Title: Lecture - Relativity, PHYS 604

Speakers: Ghazal Geshnizjani

Collection/Series: Relativity (Core), PHYS 604, November 12 - December 11, 2024

Subject: Cosmology, Strong Gravity

Date: November 12, 2024 - 2:00 PM

URL: https://pirsa.org/24110024

Abstract:

General Relativity

Energy / Matter $\longleftrightarrow$ Gravity $\longleftrightarrow$ Geometry

Gravity $\longleftrightarrow$ acceleration

Geometry

general covariance, curvature

diff Geometry, Tensor Algebra

$$\boxed{G_{\mu\nu} = 8\pi G\, T_{\mu\nu}}$$

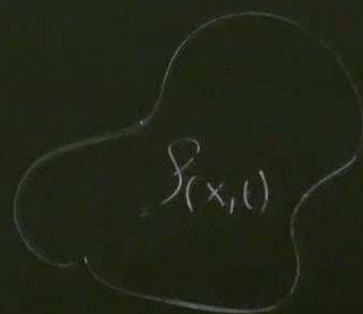
applications

Black holes

Cosmology

Gravitational waves

What is wrong with Newtonian Gravity?



$$f_g = M a$$

$$F_g = -M \nabla \phi$$

$$a = -\nabla \phi$$

$$\nabla^2 \phi = 4 \pi G \rho$$

$$\phi_{(x,t)} = -\int dx'^3 \, \frac{\rho_{(x,t)}}{|x - x'|}$$

$$\phi(x,t) = -\int dx'^3 \frac{f(x,t)}{|x-x'|}$$

Wave nature

Tensor field description

$f_{\mu\nu} =$

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2$$

$$c \sim 1$$

$$\eta_{\mu\nu} = \begin{pmatrix} -c^2 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 1 \end{pmatrix}$$

$$ds^2 = \sum_{\mu,\nu} \eta_{\mu\nu} dx^\mu dx^\nu = \eta_{\mu\nu} dx^\mu dx^\nu \qquad \mu = 0, 1, 2, 3$$

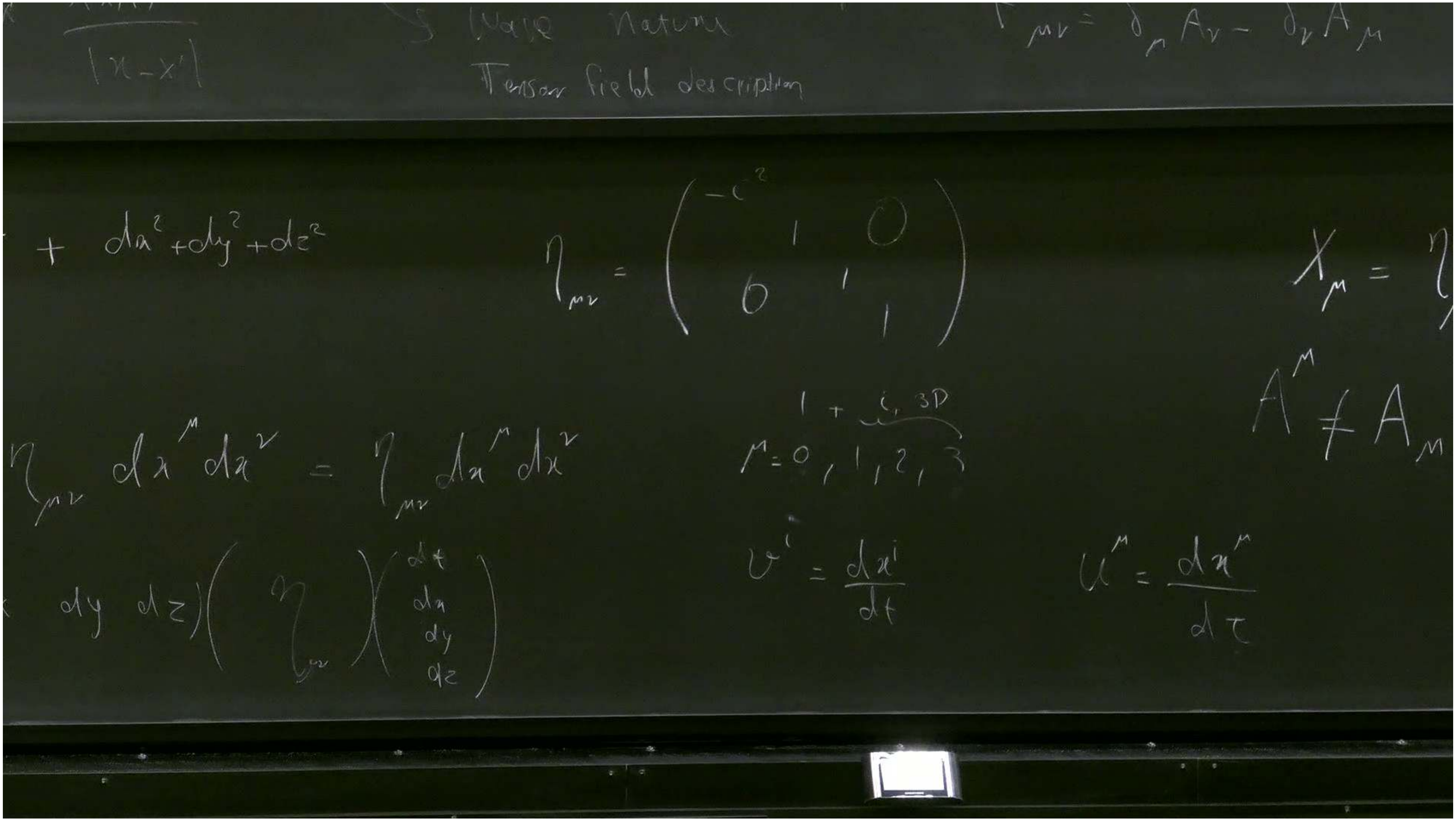
$$\begin{pmatrix} dt & dx & dy & dz \end{pmatrix} \begin{pmatrix} & & \\ & \eta_{\mu\nu} & \\ & & \end{pmatrix} \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$x_\mu = \eta_{\mu\nu} x^\nu$$

