Title: Lecture - Relativity, PHYS 604

Speakers: Ghazal Geshnizjani

Collection/Series: Relativity (Core), PHYS 604, November 12 - December 11, 2024

Subject: Cosmology, Strong Gravity

Date: November 15, 2024 - 10:45 AM

URL: https://pirsa.org/24110025

Abstract:

$$g_{tt} = -c^2\left(1 + 2\phi/c^2\right)$$

$$\nabla^2 \phi = -4\pi G \rho \qquad \text{Newton}$$

$$\nabla^2 g_{tt} = -2\,\nabla^2 \phi$$

$$\cancel{-}\frac{1}{2}\nabla^2 g_{tt} = \cancel{-}4\pi G \rho$$

$$\nabla^2 g_{tt} = 8\pi G \rho$$

$$\nabla^2 g_{00} = 8\pi G\, T_{00}$$

$$G_{00} = 8\pi G\, T_{00}$$

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

$$g_{tt} = -c^2\left(1 + 2\phi/c^2\right)$$

$$\nabla^2 \phi = -4\pi G \rho \qquad \text{Newton}$$

$$\nabla^2 g_{tt} = -2\nabla^2\phi$$

$$-\frac{1}{2}\nabla^2 g_{tt} = -4\pi G \rho$$

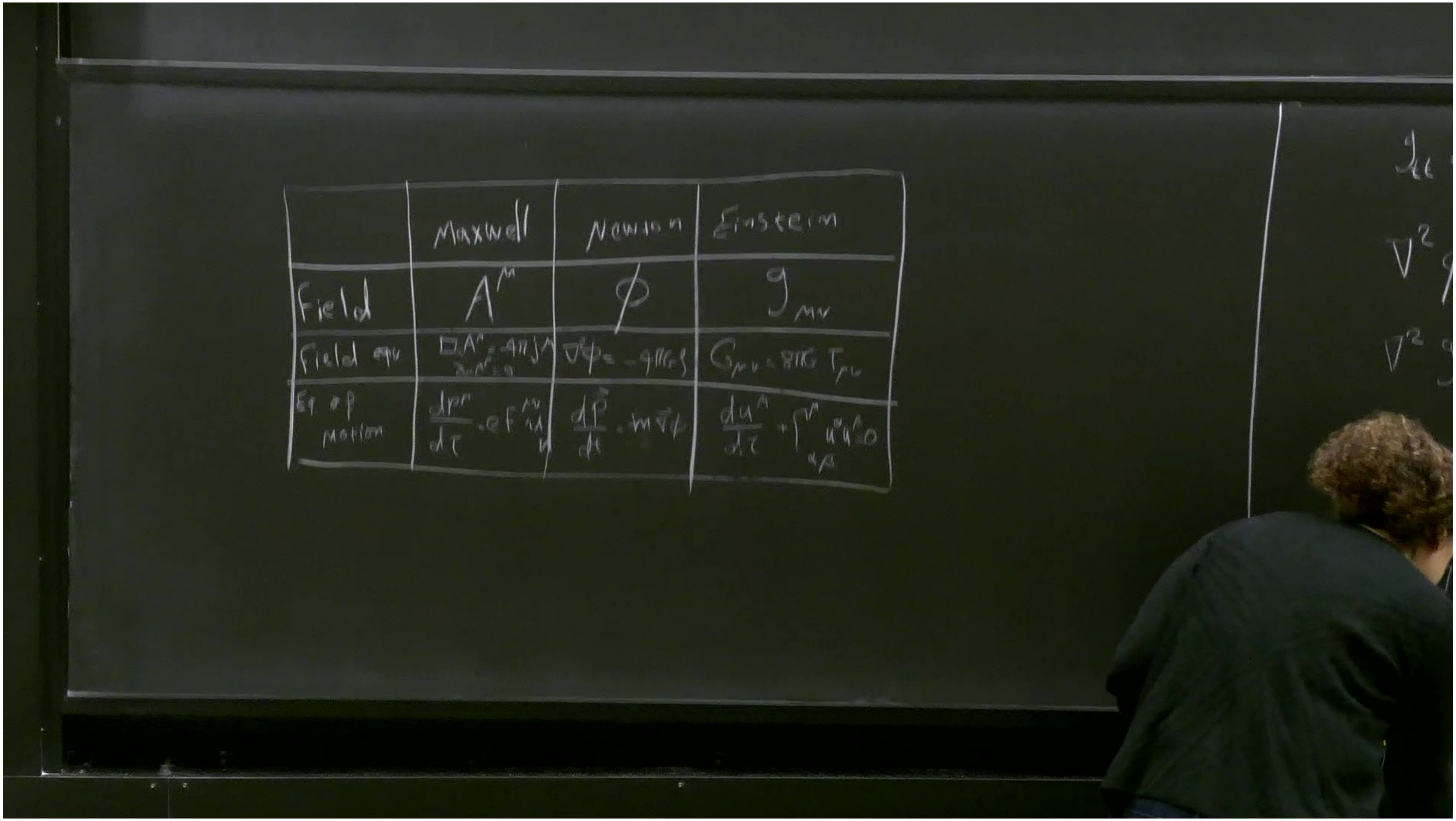
$$\nabla^2 g_{tt} = 8\pi G \rho$$

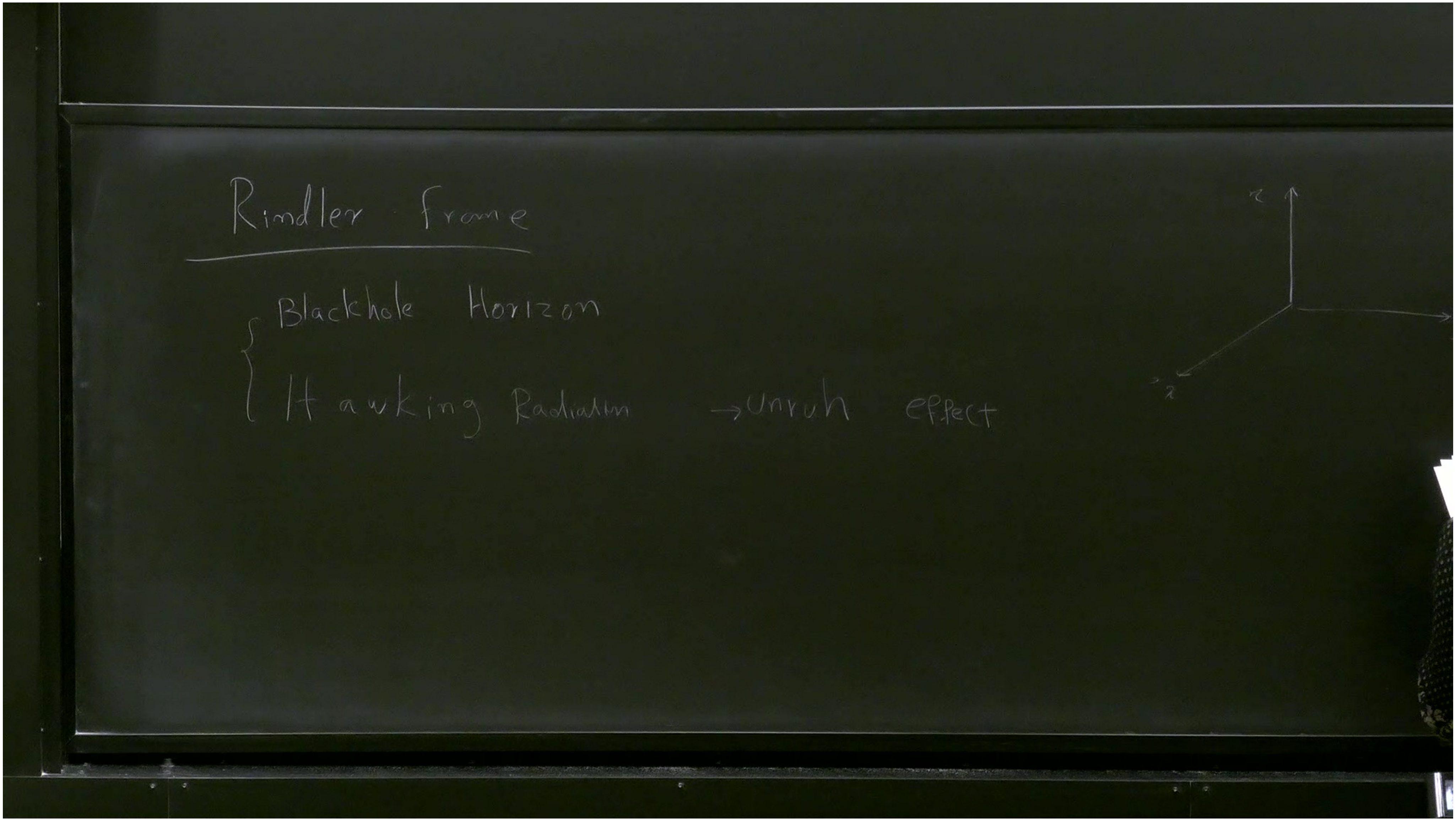
$$\nabla^2 g_{00} = 8\pi G T_{00}$$

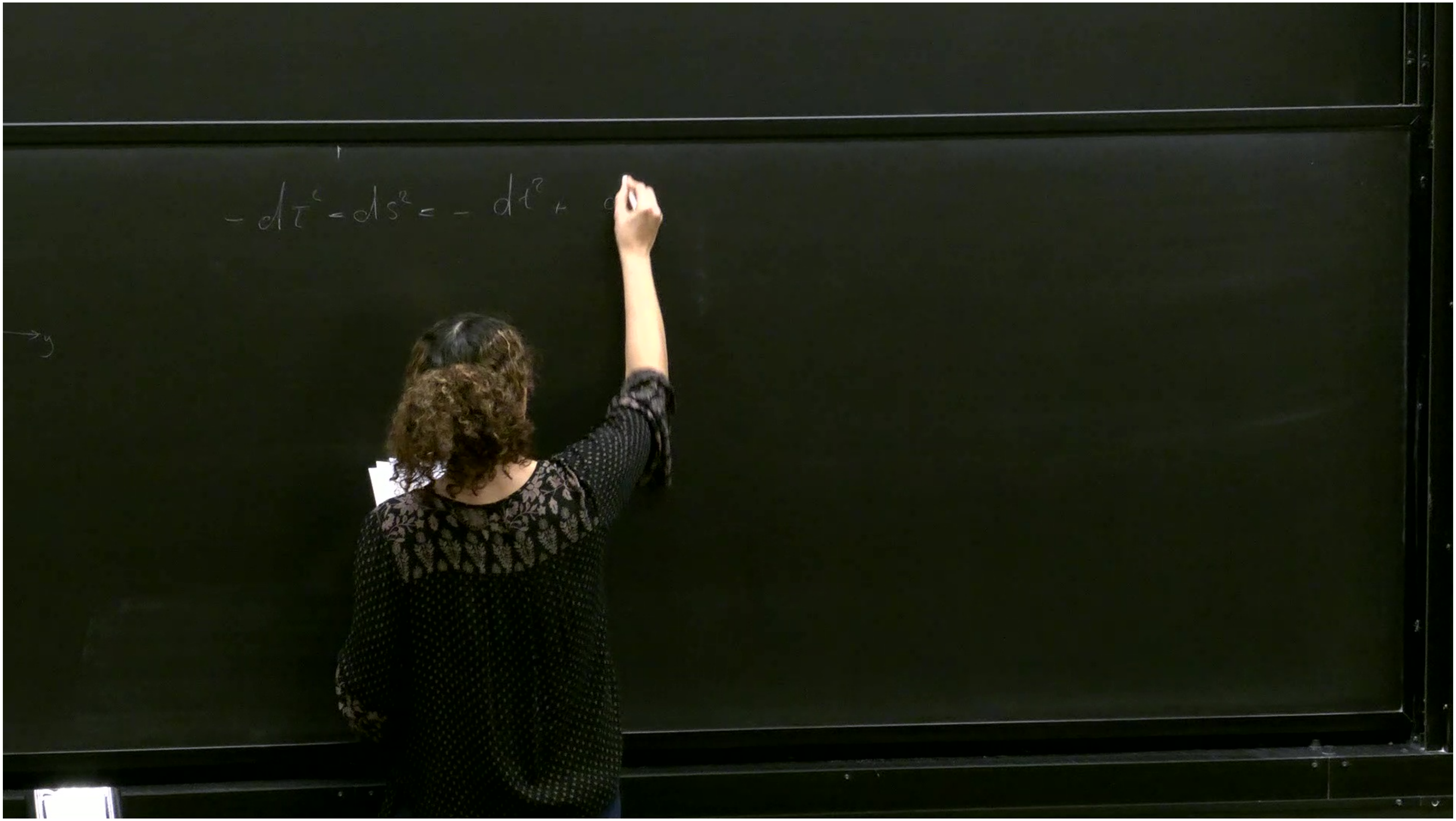
$$G_{00} = 8\pi G T_{00} \qquad \nabla^\mu T_{\mu\nu} = 0$$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \qquad \nabla^\mu G_{\mu\nu} = 0$$

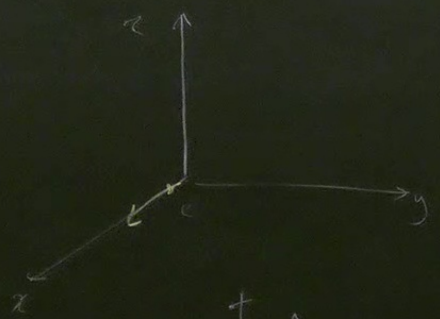
$$G_{\mu\nu}(\partial^2 g, \partial g, g)$$


Maxwell
Newton
Einstein
Field
A^μ
ϕ
$g_{\mu\nu}$
Field eqn
$\Box A^\mu = 4\pi j^\mu$
$\nabla^2 \phi = 4\pi G \rho$
$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$
Eq of motion
$\frac{dp^\mu}{d\tau} = e F^{\mu}{}_{\nu} u^\nu$
$\frac{d\vec{p}}{dt} = m \vec{\nabla}\phi$
$\frac{du^\mu}{d\tau} + \Gamma^\mu_{\alpha\beta} u^\alpha u^\beta = 0$


