Title: Lecture - Relativity, PHYS 604

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Collection/Series: Relativity (Core), PHYS 604, November 12 - December 11, 2024

Subject: Cosmology, Strong Gravity

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Abstract:

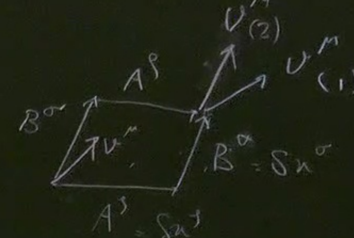
$$v \rightarrow P.T \rightarrow v + \delta v$$

$$u^\mu = \frac{dx^\mu}{d\lambda}$$

$$\delta v^\mu = \frac{dv^\mu}{d\lambda}\delta\lambda = -\Gamma^\mu_{\alpha\beta}\, v^\alpha \underbrace{u^\beta \delta\lambda}_{\delta x^\beta}$$

$$PT: \quad \frac{dv^\mu}{d\lambda} + \Gamma^\mu_{\alpha\beta}\, v^\alpha u^\beta = 0$$

$$\boxed{\frac{\partial v^\mu}{\partial x^\beta} = -\Gamma^\mu_{\alpha\beta}\, v^\alpha}$$



$\delta v_{(1)}$

$$\delta v_{(1)}{}^{\mu} = -\Gamma^{\mu}_{\nu\rho}(x)\, v^{\nu}(x)\, A^{\rho} - \Gamma^{\mu}_{\nu\rho}(x+A^{\rho})\, v^{\nu}(x+A)\, B^{\sigma}$$

$$\delta v_{(2)}{}^{\mu} = -\Gamma^{\mu}_{\nu\sigma}(x)\, v^{\nu}(x)\, B^{\sigma} - \Gamma^{\mu}_{\nu\rho}(x+B^{\sigma})\, v^{\nu}(x+B^{\sigma})\, A^{\rho}$$

$$\delta v_{(1)}{}^{\mu} - \delta v_{(2)}{}^{\mu} = \frac{\partial(\Gamma^{\mu}_{\nu\rho} v^{\nu})}{\partial x^{\sigma}} B^{\sigma} A^{\rho} - \frac{\partial \Gamma^{\mu}_{\nu\sigma} v^{\nu}}{\partial x^{\rho}} A^{\rho} B^{\sigma} + O(\delta x^{2})$$

$$= \underbrace{\left(\Gamma^{\mu}_{\nu\rho,\sigma} - \Gamma^{\mu}_{\nu\sigma,\rho} + \Gamma^{\mu}_{\alpha\sigma}\Gamma^{\alpha}_{\rho\nu} - \Gamma^{\mu}_{\alpha\rho}\Gamma^{\alpha}_{\sigma\nu}\right)}_{R^{\mu}{}_{\nu\rho\sigma}\ \text{Riemann tensor}} A^{\rho} B^{\sigma} v^{\nu}$$

$$[\nabla_\mu, \nabla_\nu] V^\rho = R^\rho{}_{\sigma\mu\nu} V^\sigma \cancel{- \underbrace{T^\lambda{}_{\mu\nu} \nabla_\lambda V^\rho}_{T=0}}$$

flat Manifold:

M is flat iff Riemann tensor vanishes for $\forall\, p \in M$

$g \to \eta \to R_{iem} \to 0$

$p \in M$

$R^{\mu}{}_{\nu\rho\sigma} \leftarrow \partial g, \partial^2 g$

$R_{\mu\nu\lambda\rho} \equiv g_{\mu\alpha} R^{\alpha}{}_{\nu\lambda\lambda\rho} = \frac{1}{2}\left(g_{\mu\rho,\nu\lambda} - g_{\mu\lambda,\nu\rho} - g_{\nu\rho,\mu\lambda} + g_{\lambda\nu,\rho,\mu} \right) + g_{\alpha\beta}\left(\Gamma^{\alpha}_{\mu\rho}\Gamma^{\beta}_{\nu\lambda} - \Gamma^{\alpha}_{\mu\lambda}\Gamma^{\beta}_{\nu\rho} \right)$

$R_{\mu\nu\lambda\rho} = -R_{\nu\mu\lambda\rho} = -R_{\mu\nu\rho\lambda} = R_{\lambda\rho\mu\nu}$