$$A^\mu \neq A_\mu$$

$$A^0 = \phi \qquad A_0 = -c^2 \phi$$


$|x - x'|$
Wave nature
Tensor field description
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
$+ dx^2 + dy^2 + dz^2$
$\eta_{\mu\nu} = \begin{pmatrix} -c^2 & & & \\ & 1 & & 0 \\ 0 & & 1 & \\ & & & 1 \end{pmatrix}$
$X_\mu = \eta$
$A^\mu \neq A_\mu$
$1 + \underbrace{i,}_{3D}$
$\mu = 0, 1, 2, 3$
$\eta_{\mu\nu} dx^\mu dx^\nu = \eta_{\mu\nu} dx^\mu dx^\nu$
$v^i = \frac{dx^i}{dt}$
$u^\mu = \frac{dx^\mu}{d\tau}$
$dy \; dz) \left(\eta_{\mu\nu} \right) \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix}$

$\Gamma(x,t) = -\int$ ... $\frac{}{|x-x'|}$ → Wave nature

Tensor field descri...

$$-c^2 d\tau^2 = ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2$$

$$c \sim 1$$

$$\eta_{\mu\nu} = \Big(-c \ldots$$

$$ds^2 = \sum_{\mu,\nu} \eta_{\mu\nu}\, dx^\mu dx^\nu = \eta_{\mu\nu}\, dx^\mu dx^\nu$$

$$\begin{pmatrix} dt & dx & dy & dz \end{pmatrix} \begin{pmatrix} \eta_{\mu\nu} \end{pmatrix} \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

3D

$$X_\mu = \eta_{\mu\nu} X^\nu$$

$$A^\mu \neq A_\mu$$

$$A^0 = \phi \qquad A_0 = -c^2 \phi$$

$$d\tau^2 = dt^2 \underbrace{\left(1 - \frac{v^2}{c^2}\right)}_{1/\gamma^2}$$

$$u^\mu = \frac{dx^\mu}{d\tau} \qquad u^0 = \frac{dt}{d\tau} = \gamma_u \qquad u^i = \quad v^i$$