Rindler frame
Blackhole Horizon
Hawking Radiation → Unruh effect


-dτ² = ds² = - dt² +

$$-d\tau^2 = ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

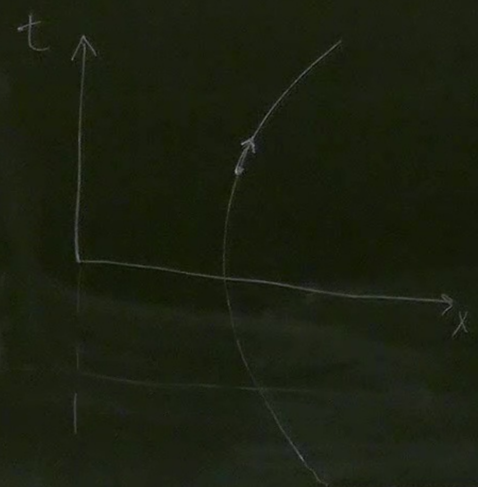


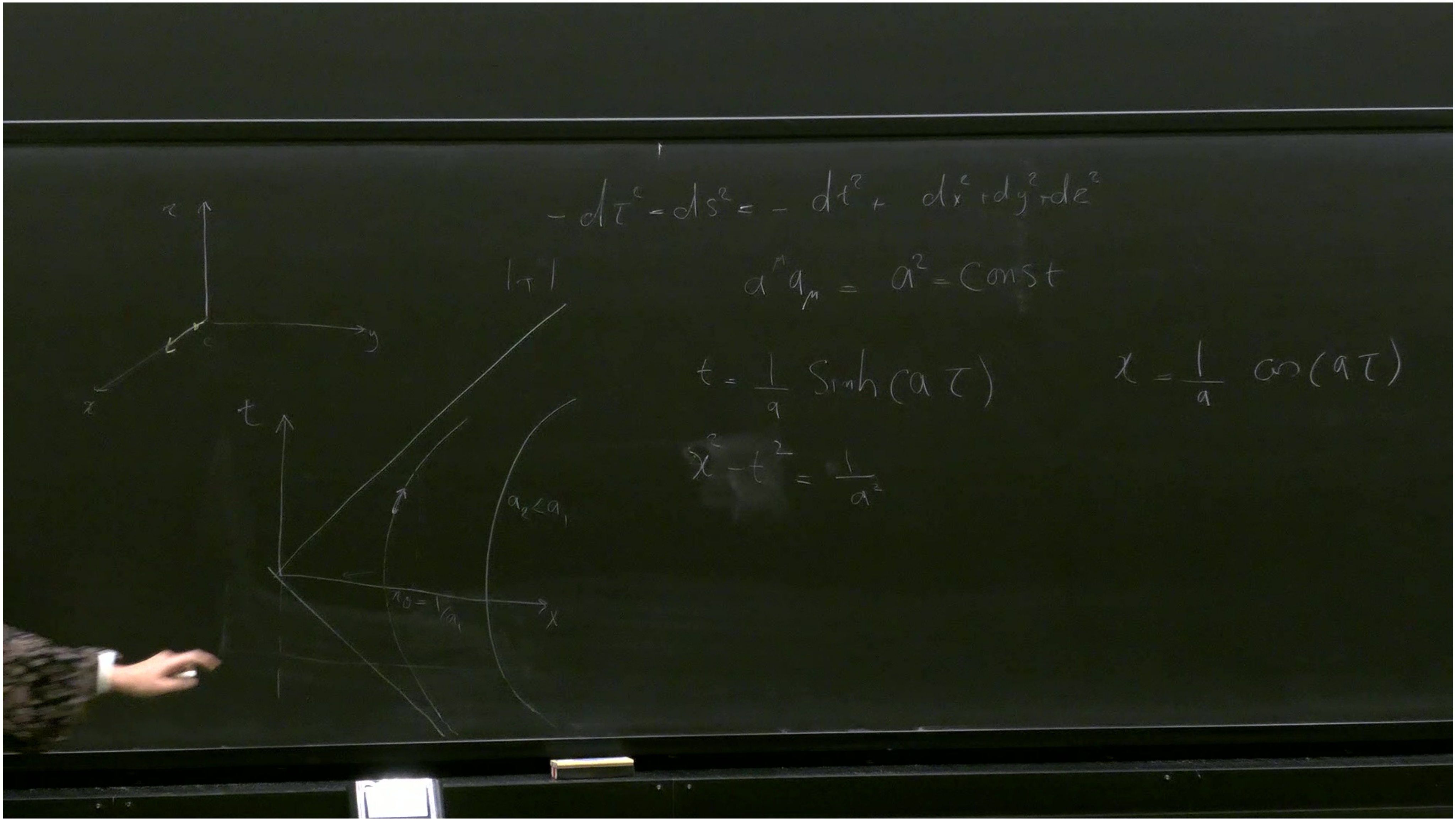
1+1

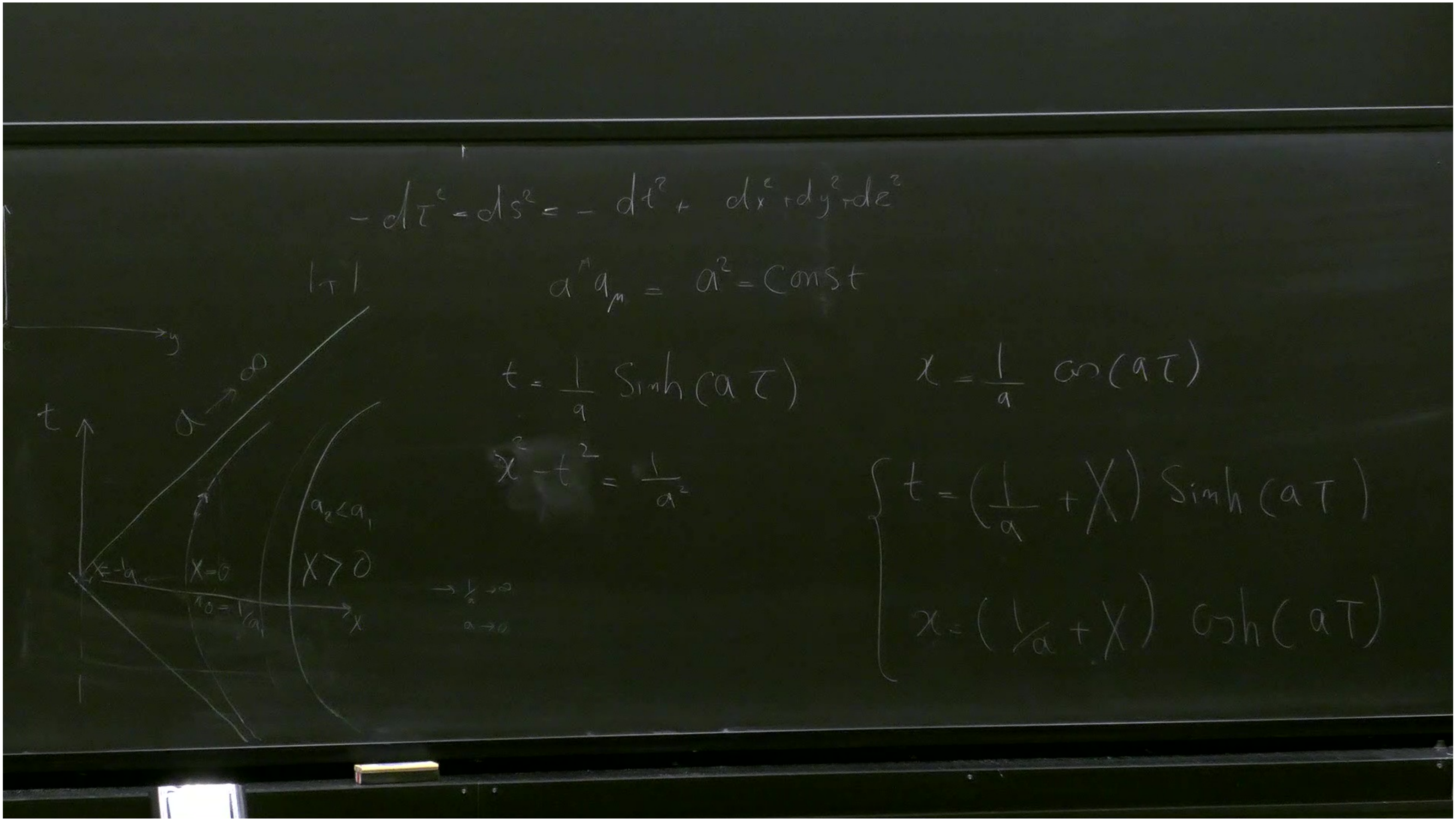
$$a^\mu a_\mu = a^2 = \text{Const}$$

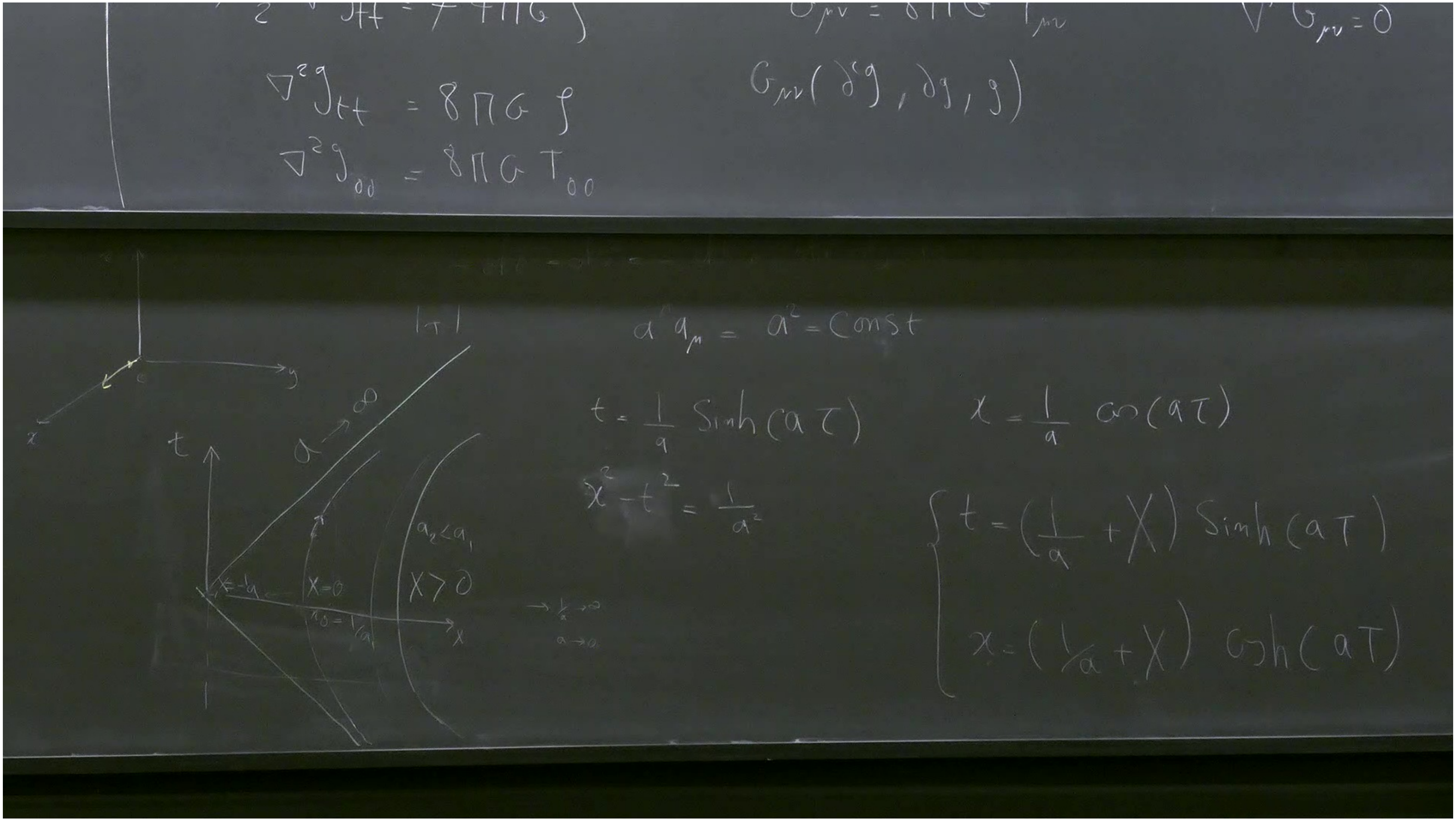
$$t = \frac{1}{a}\sinh(a\tau) \qquad x = \frac{1}{a}\cosh(a\tau)$$

