

**Title:** Lecture - Relativity, PHYS 604

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**Collection/Series:** Relativity (Core), PHYS 604, November 12 - December 11, 2024

**Subject:** Cosmology, Strong Gravity

**Date:** November 12, 2024 - 2:00 PM

**URL:** <https://pirsa.org/24110024>

**Abstract:**

# General Relativity

Energy  
Matter

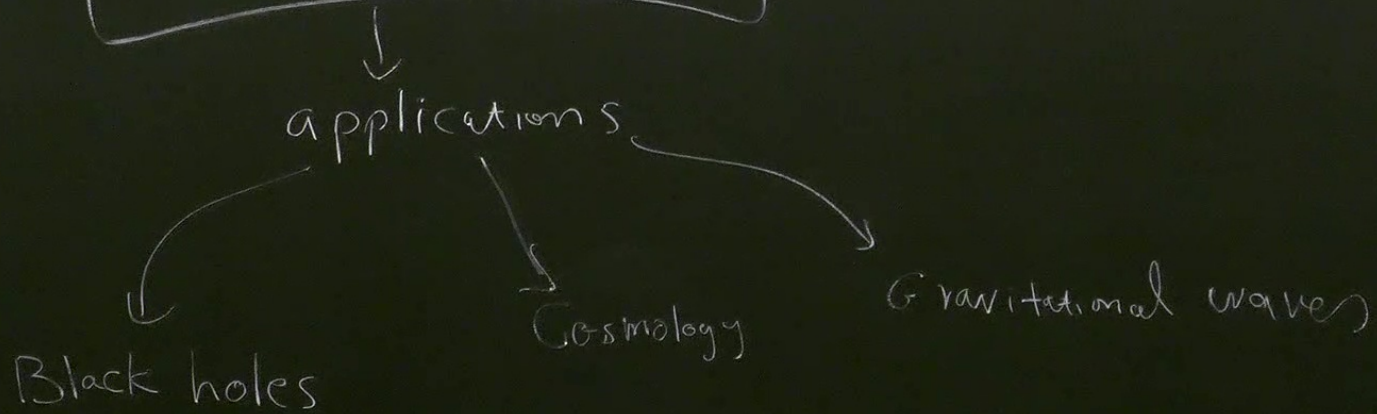
↔ Gravity ↔ Geometry

Gravity ↔ acceleration  
↓ ↓  
Geometry

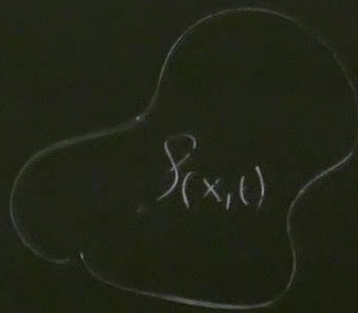
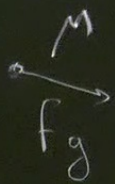
general covariance, curvature

diff Geometry, Tensor Algebra

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$



What is wrong with Newtonian Gravity?



$$f_g = M a$$

$$F_g = -M \nabla \phi$$

$$a = -\nabla \phi$$

$$\nabla^2 \phi = 4\pi G \rho$$

$$\phi(x,t) = - \int d^3x' \frac{\rho(x',t)}{|x-x'|}$$

$$\phi(x, t) = \int dx' \frac{j(x', t')}{|x-x'|}$$

Wave nature  
Tensor field description

$$t_{\mu\nu} =$$

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2$$

$$c \sim 1$$

$$\eta_{\mu\nu} = \begin{pmatrix} -c^2 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$ds^2 = \sum_{\mu, \nu} \eta_{\mu\nu} da^\mu da^\nu = \eta_{\mu\nu} da^\mu dx^\nu$$

$$\mu = 0, 1, 2, 3$$

$$\begin{pmatrix} dt & dx & dy & dz \end{pmatrix} \begin{pmatrix} \eta_{\mu\nu} \end{pmatrix} \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$x'_\mu = \eta_{\mu\nu} x^\nu$$