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & & 0 \\ & 1 & \\ 0 & & 1 \end{pmatrix}$$

$$x_\mu = \eta_{\mu}$$

$$A^\mu \neq A_\mu$$

$$\mu = 0, \underbrace{1, 2, 3}_{i,\ 3D}$$

$1 +$

$${}_{\mu\nu}\, dx^\mu dx^\nu$$

$$\begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix}$$

$$v^i = \frac{dx^i}{dt}$$

$$u^\mu = \frac{dx^\mu}{d\tau}$$

$$u^0 =$$

$$u^i = \frac{dx^i}{d\tau} = \frac{dt}{d\tau}\frac{dx^i}{dt} = \gamma_v v^i$$

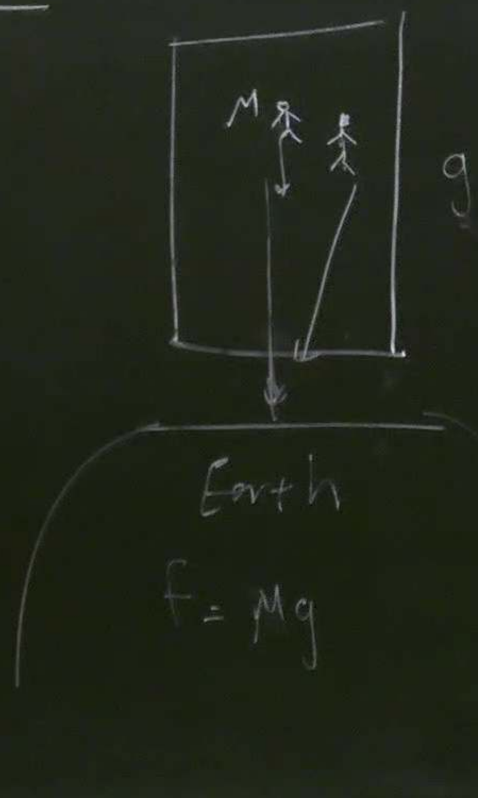
Equivalence principle

$$\cancel{m}_I \, a = \cancel{m}_g \nabla\phi$$

$$a = \nabla\phi$$

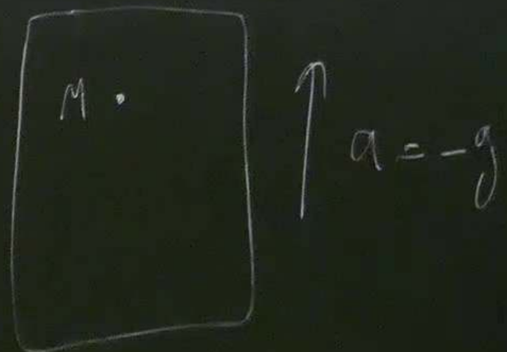
$$a = g$$

$$\frac{m_I}{m_G} = 1 \pm 10^{-13}$$



Weak Equivalence principle

A uniform grav. field is indistinguishable from uniform acceleration.