$$t^2 - x^2 = \frac{-1}{a^2}$$

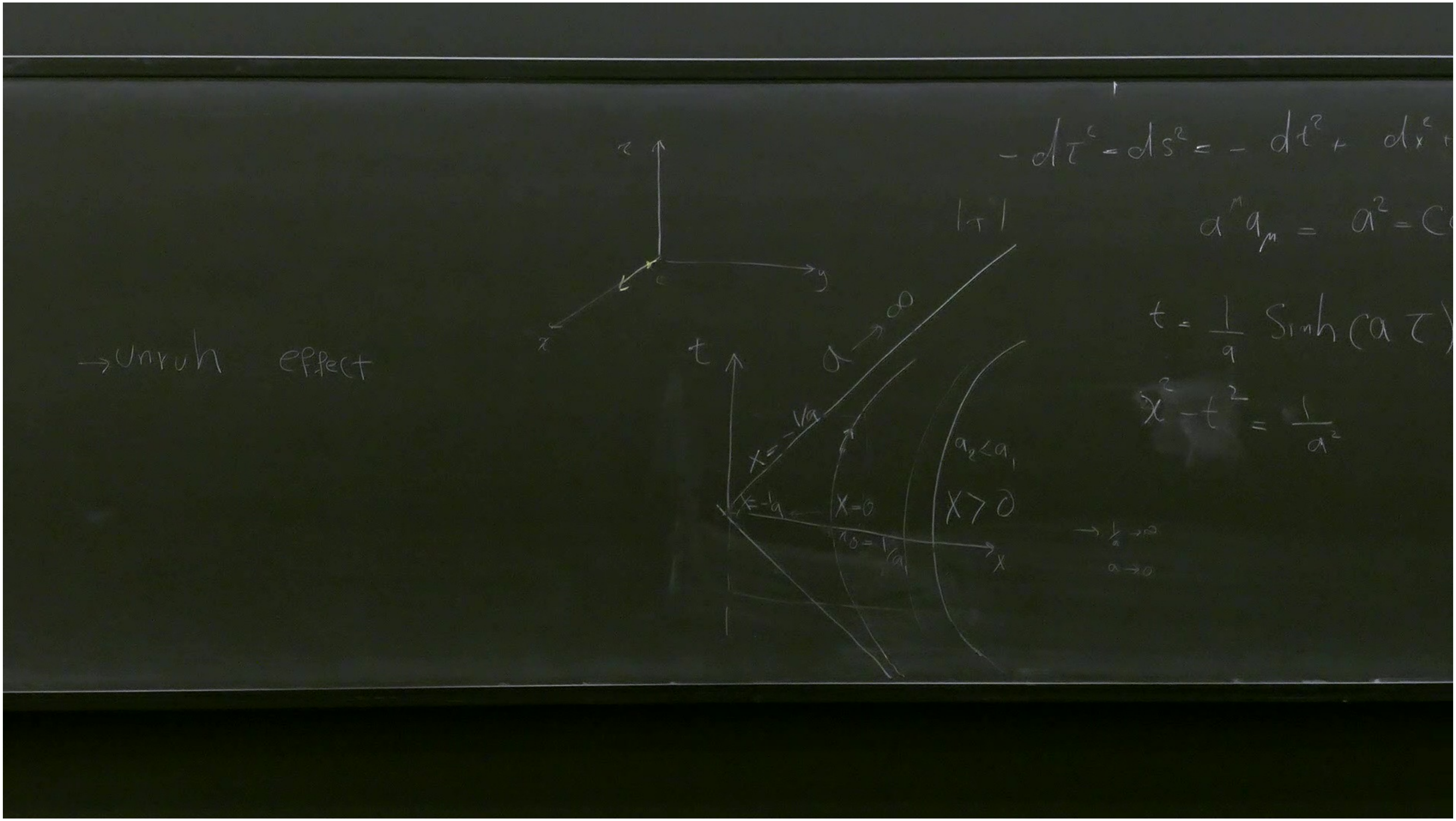


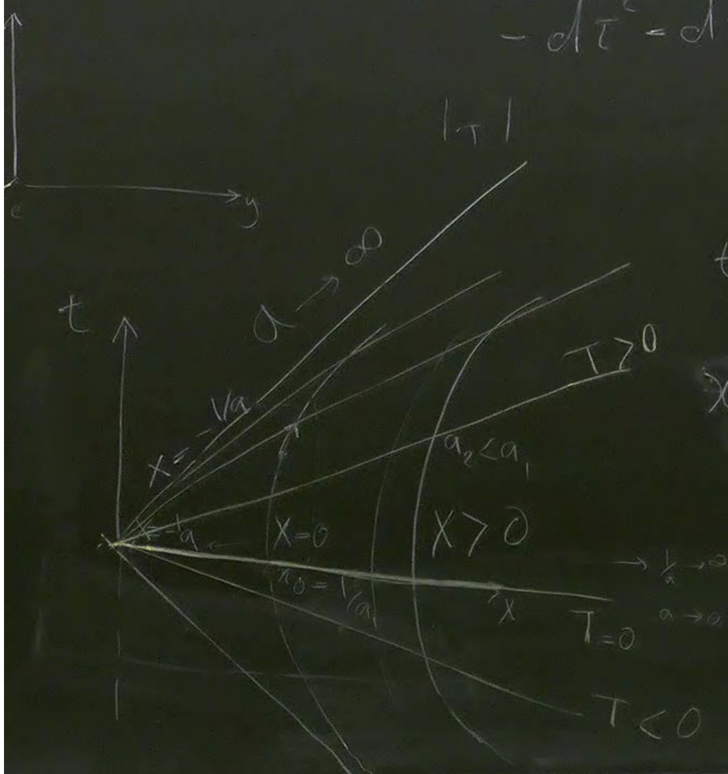

-dτ² = ds² = -dt² + dx² + dy² + dz²
a^μ a_μ = a² = const
t = 1/a sinh(aτ)
x = 1/a cos(aτ)
x² - t² = 1/a²
a₂ < a₁
x₀ = 1/a₁


-dτ² = ds² = -dt² + dx² + dy² + dz²
a^μ a_μ = a² = Const
t = 1/a Sinh(aτ)
x = 1/a cosh(aτ)
x² - t² = 1/a²
t = (1/a + X) Sinh(aT)
x = (1/a + X) Cosh(aT)
a → ∞
a₂ < a₁
X = 0
X > 0
x₀ = 1/a₁
X = -1/a
t
x
y



∇²g_tt = 8πG ρ
∇²g_00 = 8πG T_00
G_μν(∂²g, ∂g, g)
∇^μ G_μν = 0
a^μ a_μ = a² = const
t = 1/a sinh(aτ)
x = 1/a cosh(aτ)
x² − t² = 1/a²
t = (1/a + X) sinh(aT)
x = (1/a + X) cosh(aT)
t
x
y
a → ∞
a₂ < a₁
X = 0
X > 0


→ Unruh effect
t
x
y
z
X=0
X>0
X



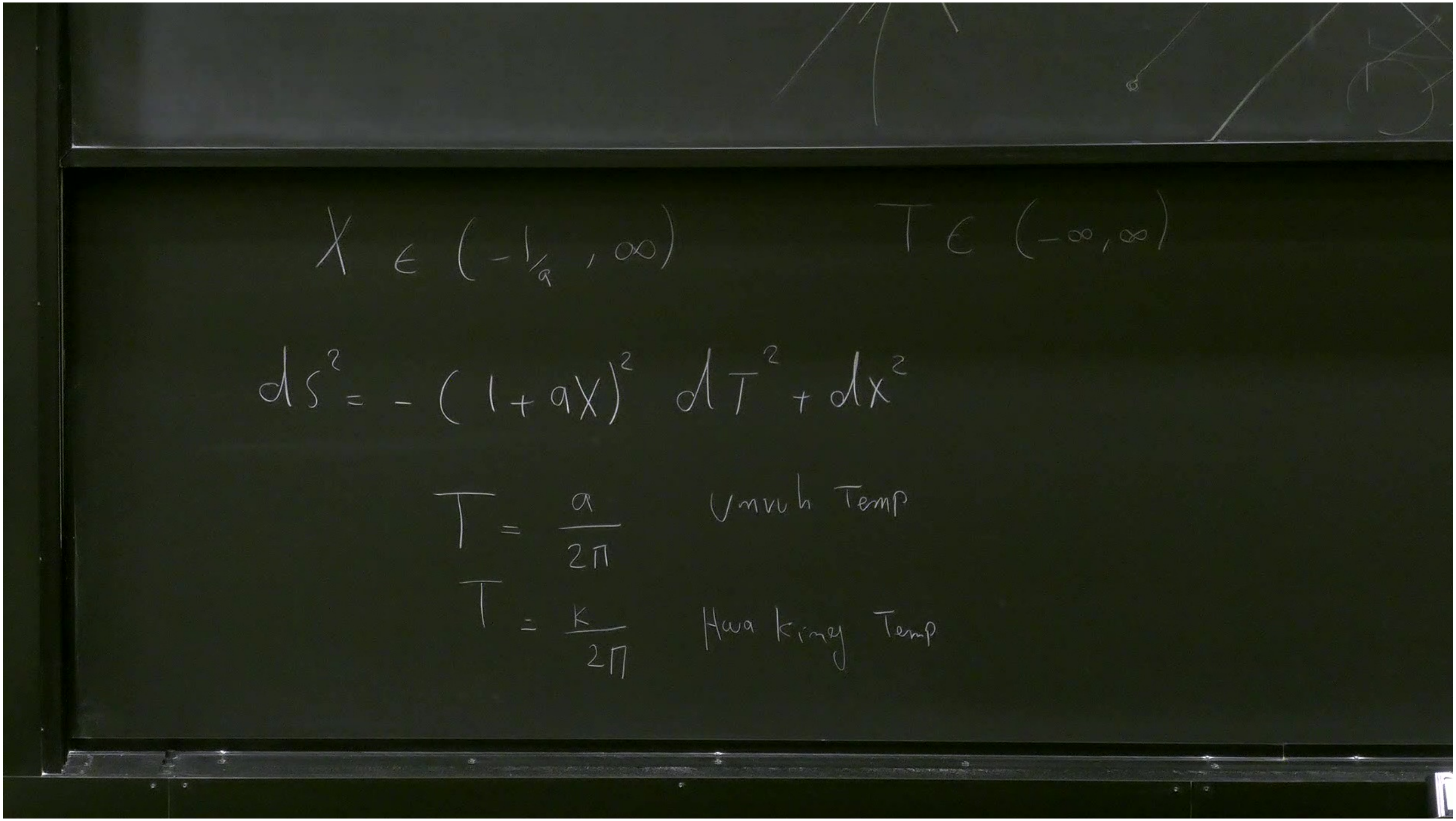
$$-d\tau^2 = ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