$R_{\mu[\nu\lambda\rho]_c} = R_{\mu\nu\lambda\rho} + R_{\mu\lambda\rho\nu} + R_{\mu\rho\nu\lambda} = 0$

$4^4 = 256$ $\qquad \# = \frac{d^2(d^2-1)}{12}$

$$R_{\underbrace{\alpha\beta}_{A \to n}\ \underbrace{\gamma\sigma}_{B \to n}}$$

$A: \alpha \neq \beta \qquad \binom{d}{2} = \frac{d!}{(d-2)!\,2!} = \frac{d(d-1)}{2} = n$

$B: \gamma \neq \sigma \qquad \to n = \frac{d(d-1)}{2}$

$AB: \quad \frac{n^2-n}{2} + n = \frac{n(n+1)}{2}$

$\mu = \alpha \quad \to \quad R_{\cancel{\mu}\mu\alpha\beta} + R_{\mu\alpha\beta\mu} + \underbrace{R_{\mu\beta\mu\alpha}}_{-R_{\beta\mu\mu\alpha}} = 0$

$-R_{\mu\alpha\beta\mu}$

$\binom{d}{4} \to$

$$R_{\mu[\nu\lambda\rho]_c} = R_{\mu\nu\lambda\rho} + R_{\mu\lambda\rho\nu} + R_{\mu\rho\nu\lambda} = 0$$

$$4^4 = 256 \qquad \# = d^2(d^2-1)$$

$$\#\,\text{Riem} = \frac{n(n+1)}{2} - \binom{d}{4} = \frac{d^2(d^2-1)}{12} \qquad d=4 \Rightarrow \#\text{Riem} = 20$$

$$n = \frac{d(d-1)}{2}$$

Ricci Tensor $\quad R_{\mu\nu} = R^{\lambda}{}_{\mu\lambda\nu} = \partial_\lambda \Gamma^\lambda_{\mu\nu} - \partial_\nu \underbrace{\Gamma^\lambda_{\mu\lambda}} + \Gamma^\lambda_{\lambda\rho}\Gamma^\rho_{\mu\nu} - \Gamma^\rho_{\mu\lambda}\underbrace{\Gamma^\lambda_{\nu\rho}}$

Ricci Scalar $\quad R = R^{\mu}{}_{\nu} = g^{\mu\nu}R_{\mu\nu}$

Einstein Tensor $\quad G_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2} R\, g_{\mu\nu}$

Weyl Tensor $C^{\alpha}{}_{\beta\gamma\delta}$

Riemann Symmetries & $C_{\mu\nu} = C^{\gamma}{}_{\mu\gamma\nu} = 0$

$C^{\alpha}{}_{\beta\gamma\delta} = 0 \iff g_{\mu\nu} = \Omega^2 \eta_{\mu\nu}$

Conformall flat

in Vacuum $R_{\mu\nu} = 0$ $R^{\alpha}{}_{\beta\gamma\delta} = C^{\alpha}{}_{\beta\gamma\delta}$