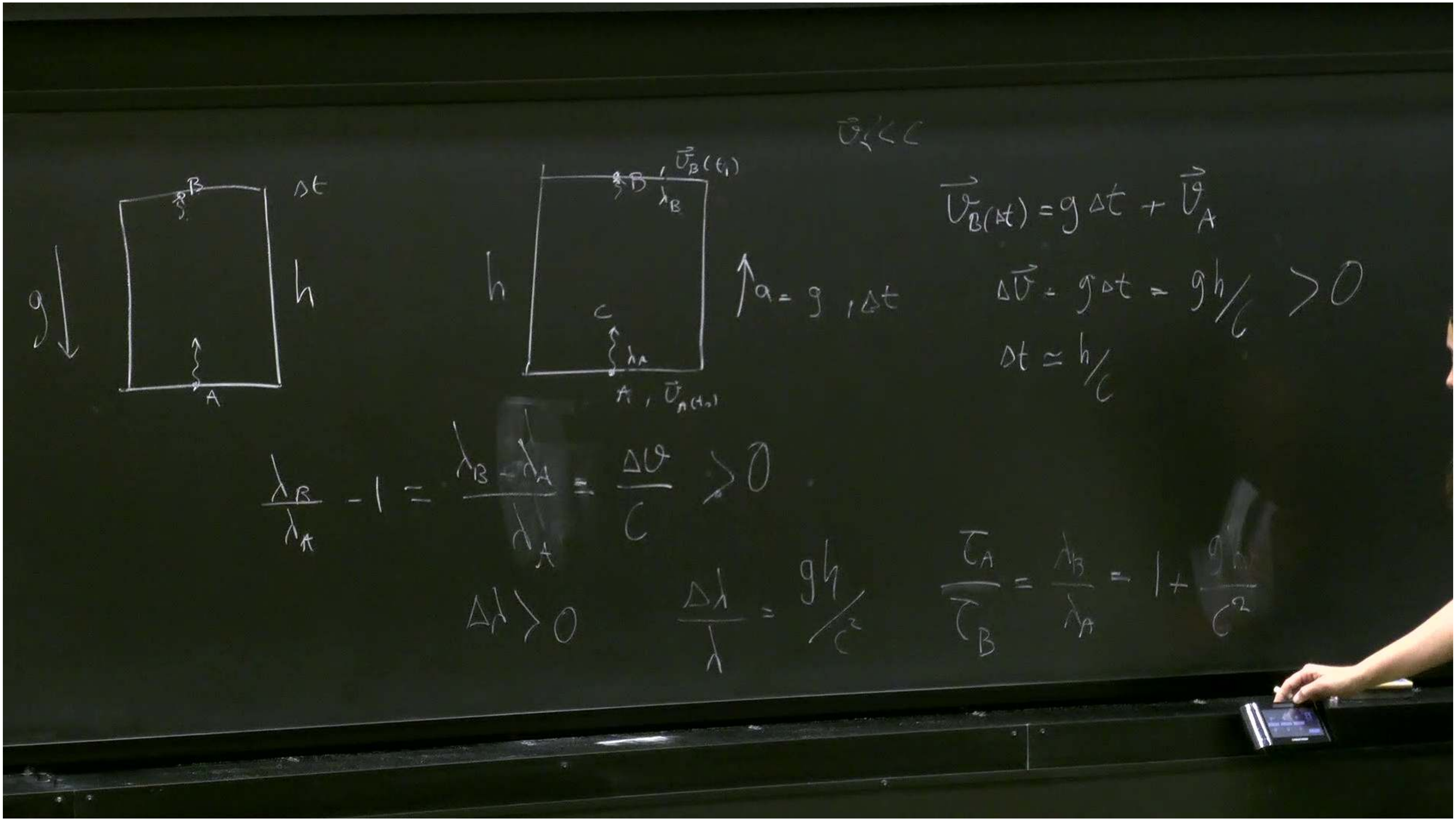
$\overline{\text{WEP}}$

Eistein Equivalence Principle

There exists a local inertial

↳ locally you can get special Relativity

+ WEP → EEP


v << c
Δt
h
g
B
A
h
v_B(t_1)
λ_B
a = g, Δt
C
λ_A
A, v_A(t_0)
v_B(Δt) = g Δt + v_A
Δv = gΔt = gh/c > 0
Δt ≃ h/c
λ_B/λ_A − 1 = (λ_B − λ_A)/λ_A = Δv/c > 0
Δλ > 0
Δλ/λ = gh/c²
τ_A/τ_B = λ_B/λ_A = 1 + gh/c²