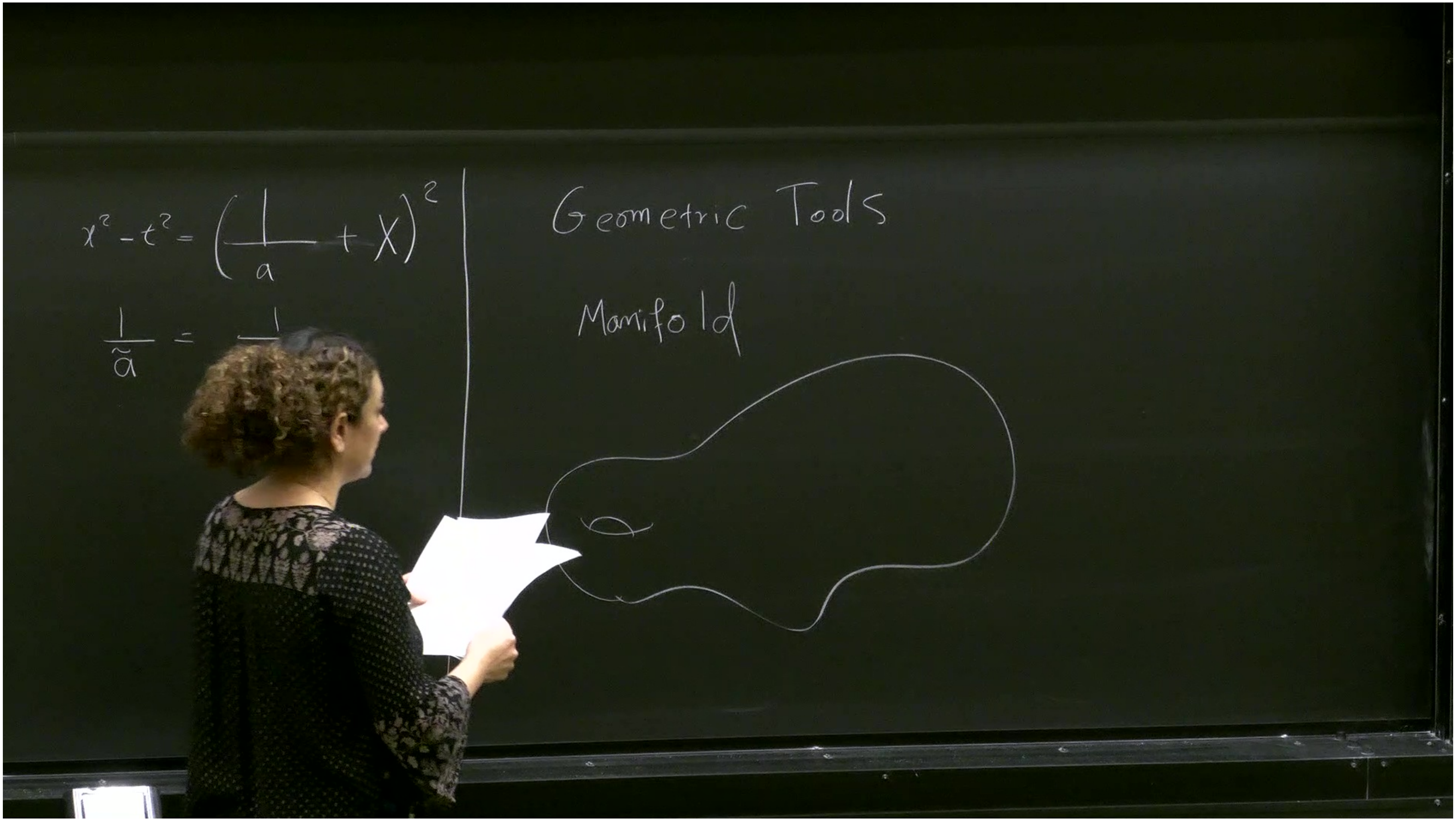
$$a^\mu a_\mu = a^2 = \text{Const}$$

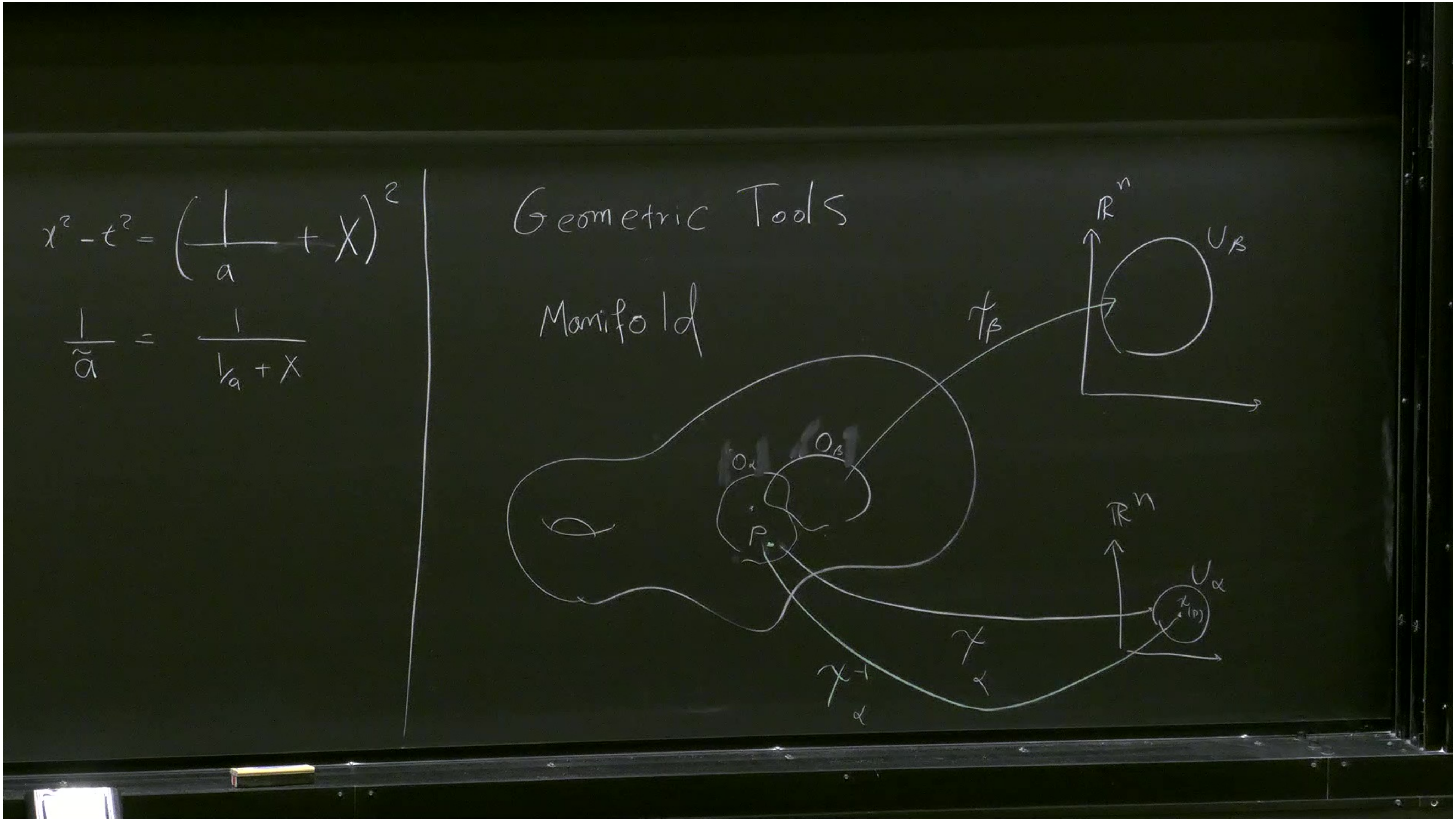
$$t = \frac{1}{a}\sinh(a\tau) \qquad x = \frac{1}{a}\cosh(a\tau)$$