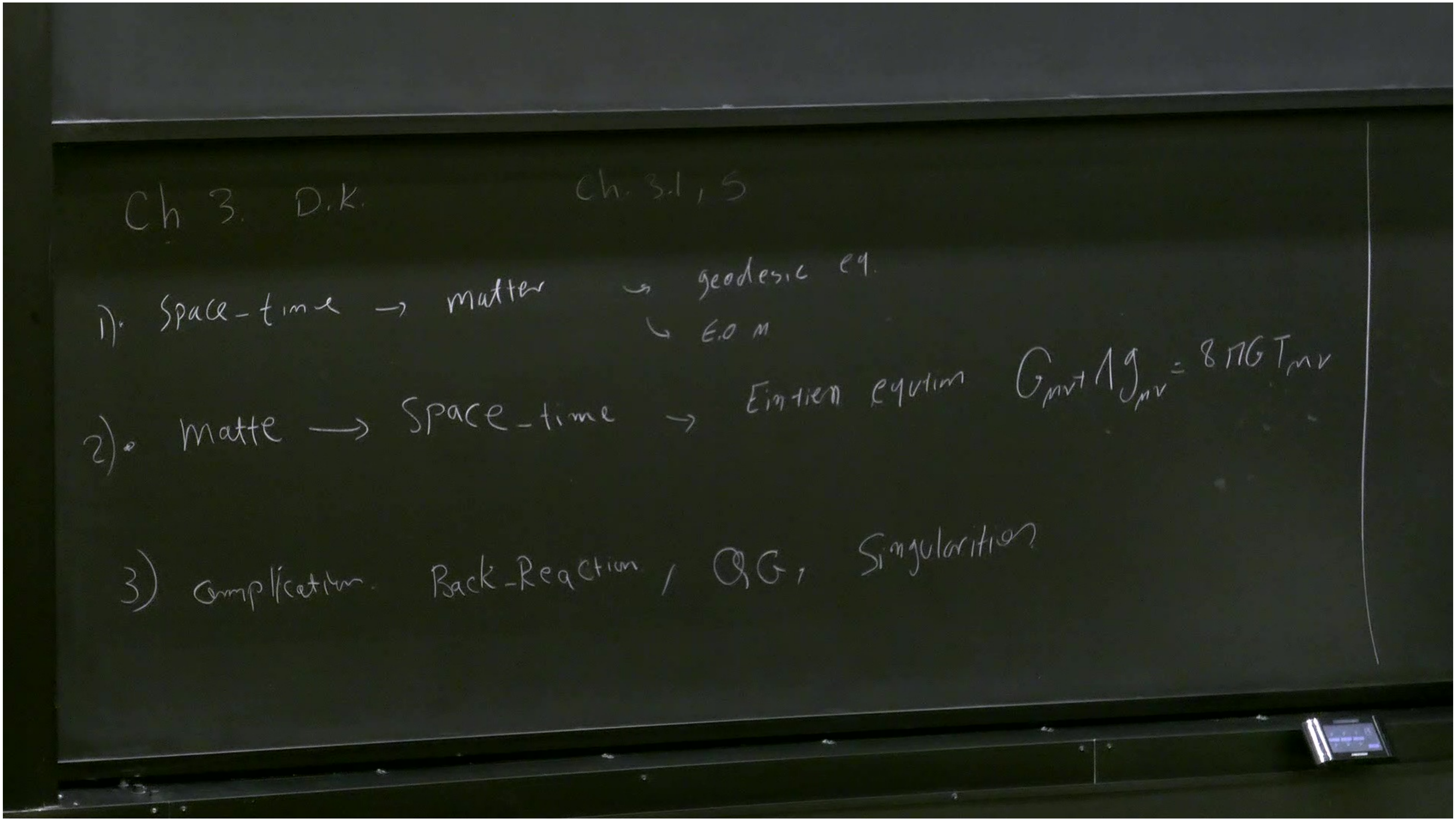
$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu}$

$R_{\alpha\beta[\gamma\delta;\epsilon]_\sigma} = 0$

$\nabla_\mu G^{\mu\nu} = 0$ Bianchi identity

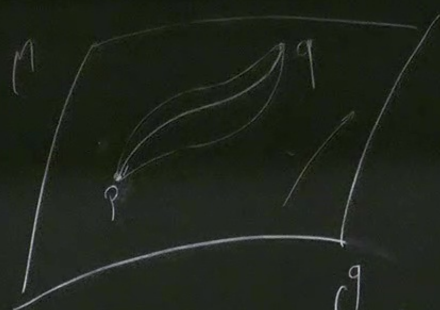
$G_{\mu\nu} \to$ Rank 2 & $G(g, \partial g, \partial^2 g)$ & Bianch identity

$G^{\mu\nu} + \Lambda g^{\mu\nu}$



Ch 3. D.K.
Ch. 3.1, 5
1). Space-time → matter → geodesic eq.
↳ E.O M
2). matte → Space-time → Eintien equtim $G_{\mu\nu}+\Lambda g_{\mu\nu}=8\pi G T_{\mu\nu}$
3) Complication. Back-Reaction, QG, Singularities