$$A'^\mu \neq A_\mu$$

$$A^0 = \phi$$

$$A_0 = -c^2 \phi$$

$$\frac{1}{|x-x'|}$$

Wave Nature  
Tensor field description

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$+ dx^2 + dy^2 + dz^2$$

$$\eta_{\mu\nu} = \begin{pmatrix} -c^2 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$x^\mu = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

$$A^\mu \neq A_\mu$$

$$\eta_{\mu\nu} dx^\mu dx^\nu = \eta_{\mu\nu} da^\mu da^\nu$$

1 + 3D  
 $\mu = 0, 1, 2, 3$

$$dy \ dz) \begin{pmatrix} \eta_{\mu\nu} \end{pmatrix} \begin{pmatrix} dt \\ da \\ dy \\ dz \end{pmatrix}$$

$$u^\mu = \frac{dx^\mu}{dt}$$

$$u^\mu = \frac{da^\mu}{d\tau}$$

$$f(x,t) = \frac{1}{|x-x'|}$$

Wave Nature  
Tensor field description

$$-c^2 d\tau^2 = ds^2 = -c^2 dt^2 + da^2 + dy^2 + dz^2$$

$c \sim 1$

$$\eta_{\mu\nu} = \begin{pmatrix} -c^2 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$ds^2 = \sum_{\mu,\nu} \eta_{\mu\nu} da^\mu da^\nu = \eta_{\mu\nu} da^\mu dx^\nu$$

$$(dt \quad dx \quad dy \quad dz) \begin{pmatrix} \eta_{\mu\nu} \end{pmatrix} \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix}$$



$$T_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$x_\mu = \eta_{\mu\nu} x^\nu$$

$$A^\mu \neq A_\mu$$

$$A^0 = \phi \quad A_0 = -c^2 \phi$$

$$d\tau^2 = dt^2 \left( 1 - \frac{v^2}{c^2} \right)$$

$$u^\mu = \frac{dx^\mu}{d\tau}$$

$$u^0 = \frac{dt}{d\tau} = \gamma_u$$

$$u^i = \gamma_u v^i$$

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$X^{\mu} = \eta^{\mu\nu} X_{\nu}$$