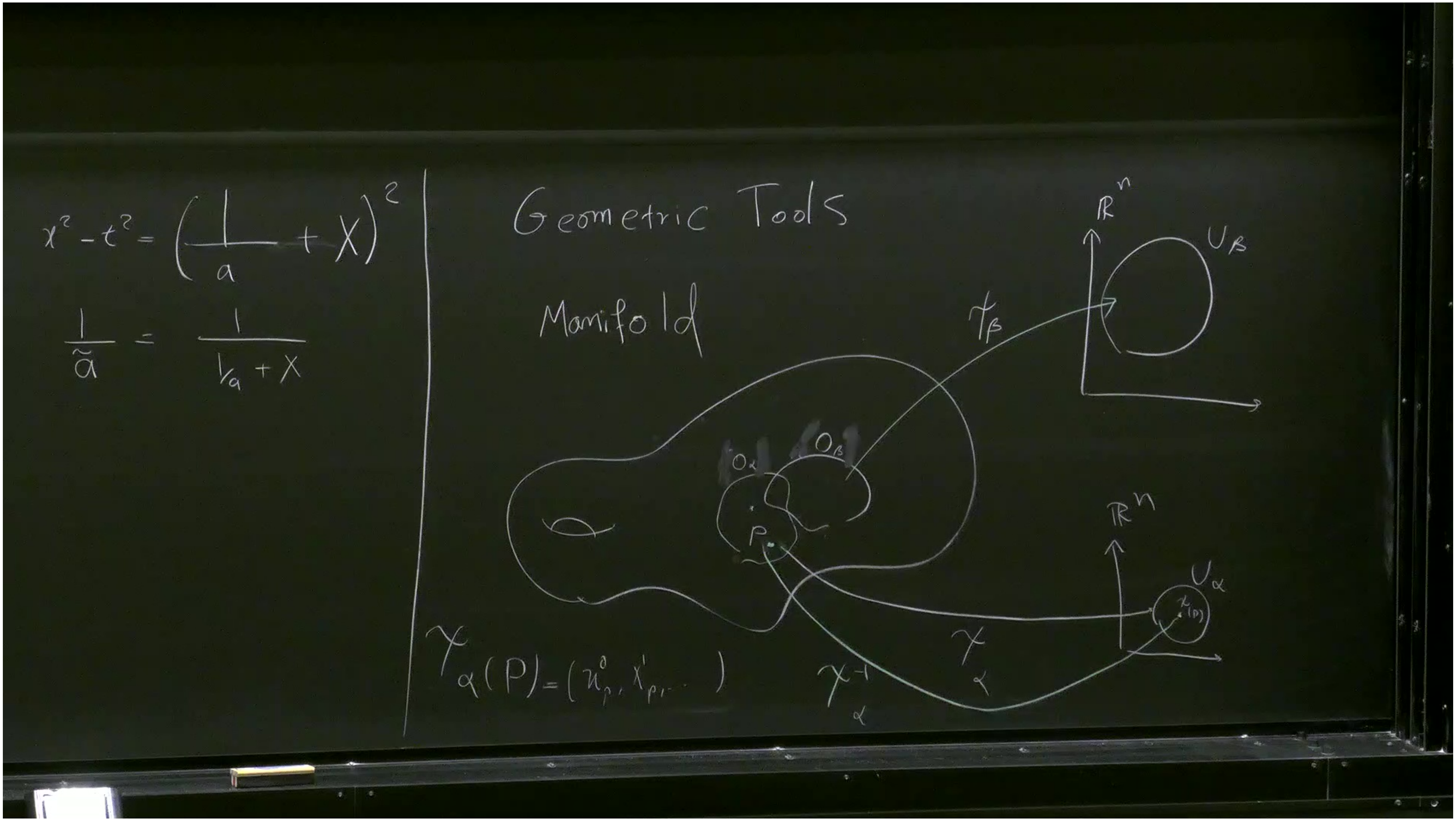
$$x^2 - t^2 = \frac{1}{a^2}$$

$$\begin{cases} t = \left(\frac{1}{a} + X\right)\sinh(aT) \\ x = \left(\frac{1}{a} + X\right)\cosh(aT) \end{cases}$$


$X \in (-1/a, \infty)$
$T \in (-\infty, \infty)$
$ds^2 = -(1+aX)^2 dT^2 + dX^2$
$T = \frac{a}{2\pi}$ Unruh Temp
$T = \frac{\kappa}{2\pi}$ Hwaking Temp


$x^2 - t^2 = \left(\frac{1}{a} + X\right)^2$
Geometric Tools
Manifold


$x^2 - t^2 = \left(\frac{1}{a} + X\right)^2$
$\frac{1}{\tilde{a}} = \frac{1}{1/a + X}$
Geometric Tools
Manifold
$\mathbb{R}^n$
U_β
ψ_β
O_α
O_β
p
$\mathbb{R}^n$
U_α
$x_\alpha(p)$
χ_α
χ_α^{-1}


$x^2 - t^2 = \left(\frac{1}{a} + X\right)^2$
$\frac{1}{\tilde{a}} = \frac{1}{1/a + X}$
Geometric Tools
Manifold
$\mathbb{R}^n$
U_β
ψ_β
O_α
O_β
P
$\mathbb{R}^n$
U_α
$x_{(P)}$
χ_α
χ_α^{-1}
$\chi_\alpha(P) = (x^0_p, x^1_p, \ldots)$

Manifold: An n-dimensional manifold M is a "set of points" with a co

i) $\forall\, p \in M \;\; \exists\, O_\alpha$ such that $p \in O_\alpha$

ii) $\forall\, O_\alpha \;\; \exists$ a 1-1 & onto map $\psi_\alpha : O_\alpha \to U_\alpha$ where U_α is open

iii) If $O_\alpha \cap O_\beta \neq \emptyset$ the maps $\psi_\beta \circ \psi_\alpha^{-1}$ is C^∞

Manifold: An n-dimensional manifold M is a "set of points" with a co

i) $\forall\, p \in M \;\exists\; O_\alpha$ such that $p \in O_\alpha$

ii) $\forall\, O_\alpha \;\exists$ a 1-1 & onto map $\psi_\alpha : O_\alpha \to U_\alpha$ where U_α is open

iii) If $O_\alpha \cap O_\beta \neq \emptyset$ the maps $\psi_\beta \circ \psi_\alpha^{-1}$ is C^∞

$\psi_\beta \circ \psi_\alpha^{-1}(\;)$

Manifold: An n-dimensional manifold M is a "set of points" with a co

i) $\forall\, p \in M \;\exists\, O_\alpha$ such that $p \in O_\alpha$

ii) $\forall\, O_\alpha \;\exists$ a 1-1 & onto map $\psi_\alpha : O_\alpha \to U_\alpha$ where U_α is open

iii) If $O_\alpha \cap O_\beta \neq \emptyset$ the maps $\psi_\beta \circ \psi_\alpha^{-1}$ is C^∞

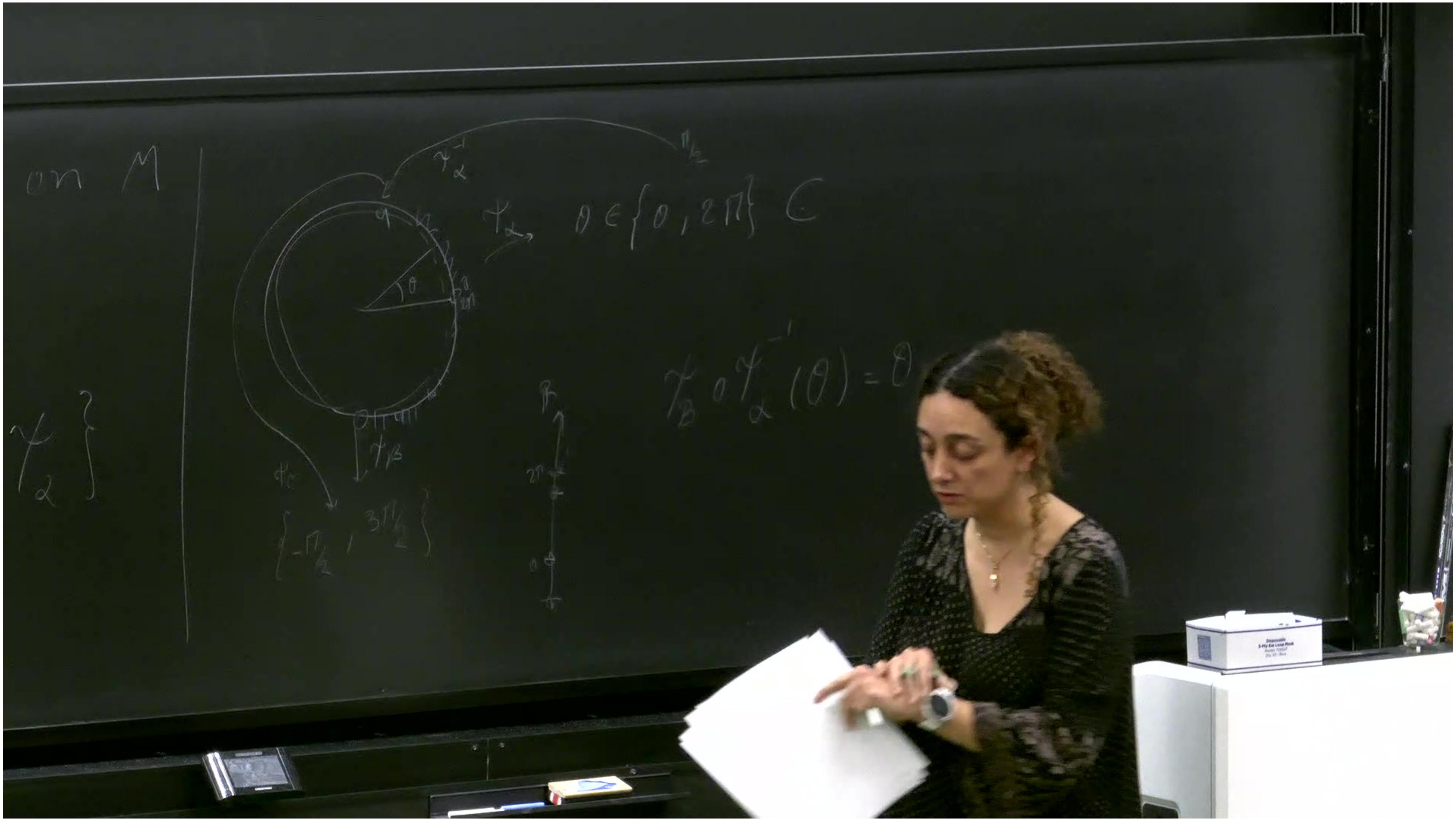
$\psi_\beta \circ \psi_\alpha^{-1}(x^\mu) = (x'^\mu, x'^\nu, \ldots)$ $\quad \psi_\beta \circ \psi_\alpha^{-1} : \mathbb{R}^n \to \mathbb{R}^n$