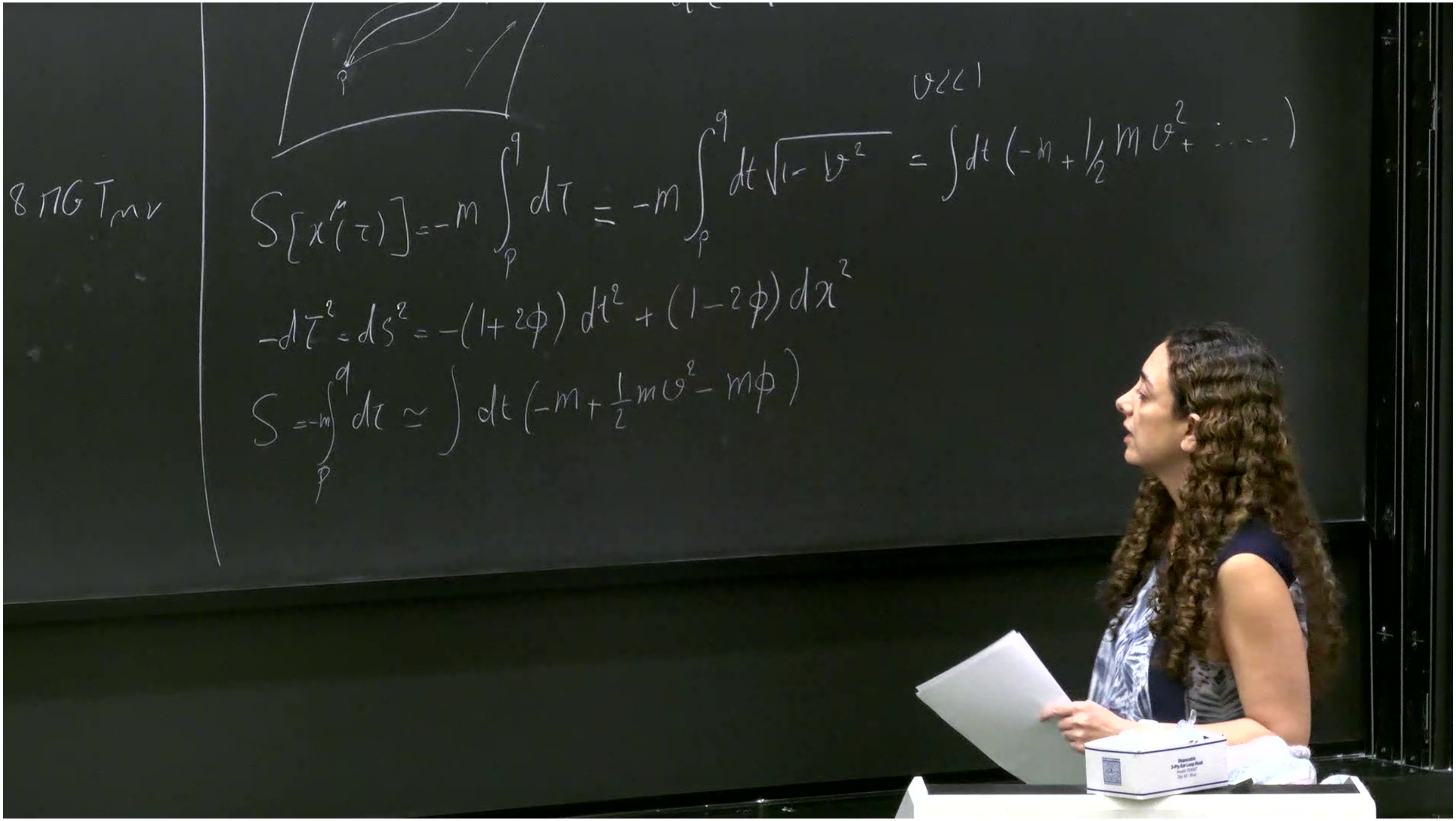
$8\pi G T_{\mu\nu}$



$$d\tau = \sqrt{-ds^2} = \sqrt{dt^2 - dx^2} = dt\sqrt{1 - v^2}$$

$v \ll 1$

$$S[x^\mu(\tau)] = -m\int_p^q d\tau = -m\int_p^q dt\sqrt{1 - v^2} = \int dt\left(-m + \tfrac{1}{2} m v^2 + \dots\right)$$