$$A^{\mu} \neq A_{\mu}$$

$$\eta_{\mu\nu} dx^{\mu} dx^{\nu}$$

$$1 + \underbrace{c^2}_{3D}$$

$$M = 0, 1, 2, 3$$

$$\left. \begin{array}{l} dt \\ dx \\ dy \\ dz \end{array} \right)$$

$$u^i = \frac{dx^i}{dt}$$

$$u^{\mu} = \frac{dx^{\mu}}{d\tau}$$

$$u^0 =$$

$$u^i = \frac{dx^i}{d\tau} = \frac{dt}{d\tau} \frac{dx^i}{dt} = \gamma u^i dt$$

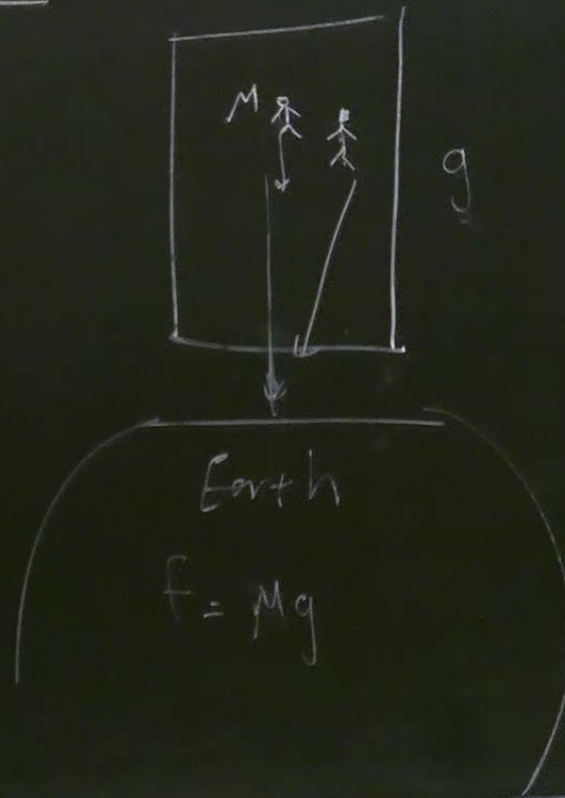
# Equivalence principle

$$\cancel{m}_I a = \cancel{m}_g \nabla \phi$$

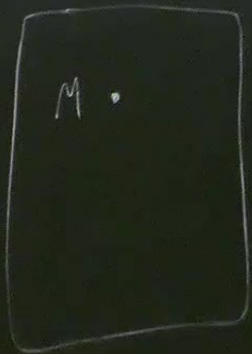
$$a = \nabla \phi$$

$$a = g$$

$$\frac{m_I}{m_G} = 1 \pm 10^{-13}$$



Weak Equivalence Principle  
A uniform grav. field is indistinguishable  
from uniform acceleration.



$$\uparrow a = -g$$

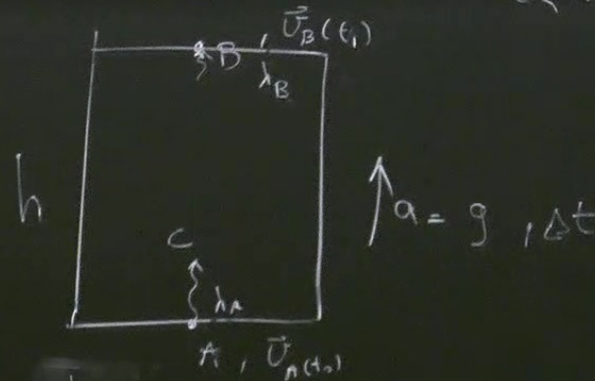
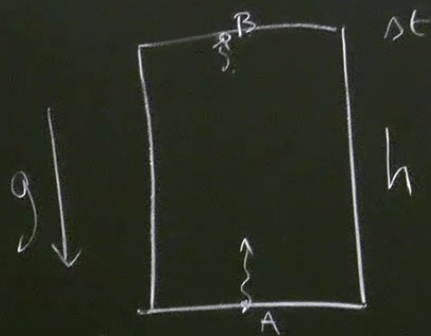
WEP

## Einstein Equivalence Principle

There exists a local inertial

↳ locally you can get Special Relativity

+ WEP → EEP



$v_B \ll c$

$$\vec{v}_B(\Delta t) = g \Delta t + \vec{v}_A$$

$$\Delta \vec{v} = g \Delta t = gh/c > 0$$

$$\Delta t \approx h/c$$

$$\frac{\lambda_B}{\lambda_A} - 1 = \frac{\lambda_B - \lambda_A}{\lambda_A} = \frac{\Delta \lambda}{\lambda_A} > 0$$

$$\Delta \lambda > 0$$

$$\frac{\Delta \lambda}{\lambda} = \frac{gh}{c^2}$$

$$\frac{\tau_A}{\tau_B} = \frac{\lambda_B}{\lambda_A} = 1 + \frac{gh}{c^2}$$

$$+ \vec{v}_A$$
$$= gh/c > 0$$

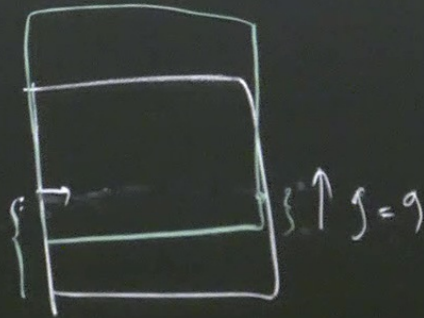
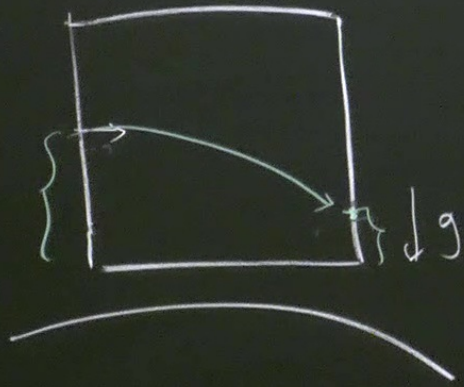
$$\nabla\phi = g \hat{z}$$