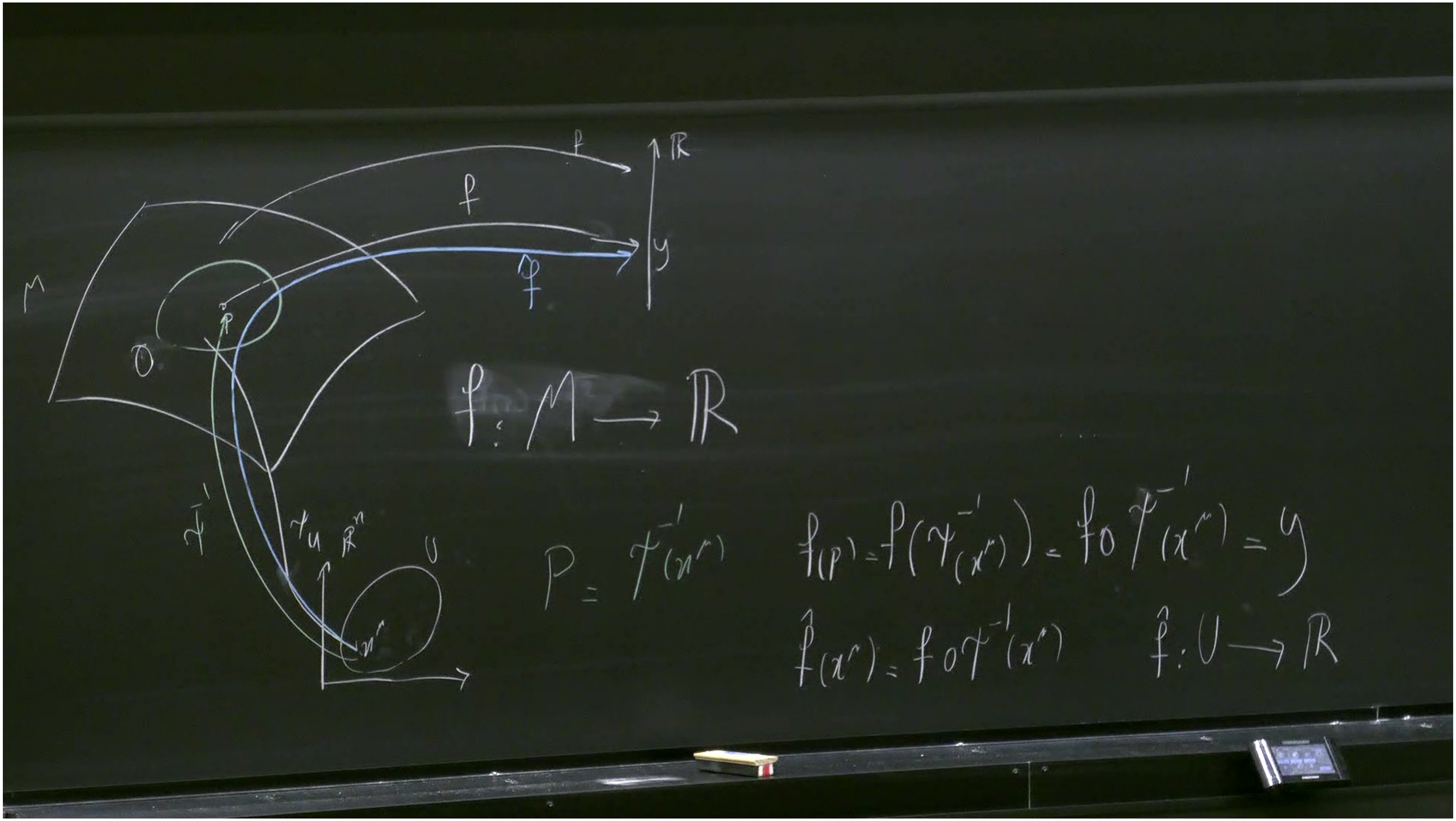
Coordinates: maps between open set O_α on M

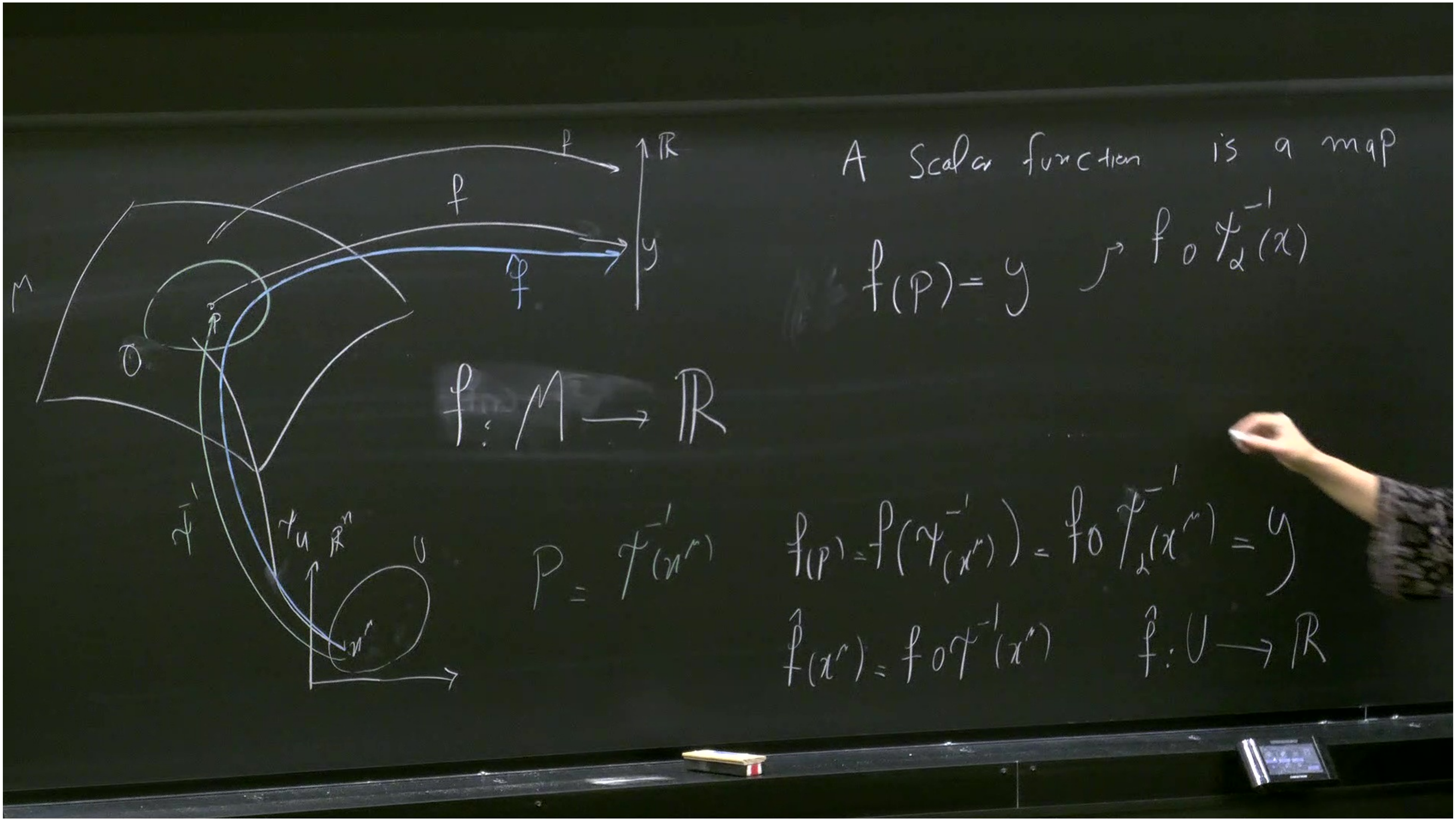
$$\psi_\alpha : O_\alpha \longrightarrow \mathbb{R}^n$$