$8\pi G T_{\mu\nu}$
$v \ll 1$
$S[x^\mu(\tau)] = -m\int_p^q d\tau = -m\int_p^q dt\sqrt{1-v^2} = \int dt\left(-m + \frac{1}{2}mv^2 + \dots\right)$
$-d\tau^2 = ds^2 = -(1+2\phi)\,dt^2 + (1-2\phi)\,dx^2$
$S = -m\int_p^q d\tau \simeq \int dt\left(-m + \frac{1}{2}mv^2 - m\phi\right)$

$$S_p = -m \int_p^q d\lambda \sqrt{-g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}}$$

$$\lambda \to \lambda' \qquad (\ )^{\cdot} = \frac{d}{d\lambda}$$

$$\Rightarrow \frac{d\tau^2}{d\lambda^2} = \frac{-g_{\mu\nu} dx^\mu dx^\nu}{d\lambda^2} \Rightarrow \frac{d\tau}{d\lambda} = \sqrt{-g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu}$$

$$L(x^\mu, \dot{x}^\mu) = -m \sqrt{-g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu}$$

$$f(\lambda) = \frac{d\lambda}{d\tau} = \frac{1}{\sqrt{-g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu}}$$

E-L

$$\frac{d}{d\lambda} \frac{\partial L}{\partial \dot{x}^\mu} - \frac{\partial L}{\partial x^\mu} = 0$$

$$\frac{\partial L}{\partial \dot{x}^\mu} = \frac{-m}{2\sqrt{\ldots}} \left(-2 g_{\mu\nu} \dot{x}^\nu \right) = f(\lambda)\, m\, g_{\mu\nu} \dot{x}^\nu$$

$$\frac{\partial L}{\partial x^\mu} = \frac{-m}{2\sqrt{\ldots}} \left(-g_{\alpha\beta,\mu} \dot{x}^\alpha \dot{x}^\beta \right) = \frac{1}{2} f(\lambda)\, g_{\alpha\beta,\mu} \dot{x}^\alpha \dot{x}^\beta$$

$$\Rightarrow \frac{d\tau}{d\lambda} = \sqrt{-g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}$$

$$f(\lambda) = \frac{d\lambda}{d\tau} = \frac{1}{\sqrt{-g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}}$$

$$= f(\lambda)\, m\, g_{\mu\nu}\dot{x}^\nu$$

E-L: $$-f' g_{\alpha\beta}\dot{x}^\beta - f g_{\alpha\beta,\nu}\dot{x}^\nu \dot{x}^\beta - f g_{\alpha\beta}\ddot{x}^\beta + \frac{1}{2} g_{\mu\nu,\alpha}\dot{x}^\mu \dot{x}^\nu f = 0$$

$$\Rightarrow \ddot{x}^\mu + \underbrace{\frac{1}{2} g^{\mu\alpha}\left(g_{\alpha\beta,\nu} + g_{\alpha\nu,\beta} - g_{\beta\nu,\alpha}\right)}\dot{x}^\beta \dot{x}^\nu = \frac{f'(\lambda)}{f}\dot{x}^\mu$$

$\Rightarrow \frac{d\tau}{d\lambda} = \sqrt{-g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}$

$(\lambda) = \frac{d\lambda}{d\tau} = \frac{1}{\sqrt{-g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}}$

$= f(\lambda) m g_{\mu\nu}\dot{x}^\nu$

E-L: $-f' g_{\alpha\beta} \dot{x} - f(\Gamma_{\alpha\beta,\nu}) \ldots$

$$\Rightarrow \ddot{x}^\mu + \underbrace{\frac{1}{2} g^{\mu\alpha}\left(g_{\alpha\beta,\nu} + g_{\alpha\nu,\beta} - g_{\beta\nu,\alpha}\right)}_{\Gamma^\mu_{\beta\nu}} \dot{x}^\beta \dot{x}^\nu = \frac{f'(\lambda)}{f}\dot{x}^\mu$$

RHS $=0$ if

$$\frac{d\lambda}{d\tau} = 0 \Rightarrow \lambda = \alpha(\tau - \tau_0)$$

$$\rightarrow u^\alpha \nabla_\alpha u^\mu = 0$$