$+ \vec{v}_A$

$= -gh/c > 0$

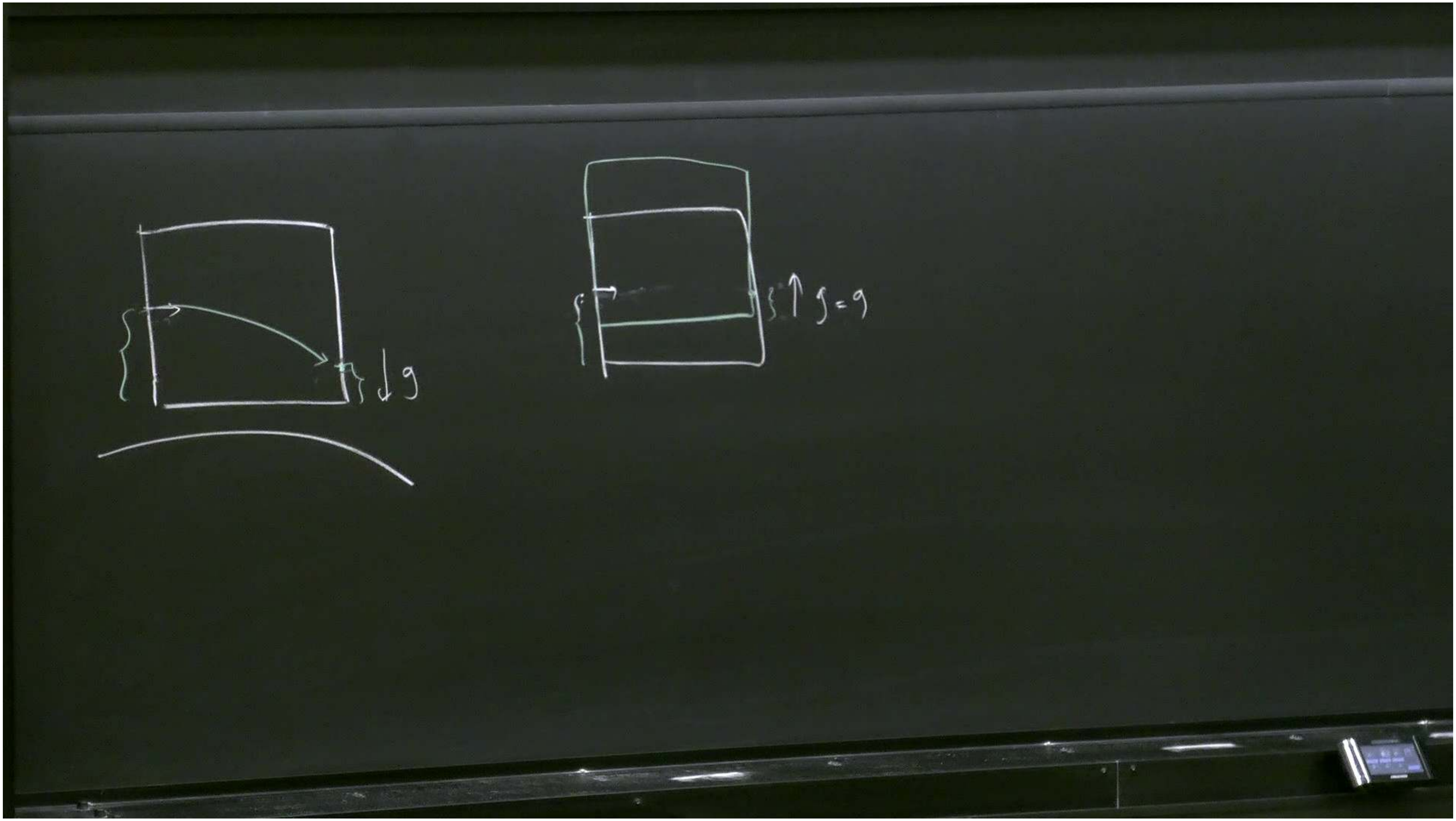
$$\nabla \phi = g \hat{z}$$

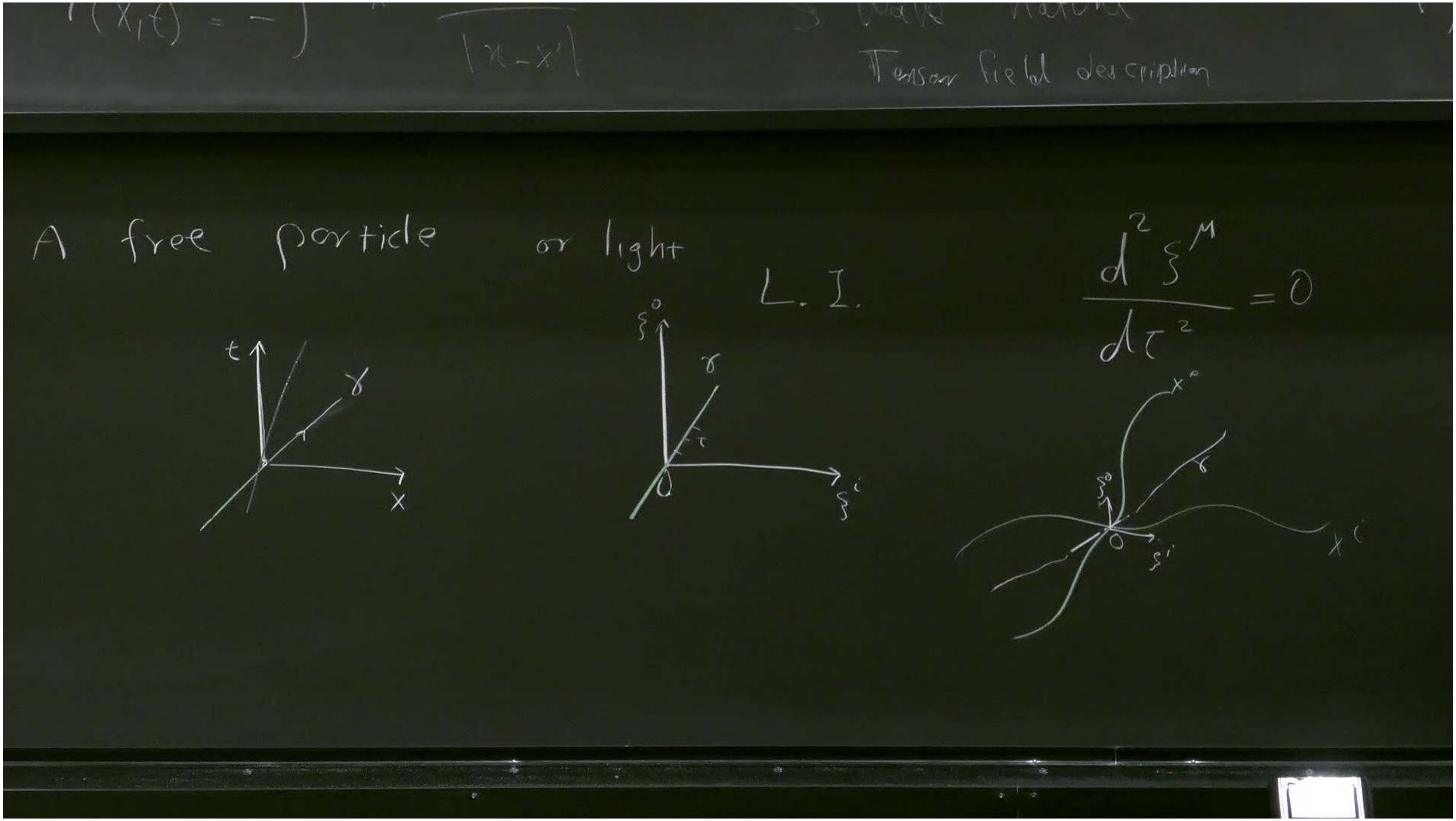
$$\frac{\Delta \phi}{\Delta z} = \frac{\phi_B - \phi_A}{h} = g$$

$$\boxed{\frac{\tau_B}{\tau_A} = 1 + \frac{\phi_B - \phi_A}{c^2}}$$

$$\frac{\Delta \tau}{\tau} = \frac{\Delta \lambda}{\lambda} = \frac{\Delta \phi}{c^2}$$

$$1 + \frac{gh}{c^2} \Rightarrow \tau_B > \tau_A$$


g
g = g


Tensor field description
A free particle or light
L. I.
$\frac{d^2 \xi^\mu}{d\tau^2} = 0$
t
x
γ
ξ^0
ξ^i
O
x^0
x^i

$$x^\alpha = x^\alpha(\xi^\mu)$$

$$\delta^\nu_\alpha$$

$$\frac{d^2 \xi^\mu(x^\alpha)}{d\tau^2} = \frac{d}{d\tau}\left(\frac{\partial \xi^\mu(x)}{\partial x^\alpha}\frac{dx^\alpha}{d\tau}\right) = \frac{\partial^2 \xi^\mu}{\partial x^\alpha \partial x^\beta}\underbrace{\frac{dx^\beta}{d\tau}}_{u^\beta}\underbrace{\frac{dx^\alpha}{d\tau}}_{u^\alpha} + \frac{\partial \xi^\mu}{\partial x^\alpha}\frac{d^2 x^\alpha}{d\tau^2} = 0$$

$$\Gamma^\nu_{\alpha\beta}$$

$$\frac{du^\alpha}{d\tau}$$

$$\frac{\partial x^\nu}{\partial \xi^\mu}\frac{\partial \xi^\mu}{\partial x^\alpha} = \delta^\nu_\alpha$$

$$\frac{du^\nu}{d\tau} + \frac{\partial x^\nu}{\partial \xi^\mu}\frac{\partial^2 \xi^\mu}{\partial x^\alpha \partial x^\beta} u^\beta u^\alpha = 0$$