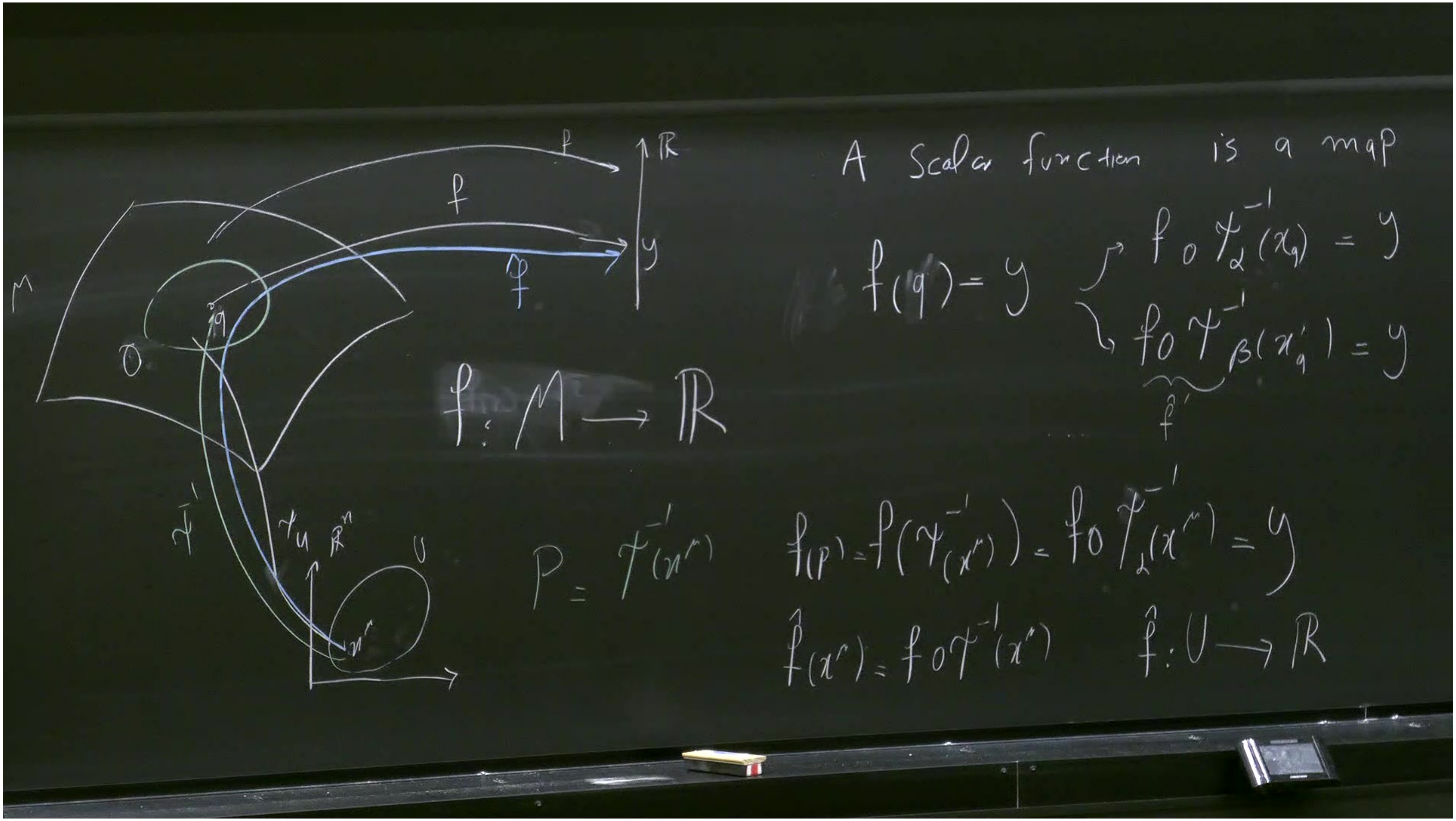
Atlas: collection of all the charts $\{\psi_\alpha\}$

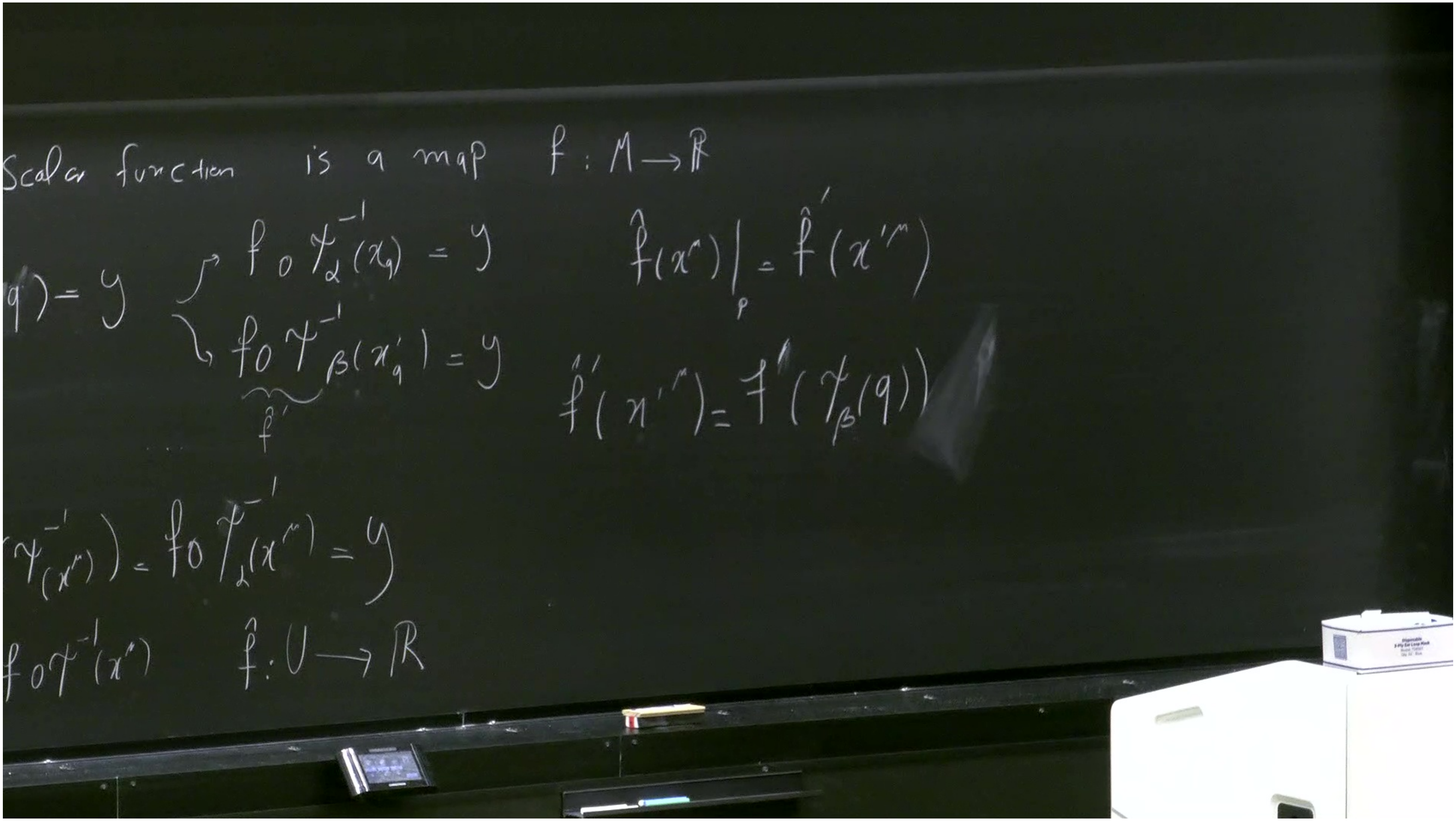
$$\psi_\alpha(p) = x^\mu(p)$$


on M
θ ∈ {0, 2π} C
{-π/2, 3π/2}


M
O
p
f
$\mathbb{R}$
y
φ
φ^{-1}
φ_U
$\mathbb{R}^n$
U
x^μ
$f : M \longrightarrow \mathbb{R}$
$p = \varphi^{-1}(x^\mu)$
$f(p) = f(\varphi^{-1}(x^\mu)) = f \circ \varphi^{-1}(x^\mu) = y$
$\hat{f}(x^\mu) = f \circ \varphi^{-1}(x^\mu)$
$\hat{f} : U \longrightarrow \mathbb{R}$


A Scalar function is a map
$f(p) = y \mapsto f \circ \psi_\alpha^{-1}(x)$
$f : M \to \mathbb{R}$
$p = \psi^{-1}(x^\mu)$
$f(p) = f(\psi^{-1}(x^\mu)) = f \circ \psi_\alpha^{-1}(x^\mu) = y$
$\hat{f}(x^\mu) = f \circ \psi^{-1}(x^\mu)$
$\hat{f} : U \to \mathbb{R}$
M
O
p
f
y
$\mathbb{R}$
ψ
ψ^{-1}
ψ_U
$\mathbb{R}^n$
U
x^μ


A Scalar function is a map
f(q) = y
f : M → R
P = ψ⁻¹(x^μ)
f(x^μ) = f ∘ ψ⁻¹(x^μ)
f̂ : U → R



Scalar function is a map f : M → ℝ
$\hat{f}(x^\mu)|_p = \hat{f}'(x'^\mu)$
q) = y
$f \circ \psi_\alpha^{-1}(x_q) = y$
$f \circ \psi_\beta^{-1}(x'_q) = y$
$\hat{f}'$
$\hat{f}'(x'^\mu) = \hat{f}'(\psi_\beta(q))$
$\psi^{-1}(x^\mu)) = f \circ \psi_\alpha^{-1}(x^\mu) = y$
$f \circ \psi^{-1}(x^\mu)$
$\hat{f} : U \to \mathbb{R}$