$$\frac{\Delta\phi}{\Delta z} = \frac{\phi_B - \phi_A}{h} = g$$

$$\boxed{\frac{T_B}{T_A} = 1 + \frac{\phi_B - \phi_A}{c^2}}$$

$$\frac{\Delta T}{T} = \frac{\Delta\lambda}{\lambda} = \frac{\Delta\phi}{c^2}$$

$$1 + \frac{gh}{c^2} \Rightarrow T_B > T_A$$





$$f(x,t) = -$$

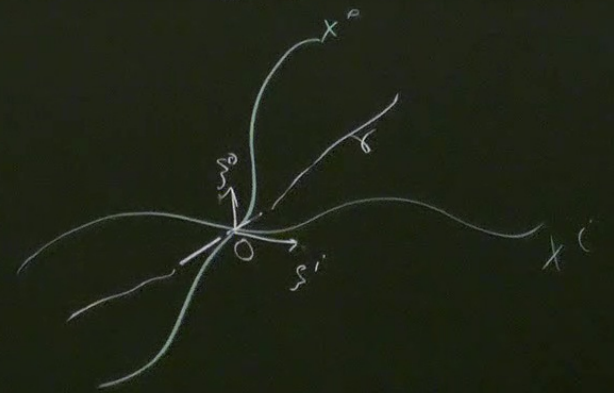
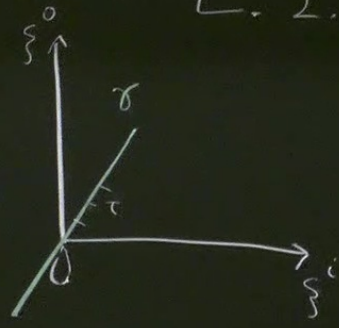
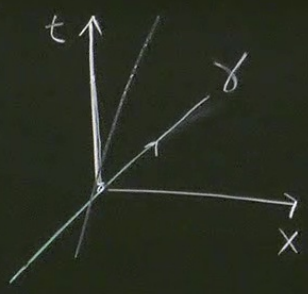
$$|x-x'|$$

Tensor field description

A free particle or light

L. I.

$$\frac{d^2 \xi^M}{d\tau^2} = 0$$



$$x^\alpha = x^\alpha(\xi^M)$$

$$\frac{d^2 \xi^M(x^\alpha)}{d\tau^2} = \frac{d}{d\tau} \left( \frac{\partial \xi^M}{\partial x^\alpha} \frac{dx^\alpha}{d\tau} \right) = \frac{\partial^2 \xi^M}{\partial x^\alpha \partial x^\beta} \frac{dx^\beta}{d\tau} \frac{dx^\alpha}{d\tau} + \frac{\partial \xi^M}{\partial x^\alpha} \frac{d^2 x^\alpha}{d\tau^2} = 0$$

$\downarrow \delta_{\alpha}^{\nu}$   
 $\frac{d^2 x^\alpha}{d\tau^2} = 0$

$$\frac{\partial x^\nu}{\partial \xi^M} \frac{\partial \xi^M}{\partial x^\alpha} = \delta_{\alpha}^{\nu}$$

$$\frac{d u^\nu}{d\tau} + \frac{\partial x^\nu}{\partial \xi^M} \frac{\partial^2 \xi^M}{\partial x^\alpha \partial x^\beta} u^\beta u^\alpha = 0$$

$\frac{d u^\alpha}{d\tau}$