delta R_{\mu\nu} = \frac{1}{16\pi G} \int dx^d \sqrt{-g}\, G_{\mu\nu} \delta g^{\mu\nu} + B.T.$
$G_{\mu\nu}$
$g^{\mu\nu} \delta R_{\mu\nu} = \nabla_\mu X^\mu$
$X^\mu = g^{\rho\nu} \delta \Gamma^\mu_{\rho\nu} - g^{\mu\nu} \delta \Gamma^\rho_{\nu\rho}$